PHYSICAL REVIEW D **73,** 055012 (2006)

Nonminimal supersymmetric standard model with lepton number violation

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(Received 11 January 2006; published 24 March 2006)

We carry out a detailed analysis of the nonminimal supersymmetric standard model with lepton number violation. The model contains a unique trilinear lepton number violating term in the superpotential which can give rise to neutrino masses at the tree level. We search for the gauged discrete symmetries realized by cyclic groups Z_N which preserve the structure of the associated trilinear superpotential of this model, and which satisfy the constraints of the anomaly cancellation. The implications of this trilinear lepton number violating term in the superpotential and the associated soft supersymmetry breaking term on the phenomenology of the light neutrino masses and mixing is studied in detail. We evaluate the tree and one-loop contributions to the neutrino mass matrix in this model. We search for possible suppression mechanism which could explain large hierarchies and maximal mixing angles.

DOI: 10.1103/PhysRevD.73.055012 PACS numbers: 12.60.Jv, 11.30.Fs, 14.60.Pq

I. INTRODUCTION

Supersymmetry [1] is at present the only known framework in which the Higgs sector of the standard model (SM), so crucial for its internal consistency, is natural. A much favored implementation of the idea of supersymmetry at low energies is the minimal supersymmetric standard model (MSSM), which is obtained by doubling the number of states of SM, and introducing a second Higgs doublet (with opposite hypercharge to the standard model Higgs doublet) to generate masses for all the SM fermions and to cancel the triangle gauge anomalies. However, the MSSM suffers from the so-called μ problem associated with the bilinear term connecting the two Higgs doublet superfields H_u and H_d in the superpotential. An elegant solution to this problem is to postulate the existence of a chiral electroweak gauge singlet superfield S, and couple it to the two Higgs doublet superfields H_u and H_d via a dimensionless trilinear term $\lambda H_u H_d S$ in the superpotential. When the scalar component of the singlet superfield S obtains a vacuum expectation value (VEV), a bilinear term $\lambda H_u H_d \langle S \rangle$ involving the two Higgs doublets is naturally generated. Furthermore, when this scalar component of the chiral singlet superfield S acquires a vacuum expectation value of the order of the $SU(2)_L \times U(1)_Y$ breaking scale, it gives rise to an effective value of μ, $\mu_{\rm eff} = \lambda\langle S \rangle$, of the order of electroweak scale. However, the inclusion of the singlet superfield leads to additional trilinear superpotential coupling $(\kappa/3)S^3$ in the model, the so-called nonminimal, or next-to-minimal [2–6], supersymmetric standard model (NMSSM). The absence of $H_u H_d$ term, and the absence of tadpole and mass couplings, S and S^2 in the NMSSM is made natural by postulating a suitable discrete symmetry. The NMSSM is attractive on account of the simple resolution it offers to the μ-problem, and of the scale invariance of its classical action in the supersymmetric limit. Since no dimensional supersymmetric parameters are present in the superpotential of NMSSM, it is the simplest supersymmetric extension of the standard model in which the electroweak scale originates from the supersymmetry breaking scale only. Its enlarged Higgs sector may help in relaxing the fine-tuning and little hierarchy problems of the MSSM [7], thereby opening new perspectives for the Higgs boson searches at high energy colliders [8–11], and for dark matter searches [12].

Since supersymmetry requires the introduction of superpartners of all known particles in the SM, which transform in an identical manner under the gauge group, there are additional Yukawa couplings in supersymmetric models which violate [13] baryon number (B) or lepton number (L). In the minimal supersymmetric standard model there are both bilinear and trilinear lepton number violating Yukawa terms in the superpotential. There are also trilinear baryon number violating Yukawa terms in the superpotential. All these terms are allowed by renormalizability, gauge invariance, and supersymmetry. In MSSM, a discrete symmetry [14] called R-parity (R_p) is invoked to eliminate these B and L violating Yukawa couplings. However, the assumption of R-parity conservation at the level of low energy supersymmetry appears to be *ad hoc*, since it is not required for the internal consistency of supersymmetric models.

If we do not postulate R_p conservation, then there are baryon and lepton number violating terms in the superpotential of NMSSM as well. What is perhaps interesting is the presence of an additional lepton number violating trilinear superpotential coupling [15,16] in this model which

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has no analog in the MSSM with baryon and lepton number violation. It is, therefore, important to study the implications of this additional lepton number violating trilinear interaction term in the superpotential of NMSSM, contrast the situation with MSSM with lepton number violation, and pin down the possible differences with its predictions.

One of the far reaching implications of the lepton number violating couplings in NMSSM concerns the physics of light neutrino states. In identifying the dominant contributions to the neutrino masses, and suppression mechanisms, one must compare with the situation that obtains in MSSM with bilinear lepton number violation. In NMSSM, the three light neutrinos mix with $SU(2)_L \times U(1)_Y$ nonsinglet gaugino and Higgsino fields as well as the gauge singlet fermionic component of S, the singlino ($\tilde{S}$). The resulting 8×8 mass matrix of the neutrino-gaugino-Higgsino-singlino has a seesaw structure, which leads to a separable rank one effective mass matrix for the light neutrinos, implying the presence of a single massive Majorana neutrino. At one-loop order, there occur two main mechanisms for generating masses for the Majorana neutrinos. One of these involves only the matter interactions. The second mechanism involves matter interactions in combination with the gauge interactions and propagation of neutralinos and mixed sneutrino-Higgs boson system, whose contribution depends sensitively on the soft supersymmetry breaking couplings. While both these mechanisms have the ability to generate masses for the Majorana neutrinos, the latter one, initially proposed in the context of MSSM by Grossman and Haber [17], is expected to dominate. In the case of MSSM with bilinear lepton number (or R_p) violation, the tree and one-loop contributions to the neutrino masses, and their ability to reproduce the experimental observations, have been extensively discussed in the literature [18–26].

In this paper we carry out a detailed investigation of the nonminimal supersymmetric standard model with lepton number violation. Since the NMSSM has a unique lepton number violating trilinear coupling term in the superpotential, one of the issues we want to address concerns the implications of the neutrino masses and mixing for the NMSSM with such a lepton number violating term. Our purpose is to pin down the features specific to this version of the NMSSM and extract constraints implied by the comparison with experimental data. We compare and contrast the situation in NMSSM with that of MSSM with bilinear R-parity violation (RPV) [24]. Despite the presence of the singlino, and its mixing with the neutrinos, a light mass Majorana neutrino appears at the tree level. This is due to the constrained nature of the couplings in the model. Nevertheless, as in the MSSM with lepton number violation, one-loop contributions play an important role in determining the neutrino mass spectrum. The ability to reproduce experimental observations is expected to set useful constraints on the Higgs sector parameters of the NMSSM. The situation differs from the one that arises in the seesaw mechanism or the bilinear lepton number violation in MSSM in that no dimensional mass parameters (large or small) are introduced. The neutrino Majorana masses arise from dimensionless Yukawa couplings. However, despite the presence of a gauge singlet fermion that could play the role of a sterile neutrino, whether an ultra light singlino mode, compatible with the cosmological bound on the summed mass of light neutrinos, $\sum_\nu m_\nu <$ 10 eV, does indeed occur is at variance with the physical constraints on the NMSSM which rule out the possibility that the lightest mode in the massive neutralino sector lies below $O(50)$ GeV. Thus, the understanding of neutrino physics provided by the NMSSM with lepton number violation contrasts with that proposed in models using the compactification moduli superfields [27] or axion superfields [28] coupled gravitationally to the observable sector modes.

This paper is organized as follows. We begin in Section II with a discussion of the general structure of the superpotential and soft supersymmetry breaking interactions in the nonminimal supersymmetric standard model with lepton number violation. In this section we also discuss the local Z_N cyclic symmetries which can protect the NMSSM against B or L, or combined B and L, number violating superpotential couplings (subsection II B). We further elaborate on the general approach to analyze the gauged cyclic group symmetries in the Appendix A to the paper. In Section III we derive the tree-level light neutrino mass spectrum that arises in this model (subsection III A), and then obtain the one-loop radiative corrections to the mass spectrum in subsection III B. In Section IV we present a general discussion of the predictions from this model, which are based on the consideration of Abelian horizontal symmetries for the flavor structure of effective couplings. Finally in Section V we summarize our results and conclusions.

II. NMSSM WITH BARYON AND LEPTON NUMBER VIOLATION

A. The superpotential

In this section we recall the basic features of NMSSM with baryon and lepton number violation, and establish our notations and conventions. The superpotential of the model is written as

$$W_{\mathrm{NMSSM}} = (h_U)_{ab} Q_L^a \bar{U}_R^b H_u + (h_D)_{ab} Q_L^a \bar{D}_R^b H_d + (h_E)_{ab} L_L^a \bar{E}_R^b H_d + \lambda S H_d H_u - \frac{\kappa}{3} S^3, \quad (2.1)$$

where L, Q, $\bar{E}$, $\bar{D}$, $\bar{U}$ denote the lepton and quark doublets, and antilepton singlet, d-type antiquark singlet, and u-type antiquark singlet, respectively. In Eq. (2.1), $(h_U)_{ab}$, $(h_D)_{ab}$, and $(h_E)_{ab}$ are the Yukawa coupling matrices, with a, b, c as the generation indices. Gauge invariance, supersymmetry, and renormalizability allow the addition of the following L and B violating terms to the superpotential (2.1):

$$W_L = \tilde{\lambda}_a L_a H_u S + \frac{1}{2}\lambda_{abc} L_L^a L_L^b \bar{E}_R^c + \lambda'_{abc} L_L^a Q_L^b \bar{D}_R^c, \quad (2.2)$$

$$W_B = \frac{1}{2}\lambda''_{abc} \bar{D}_R^a \bar{D}_R^b \bar{U}_R^c, \quad (2.3)$$

where the notation [15,16] is standard. We note that there is an additional L-violating term with the dimensionless Yukawa coupling $\tilde{\lambda}_a$ in (2.2) which does not have an analogue in the MSSM. This term can be rotated away into the R-parity conserving term $\lambda S H_u H_d$ via an $SU(4)$ rotation between the superfields H_d and L_a. However, this rotation must be performed at some energy scale, and the term is regenerated through the renormalization group equations. The Yukawa couplings λ_{abc} and λ''_{abc} are antisymmetric in their first two indices due to $SU(2)_L$ and $SU(3)_C$ group symmetries, respectively.

The supersymmetric part of the Lagrangian of NMSSM with baryon and lepton number violation can be obtained from the superpotential (2.1), (2.2), and (2.3) by the standard procedure. In addition to this supersymmetric Lagrangian, there are soft supersymmetry breaking terms which include soft masses for all scalars, gaugino masses, and trilinear scalar couplings, respectively. These can be written as

$$\begin{aligned} V_{\text{soft}} = -\mathcal{L}_{\text{soft}} = {} & [M_Q^{ab2}\tilde{Q}^{a*}\tilde{Q}_L^b + M_U^{ab2}\tilde{\bar{U}}_R^a\tilde{\bar{U}}_R^{b*} + M_D^{ab2}\tilde{\bar{D}}_R^a\tilde{\bar{D}}_R^{b*} + M_L^{ab2}\tilde{L}_L^{a*}\tilde{L}_L^b + M_E^{ab2}\tilde{\bar{E}}_R^a\tilde{\bar{E}}_R^{b*} + m_{H_d}^2 H_d^* H_d + m_{H_u}^2 H_u^* H_u \\ & + m_S^2 S^* S] + \left[\frac{1}{2}M_s\lambda^s\lambda^s + \frac{1}{2}M_2\lambda^w\lambda^w + \frac{1}{2}M_1\lambda'\lambda'\right] \\ & + \left[(A_U)_{ab}(h_U)_{ab}\tilde{Q}_L^a\tilde{\bar{U}}_R^b H_u + (A_D)_{ab}(h_D)_{ab}\tilde{Q}_L^a\tilde{\bar{D}}_R^b H_d + (A_E)_{ab}(h_E)_{ab}\tilde{L}_L^a\tilde{\bar{E}}_R^b H_d - A_\lambda \lambda S H_d H_u - \frac{A_\kappa}{3}\kappa S^3\right] \\ & + \left[-(A_{\tilde{\lambda}_a})\tilde{\lambda}_a\tilde{L}_L^a H_u S + \frac{1}{2}(A_\lambda)_{abc}\lambda_{abc}\tilde{L}_L^a\tilde{L}_L^b\tilde{\bar{E}}_R^c + (A_{\lambda'})_{abc}\lambda'_{abc}\tilde{L}_L^a\tilde{Q}_L^b\tilde{\bar{D}}_R^c\right] \\ & + \left[\frac{1}{2}(A_{\lambda''})_{abc}\lambda''_{abc}\tilde{D}_R^a\tilde{D}_R^b\tilde{\bar{U}}_R^c\right] + H.c., \end{aligned} \quad (2.4)$$

where a tilde over a matter chiral superfield denotes its scalar component, and the notation for the scalar component of the Higgs superfield is the same as that of the corresponding superfield. We note that the soft supersymmetry breaking gaugino masses have been denoted by M_1, M_2, and M_s corresponding to the gauge groups $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively. We have chosen the sign conventions for the soft trilinear couplings involving the gauge singlet field in Eq. (2.4) which are different from those used in Ref. [16].

The dimension-4 terms in the superpotentials (2.2) and (2.3) are the most dangerous terms for nucleon decay, and some of them must be suppressed. This leads to constraints [29] on the different couplings λ_{abc}, λ'_{abc}, and λ''_{abc}, but considerable freedom remains for the various B and L violating couplings. Furthermore, there are dimension-5 operators [16] which may lead to nucleon decay suppressed by $1/M$, where M is some large mass scale at which the B and L violation beyond that of NMSSM (and MSSM) comes into play. Some of these dimension-5 operators may also lead to unacceptable nucleon decay if their coefficients are of order unity, and therefore must be suppressed. We shall not consider here the dimension-5 operators, but instead concentrate on the dimension-4 lepton number violating terms (2.2) only.

As noted above, rotating away at some momentum scale Q_0 the dimensionless Yukawa coupling $\tilde{\lambda}_a$ in (2.2) by setting $\tilde{\lambda}_a(Q_0) = 0$, a calculable finite coupling will be regenerated at different scales upon integrating the renormalization group equation [15,16]

$$\frac{d\tilde{\lambda}_3}{d\ln Q} = \frac{1}{16\pi^2}\left[(3h_t^2 + h_\tau^2 + 4\lambda^2 + 2\kappa^2 + 4\tilde{\lambda}_3^2 + \lambda_{233}^2 + 3\lambda'^2_{333})\tilde{\lambda}_3 + 3h_b\lambda\lambda'_{333} - \left(3g_2^2 + \frac{3}{5}g_1^2\right)\tilde{\lambda}_3\right], \quad (2.5)$$

where $(h_U)_{33} \sim h_t$, $(h_D)_{33} \sim h_b$, $(h_L)_{33} \sim h_\tau$. Here g_2, g_1 are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, and the meaning of other quantities is obvious. For simplicity, we have retained only the highest generation trilinear couplings in (2.5).

It is important to point out two distinctive features of the present model relative to the MSSM. First, in NMSSM no distinction is made between the bilinear and trilinear lepton number violation, since the bilinear Lagrangian terms, $\mu H_d H_u + \mu_i L_i H_u$, $[\mu = \lambda\langle S\rangle$, $\mu_i = \tilde{\lambda}_i\langle S\rangle]$, can arise as effective couplings once the singlet scalar field component of S acquires a finite VEV. The wide hierarchies between the lepton number conserving and violating couplings, of the expected size $\tilde{\lambda}_i/\lambda = \mu_i/\mu \sim 10^{-6}$, arise from the hierarchies of the dimensionless Yukawa couplings. Second, the lepton number violating trilinear operator $L_i H_u S$ has the ability to radiatively induce other trilinear lepton number violating couplings, which is precluded for the bilinear operator L_iH_u. This property may be used to justify a scheme where naturally suppressed trilinear couplings λ_{ijk}, λ'_{ijk} occur as a result of being set to zero at some large mass scale (gauge unification scale), and receive small finite radiative corrections from the gauge singlet coupling $\tilde{\lambda}_i L_i H_u S$. This possibility can be established on a quantitative basis by examining the one-loop renormalization group equations for the trilinear coupling with maximal number of third generation indices [16]

$$(4\pi)^2 \frac{\partial \lambda_{233}}{\partial \log Q} = \lambda_{233}\left[4h_\tau^2 + 4\lambda_{233}^2 + 3\lambda_{333}'^2 + \tilde{\lambda}_3^2 - \left(\frac{9}{5}g_1^2 + 3g_2^2\right)\right],$$
$$(4\pi)^2 \frac{\partial \lambda'_{333}}{\partial \log Q} = \lambda'_{333}\left[h_t^2 + 6h_b^2 + h_\tau^2 + \tilde{\lambda}_3^2 + \lambda_{233}^2 + 6\lambda_{333}'^2 - \left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2\right)\right] + h_b\lambda\tilde{\lambda}_3. \tag{2.6}$$

Looking for solutions of these equations with suitably large values of the trilinear couplings at some large mass scale, for instance, gauge coupling unification scale, might reveal the presence of infrared fixed points which would then serve as upper bounds on the weak scale values of these couplings.

To establish the lepton number violating nonminimal supersymmetric standard model on a firmer basis, it is important to determine whether there are discrete symmetries respecting the postulated interaction superpotential which can be regarded as local or gauged symmetries [30] obeying the anomaly cancellation conditions. As is known, the gauged discrete symmetries enjoy a natural protection against breaking by nonperturbative quantum effects initiated by the gravitational interactions, and against the emergence of massless Nambu-Goldstone bosons from the spontaneous symmetry breaking. Another advantage lies in evading the cosmological domain wall problem by the removal of classical domain wall solutions as a result of the gauge equivalence of degenerate vacua. We recall that if stable domain wall solutions were present, the production of cosmic domain walls in the early Universe would result in a contribution to the present day mass density of the Universe which exceeds the critical density. The case of Green-Schwarz (GS) anomalous gauged discrete symmetries is special in that although domain wall solutions do exist in gauge field theories satisfying such global type symmetries, the instanton tunneling effects present in these theories lift the degeneracy of vacua so as to render the solutions unstable [31,32]. The above resolution applies independently of the familiar one invoking the domain wall dilution during a cosmic inflation era. We also note that the discrete gauge symmetries have been used in connection with various naturalness problems, such as the doublet-triplet splitting in unified gauge models, the stabilization of axion symmetries, or the construction of flavor symmetries realized via Froggatt-Nielsen mechanism [33,34]. Following the work by Ibáñez and Ross [35,36], we are led here to consider the so-called generalized parities (GPs) of the NMSSM which forbid part or all of the dangerous couplings at the renormalizable level [38]. In the following subsection we shall describe the construction of the generalized parities for the NMSSM in detail.

B. Discrete symmetries

In this subsection we consider the Z_N cyclic local symmetries which can protect the NMSSM against B, or L, or combined B and L number violating superpotential couplings. Demanding consistency with the anomaly cancellation conditions sets highly nontrivial constraints on the generalized baryon, lepton, and matter parity symmetries (designated as GBP, GLP, and GMP) for the appropriate superpotential. These are examined by making use of the approach of Ibáñez and Ross [35,36], which is developed in Appendix A for the NMSSM. Before presenting our results, we shall outline the problem in general and introduce our notations.

Besides the regular R-parity conserving (RPC) trilinear matter-Higgs Yukawa couplings, QH_uU^c, QH_dD^c, LH_dE^c, and the dangerous R-parity violating (RPV), and B and L violating, couplings $U^cD^cD^c$ and LLE^c, LQD^c, the renormalizable superpotential of the NMSSM includes the trilinear couplings H_dH_uS and LH_uS, but excludes the dimensional superpotential couplings S, S^2, and H_dH_u. Thus, in addition to the weak hypercharge Y, the regular couplings conserve three $U(1)$ charges. A convenient basis for the generators of the corresponding 3-dimensional vector space is given by the $U(1)$ charges $\hat{R}$, $\hat{A}$, $\hat{L}$, which are defined in the table of Appendix A. The cyclic Z_N group elements $R = e^{i\alpha_R\hat{R}}$, $A = e^{i\alpha_A\hat{A}}$, and $L = e^{i\alpha_L\hat{L}}$ are defined by restricting the complex phase rotation angles to the values $\alpha_R = 2\pi m/N$, $\alpha_A = 2\pi n/N$, $\alpha_L = 2\pi p/N$, with m, n, p being integers. The generators of the independent Z_N multiplicative symmetry groups are thus of the form, $g = R^mA^nL^p = g_{PQ}^nR^{m-2n}L^p$, $[g_{PQ} = R^2A]$ with specific (modulo N) relations linking the three integers m, n, p, depending on the allowed subset of dangerous couplings. For the ordinary symmetries (**O**), the charges are given explicitly by $\hat{g}(Q) = 0$, $\hat{g}(U^c) = -m$, $\hat{g}(D^c) = m - n$, $\hat{g}(L) = -n - p$, $\hat{g}(E^c) = m + p$, $\hat{g}(H_d) = -m + n$, $\hat{g}(H_u) = m$, $\hat{g}(S) = mx + ny + pz$. The charges x, y, z assigned to the gauge singlet S must obey the selection rules, $x + y + z \neq 0$, $2(x + y + z) \neq 0$, $3(x + y + z) = 0$, $x + y + z + n = 0$. A special role is played by the Peccei-Quinn like charge, $(PQ) = 2\hat{R} + \hat{A}$, which has a finite color group anomaly and is conserved by all the

NMSSM couplings except those involving the gauge singlet superfield S. The general form of the GBP, GLP, and GMP generators is displayed in Eq. (A2) of Appendix A. The soft supersymmetry breaking terms, which are generated via couplings to the Goldstino spurion superfield X of form, $V_{\rm soft} \sim [XW]_F + \text{H.c.} = F_X W + \text{H.c.}$, are automatically protected for ordinary symmetries (**O**), and this protection remains valid for the R symmetries (**R**) as well, provided one assigns the R charge $Q(X) = 0$, and hence $Q(F_X) = 2$.

Having classified the Abelian GP generators of fixed order N in terms of the three integers (m, n, p), our next task is to select the solutions satisfying the quantum anomaly conditions. These are expressed in terms of an overdetermined system of linear and nonlinear equations for m, n, p. We have developed a numerical program to solve the anomaly cancellation conditions for the generalized parity generators (GBP, GLP, GMP) of four types. Unless stated otherwise, the search has been restricted to the case involving three quark and lepton generations, $N_g = 3$, and a single pair of Higgs boson doublets, $N_{2h} = 1$. While our presentation of results will be limited to cyclic groups of order $N \leq 15$, we note that higher order solutions occur at integer multiples of the low order solutions, and as such they do not reveal novel features.

If we demand that all the anomaly constraints are satisfied, then we find that, in general, no solutions exist, even if one is willing to push the search to high enough group orders, say $N \leq 30$. However, one must realize that the various anomaly cancellation conditions need not all be placed on the same footing. The linear anomalies $\mathcal{A}_3$, $\mathcal{A}_2$, $\mathcal{A}_1$, $\mathcal{A}_{\rm grav}$ have an obvious priority over the others, to the extent that these identify with the selection rules obeyed by the determinant interactions of fermions mediated by the classical instanton solutions [31,32] of the non-Abelian gauge theory factors. More importantly, these conditions are independent of the Z_N charge normalization, in contrast to the less physically motivated nonlinear anomalies, $\mathcal{A}_{Z^2}$ and $\mathcal{A}_{Z^3}$, which thus depend on the spectrum of massive decoupled modes. Among the linear anomalies, the gravitational and Abelian gauge $U(1)_Y$ anomalies, $\mathcal{A}_{\rm grav}$ and $\mathcal{A}_1$, are believed to be less robust than the non-Abelian ones, $\mathcal{A}_3$, $\mathcal{A}_2$. Indeed, $\mathcal{A}_1$ is sensitive to the normalization of the hypercharge which remains a free parameter as long as one is not concerned with the gauge group unification. Also, $\mathcal{A}_{\rm grav}$, $\mathcal{A}_{Z^3}$ are sensitive to contributions from additional gauge singlet fields, which could either belong to the observable sector, such as the singlet S, or to the hidden sector, and hence coupled only through the gravitational interactions. While the nonlinear anomalies $\mathcal{A}_{Z^2}$, $\mathcal{A}_{Z^3}$ are both sensitive to the ambiguity which arises due to the arbitrary Z_N charge normalization, $\mathcal{A}_{Z^3}$ is also sensitive to presence of gauge singlets.

We now discuss our results. These are displayed in Table I for the four different realizations of the three generalized parity symmetries in terms of the generator indices (m, n, p). Let us start first with the ordinary symmetries **O**. While it proves impossible to solve the complete set of equations, as already indicated, solutions do arise in very large numbers if one chooses to cancel the non-Abelian gauge anomalies $\mathcal{A}_3$, $\mathcal{A}_2$ only. At this point, we mention that in the case of ordinary symmetries, and only for this case, the cubic anomaly $\mathcal{A}_{Z^3}$ automatically cancels once the equations for $\mathcal{A}_3$, $\mathcal{A}_2$ are satisfied. The option of cancelling only the mixed gauge anomalies, $\mathcal{A}_3$, $\mathcal{A}_2$, $\mathcal{A}_1$, including $\mathcal{A}_{Z^3}$ for the **O** type symmetries is much more restrictive, but still yields solutions. The first realization of **O** symmetry occurs at the group order $N = 9$ with 3 GBP and GLP solutions and some 22 GMP solutions. We have quoted in Table I the values for the gravi-

TABLE I. The solutions for the generalized flavor blind Z_N parity symmetries of the NMSSM which cancel the mixed gauge anomalies $\mathcal{A}_{3,2,1}$ only. We have made the choice $N_g = 3$, $N_{2h} = 1$ for the number of quark and lepton generations and Higgs boson pairs, respectively, and $k_1 = \frac{5}{3}$ for the hypercharge normalization. The cancellation conditions for the gravitational and nonlinear anomalies, $\mathcal{A}_{\rm grav}$, $\mathcal{A}_{Z^2}$, $\mathcal{A}_{Z^3}$ are not obeyed in general. The four rows give the solutions for the GBP, GLP, and GMP generators for symmetries of four distinct types: ordinary anomaly free (**O**), ordinary Green-Schwarz anomalous (**O/GS**), R symmetry anomaly free (**R**), and anomalous R symmetry (**R/GS**), respectively. The entries display integer (modulo N) parameters (m, n, p) for the Z_N generator, and in the suffices we have given the values of the finite anomaly coefficients $\mathcal{A}_{\rm grav}$, $\mathcal{A}_{Z^2}$. For simplicity, we have not quoted the generally finite chiral anomaly coefficient $\mathcal{A}_{Z^3}$. For the GMP symmetries, additional solutions of same order N as those quoted in the Table below also arise, as discussed in the text.

Z_N **Type**	N	**GBP**	**GLP**	**GMP**
O	9	$(1,3,4)_{-51,-87}$	$(6,3,4)_{-36,-117}$	$(0,3,4)_{-54,-81}$
		$(5,6,2)_{-75,-348}$	$(3,6,2)_{-81,-252}$	$(2,3,4)_{-48,-93}$
		$(2,6,8)_{-102,-348}$	$(3,6,8)_{-99,-360}$	$(3,3,4)_{-45,-99}$
O/GS	7	$(0,6,2)_{-126,-108}$	$(5,6,2)_{-141,-348}$	$(1,6,2)_{-129,-156}$, $(2,6,2)_{-132,-240}$
				$(3,6,2)_{-135,-252}$, $(4,6,2)_{-138,-300}$
R	12	$(8,6,6)_{-131,-560}$	$(2,6,6)_{-149,-320}$	$(0,6,6)_{-155,-240}$, $(1,6,6)_{-152,-280}$
R/GS	11	$(5,6,2)_{-160,-368}$	$(3,6,2)_{-154,-240}$	$(0,6,2)_{-145,-48}$, $(1,6,2)_{-148,-112}$

tational anomalies and $\mathcal{A}_{Z^2}$ which remain generally uncanceled. For instance, the GBP generator $(m, n, p) = (1, 3, 4) \bmod (9)$, has $\mathcal{A}_{\rm grav} = -81 = 0 \bmod (9)$, and hence a single uncanceled anomaly, $\mathcal{A}_{Z^2} = -87 = -6 \bmod (9)$. At higher group orders, a restricted number of similar **O** solutions arise at orders $N = 18$ and $N = 27$. For the GMP case we have displayed in the table only a subset of the solutions. In fact, the **O** GMP solutions for the Z_9 group can be grouped into the three families of generators, $(m, 3, 4)$, $[m = 0, 2, 3, 4, 5, 7]$; $(m, 6, 2)$, $[m = 0, 1, 2, 4, 6, 7]$; $(m, 6, 8)$, $[m = 0, 1, 4, 5, 6, 7]$.

Having a fixed N solution is not enough, since we must still solve the equations for the S field charges. A large number of solutions exist, in general, for the **O** symmetries under discussion. For instance, with the GBP solution $(m, n, p) = (1, 3, 4) \bmod (9)$, the equations $x + 3y + 4z = -3 \bmod (9)$, $3(x + 3y + 4z) = 0 \bmod 9$ admit about 80 different solutions of which we quote an illustrative sample: $(x, y, z) = (0, 0, 6), (0, 1, 3), (0, 2, 0), (0, 3, 6), (0, 4, 3)$. The two other solutions, namely $(m, n, p) = (5, 6, 2), (2, 6, 8) \bmod 9$, have similar features.

The case of Green-Schwarz anomalous symmetries **O/GS** is more constrained than the nonanomalous case discussed above. Unless one excludes the model-dependent anomalies $\mathcal{A}_{Z^2}$, $\mathcal{A}_{Z^3}$, and $\mathcal{A}_{\rm grav}$, no solutions exist. However, by restricting again to the mixed gauge anomaly cancellation conditions only, which we express in terms of vanishing linear combinations modulo N, $\mathcal{A}_3/k_3 - \mathcal{A}_2/k_2$, $k_1\mathcal{A}_3/k_3 - \mathcal{A}_1$, $k_1\mathcal{A}_2/k_2 - \mathcal{A}_1$, and setting the normalization parameters for the SM gauge group factors at the rational values, $k_3 = k_2 = 1$, $k_1 = \frac{5}{3}$, we find a single GBP solution appearing first for the group Z_7 with the generator $(m, n, p) = (0, 6, 2)$. While this generator turns out to have a vanishing gravitational anomaly, $k_{\rm grav}\mathcal{A}_3/k_3 - \mathcal{A}_{\rm grav} = -126 = 0 \bmod 7$, it still exhibits an uncanceled nonlinear anomaly, $\mathcal{A}_{Z^2} = -108 = -3 \bmod 7$. Note that the next group order at which solutions appear is the integer multiple of the above with $N = 14$. Unfortunately, proceeding to the next stage of solving for the S field charges, we find it impossible to solve the relevant equations for the integers x, y, z. It is possible that this feature may be cured by adding an extra gauge singlet field.

The cyclic R parity discrete symmetries, **R** and **R/GS**, are more severely constrained than the ordinary ones. As seen on Table I, unique solutions are found at orders 12 and 11, respectively, if one chooses to cancel the gauge anomalies only, while leaving $\mathcal{A}_{\rm grav}$ and the nonlinear anomalies uncanceled. The **R** GMP solutions for the group Z_{12} include the family $(m, 6, 6)$, $[m = 0, 1, 3, 4, 5, 7, 7, 9, 10]$, and the **R/GS** GMP solutions for the group Z_{11} include the family $(m, 6, 2)$, $[m = 0, 1, 2, 4, 6, 7, 8, 9]$. For both anomaly free and anomalous cases, we fail to find solutions for the equations for the S field charges x, y, z.

In closing the discussion of results, we note that practically all the GMP solutions in Table I forbid the dimension-5 dangerous operators $QQQL$ and $U^cD^cU^cE^c$. This is easily established by noting the corresponding selection rules, which require for **O** and **R** symmetries vanishing values for the total charges, $\hat{g}(QQQL) = -(n + p)$, $\hat{g}(U^cD^cU^cE^c) = -(n - p)$ and $\Delta(QQQL) = -(2 + n + p)$, $\Delta(U^cD^cU^cE^c) = -(2 + n - p)$, where $\Delta(\Phi^M) = \hat{\hat{g}}(W) - (2 - M)$.

It is of interest to find out whether, by slightly modifying our search strategy, alternative options could exist. One might first consider solving the anomaly cancellation equations by setting the number of generations at the smaller values, $N_g = 1$ or $N_g = 2$. These solutions can be combined into quark and lepton generation dependent direct products, $(Z_N)^3_{N_g=1}$ or $(Z_N)_{N_g=2} \times (Z_N)_{N_g=1}$. This option is not promising, however, since the solutions for $N_g < 2$ are even more scarce than for $N_g = 3$. Thus, for GBP with $N_g = 2$, the first solution for **O** symmetries occurs at $N = 16$, with $(m, n, p) = (14, 12, 6)$, for **R** symmetries at $N = 12$, with $(m, n, p) = (8, 6, 6)$, and for **R/GS** symmetries at $N = 5$ with $(m, n, p) = (2, 4, 2)$. The case with $N_g = 1$ does not have any solutions.

Another option consists in enforcing the modulo N cancellation of $\mathcal{A}_1$ by adjusting the hypercharge normalization. As already noted, the freedom gained in relaxing $\mathcal{A}_1$ anomaly cancellation constraint vastly increases the space of solutions. Changing the hypercharge normalization $Y \to cY$ induces the modifications $\mathcal{A}_1 \to c^2\mathcal{A}_1 \longrightarrow k_1 \to k'_1 = c^2 k_1$. Specifically, given an **O** charge generator $\hat{g}$ with an uncanceled hypercharge anomaly, $\mathcal{A}_1 \neq 0 \bmod (N)$, we can salvage the situation by transforming $Y \to cY$ so that $c^2\mathcal{A}_1 = 0 \bmod (N)$. For the anomalous **GS** symmetry, the same reasoning applies to the linear combination $c^2\mathcal{A}_1 - 2k'_1 = 0 \bmod N$. Of course, within a nonminimal grand unification or a string theory model in which some freedom is left for the hypercharge normalization, one must still consider how well the asymptotic prediction for the weak angle parameter, $\sin^2\theta_W \simeq k_2/(k_2 + k'_1)$, fits in with the observed value. Considering, for concreteness, the illustrative case where $\mathcal{A}_1 = -pN + \nu$, $[p, \nu \in Z^+]$ and changing $Y \to cY$, so that $\mathcal{A}'_1 = c^2\mathcal{A}_1 = -pN$, fixes the rescaling factor as $c^2 = \frac{k'_1}{k_1} = \frac{-pN}{-pN+\nu} \simeq 1 + \frac{\nu}{pN}$, and hence the modified asymptotic value of the weak angle as $\sin^2\theta_W \simeq \frac{3/8}{1+5\nu/8pN}$, where we have assumed, for simplicity, $\nu \ll N$.

The case of an uncanceled $\mathcal{A}_{\rm grav}$ anomaly may be treated by adding observable or hidden sector chiral supermultiplet singlets, as already hinted above. Both kinds of singlets affect only $\mathcal{A}_{\rm grav}$, $\mathcal{A}_{Z^3}$. Thus, given some generator with uncanceled gravitational and chiral anomalies, $\mathcal{A}_{\rm grav} \neq 0 \bmod N$, $\mathcal{A}_{Z^3} \neq 0 \bmod N$, one can attempt to rescue this solution by including an extra hidden sector singlet S_1 with $\hat{R}$, $\hat{A}$, $\hat{L}$ charges x_1, y_1, z_1, and solving the equations, $\mathcal{A}_{\rm grav} + \mathcal{S}_1 = 0 \mod N$, $\mathcal{A}_{Z^3} + \mathcal{S}_1^3 = 0 \mod N$ with $\mathcal{S} = mx + ny + pz$. Obvious modifications hold for the Green-Schwarz (GS) anomalous symmetries and for R symmetries.

The option of extending the matter field content of the low energy theory by adding extra vector multiplets is also very efficient in relaxing the anomaly constraints. Indeed, we recall that the string theory models achieve consistency thanks to the presence of extra charged or singlet modes in the massless particle spectrum. To conclude, we note that interesting generalizations of our discussion would be to consider direct product of cyclic groups, $Z_N \times Z_M$, or lepton flavor dependent cyclic groups.

III. PHENOMENOLOGICAL IMPLICATIONS AND NEUTRINO MASSES

A. Tree-level neutrino masses

After spontaneous breaking of the $SU(2)_L \times U(1)_Y$ gauge symmetry via the vacuum expectation values of the scalar components of H_u, H_d, and S, the gauginos and Higgsinos mix with neutrinos. The resulting lepton number violating neutrino-gaugino-Higgsino mass matrix receives contributions from gauge interactions and the superpotential

$$W_\nu = \lambda S H_d H_u + \tilde{\lambda}_a L_a H_u S - \frac{\kappa}{3} S^3, \tag{3.1}$$

which arises from the last two terms of (2.1) and the first term of (2.2), respectively. The resulting mass terms of the neutrino-gaugino-Higgsino system can be written as

$$\begin{aligned}\mathcal{L}_{\rm mass} = {}& [\lambda x \tilde{H}_u \tilde{H}_d + \lambda v_u \tilde{H}_d \tilde{S} + \lambda v_d \tilde{H}_u \tilde{S} - \kappa x \tilde{S}\tilde{S} + \text{H.c.}] + \frac{i g_2 \lambda^3}{\sqrt{2}}[v_d \tilde{H}_d - v_u \tilde{H}_u + \text{H.c.}] \\ & - \frac{i g_1 \lambda'}{\sqrt{2}}[v_d \tilde{H}_d - v_u \tilde{H}_u + \text{H.c.}] + \sum_a \tilde{\lambda}_a [x \nu_a \tilde{H}_u + v_u \nu_a \tilde{S} + v_a \tilde{H}_u \tilde{S} + \text{H.c.}] + \frac{i g_2 \lambda^3}{\sqrt{2}}\Big[\sum_a v_a \nu_a + \text{H.c.}\Big] \\ & - \frac{i g_1 \lambda'}{\sqrt{2}}\Big[\sum_a v_a \nu_a + \text{H.c.}\Big],\end{aligned} \tag{3.2}$$

where λ^3 is the third component of the $SU(2)_L$ gaugino λ^w, and λ' is the $U(1)$ gaugino. In (3.2) we have used the following notation

$$v_u = \langle H_u^0 \rangle, \qquad v_d = \langle H_d^0 \rangle, \qquad \tan\beta = \frac{v_u}{v_d}, \qquad x = \langle S \rangle, \qquad v_a = \langle \tilde{\nu}_a \rangle, \tag{3.3}$$

while the rest of the symbols have their usual meaning. Using (3.2), we find the resulting 8×8 neutralino-neutrino mass matrix in the field basis $(-i\lambda', -i\lambda^3, \tilde{H}_u, \tilde{S}, \tilde{H}_d, \nu_e, \nu_\mu, \nu_\tau)$ as

$$\mathbf{M_N} = \begin{bmatrix} M_1 & 0 & \frac{g_1 v_u}{\sqrt{2}} & 0 & \frac{-g_1 v_d}{\sqrt{2}} & \frac{-g_1 v_1}{\sqrt{2}} & \frac{-g_1 v_2}{\sqrt{2}} & \frac{-g_1 v_3}{\sqrt{2}} \\ 0 & M_2 & \frac{-g_2 v_u}{\sqrt{2}} & 0 & \frac{g_2 v_d}{\sqrt{2}} & \frac{g_2 v_1}{\sqrt{2}} & \frac{g_2 v_2}{\sqrt{2}} & \frac{g_2 v_3}{\sqrt{2}} \\ \frac{g_1 v_u}{\sqrt{2}} & \frac{-g_2 v_u}{\sqrt{2}} & 0 & Y & -\lambda x & -\tilde{\lambda}_1 x & -\tilde{\lambda}_2 x & -\tilde{\lambda}_3 x \\ 0 & 0 & Y & 2\kappa x & -\lambda v_u & -\tilde{\lambda}_1 v_u & -\tilde{\lambda}_2 v_u & -\tilde{\lambda}_3 v_u \\ \frac{-g_1 v_d}{\sqrt{2}} & \frac{g_2 v_d}{\sqrt{2}} & -\lambda x & -\lambda v_u & 0 & 0 & 0 & 0 \\ \frac{-g_1 v_1}{\sqrt{2}} & \frac{g_2 v_1}{\sqrt{2}} & -\tilde{\lambda}_1 x & -\tilde{\lambda}_1 v_u & 0 & 0 & 0 & 0 \\ \frac{-g_1 v_2}{\sqrt{2}} & \frac{g_2 v_2}{\sqrt{2}} & -\tilde{\lambda}_2 x & -\tilde{\lambda}_2 v_u & 0 & 0 & 0 & 0 \\ \frac{-g_1 v_3}{\sqrt{2}} & \frac{g_2 v_3}{\sqrt{2}} & -\tilde{\lambda}_3 x & -\tilde{\lambda}_3 v_u & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{3.4}$$

where

$$Y = -\lambda v_d - \sum_a \tilde{\lambda}_a v_a. \tag{3.5}$$

The mass matrix (3.4) can be written in the form

$$\mathbf{M_N} = \begin{bmatrix} \mathcal{M}_{\chi^0} & m^T \\ m & 0_{3\times 3} \end{bmatrix}, \tag{3.6}$$

where

$$\mathcal{M}_{\chi^0} = \begin{bmatrix} M_1 & 0 & \frac{g_1 v_u}{\sqrt{2}} & 0 & \frac{-g_1 v_d}{\sqrt{2}} \\ 0 & M_2 & \frac{-g_2 v_u}{\sqrt{2}} & 0 & \frac{g_2 v_d}{\sqrt{2}} \\ \frac{g_1 v_u}{\sqrt{2}} & \frac{-g_2 v_u}{\sqrt{2}} & 0 & Y & -\lambda x \\ 0 & 0 & Y & 2\kappa x & -\lambda v_u \\ \frac{-g_1 v_d}{\sqrt{2}} & \frac{g_2 v_d}{\sqrt{2}} & -\lambda x & -\lambda v_u & 0 \end{bmatrix}, \tag{3.7}$$

and

$$m^T = \begin{bmatrix} \frac{-g_1 v_1}{\sqrt{2}} & \frac{-g_1 v_2}{\sqrt{2}} & \frac{-g_1 v_3}{\sqrt{2}} \\ \frac{g_2 v_1}{\sqrt{2}} & \frac{g_2 v_2}{\sqrt{2}} & \frac{g_2 v_3}{\sqrt{2}} \\ -\tilde{\lambda}_1 x & -\tilde{\lambda}_2 x & -\tilde{\lambda}_3 x \\ -\tilde{\lambda}_1 v_u & -\tilde{\lambda}_2 v_u & -\tilde{\lambda}_3 v_u \\ 0 & 0 & 0 \end{bmatrix}. \tag{3.8}$$

The block form displayed in (3.6) clearly demonstrates the "seesaw structure" of the mass matrix (3.4). We note that the block matrix m characterizes the lepton number violation in the model. Furthermore, the NMSSM with lepton number violation is invariant under the $SU(4)$ group acting on the set of superfields (H_d, L_i), in the sense that the action of $SU(4)$ transformations on (H_d, L_i) leaves the superpotential form invariant up to corresponding transformations of Yukawa couplings. We can use this freedom to choose a basis which is characterized by vanishing sneutrinos vacuum expectation values, $v_a = 0$. We shall choose such a basis in the following whenever it is convenient.

The masses of the neutralinos and neutrinos can be obtained by the diagonalization of the mass matrix (3.4)

$$\mathcal{N}^* \mathbf{M_N} \mathcal{N}^{-1} = \mathrm{diag}(m_{\chi_i^0}, m_{\nu_j}), \tag{3.9}$$

where $m_{\chi_i^0}$, $(i = 1, \ldots, 5)$ are the neutralino masses, and m_{ν_j}, $(j = 1, 2, 3)$ are the neutrino masses, respectively. The matrix (3.4) cannot, in general, be diagonalized analytically. However, we are interested in the case where the tree-level neutrino masses as determined from the mass matrix (3.4) are small. In this case we can find approximate analytical expression for the neutrino masses which are valid in the limit of small lepton number violating couplings. To do so, we define the matrix [39]

$$\xi = m \cdot \mathcal{M}_{\chi^0}^{-1}. \tag{3.10}$$

If all the elements of this matrix are small, i. e.

$$\xi_{ij} \ll 1, \tag{3.11}$$

then we can use it as an expansion parameter for finding an approximate solution for the mixing matrix $\mathcal{N}$. Calculating the matrix elements of ξ_{ij} we find

$$\begin{aligned}
\xi_{i1} &= \frac{g_1 M_2(\lambda x)}{\sqrt{2}\det(\mathcal{M}_{\chi^0})}[(2\kappa x)x - 2Yv_u]\Lambda_i, \\
\xi_{i2} &= -\frac{g_2 M_1(\lambda x)}{\sqrt{2}\det(\mathcal{M}_{\chi^0})}[(2\kappa x)x - 2Yv_u]\Lambda_i, \\
\xi_{i3} &= -\frac{(g_2^2 M_1 + g_1^2 M_2)}{2\det(\mathcal{M}_{\chi^0})}[(2\kappa x)v_d x + (\lambda v_u^2 - Yv_d)v_u]\Lambda_i, \\
\xi_{i4} &= \frac{(g_2^2 M_1 + g_1^2 M_2)}{2\det(\mathcal{M}_{\chi^0})}(\lambda v_u^2 + Yv_d)x\Lambda_i, \\
\xi_{i5} &= -\frac{\tilde{\lambda}_i x}{\lambda x}\left[1 + \frac{(g_2^2 M_1 + g_1^2 M_2)}{2\det(\mathcal{M}_{\chi^0})}(\lambda v_u^2 - Yv_d)^2\right] \\
&\quad + \frac{(g_2^2 M_1 + g_1^2 M_2)}{2\det(\mathcal{M}_{\chi^0})}[(2\kappa x)v_u x\Lambda_i \\
&\quad + (\tilde{\lambda}_i v_u^2 - Yv_i)(\lambda v_u^2 - Yv_d)],
\end{aligned} \tag{3.12}$$

where we have used the notation

$$\Lambda_i = \lambda v_i - \tilde{\lambda}_i v_d. \tag{3.13}$$

We note that ξ_{i5} is not proportional to Λ_i. From Eqs. (3.12) and (3.13), we see that $\xi = 0$ in the MSSM limit where $\tilde{\lambda}_i = 0$, $v_i = 0$. The matrix $\mathcal{N}^*$ which diagonalizes the neutralino-neutrino mass matrix $\mathbf{M_N}$ can now be written as

$$\mathcal{N}^* = \begin{pmatrix} N^* & 0 \\ 0 & V_\nu^T \end{pmatrix}\begin{pmatrix} 1 - \frac{1}{2}\xi^\dagger\xi & \xi^\dagger \\ -\xi & 1 - \frac{1}{2}\xi\xi^\dagger \end{pmatrix}, \tag{3.14}$$

where we have retained only the leading order terms in ξ. The second matrix in (3.14) block diagonalizes the neutralino-neutrino mass matrix $\mathbf{M_N}$ to the form $\mathrm{diag}(\mathcal{M}_{\chi^0}, m_{\mathrm{eff}})$, with

$$\begin{aligned}
m_{\mathrm{eff}} &= -m \cdot \mathcal{M}_{\chi^0}^{-1} \cdot m^T \\
&= \frac{(M_1 g^2 + M_2 g'^2)(\kappa x^2 - Yv_u)x}{\det(\mathcal{M}_{\chi^0})} \\
&\quad \times \begin{pmatrix} \Lambda_e^2 & \Lambda_e\Lambda_\mu & \Lambda_e\Lambda_\tau \\ \Lambda_e\Lambda_\mu & \Lambda_\mu^2 & \Lambda_\mu\Lambda_\tau \\ \Lambda_e\Lambda_\tau & \Lambda_\mu\Lambda_\tau & \Lambda_\tau^2 \end{pmatrix},
\end{aligned} \tag{3.15}$$

where the tree-level contribution to the light neutrino mass matrix admits the Feynman diagram representation of Fig. 1(a). The eigenvalues of m_{eff} give the tree-level neutrino masses. These eigenvalues are

$$m_{\nu_3} = \frac{(M_1 g^2 + M_2 g'^2)(\kappa x^2 - Yv_u)x}{\det(\mathcal{M}_{\chi^0})}\sum_i \Lambda_i^2, \tag{3.16}$$

$$m_{\nu_1} = m_{\nu_2} = 0, \tag{3.17}$$

where we have used $m_{\nu_3} \geq m_{\nu_2} \geq m_{\nu_1}$. Thus, at the tree level only one neutrino is massive. Its mass is proportional to the lepton number (and R-parity) violating parameter $\sum_i \Lambda_i^2$. A single lepton number violating coupling $\tilde{\lambda}_i$ can

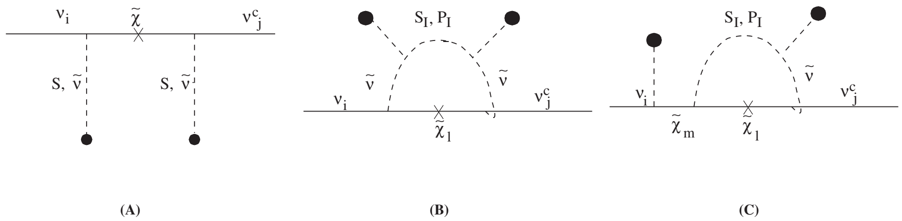

FIG. 1. The two-point amplitudes for the process $\nu_i \to \nu_j^c$ associated with contributions to the neutrino Majorana mass matrix at tree level (graph (A)) due to exchange of neutralino $\tilde{\chi}_l$, and at one-loop level (graphs (B) and (C)) due to intermediate propagation of neutralino $\tilde{\chi}_l$, sneutrino $\tilde{\nu}_i$, and Higgs boson sector mass basis modes S_I, P_I. The amplitude in (A) is initiated by tadpoles (VEVs) of H_u, S, $\tilde{\nu}$, and the amplitudes in (B) and (C) by double and single tadpoles of H_u, H_d, S, $\tilde{\nu}$. The cross on the neutralino propagator indicates a mass insertion term.

lead to a nonzero neutrino mass. Furthermore, we note that the submatrices N and V_ν diagonalize $\mathcal{M}_{\chi^0}$ and $m_{\rm eff}$:

$$N^* \mathcal{M}_{\chi^0} N^\dagger = \mathrm{diag}(m_{\chi_i^0}), \tag{3.18}$$

$$V_\nu^T m_{\rm eff} V_\nu = \mathrm{diag}(0, 0, m_{\nu_3}). \tag{3.19}$$

Since only one of the neutrinos obtains mass, we can rotate away one of the three angles in the matrix V_ν. We can then write V_ν as a product of two matrices [40]

$$V_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & -\sin\theta_{23} \\ 0 & \sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \times \begin{pmatrix} \cos\theta_{13} & 0 & -\sin\theta_{13} \\ 0 & 1 & 0 \\ \sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}, \tag{3.20}$$

where the mixing angles can be written in terms of Λ_i as

$$\tan\theta_{13} = \frac{\Lambda_e}{(\Lambda_\mu^2 + \Lambda_\tau^2)^{\frac{1}{2}}}, \tag{3.21}$$

$$\tan\theta_{23} = -\frac{\Lambda_\mu}{\Lambda_\tau}. \tag{3.22}$$

Finally, using the expression

$$\begin{aligned} \det(\mathcal{M}_{\chi^0}) = {} & (2\kappa x)(\lambda x)[(g_1^2 M_2 + g_2^2 M_1) v_u v_d \\ & - M_1 M_2 (\lambda x)^2] + 2 M_1 M_2 (\lambda v_u)(\lambda x) Y \\ & + \tfrac{1}{2}(g_1^2 M_1 + g_2^2 M_2)(\lambda v_u^2 - v_d Y)^2, \end{aligned} \tag{3.23}$$

for the determinant of $\mathcal{M}_{\chi^0}$ in (3.16), we have for the tree-level mass of the neutrino

$$m_{\nu_3} \sim \frac{\cos^2\beta}{\tilde{m}} \sum_i (\tilde{\lambda}_i x)^2, \tag{3.24}$$

where we have assumed that all the relevant masses (and the relevant vacuum expectation values) are at the electroweak (or supersymmetry breaking scale) scale denoted by $\tilde{m}$. For simplicity we have chosen the basis in which the sneutrino vacuum expectation values $v_i = 0$ to write the result (3.24). We, thus, see that apart from the R-parity violating parameter $\sum_i (\tilde{\lambda}_i x)^2$, the tree-level neutrino mass is proportional to $\cos^2\beta$. For large values of $\tan\beta$, this could lead to a suppression of m_{ν_3}, which could be important.

It is now important to calculate the admixture of the singlet component (arising from the fermionic component of the Higgs singlet superfield S) in the three light neutrino states. From (3.14) we can write the matrix $\mathcal{N}^*$ which diagonalizes the neutralino-neutrino mass matrix as

$$\mathcal{N}^* = \begin{pmatrix} N^*(1 - \frac{1}{2}\xi^\dagger \xi) & N^* \xi^\dagger \\ -V_\nu^T \xi & V_\nu^T (1 - \frac{1}{2}\xi \xi^\dagger) \end{pmatrix}. \tag{3.25}$$

The eigenvectors of the neutralino-neutrino mass matrix are then given by

$$F_i^0 = \mathcal{N}_{ij} \psi_j, \tag{3.26}$$

where as indicated above we use the basis $\psi_j = (-i\lambda', -i\lambda^3, \tilde{H}_u, \tilde{S}, \tilde{H}_d, \nu_e, \nu_\mu, \nu_\tau)$. The singlet component in the three neutrino states is then given by $|\mathcal{N}_{64}|^2$, $|\mathcal{N}_{74}|^2$, and $|\mathcal{N}_{84}|^2$, respectively. For calculating these components we require the submatrix $V_\nu^T \xi$ of the matrix (3.25). It is straightforward to calculate this submatrix, and the result is ($\vec{\Lambda} = (\Lambda_e, \Lambda_\mu, \Lambda_\tau)$)

$$V_\nu^T \xi = \begin{pmatrix} 0 & 0 & 0 & 0 & \tilde{\epsilon}_1 \\ 0 & 0 & 0 & 0 & \tilde{\epsilon}_2 \\ a_1 |\vec{\Lambda}| & a_2 |\vec{\Lambda}| & a_3 |\vec{\Lambda}| & a_4 |\vec{\Lambda}| & \tilde{\epsilon}_3 + a_5 |\vec{\Lambda}| \end{pmatrix}, \tag{3.27}$$

where

$$\tilde{\epsilon}_1 = \frac{\tilde{\epsilon}_e (\Lambda_\mu^2 + \Lambda_\tau^2) - \Lambda_e (\Lambda_\mu \tilde{\epsilon}_\mu + \Lambda_\tau \tilde{\epsilon}_\tau)}{\sqrt{(\Lambda_\mu^2 + \Lambda_\tau^2)} \sqrt{(\Lambda_e^2 + \Lambda_\mu^2 + \Lambda_\tau^2)}}, \tag{3.28}$$

$$\tilde{\epsilon}_2 = \frac{-\tilde{\epsilon}_\mu \Lambda_\tau + \tilde{\epsilon}_\tau \Lambda_\mu}{\sqrt{(\Lambda_\mu^2 + \Lambda_\tau^2)}}, \tag{3.29}$$

$$\tilde{\epsilon}_3 = -\frac{\vec{\Lambda}\cdot\vec{\tilde{\epsilon}}}{\sqrt{(\Lambda_e^2+\Lambda_\mu^2+\Lambda_\tau^2)}}, \tag{3.30}$$

$$\tilde{\epsilon}_e = -\frac{\tilde{\lambda}_1 x}{\lambda x}\left[1+\frac{(g_2^2M_1+g_1^2M_2)}{2\det(\mathcal{M}_{\chi^0})}(\lambda v_u^2 - Y v_d)^2\right] + \frac{(g_2^2M_1+g_1^2M_2)}{2\det(\mathcal{M}_{\chi^0})}[(\tilde{\lambda}_i v_u^2 - Y v_1)(\lambda v_u^2 - Y v_d)], \tag{3.31}$$

$$\tilde{\epsilon}_\mu = -\frac{\tilde{\lambda}_2 x}{\lambda x}\left[1+\frac{(g_2^2M_1+g_1^2M_2)}{2\det(\mathcal{M}_{\chi^0})}(\lambda v_u^2 - Y v_d)^2\right] + \frac{(g_2^2M_1+g_1^2M_2)}{2\det(\mathcal{M}_{\chi^0})}[(\tilde{\lambda}_2 v_u^2 - Y v_2)(\lambda v_u^2 - Y v_d)], \tag{3.32}$$

$$\tilde{\epsilon}_\tau = -\frac{\tilde{\lambda}_3 x}{\lambda x}\left[1+\frac{(g_2^2M_1+g_1^2M_2)}{2\det(\mathcal{M}_{\chi^0})}(\lambda v_u^2 - Y v_d)^2\right] + \frac{(g_2^2M_1+g_1^2M_2)}{2\det(\mathcal{M}_{\chi^0})}[(\tilde{\lambda}_3 v_u^2 - Y v_3)(\lambda v_u^2 - Y v_d)], \tag{3.33}$$

and

$$a_1 = -\frac{g_1M_2(\lambda x)}{\sqrt{2}\det(\mathcal{M}_{\chi^0})}[(2\kappa x)x - 2Y v_u], \tag{3.34}$$

$$a_2 = \frac{g_2M_1(\lambda x)}{\sqrt{2}\det(\mathcal{M}_{\chi^0})}[(2\kappa x)x - 2Y v_u], \tag{3.35}$$

$$a_3 = \frac{(g_2^2M_1+g_1^2M_2)}{2\det(\mathcal{M}_{\chi^0})}[(2\kappa x)v_d x + (\lambda v_u^2 - Y v_d)v_u], \tag{3.36}$$

$$a_4 = \frac{(g_2^2M_1+g_1^2M_2)}{2\det(\mathcal{M}_{\chi^0})}(\lambda v_u^2 + Y v_d)x, \tag{3.37}$$

$$a_5 = -\frac{(g_2^2M_1+g_1^2M_2)}{2\det(\mathcal{M}_{\chi^0})}[(2\kappa x)v_u x]. \tag{3.38}$$

From (3.25) and (3.27) we obtain the important result

$$|\mathcal{N}_{64}|^2 = |\mathcal{N}_{74}|^2 = 0, \tag{3.39}$$

$$|\mathcal{N}_{84}|^2 = a_4^2|\vec{\Lambda}|^2. \tag{3.40}$$

Thus, at the tree level two light neutrinos do not have a singlet component, whereas the heaviest neutrino has a singlet component with a strength proportional to the square of the lepton number violating parameter $|\vec{\Lambda}|$.

B. One-loop supersymmetry breaking contributions

As shown above, at the tree level only one of the neutrinos obtains a mass through the lepton number violating Yukawa coupling $\tilde{\lambda}_i$, so that the tree-level neutrino mass matrix can be written as

$$m_\nu^0 \equiv V_\nu^T m_{\rm eff} V_\nu = {\rm diag}(0, 0, m_{\nu_3}). \tag{3.41}$$

However, the neutrino mass matrix can receive contributions from loop effects. The supersymmetry breaking parameters are expected to play a crucial role through the one-loop corrections involving gauge interactions with exchange of sneutrinos [17,20]. At one-loop level the needed suppression of neutrino masses can arise from cancellations between contributions involving the Higgs sector, and from possible mass degeneracies among the sneutrinos. In the context of MSSM, this has been discussed in [24].

At one-loop level, finite Majorana neutrino masses can be generated through two classes of mechanisms involving either the gauge or superpotential interactions in combination with the soft supersymmetry breaking interactions. These mechanisms have been discussed in detail for the MSSM [29]. The loop amplitudes in the former class propagate matter particles and contribute at orders $\tilde{\lambda}_i\tilde{\lambda}_j$, $\tilde{\lambda}_i\lambda_{ijk}$, and $\tilde{\lambda}_i\lambda'_{ijk}$, and those in the latter class propagate sleptons and gauginos and contribute at orders $A_{\tilde{\lambda}_i}\tilde{\lambda}_i A_{\tilde{\lambda}_j}\tilde{\lambda}_j$, $A_{\tilde{\lambda}_i}\tilde{\lambda}_i\tilde{\lambda}_j$. These are associated with the mixing of sneutrinos and Higgs bosons, and are also responsible for the mass splittings between sneutrinos and antisneutrinos [17]. The possibility that the combined tree and one-loop contributions in the MSSM could account for the observed flavor hierarchies in the masses and mixing angles of light neutrinos has been studied in several recent works [20–25]. The supersymmetry breaking interactions, the cancellations between contributions involving the Higgs sector modes, and mass splittings among sneutrinos of different flavors, are expected to play a crucial role.

The present section is aimed at studying the one-loop contributions to the neutrino mass matrix, and the extent to which these constitute a sensitive probe of the Higgs boson sector of the NMSSM. The finite VEVs for the components of the scalar fields H_d, H_u, S, $\tilde{\nu}_i$ can result in one-loop contributions for the two-point amplitude represented by the Majorana mass term $L_{\rm EFF} = -\frac{1}{2}(m_\nu)_{ij}\bar{\nu}_j^c\nu_i + {\rm H.c.}$ These contributions are displayed by the Feynman diagrams in Fig. 1(b) and 1(c) with double and single mass sneutrino-scalar mass mixing insertion terms. These are the analogs of MSSM [17] for the case of NMSSM. It is important to carefully treat these seesaw-like contributions by expressing the intermediate scalar and pseudoscalar propagators in the mass eigenbasis. We shall first obtain the scalar potential for the sneutrinos and Higgs bosons, minimize it with respect to the corresponding VEVs, v_d, v_u, x, v_i, extract the squared mass matrix whose off-diagonal blocks represent the sneutrino-Higgs mass mixing terms, and finally evaluate the one-loop contributions to the neutrino Majorana mass terms, $(m_\nu)_{ij}$.

1. Coupling of Higgs boson and sneutrino sectors

Using the standard procedure, we can write down the scalar potential of the NMSSM involving the relevant components of the complex scalar fields in terms of the F-terms, arising from the superpotential, the D-terms, arising from the gauge interactions, and the soft supersymmetry breaking terms as follows:

$$\begin{aligned} W_\nu &= \lambda H_d H_u S + \tilde{\lambda}_i L_i H_u S - \frac{\kappa}{3} S^3, \\ V &= V_F + V_D + V_{\mathrm{soft}}, \\ V_F &= \sum_i \left| \frac{\partial W_\nu}{\partial \phi_i} \right|^2, \\ V_D &= \frac{g_1^2 + g_2^2}{8} (|v_u|^2 - |v_A|^2)^2, \\ V_{\mathrm{soft}} &= \sum_{AB} M^2_{\tilde{\nu}_B \tilde{\nu}_A} \tilde{\nu}_B^\star \tilde{\nu}_A + m^2_{H_u} |v_u|^2 + m_S^2 |x|^2 \\ &\quad - \left[A_{\tilde{\lambda}_A} \tilde{\lambda}_A v_A v_u + \frac{A_\kappa \kappa}{3} x^3 + H.c. \right]. \end{aligned} \tag{3.42}$$

In (3.43), ϕ_i stand for all the relevant scalar fields, and we have used the convenient four-vector notations $\tilde{\lambda}_A = (\lambda, \tilde{\lambda}_i)$, $v_A = (v_d, v_i)$, with the summation convention over repeated indices understood. The VEVs of different fields are temporarily extended to complex numbers, $v_d = \langle H_d \rangle = v_{d1} + i v_{d2}$, $v_u = \langle H_u \rangle = v_{u1} + i v_{u2}$, $x = \langle S \rangle = x_1 + i x_2$, $v_i = \langle \tilde{\nu}_i \rangle = v_{i1} + i v_{i2}$, corresponding to the decomposition of Higgs boson and sneutrino fields into real scalar CP-even and imaginary pseudoscalar CP-odd components [3]

$$\begin{aligned} H_d &= \frac{H_{dR} + i H_{dI}}{\sqrt{2}}, \qquad H_u = \frac{H_{uR} + i H_{uI}}{\sqrt{2}}, \\ S &= \frac{S_R + i S_I}{\sqrt{2}}, \qquad \tilde{\nu}_i = \frac{\nu_{iR} + i \nu_{iI}}{\sqrt{2}}. \end{aligned} \tag{3.43}$$

This basis for the scalar fields (H_{dR}, H_{uR}, S_R), (H_{dI}, H_{uI}, S_I) is related to the mass eigenstate basis (S_I), (P_I), $[I = 1, 2, 3]$ by the linear transformations

$$\begin{aligned} \begin{pmatrix} H_{dR} \\ H_{uR} \\ S_R \end{pmatrix} - \sqrt{2}\Re \begin{pmatrix} v_d \\ v_u \\ x \end{pmatrix} &= U_s^T \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}, \\ \begin{pmatrix} H_{dI} \\ H_{uI} \\ S_I \end{pmatrix} - \sqrt{2}\Im \begin{pmatrix} v_d \\ v_u \\ x \end{pmatrix} &= U_p^T \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}, \end{aligned} \tag{3.44}$$

where U_s, U_p denote the unitary matrices which diagonalize the mass squared matrices in the interaction basis, $(M^2)_{H_{dR}, H_{uR}, S_R} \equiv M^2_{s,ij}$ and $(M^2)_{H_{dI}, H_{uI}, S_I} \equiv M^2_{p,ij}$, using the definition $U^T_{s,p} M^2_{s,p} U_{s,p} = (M^2_{s,p})_{\mathrm{diag}}$. We then minimize the scalar potential with respect to the VEVs of various fields, and eliminate the soft supersymmetry breaking mass parameters $m^2_{H_u}$, m_S^2, $m^2_{H_d}$ and the product $M^2_{\tilde{\nu}_i \tilde{\nu}_j} v_j^\star$ through the equations

$$\begin{aligned} \frac{\partial V}{\partial v_u} = 0, \qquad \frac{\partial V}{\partial x} = 0, \\ \frac{\partial V}{\partial v_A} \equiv \frac{1}{2} \left(\frac{\partial V}{\partial v_{AR}} - i \frac{\partial V}{\partial v_{AI}} \right) = 0. \end{aligned} \tag{3.45}$$

The mass squared matrices for the CP-even and CP-odd sector fields $(H_d, H_u, S, \tilde{\nu}_i)_{R,I}$ are then evaluated by applying the definitions

$$M^2_{s,ij} = \frac{d^2 V}{\partial \phi_{iR} \partial \phi_{jR}}, \qquad M^2_{p,ij} = \frac{d^2 V}{\partial \phi_{iI} \partial \phi_{jI}}. \tag{3.46}$$

Finally, we restrict our considerations to the physical vacuum solutions with vanishing imaginary parts of the field VEVs, $\Im(v_u) = 0$, $\Im(v_d) = 0$, $\Im(x) = 0$, $\Im(v_i) = 0$. For convenience, and without loss of generality, we shall also specialize to the choice of L_A field basis characterized by vanishing complex sneutrino VEVs, $v_i = 0$. While feasible, the basis independent analysis in the supersymmetry breaking case [19,20] is significantly complicated by the need to account for several independent algebraic invariants. As a function of the dimensionless parameters (λ, $\tilde{\lambda}_i$, κ), of the dimensional supersymmetry breaking parameters (A_λ, $A_{\tilde{\lambda}_i}$, A_κ), which include the gravitino mass parameter $m_{3/2}$, and of the scalar field VEVs (v_d, v_u, x), the scalar and pseudoscalar mass squared matrices $M^2_{s,ij}$, $M^2_{p,ij}$, $[i, j = d, u, S, \tilde{\nu}_k]$ are given by the symmetric matrices

$$\begin{aligned} M^2_{s,dd} &= \frac{1}{v_d} \left[\frac{g_2^2 + g_1^2}{2} v_d^3 + v_u x (A_\lambda \lambda + \kappa \lambda x) \right], \\ M^2_{s,du} &= -\frac{g_2^2 + g_1^2}{2} v_d v_u + 2\lambda^2 v_d v_u - A_\lambda \lambda x - \kappa \lambda^2 x^2, \\ M^2_{s,dS} &= -(A_\lambda \lambda v_u) + 2\lambda(\lambda v_d - \kappa v_u) x, \\ M^2_{s,d\tilde{\nu}_i} &= \frac{v_u x}{v_d} (A_{\tilde{\lambda}_i} \tilde{\lambda}_i + \kappa \tilde{\lambda}_i x), \\ M^2_{s,uu} &= \frac{1}{v_u} \left[\frac{g_2^2 + g_1^2}{2} v_u^3 + v_d x (A_\lambda \lambda + \kappa \lambda x) \right], \\ M^2_{s,uS} &= -A_\lambda \lambda v_d + 2[-\kappa \lambda v_d + (\lambda^2 + \tilde{\lambda}_i^2) v_u] x, \\ M^2_{s,u\tilde{\nu}_i} &= 2\lambda \tilde{\lambda}_i v_d v_u - x(A_{\tilde{\lambda}_i} \tilde{\lambda}_i + \kappa \tilde{\lambda}_i x), \\ M^2_{s,SS} &= \frac{A_\lambda \lambda v_d v_u}{x} + x(-A_\kappa \kappa + 4\kappa^2 x), \\ M^2_{s,S\tilde{\nu}_i} &= -A_{\lambda_i} \tilde{\lambda}_i v_u + 2\tilde{\lambda}_i (\lambda v_d - \kappa v_u) x, \\ M^2_{s,\tilde{\nu}_i \tilde{\nu}_j} &= M^2_{s,\tilde{\nu}\tilde{\nu}} + \frac{g_2^2 + g_1^2}{4} (v_d^2 - v_u^2) + \tilde{\lambda}_i^2 (v_u^2 + x^2), \end{aligned} \tag{3.47}$$

for the CP-even scalars, and the symmetric matrices

$$
\begin{aligned}
M^2_{p,dd} &= \frac{v_u x}{v_d}(A_\lambda \lambda + \kappa\lambda x),\\
M^2_{p,du} &= x(A_\lambda \lambda + \kappa\lambda x),\\
M^2_{p,dS} &= v_u(A_\lambda \lambda - 2\kappa\lambda x),\\
M^2_{p,d\tilde{\nu}_i} &= \frac{v_u x}{v_d}(A_{\tilde{\lambda}_i}\tilde{\lambda}_i + \kappa\tilde{\lambda}_i x),\\
M^2_{p,uu} &= \frac{v_d x}{v_u}(A_\lambda \lambda + \kappa\lambda x),\\
M^2_{p,uS} &= v_d(A_\lambda \lambda - 2\kappa\lambda x),\\
M^2_{p,u\tilde{\nu}_i} &= x(A_{\tilde{\lambda}_i}\tilde{\lambda}_i + \kappa\tilde{\lambda}_i x),\\
M^2_{p,SS} &= 4\kappa\lambda v_d v_u + \frac{A_\lambda \lambda v_d v_u}{x} + 3A_\kappa \kappa x,\\
M^2_{p,S\tilde{\nu}_i} &= v_u(A_{\tilde{\lambda}_i}\tilde{\lambda}_i - 2\kappa\tilde{\lambda}_i x),\\
M^2_{p,\tilde{\nu}_i\tilde{\nu}_i} &= M^2_{\tilde{\nu}_i\tilde{\nu}_j} + \frac{g_2^2 + g_1^2}{4}(v_d^2 - v_u^2) + \tilde{\lambda}_i^2(v_u^2 + x^2),
\end{aligned}
\tag{3.48}
$$

for the CP-odd scalars. The orthogonal linear combinations of CP-odd scalar fields, $G^0(x) = \cos\beta H_{dI}(x) - \sin\beta H_{uI}(x)$, $A(x) = \sin\beta H_{dI}(x) + \cos\beta H_{uI}(x)$, identify with the decoupled Goldstone field which is absorbed as the longitudinal polarization mode of Z^0 and with the axionic symmetry pseudo-Goldstone boson mode A, respectively. The mass squared matrix in the field basis $[A, S_I, \tilde{\nu}_i]$ is obtained by first applying the similarity transformation $(G^0, A, S_I)^T = \mathcal{R}^T(H_{dI}, H_{uI}, S_I)^T$, $[\mathcal{R} = \mathrm{diag}(\mathcal{R}_\beta, 1)]$ with $\mathcal{R}_\beta$ denoting the $SO(2)$ rotation of angle β, and next by removing the decoupled Goldstone mode G^0. The mass squared matrix in the transformed basis, $(M^2)_{G,A,S_I} = \mathcal{R}^T(M^2)_{H_{dI},H_{uI},S_I}\mathcal{R}$, $[\mathcal{R} = \mathrm{diag}(\mathcal{R}_\beta, 1_4)]$ can then be written as

$$
\begin{aligned}
M^2_{p,AA} &= \sin^2\beta M^2_{p,dd} + \cos^2\beta M^2_{p,uu} + 2\sin\beta\cos\beta M^2_{p,ud}\\
&= \frac{1}{\cos\beta\sin\beta}x(A_\lambda\lambda + \kappa\lambda x),\\
M^2_{p,AS} &= \sin\beta M^2_{p,dS} + \cos\beta M^2_{p,uS} = v(A_\lambda\lambda - 2\kappa\lambda x),\\
M^2_{p,A\tilde{\nu}_i} &= \frac{x}{\cos\beta}(A_{\tilde{\lambda}_i}\tilde{\lambda}_i + \kappa\tilde{\lambda}_i x).
\end{aligned}
\tag{3.49}
$$

These results, with finite $\tilde{\lambda}_i x$, are a generalization of the results of NMSSM [3] to the case when there is lepton number violation induced by trilinear couplings. These results reduce to the corresponding results of MSSM with lepton number violation [17] in the limit $x \to \infty$ with fixed $\lambda x = -\mu$, $\tilde{\lambda}_i x = -\mu_i$, and κx^3.

Since the off-diagonal entries in the sneutrino-Higgs mass matrix can be safely assumed to be small in comparison to the diagonal entries, one may evaluate the contributions to the sneutrino mass splittings by making use of second order matrix perturbation theory. The same approximation is also used in evaluating the modified sneutrino propagators in the mass insertion approximation. Specifically, the Higgs boson sector propagator, modified by the two mass mixing terms $\tilde{\nu}_i \times (S_I \oplus P_I) \times \tilde{\nu}_j$ in the Feynman diagram of Fig. 1(b), can be represented by the weighted propagator

$$
P_{\tilde{\nu}_i\tilde{\nu}_j}(q) \equiv \sum_{J=1,2,3}\frac{s^J_{ij}}{q^2 - M^2_{S_J}} - \sum_{J=1,2}\frac{p^J_{ij}}{q^2 - M^2_{P_J}}, \tag{3.50}
$$

where

$$
s^J_{ij} = \Big(\sum_{k=d,u,S} M^2_{s,\tilde{\nu}_i k}(U_s^T)_{kJ}\Big)\Big(\sum_{k=d,u,S} M^2_{s,\tilde{\nu}_j k}(U_s^T)_{kJ}\Big), \tag{3.51}
$$

$$
p^J_{ij} = \Big(\sum_{k=d,u,S} M^2_{p,\tilde{\nu}_i k}(U_p^T)_{kJ}\Big)\Big(\sum_{k=d,u,S} M^2_{p,\tilde{\nu}_i k}(U_p^T)_{kJ}\Big). \tag{3.52}
$$

Evaluating the transition amplitude for the double mass insertion one-loop Feynman graph of Fig. 1(b) with the above formula for the weighted Higgs boson propagator, one obtains the contribution to the light neutrino mass matrix

$$
\begin{aligned}
(m_\nu)^B_{ij} = \frac{g_2^2}{4}\sum_l M_{\tilde{\chi}_l}(N_{l2} - \tan\theta_W N_{l1})^2
&\times\Big[\sum_{J=1,2,3} s^J_{ij} I_4(m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j} M_{\tilde{\chi}_l}, M_{S_J})\\
&- \sum_{J=1,2} p^J_{ij} I_4(m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, M_{\tilde{\chi}_l}, M_{P_J})\Big],
\end{aligned}
\tag{3.53}
$$

with

$$
\Big[I_4(m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, M_{\tilde{\chi}_l}, M_{X_J}) \equiv \frac{1}{(4\pi)^2}C(m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, M_{\tilde{\chi}_l}, M_{X_J}) = \int\frac{d^4q}{i(2\pi)^4}\frac{1}{(q^2 - m^2_{\tilde{\nu}_i})(q^2 - m^2_{\tilde{\nu}_j})(q^2 - M^2_{\tilde{\chi}_l})(q^2 - M^2_{X_J})}\Big], \tag{3.54}
$$

where $\tan\theta_W = g_1/g_2$, and we have used the matrix N to denote the unitary transformation linking the interaction and mass eigenstates of massive neutralinos, $(\tilde{\chi}_m)_{\mathrm{mass}} = (N^\dagger)_{ml}\tilde{\chi}_l$. The momentum integral I_4 admits the analytic representation [41]

$$I_4(m_1, m_2, m_3, m_4) = \frac{1}{m_3^2 - m_4^2}[I_3(m_1, m_2, m_3) - I_3(m_1, m_2, m_4)],$$
$$I_3(m_1, m_2, m_3) = \frac{1}{m_2^2 - m_3^2}[I_2(m_1, m_2) - I_3(m_1, m_3)], \qquad I_2(m_1, m_2) = \frac{1}{(4\pi)^2}\frac{m_1^2}{m_2^2 - m_1^2}\log\frac{m_1^2}{m_2^2}. \tag{3.55}$$

The single mass insertion one-loop amplitude, displayed in Feynman graph of Fig. 1(c), yields the following contribution to the light neutrino mass matrix

$$(m_\nu)^C_{ij} = \frac{g_2^2}{4}\sum_{l,m}\frac{M_{\tilde\chi_m}}{M_{\tilde\chi_l}}\tilde\lambda_i x\Bigg[(N_{l4}(N_{m2} - \tan\theta_W N_{m1}) + N_{m4}(N_{l2} - \tan\theta_W N_{l1}))\times\Bigg[\sum_{J=1}^{3}(U_s^T)_{dJ}Q^s_{\tilde\nu_j J} - \sum_{J=1}^{2}(U_p^T)_{dJ}Q^p_{\tilde\nu_j J}\Bigg]$$
$$- N_{l3}(N_{m2} - \tan\theta_W N_{m1})\Bigg[\sum_{J=1}^{3}(U_s^T)_{uJ}Q^s_{\tilde\nu_j J} - \sum_{J=1}^{2}(U_p^T)_{uJ}Q^p_{\tilde\nu_j J}\Bigg]\Bigg] + (i\leftrightarrow j), \tag{3.56}$$

where various quantities in the above equation are defined as

$$Q^s_{\tilde\nu_j J} = \sum_{k=d,u,S} M^2_{s.\tilde\nu_j k}(U_s^T)_{kJ} I_3(m_{\tilde\nu_j}, M_{\tilde\chi_m}, M_{S_J}), \tag{3.57}$$

$$Q^p_{\tilde\nu_j J} = \sum_{k=d,u,S} M^2_{p.\tilde\nu_j k}(U_p^T)_{kJ} I_3(m_{\tilde\nu_i}, M_{\tilde\chi_m}, M^2_{P_J}), \tag{3.58}$$

$$I_3(m_{\tilde\nu_i}, M_{\tilde\chi_m}, M_{X_J}) \equiv \frac{1}{(4\pi)^2}C_3(m_{\tilde\nu_i}, M_{\tilde\chi_m}, M_{X_J}) \tag{3.59}$$

$$= \int\frac{d^4q}{i(2\pi)^4}\frac{1}{(q^2 - m^2_{\tilde\nu_i})(q^2 - M^2_{\tilde\chi_m})(q^2 - M^2_{X_J})}. \tag{3.60}$$

An examination of the off-diagonal matrix elements of the sneutrino-Higgs squared mass matrix given by Eqs. (3.47) and (3.48) shows that the above one-loop contributions to the neutrino mass matrix consist of sums of two separate matrices involving the three-vectors, $\tilde\lambda_i$ and $A_{\tilde\lambda_i}\tilde\lambda_i$ in the space of fields L_i. Combining these with the tree contribution discussed in subsection III A, one can now write the following representation of the effective light neutrino mass matrix as a sum of three contributions

$$(m_\nu)_{ij} = (X_A^t + X_B^l + X_C^l)\tilde\lambda_i\tilde\lambda_j + Y_B^l A_{\tilde\lambda_i}\tilde\lambda_i A_{\tilde\lambda_j}\tilde\lambda_j + (Z_B^l + Z_C^l)(\tilde\lambda_i A_{\tilde\lambda_j}\tilde\lambda_j + A_{\tilde\lambda_i}\tilde\lambda_i\tilde\lambda_j), \tag{3.61}$$

where the lower suffix labels A, and B, C in the coefficients X, Y, Z refer to the tree and one-loop contributions coming from the Feynman diagrams (A), and (B), (C) in Fig. 1 and we have appended the upper suffix labels t, l to emphasize the distinction between tree and one-loop contributions. We note the absence of the coefficient Y_C^l, and the relation $X_B^l, X_C^l \ll X_A^t$ expected from the loop suppression factor, which allows us to ignore the coefficients X_B^l and X_C^l. The single mass insertion contributions Z_B^l, Z_C^l, and the double mass insertion term Y_B^l have the ability, either separately or in combination, to produce a second nonvanishing mass eigenvalue, provided only that the three-vector $A_{\tilde\lambda_i}\tilde\lambda_i$ is not aligned with λ_i. We recall that the three-vector proportionality, $A_{\tilde\lambda_i}\tilde\lambda_i \propto \lambda_i$, would hold if supersymmetry breaking were flavor universal. Moreover, as was first observed by Chun *et al.* [23] in the context of MSSM, application of matrix perturbation theory to the additively separable neutrino mass matrix $(m_\nu)_{ij} = x\mu_i\mu_j + yb_ib_j + z(\mu_ib_j + \mu_jb_i)$ indicates that the two finite eigenvalues present in the limit $y, z \ll x$ are given by $x\mu_i^2 + 2yb_i\mu_i + O(y^2, z^2)$ and $yb_i^2 + O(y^2, z^2)$. Hence, assuming in the above Majorana neutrino mass matrix $(m_\nu)_{ij}$ that the coefficients Y_B^l, $Z_{B,C}^l$ are of subleading order relative to X_A^t, one concludes that the second nonvanishing eigenvalue is of first order in Y_B^l but of second order in Z_B^l, Z_C^l, namely, $m_{\nu_2} \simeq O(Y_B^l) + O(Z_B^{l2}, Z_C^{l2})$. Thus, as far as the second mass eigenvalue is concerned, this implies that the single mass insertion amplitude (C) is subdominant, so that we can restrict consideration to the double mass insertion contribution (B) only.

A rough estimate of various contributions can now be obtained by isolating the stronger dependence on $\tan\beta$, while assuming that all the mass parameters take values of same order of magnitude as the supersymmetry breaking mass scale $\tilde m_0$. This yields the approximate formulas for the coefficients representing the tree and one-loop contributions, X_A^t and Y_B^l, Z_B^l, Z_C^l, respectively:

$$X_A^t \simeq \frac{x^2\cos^2\beta}{\tilde m_0}, \qquad Y_B^l \simeq \frac{x^2}{\tilde m_0\cos^2\beta}\epsilon_L\epsilon_H,$$
$$Z_B^l \simeq \frac{\kappa x^2}{\tilde m_0\cos^2\beta}\epsilon_L\epsilon_H, \qquad Z_C^l \simeq \frac{\kappa x^2}{\tilde m_0\cos\beta}\epsilon_L\epsilon_H', \tag{3.62}$$

where we have included the suppression effect from the loop in the factor $\epsilon_L \simeq 1/(4\pi)^2 \sim 10^{-2}$ and that from the Higgs sector in the factors ϵ_H and ϵ_H'. Note that we have omitted the one-loop contributions to the component $\tilde\lambda_i\tilde\lambda_j$, which are associated with the suppressed coefficients, $X_B^l \simeq Z_B^l$, $X_C^l \simeq Z_C^l$. The Higgs sector decoupling effect arises from the cancellation between the contributions from CP-even and CP-odd scalars, and is most effective in the case where the lightest scalar mass is well separated from the other modes, and the mass spectrum is ordered as, $m_{S_1} \simeq m_Z \ll m_{S_2}, m_{S_3}, m_{P_1}, m_{P_2}$. The dominant contribution to the second finite neutrino mass eigenvalue is, then, of order $m_{\nu_2} \simeq \langle A_{\tilde{\chi}^2} \tilde{\lambda}^2 \rangle \cos^2\beta \epsilon_L \epsilon_H / \tilde{m}_0$. Moreover, a third finite mass eigenvalue may be generated from the one-loop amplitudes under study by taking into account the flavor nondegeneracy in the sneutrino mass spectrum. The presence of a small relative mass splitting for the sneutrinos, say, $\tilde{\nu}_1$, $\tilde{\nu}_2$, has the ability to produce a third nonzero neutrino mass eigenvalue. The relationship between the ratio of nonzero neutrino masses and the sneutrino mass splitting is given by [24]

$$\epsilon_D = \frac{m_{\nu_1}}{m_{\nu_2}} \simeq 10^{-1} \Delta_\epsilon^2, \qquad \Delta_\epsilon = \frac{|m_{\tilde{\nu}_1}^2 - m_{\tilde{\nu}_2}^2|}{2m_{\tilde{\nu}_1}^2}. \quad (3.63)$$

2. Higgs boson decoupling

Our next task is to estimate semiquantitatively the Higgs sector suppression factor ϵ_H. Following MSSM [24], we consider the definition

$$\epsilon_H = \frac{|\sum_I s_{ij}^I I_4(m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, M_{\tilde{\chi}}, M_{S_I}) - \sum_J p_{ij}^J I_4(m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, M_{\tilde{\chi}}, M_{P_J})|}{\sum_I |s_{ij}^I I_4(m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, M_{\tilde{\chi}}, M_{S_I})| + \sum_J |p_{ij}^J I_4(m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, M_{\tilde{\chi}}, M_{P_J})|}. \quad (3.64)$$

One can easily verify that ϵ_H vanishes in the limit of mass degenerate scalars and pseudoscalars. The Higgs boson mass eigenvalues $m_{S_I}^2$ $(I = 1, 2, 3)$ and $m_{P_J}^2$ $(J = 1, 2)$ and mixing matrices U_s and U_p, where the latter is expressed in the basis (A, S) as $U_p = \mathcal{R}_\gamma$ in terms of the $SO(2)$ rotation matrix of angle γ, are determined once one substitutes the values of the free parameters λ, κ, A_λ, A_κ, $\tan\beta = v_u/v_d$, while using the observed value $v = (v_d^2 + v_u^2)^{1/2} = 174$ GeV. Ensuring a vacuum solution with electroweak symmetry breaking at the appropriate scale, and without tachyonic scalar modes, is known to impose strong constraints on the NMSSM [3]. However, a systematic exploration of entire parameter space consistent with all the physical constraints is beyond the scope of the present work. For a semiquantitative estimate, which is adequate for the purpose of illustrating the typical order of magnitude values assumed by ϵ_H, we only explore a small region of parameter space. For this purpose, we shall consider a modest numerical study confined to large and small values of x, respectively, with the coupling constants held fixed, where one expects to find the largest departures from the MSSM. The mixing matrices in these two regimes are described by the approximate formulas

$$x \gg v_1, v_2\colon\ U_s \simeq \mathrm{diag}(\mathcal{R}_\beta, 1), \qquad \gamma \simeq \pi/2, \quad (3.65)$$

$$x \ll v_1, v_2\colon\ U_s \simeq \begin{pmatrix} -\sin\alpha + C\cos\alpha & \cos\alpha + C\sin\alpha & 0 \\ \cos\alpha + C\sin\alpha & \sin\alpha - C\cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$\gamma \simeq 0, \quad (3.66)$$

where $C = 2\lambda A_\lambda x \cos(2\alpha)\sin 2(\beta - \alpha)/[m_Z^2 \sin(4\beta)]$.

We shall study the dependence of the function ϵ_H on various parameters by means of two different prescriptions. In the first, we set the dimensionless coupling constants at the renormalization group infrared fixed point values, $\lambda = 0.87$, $\kappa = 0.63$, with the soft supersymmetry breaking parameter A_κ having the fixed value $A_\kappa = 200$ GeV, and vary the parameter A_λ. This results in the variation of the mass m_C of the charged Higgs boson, $C^+ = \cos\beta H_u^+ + \sin\beta H_d^{-*}$, which is given by the tree-level formula $m_C^2 = m_W^2 - \lambda^2(v_d^2 + v_u^2) + 2\lambda(A_\lambda + \kappa x)x/\sin(2\beta)$. The variation of ϵ_H with m_C is examined at discrete values of x and $\tan\beta$, while the sensitivity of these results with respect to the other fixed parameters is examined by considering small variations around the above reference values of the parameters. In the second prescription, we examine the dependence of ϵ_H on x for the choice of fixed parameter values $\lambda = 0.5$, $\kappa = 0.5$, $A_\kappa = 100$ GeV, $A_\lambda = 100$ GeV. In both prescriptions, we assign definite mass values to the lowest lying neutralino and the pair of lowest lying sneutrinos, namely, $m_{\tilde{\chi}_l} = 300$ GeV and $m_{\tilde{\nu}_1} = 100$ GeV, $m_{\tilde{\nu}_2} = 200$ GeV, while noting that the loop momentum integral I_4 depends very weakly on the input masses.

The plots of the ratio ϵ_H as a function of m_C and x is displayed in Fig. 2 in the frames (a), (b), and (c) for the above two prescriptions. For the first prescription using variable m_C, the plots are restricted to the physically acceptable values of the parameter A_λ in which no tachyonic scalars or pseudoscalars are present in the neutral Higgs boson sector. In the small x regime with $r \equiv 0.1$, the lowest lying Higgs boson mass lies in the interval $m_h \sim$ 50–20 GeV for $m_C \sim$ 20–100 GeV, which is excluded by the experimental limits. In the intermediate x regime with $r = 1$ and $r = 10$, it is pushed up to the interval $m_h \sim$ 70–120 GeV and $m_h \sim$ 130–140 GeV, respectively, for $m_C \sim$ 100–300 GeV and $m_C \sim$ 300–2000 GeV. The plots in the frames (a), (b) show that the variation of ϵ_H with m_C is slow except when one approaches the boundaries where tachyons appear. The typical size of the ratio is $\epsilon_H = O(10^{-1})$, irrespective of the small or large values of x, but decreases by a factor 2–3 with increasing $\tan\beta$. The discontinuous behavior of the curves for ϵ_H is explained by the fact that this is the absolute value of the difference of two amplitudes. The plot in frame (c) shows that ϵ_H has a

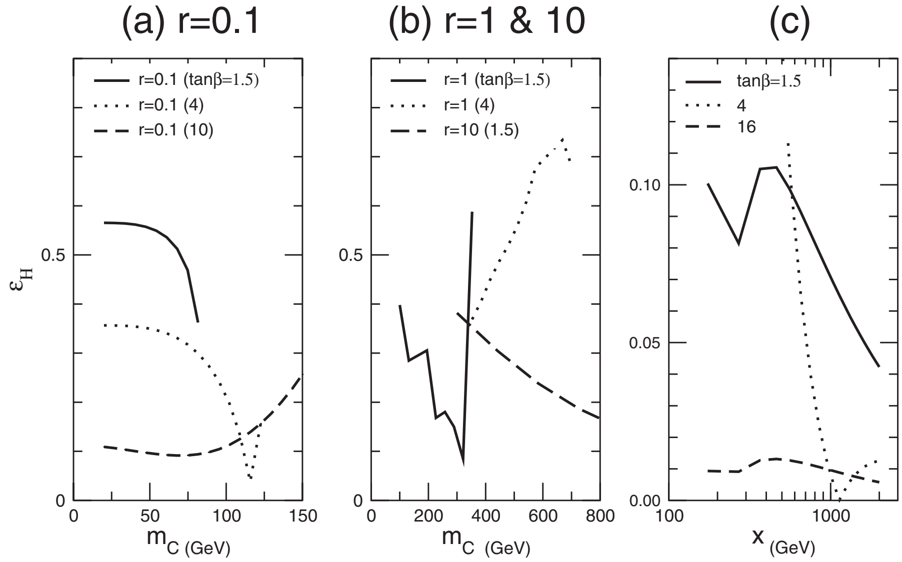

FIG. 2. The Higgs sector decoupling ratio ϵ_H in the double mass insertion approximation at one-loop order is plotted as a function of the charged Higgs boson mass m_C for the three regimes of the VEV ratio parameter, $r \equiv \frac{x}{v} = 0.1$ and 1, 10 in the frames (a) and (b), respectively, and as a function of the S field VEV, x, in the frame (c). The results in frames (a), (b) are obtained with the fixed values of parameters $\lambda = 0.87$, $\kappa = 0.63$, $A_\kappa = 200$ GeV, and with a variable parameter A_λ, which determines the charged Higgs boson mass m_C. The plots in frame (a) are for $\tan\beta = 1.5$, 4, 10, with $r = 0.1$, and those in frame (b) for $r = 1$, $\tan\beta = 1.5$, 4, and for $r = 10$, $\tan\beta = 1.5$, as indicated in the legends. The curves in frame (c) are for $\tan\beta = 1.5$, 4, 16 at the fixed values of parameters, $\lambda = 0.5$, $\kappa = 0.5$, $A_\kappa = 100$ GeV, $A_\lambda = 100$ GeV, as indicated in the legend. The variables in the 4-point amplitude I_4 are chosen by setting the masses of the lightest pair of sneutrinos at $m_{\tilde{\nu}_1} = 100$ GeV, $m_{\tilde{\nu}_2} = 200$ GeV, and that of the lightest neutralino at $m_{\tilde{\chi}_1} = 300$ GeV.

strong variation with increasing x in the interval $x > v$ with a typical size $O(10^{-1})$, decreasing by a factor 10 with increasing $\tan\beta$.

We have also examined how the ratio ϵ_H varies with small variations about the reference values of the couplings for the first prescription. Changing A_κ has a mild influence on the Higgs boson mass spectrum and hence on ϵ_H. Indeed, increasing A_κ by a factor 2–3 does not affect the prediction for ϵ_H significantly. Decreasing λ by a factor 2 reduces ϵ_H mildly at small x and more strongly, by factors of 2–5, at large values of the parameter x. A similar but weaker decrease applies when we reduce κ by a factor 2. As shown by Fig. 2, ϵ_H decreases rapidly with increasing $\tan\beta$, but undergoes very small changes when we allow for large variations of $m_{\tilde{\chi}}$, $m_{\tilde{\nu}_i}$.

Thus, the main conclusion of our analysis is that the suppression factor ϵ_H arising from the Higgs sector is typically of order 10^{-1}–10^{-2}. This is larger than the value obtained for the corresponding factor in the MSSM [24], $\epsilon_H \simeq 10^{-2}$–$10^{-3}$, at the values of parameters consistent with physical constraints. It is, however, possible that there are regions of parameter space where ϵ_H is significantly smaller in the NMSSM.

IV. FLAVOR SYMMETRIES

There are too many parameters in supersymmetric models, including the nonminimal supersymmetric model, to make any specific predictions for the neutrino spectrum. It is even difficult to identify important contributions to the neutrino masses. Here we shall study a specific framework, that of an Abelian flavor (horizontal) symmetry [42,43], where specific predictions can be made. Flavor symmetries are usually invoked to explain the pattern of fermion masses. However, any theory of fermion masses must also explain why the violations of R-parity (or lepton and baryon number) are small. This applies particularly to NMSSM with lepton number violation coming from a trilinear type superpotential coupling that we are considering here as the origin of neutrino masses.

We start by recalling the salient features of the Abelian flavor symmetry framework. The basic idea is to use an Abelian horizontal symmetry $U(1)_F$ to forbid most of the Yukawa couplings except perhaps the third generation couplings. The hierarchies of fermion masses and mixing are then generated through higher dimensional operators involving one or more electroweak singlet scalar fields. These fields acquire vacuum expectation values at some high scale and give rise to the usual Yukawa couplings. More specifically, if Θ is some such field which has charge -1 under $U(1)_F$, then X-charge allows the nonrenormalizable term in the superpotential

$$\lambda_{ij}\Phi_i\Phi_j H\left(\frac{\Theta}{M}\right)^{n_{ij}}, \tag{4.1}$$

where Φ_i is a matter superfield of flavor i, and H is a Higgs superfield with appropriate transformation properties under the gauge group. The coupling λ_{ij} is of order unity, and M is some large mass scale. The positive rational numbers n_{ij} are nothing but the sum of X-charges of Φ_i, Φ_j, and H:

$$n_{ij} = \phi_i + \phi_j + h. \tag{4.2}$$

When Θ gets a vacuum expectation value, an effective Yukawa coupling

$$Y_{ij} = \lambda_{ij}\left(\frac{\langle\Theta\rangle}{M}\right)^{n_{ij}} \equiv \lambda_{ij}\theta_C^{n_{ij}}, \tag{4.3}$$

is generated. If θ_C is a small number, and if the $U(1)_F$ charges are sufficiently diverse, one can implement various hierarchies of fermion masses and mixing. This can then be viewed as an effective low energy theory that originates from the supersymmetric version of the Froggatt-Nielsen mechanism at higher energies. From above we then have the following consequences:

(i) Terms in the superpotential that carry charge $n \geq 0$ are suppressed by $\mathcal{O}(\theta_C^n)$, whereas those which have $n < 0$ are forbidden by the holomorphy of the superpotential.

(ii) Soft supersymmetry breaking terms that carry a charge n under $U(1)_X$ are suppressed by $\mathcal{O}(\theta_C^{|n|})$.

Applying the above scheme to the neutrino mass matrix, we see that the additive separable structure of the combined tree and loop-level contributions give us the ability to account for moderate flavor hierarchies. Let us first recall that the fit to the neutrino oscillation experimental data, assuming a mass spectrum of normal kind with mild hierarchies, favors the following approximate solution for the three masses and mixing angles [26]: $m_{\nu_3} \sim 10^{-1}$ eV, $m_{\nu_2} \sim 10^{-2}$ eV, $m_{\nu_1} \sim 10^{-3}$ eV and $\sin^2\theta_{23} \sim \frac{1}{2}$, $\sin^2\theta_{12} \sim \frac{1}{3}$, $\sin^2\theta_{13} < 1.410^{-2}$. Of course, the contributions that we have discussed so far are controlled by $O(100)$ GeV weak interaction scale, which lies considerably higher than the observed neutrino mass scales. Having identified the supposedly dominant contributions, it is now necessary to find a plausible suppression mechanism which accounts for the wide $O(10^{12})$ hierarchy in mass scales. As in the familiar Froggatt-Nielsen approach [42,43], we can adjust the overall size of the contributions without an excessive fine-tuning of the free parameters by postulating that the superpotential and supersymmetry breaking couplings of the NMSSM arise from nonrenormalizable operators with effective couplings weighted by powers of the small parameter $\theta_C = \langle\Theta\rangle/M$, which we shall identify here with the Cabibbo angle parameter, $\theta_C \simeq 0.2$. With $h(L_i), h(H_{d,u}), \cdots$ denoting the Abelian horizontal group $U(1)_F$ charges assigned to the various superfields, one finds, $\tilde{\lambda}_i = \theta_C^{[h(L_i)+h(S)+h(H_u)]}\langle\tilde{\lambda}_i\rangle$, and similarly for the associated supersymmetry breaking parameters, $A_{\tilde{\lambda}_i}\tilde{\lambda}_i$, with the expectation that $\langle\tilde{\lambda}_i^2\rangle = O(1)$, $\langle A_{\tilde{\lambda}_i}^2\tilde{\lambda}_i^2\rangle = O(1)$. Using the results of subsections III A and III B, one can write the predicted finite neutrino mass eigenvalues as

$$\begin{aligned} m_{\nu_3} &\simeq \frac{x^2\cos^2\beta}{\tilde{m}_0}\langle\tilde{\lambda}^2\rangle\theta_C^{2h(L_3)}, \\ \frac{m_{\nu_3}}{m_{\nu_2}} &\simeq \frac{\cos^4\beta}{\epsilon_L\epsilon_H}\frac{\langle\tilde{\lambda}^2\rangle}{\langle A_{\tilde{\lambda}}^2\tilde{\lambda}^2\rangle}\theta_C^{2[h(L_3)-h(L_2)]}, \\ \frac{m_{\nu_1}}{m_{\nu_2}} &\simeq \epsilon_D\theta_C^{2[h(L_2)-h(L_1)]}. \end{aligned} \tag{4.4}$$

In order to obtain $m_{\nu_3} \sim 10^{-1}$, we must have $\theta_C^{2h(L_3)} \geq 10^{-12}$, and hence $h(L_3) \leq 9$. Similarly, in order to obtain $m_{\nu_2} \sim 10^{-2}$ yields $h(L_3) - h(L_2) \leq 2$. Furthermore, $m_{\nu_1} \sim 10^{-3}$ can be achieved by lifting the mass degeneracy between sneutrinos with $h(L_2) \sim h(L_1)$. Recalling the predictions for the lepton flavor mixing angles, $\sin\theta_{ij} \sim \theta_C^{h(L_i)-h(L_j)}$, it follows that, as in the case of MSSM [24], the selection of horizontal symmetries involving nearly equal horizontal charges $h(L_i)$ introduces a fine-tuning problem in order to account for the small observed mixing angle θ_{13}.

V. SUMMARY AND CONCLUSIONS

We have studied the nonminimal supersymmetric standard model with lepton number violation in detail. This model has a unique trilinear lepton number violating term in its superpotential, and a corresponding soft SUSY breaking scalar trilinear coupling. We have attempted to justify these terms on the basis of a gauged discrete symmetry. We have shown that these terms give a viable description of the light neutrino Majorana mass matrix provided one stabilizes the large mass hierarchy with respect to the weak gauge interactions scale by invoking horizontal flavor symmetries. A satisfactory feature of this extended version of the NMSSM is that the suppressed interactions are all associated with effectively renormalizable and dimensionless Yukawa couplings. This mechanism represents an economic alternative option to the familiar seesaw mechanism of generating the light neutrino mass matrix. Although qualitatively similar to the bilinear lepton number violation that occurs in MSSM, it distinctly differs from it on important quantitative grounds. We find that only one of the three neutrinos obtains mass at the tree level and that this has a finite component of the massive singlet fermion. We have also calculated the one-loop radiative corrections to the neutrino mass matrix generated by the coupling of sneutrino and Higgs boson sectors. These can contribute finite masses to the other two neutrinos in a manner favoring mild hierarchies of normal kind for the neutrino mass spectrum along with large lepton flavor mixing angles. One can reproduce a single small mixing angle, as needed for agreement with the current experimental data, at the cost of a small fine-tuning.

In an effort to put on a firmer theoretical basis the different versions of the NMSSM with renormalizable B or L number violation, we have also studied the four main gauged Z_N cyclic group (ordinary and R, free and GS anomalous) realizations of the generalized baryon, lepton, and matter parities. The constraints from the anomaly cancellation conditions are so strong that no solutions exist if one restricts to the strictly minimal matter field content. However, making the reasonable choice of retaining only the least model-dependent conditions, associated with the mixed gauge anomalies, and of admitting at least one extra gauge singlet chiral superfield (in addition to the standard one which couples to the Higgs bosons), we find interesting restricted classes of symmetry solutions at low cyclic group orders N.

ACKNOWLEDGMENTS

One of the authors (P. N. P) would like to thank the theoretical physics group at Saclay for its hospitality during the initial stages of this work. He would also like to thank the Inter-University Center for Astronomy and Astrophysics, Pune for its hospitality while this work was being completed. The work of P. N. P is supported by the University Grants Commission, India. The work is also supported by the Laboratoire de la Direction des Sciences de la Matière du Commissariat à l'Energie Atomique.

APPENDIX A: CLASSIFICATION OF CYCLIC DISCRETE GAUGE SYMMETRIES

The interest in generalized parities for the MSSM was historically motivated by the need to suppress the dimension-5 baryon and lepton number violating supersymmetric operators [35,36]. Our purpose in the present appendix is rather to classify the discrete cyclic group symmetries which protect the structure of renormalizable versions of the NMSSM superpotential with baryon or lepton number violation. For a recent discussion of discrete symmetries in MSSM, see Ref. [37]. The issue of adding gauge singlet chiral supermultiplets was considered by Lola and Ross [38], although their work was focused on applications involving nonrenormalizable couplings. Our present treatment of this problem also slightly deviates in certain technical details from that followed in this earlier work.

We wish to prove the existence of cyclic symmetries Z_N of general order N which leave invariant the trilinear interaction superpotential of the NMSSM with B or L number violation. Based on the approach initiated by Ibáñez and Ross [35,36], one distinguishes three cases of discrete symmetries designated as generalized baryon (GBP), lepton (GLP), and matter parities (GMP), respectively. For each case, there are four different realizations depending on whether the symmetry is ordinary or R-like, and whether it is Green-Schwarz (GS) anomaly free or anomalous. We discuss first the general classification of the different discrete symmetries and next the consistency conditions imposed by the cancellation of anomalies. Our considerations will be restricted to the flavor blind symmetries.

1. Ordinary symmetries

Let us start with the ordinary anomaly free symmetries. Recall first that the quark and lepton generation independent Abelian charges conserved by the renormalizable R parity conserving (RPC) superpotential couplings of the MSSM form a vector space generated by three continuous $U(1)$ symmetries. A convenient basis for the three independent charges is given by $\hat{R}$, $\hat{A}$, $\hat{L}$ where $\hat{R} = T_{3R}$ and $\hat{A} = Y_A$ identify with the Cartan generators of the right symmetry group $SU(2)_R$ and the $SU(2)$ group embedded in $SU(6) \times SU(2) \subset E_6$, and $-\hat{L}$ identifies with the usual lepton number. The charges $\hat{R}$, $\hat{A}$, $\hat{L}$ assigned to quarks, leptons, and Higgs boson superfields are displayed in the following table, along with those assigned to the singlet superfield S, which are denoted by x, y, z. (The generator $\tilde{P}$ with charge $\hat{\tilde{P}}$ will appear in the next subsection in the discussion of R symmetries.)

Mode	Q	U^c	D^c	L	E^c	H_d	H_u	S
$6Y$	1	-4	2	-3	6	-3	3	0
$\hat{R}$	0	-1	1	0	1	-1	1	x
$\hat{A}$	0	0	-1	-1	0	1	0	y
$\hat{L}$	0	0	0	-1	1	0	0	z
$\hat{\tilde{P}}$	-1	-1	-1	-1	-1	1	1	-1

The Z_N^R, Z_N^A, Z_N^L group elements are constructed in the same way as for the continuous groups, $U(1)_{R,A,L}$, by writing $R = e^{i\alpha_R \hat{R}}$, $A = e^{i\alpha_A \hat{A}}$, $L = e^{i\alpha_L \hat{L}}$ while restricting the complex phase angles to the fixed values, $\alpha_R = 2\pi m/N$, $\alpha_A = 2\pi n/N$, $\alpha_L = 2\pi p/N$, with integer charges m, n, p defined modulo N. Note that the $U(1)_{PQ}$ symmetry generated by $g_{PQ} = R^2 A$ is a chiral, color group anomalous symmetry which conserves all renormalizable (RPC and RPV) trilinear couplings of the MSSM. The pseudoscalar Higgs boson A is the pseudo-Goldstone boson of the $U(1)_{PQ}$ symmetry present in the limit $\mu \to 0$ where the explicit symmetry breaking bilinear coupling $\mu H_u H_d$ is absent.

The multiplicative Z_N symmetries of the renormalizable superpotential for the quarks, leptons, and Higgs bosons may be parametrized in terms of the generators $g = R^m A^n L^p = g_{PQ}^n R^{m-2n} L^p$ [m, n, p integers]. The symmetry solutions preserving B and L, or B alone or L alone are designated as generalized matter, baryon, and lepton parities (GMP, GBP, GLP), respectively. Thus, aside from the regular interactions with Higgs bosons, QU^cH_u, QD^cH_d, LE^cH_d the GBP are required to forbid the interactions

$U^cD^cD^c$ but to allow the interactions LH_u, LLE^c, LQD^c, the GLP acts in a manner forbidding the lepton number violating interactions, but allowing baryon number violating interactions $U^cD^cD^c$, while the GMP must forbid all the matter interactions. Noting the charges for the pure matter couplings, $\hat{g}(LLE^c) = g(LQD^c) = m - 2n - p$, $\hat{g}(U^cD^cD^c) = m - 2n$, one finds that the discrete symmetry generators preserving the MSSM trilinear superpotential in the three relevant cases are given by [35,36]

$$\begin{aligned} &GBP\colon m - 2n - p = 0;\ m - 2n \neq 0 \longrightarrow g_{GBP} = g^n_{PQ}(RL)^p, \quad [p \neq 0] \\ &GLP\colon m - 2n = 0;\ m - 2n - p \neq 0,\ \ \neq 0 \longrightarrow g_{GLP} = g^n_{PQ}L^p, \quad [p \neq 0] \\ &GMP\colon m - 2n - p \neq 0,\ m - 2n \neq 0,\ \longrightarrow g_{GMP} = g^n_{PQ}R^{m-2n}L^p, \quad [m - 2n \neq 0, p \neq 0]. \end{aligned} \tag{A1}$$

A similar analysis applies in the NMSSM, with the generators for GBP, GLP, and GMP required to forbid the matter couplings violating baryon number only ($U^cD^cD^c$), lepton number only (L_iH_uS, LLE^c, LQD^c), and both combined, respectively. The selection rules for the allowed and forbidden S field dependent trilinear couplings are given by, $\hat{g}(H_dH_uS) \equiv n + \mathcal{S} = 0$, $\hat{g}(LH_uS) \equiv m - n - p + \mathcal{S} = 0$, $\hat{g}(S^3) \equiv 3\mathcal{S} = 0$, $[\mathcal{S} = mx + ny + pz]$, and $\hat{g}(S) \equiv \mathcal{S} \neq 0$, $\hat{g}(S^2) \equiv 2\mathcal{S} \neq 0$. Except for the different conditions on the integers $(m, n, p) \in Z_N$, the cyclic symmetry generators have the same functional form as in the MSSM,

$$\begin{aligned} &GBP\colon m - 2n - p = 0,\ m - n - p + \mathcal{S} = 0;\ m - 2n \neq 0 \longrightarrow g_{GBP} = g^n_{PQ}(RL)^p, \\ &GLP\colon m - 2n = 0;\ m - 2n - p \neq 0,\ n - p + \mathcal{S} \neq 0,\ m - n - p + \mathcal{S} \neq 0 \longrightarrow g_{GLP} = g^n_{PQ}L^p, \\ &GMP\colon m - 2n - p \neq 0,\ m - 2n \neq 0,\ m - n - p + \mathcal{S} \neq 0 \longrightarrow g_{GMP} = g^n_{PQ}R^{m-2n}L^p, \end{aligned} \tag{A2}$$

where the three symmetry cases must satisfy $p \neq 0$ along with the conditions: $n + \mathcal{S} = 0$, $3\mathcal{S} = 0$; $n \neq 0$, $\mathcal{S} \neq 0$, $2\mathcal{S} \neq 0$.

Having classified the cyclic groups, we now wish to implement the condition that these belong to gauged symmetries. This means that the total contributions to the quantum anomalies from massless fermions of the low energy theory must either vanish or be compensated by those of the massive fermions of the high energy theory which decouple by acquiring large Dirac or Majorana masses. The coefficients of mixed gauge and gravitational anomaly operators $F_a\tilde{F}_a$, $R\tilde{R}$, $F_g^2F_Y$ and of chiral anomaly operator, F_g^3, acquire the following contributions from massless fermions,

$$\begin{aligned} \mathcal{A}(Z_N \times G_a^2) &= \sum_i \mu_a(\psi_i)\hat{g}(\psi_i), \\ \mathcal{A}(Z_N \times grav^2) &= \sum_i \hat{g}(\psi_i), \\ \mathcal{A}(Z_N^3) &= \sum_i \hat{g}^3(\psi_i), \\ \mathcal{A}(Z_N^2 \times U(1)_Y) &= \sum_i \hat{g}^2(\psi_i)Y_i, \end{aligned} \tag{A3}$$

where $\mu_a(\psi_i)$ denotes the Dynkin index of the fermion ψ_i representation with respect to gauge group factors G_a of the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and the label "grav" stands for the gravitational field source. The Abelian gauge anomaly $\mathcal{A}_1$ will be evaluated by setting conventionally the hypercharge normalization such that, $Y(e^c) = 1$, which implies that the trace of Y^2 over a single quark and lepton generation amounts to $\mathrm{Trace}(Y^2) \equiv 2k_1 = 10/3$. The anomaly cancellation conditions for the NMSSM are given by the formulae

$$\begin{aligned} \mathcal{A}_3 &= \mathcal{A}(SU(3)^2 \times Z_N) = -nN_g = rN, \\ \mathcal{A}_2 &= \mathcal{A}(SU(2)^2 \times Z_N) = -N_g(n + p) + N_{2h}n = rN, \\ \mathcal{A}_1 &= \mathcal{A}(U(1)_Y^2 \times Z_N) = N_g\left(-\frac{5n}{6} + \frac{p}{2}\right) + N_{2h}\frac{n}{2} = rN, \\ \mathcal{A}_{Z^2} &= \mathcal{A}(U(1)_Y \times Z_N^2) = -2N_g(2mn + p(n - m)) \\ &\quad - N_{2h}n(n - 2m) = rN, \\ \mathcal{A}_{\rm grav} &= \mathcal{A}(grav^2 \times Z_N) = -N_g(5n - m + p) + 2N_{2h}n \\ &\quad + \mathcal{S} = rN + \eta s\frac{N}{2}, \\ \mathcal{A}_{Z^3} &= \mathcal{A}(Z_N^3) = N_g[-3(m^3 + (n - m)^3) - 2(n + p)^3 \\ &\quad + (m + p)^3] + N_{2h}[(n - m)^3 + m^3] \\ &\quad + \mathcal{S}^3 = rN + \eta s\frac{N^3}{8}. \end{aligned} \tag{A4}$$

Here, N_g denotes the number of quark and lepton generations, N_{2h} the number of H_d, H_u Higgs boson supermultiplet pairs, and the symbols r, $s \in Z$ denote arbitrary integers (taking independent values for the different anomalies) so that the different equations are understood to be satisfied modulo N. The additional vanishing conditions associated with the parameter $\eta = 0, 1$ for N odd and even, respectively, are introduced to account in the even N case for the presence of massive Majorana fermions in real representations of the gauge group factor G_a. The gauge singlet charges enter only through the linear combination, $\mathcal{S} \equiv mx + ny + pz$, which is set to $\mathcal{S} = -n$.

It is straightforward to generalize the above results to the case involving several gauge singlet chiral supermultiplets,

S_i. One just needs to assign S_i the $\hat{R}$, $\hat{A}$, $\hat{L}$ charges x_i, y_i, z_i and to replace in the anomaly coefficients, $\mathcal{S} \to \sum_i \mathcal{S}_i$, $\mathcal{S}^3 \to \sum_i \mathcal{S}_i^3$. These additional contributions set conditions on the charges x_i, y_i, z_i expressing the net cancellation of the anomaly coefficients $\mathcal{A}_{\text{grav}}$, $\mathcal{A}_{Z^3}$. Each allowed coupling must also be accompanied by an additional constraint equation expressing the associated selection rule.

The following two-stage procedure may be used in solving the anomaly cancellation conditions for each fixed N generator. One first scans through the nonvanishing integers m, n, p to select those satisfying the above set of equations and next scans the nonvanishing integers $x, y, z \in Z_N$ which solve the equations $\mathcal{S} \equiv mx + ny + pz = -n \neq 0$, $2\mathcal{S} \neq 0$. The search is most easily implemented with the help of a numerical computer program.

We study next the Green-Schwarz anomalous discrete symmetries. This case differs from the anomaly free one in that the anomaly coefficients in the effective action is now allowed to take finite values, provided only that these are canceled by the additive contributions to the anomalies of universal form associated with the gauge and gravitational couplings of the model-independent axion-dilaton chiral supermultiplet. The modified anomaly cancellation conditions are then expressed in terms of the shifted anomaly coefficients, $\mathcal{A}_a - 2k_a\delta_{GS} = 0$, $\mathcal{A}_{\text{grav}} - 2k_{\text{grav}}\delta_{GS} = 0$, $[a = 3, 2, 1]$ where k_a are rational parameters (integer quantized for non-Abelian group factors G_a), $k_{\text{grav}} = 12$, and δ_{GS} denotes a universal model-dependent parameter reflecting the underlying high energy theory. These conditions can also be represented by the proportionality relations, $\mathcal{A}_3/k_3 = \mathcal{A}_2/k_2 = \mathcal{A}_1/k_1 = \mathcal{A}_{\text{grav}}/12$. The parameters k_a in the minimal gauged unified theories are set at the numerical values, $k_3 = k_2 = 1$, $k_1 = 5/3$.

2. *R* symmetries

We shall continue using the abbreviations GBP, GLP, GMP for the generalized baryon, lepton, and matter R parity discrete symmetries. A convenient representation of the Z_N group generators can be constructed by introducing the fermionic generator $\tilde{P} = e^{2\pi i \hat{\tilde{P}}/N}$ defined by its action on the superspace differential, $\tilde{P} \cdot d\theta = e^{-2i\pi/N} d\theta$ and by the charge assignments of the gauginos, $\hat{\tilde{P}}(\tilde{g}) = \hat{\tilde{P}}(\tilde{W}) = \hat{\tilde{P}}(\tilde{B}) = 1$, and of the matter and Higgs boson superfields, as displayed in the table placed at the beginning of Section A 1. With the understanding that the charge assignments displayed in the table for $\tilde{P}$ and for $\hat{R}$, $\hat{A}$, $\hat{L}$ apply to the fermion field component of the chiral superfields, the Z_N group generators preserving the MSSM matter-Higgs boson trilinear superpotential are simply given by, $\tilde{g} = \tilde{P} R^m A^n L^p$. The $\tilde{g}$ charges of fermion and scalar field components of chiral superfields ψ, ϕ are then related in the usual way, $\hat{\tilde{g}}(\psi) = \hat{\tilde{g}}(\phi) - 1$, so that a superpotential term W is conserved to the extent that it obeys the selection rule, $\tilde{g}(W) = e^{4i\pi/N} W$, corresponding to an R-charge of 2. Thus, the invariance requirement of an order M superpotential monomial, $W = \prod_{I=1}^M \Phi_I$, can be expressed by the condition,

$$\hat{\tilde{g}}(W) = \hat{\tilde{g}}\left(\prod_{I=1}^{M} \Phi_I\right) = \sum_{I=1}^{M} \hat{\tilde{g}}(\psi_I) + M = 2, \tag{A5}$$

implying the selection rule, $\sum_{I=1}^M \hat{\tilde{g}}(\psi_I) = 2 - M$.

Applying the above discussion to the S field dependent couplings, one derives the following selection rules, valid for the three symmetry cases: $3\mathcal{S} - 2 = 0$, $n + 2 + \mathcal{S} = 0$; $n + 2 \neq 0$, $\mathcal{S} - 2 \neq 0$, $2\mathcal{S} - 2 \neq 0$. It is again useful to single out the R like Peccei-Quinn symmetry, $\tilde{g}^n_{\text{PQ}} = \tilde{P}(R^2A)^n R^2$. This conserves all couplings with the exception of those involving the pair of Higgs boson superfields, for which one has the selection rules, $\Delta^n_{\text{PQ}}(H_dH_u) = 2 + n$, $\Delta^n_{\text{PQ}}(H_dH_uS) = \Delta^n_{\text{PQ}}(LH_uS) = 2 + n + (2n+2)x + ny$, $[\Delta^n_{\text{PQ}}(O_M) = \hat{\tilde{g}}^n_{\text{PQ}}(O_M) + M - 2]$. Focusing, for definiteness, on the GLP, one obtains the following selection rules for the bilinear and trilinear couplings: $\hat{\tilde{g}}(H_dH_u) = n + 2 \neq 0$, $\hat{\tilde{g}}(LH_u) = m - n - p \neq 0$, $\hat{\tilde{g}}(LLE^c) = \hat{\tilde{g}}(LQD^c) = m - 2n - p - 3 = -1$, $\hat{\tilde{g}}(U^cD^cD^c) = m - 2n - 3 \neq -1$, $\hat{\tilde{g}}(H_dH_uS) = \mathcal{S} + n + 1 = -1$, $\hat{\tilde{g}}(LH_uS) = \mathcal{S} + m - n - p - 1 = -1$, $\tilde{g}(S) = \mathcal{S} - 1 \neq 1$, $\hat{\tilde{g}}(S^2) = 2\mathcal{S} - 2 \neq 0$, $\hat{\tilde{g}}(S^3) = 3\mathcal{S} - 3 = -1$.

We can summarize the defining conditions for the GBP, GLP, and GMP generators and the resulting representations of the generators by the formulas,

$$\begin{aligned}
&GBP\colon\ m - 2n - p - 2 = 0;\ m - n - p + \mathcal{S} \neq 0,\ m - 2n - 2 \neq 0 \longrightarrow \tilde{g}_{GBP} = \tilde{g}_{\text{PQ}}(RL)^p,\\
&GLP\colon\ m - 2n - 2 = 0,\ m - n - p + \mathcal{S} = 0;\ m - 2n - p - 2 \neq 0 \longrightarrow \tilde{g}_{GLP} = \tilde{g}_{\text{PQ}}(L)^p,\\
&GMP\colon\ m - 2n - p - 2 \neq 0,\ m - 2n - 2 \neq 0,\ m - n - p + \mathcal{S} \neq 0 \longrightarrow \tilde{g}_{GMP} = \tilde{g}_{\text{PQ}} R^{m-2n-2} L^p,
\end{aligned} \tag{A6}$$

which must be complemented by the conditions $p \neq 0$ and $n + 2 + \mathcal{S} = 0$. The anomaly cancellation conditions are now readily evaluated by inspection of the table given at the beginning of Section A 1 which displays the fermion modes charges. One must include in the mixed gauge anomalies the contributions from the spin $1/2$ gauginos, and in the gravitational anomaly those from the gauginos and the spin $3/2$ gravitinos which add to the anomaly coefficient 1 and -21 per mode, respectively. The contribution from the SM gauge group gauginos amounts then to

$\sum_a \dim(G_a) = 12$. The anomaly coefficients for the gauge, gravitational, and chiral anomalies are given by the formulas

$$\begin{aligned}
\mathcal{A}_3 &= \mathcal{A}(SU(3)^2 \times Z_N) = 6 - N_g(4+n) = rN, \\
\mathcal{A}_2 &= \mathcal{A}(SU(2)^2 \times Z_N) = 4 - N_g(4+n+p) \\
&\quad + N_{2h}(n+2) = rN, \\
\mathcal{A}_1 &= \mathcal{A}(U(1)_Y^2 \times Z_N) = N_g\Big(-\frac{10}{3} - \frac{5n}{6} + \frac{p}{2}\Big) \\
&\quad + N_{2h}\Big(\frac{n}{2}+1\Big) = rN, \\
\mathcal{A}_{Z^2} &= \mathcal{A}(U(1)_Y \times Z_N^2) = N_g[1 - 2(1+m)^2 \\
&\quad + (1-m+n)^2 - (1+n+p)^2 \\
&\quad + (-1+m+p)^2] + N_{2h}[(1-m+n)^2 \\
&\quad - (1+m)^2] = rN, \\
\mathcal{A}_{\rm grav} &= \mathcal{A}({\rm grav}^2 \times Z_N) = N_g(-15 - 5n + m - p) \\
&\quad + N_{2h}(4+2n) + 12 - 21 + (-1+\mathcal{S}) \\
&= rN + \eta s\frac{N}{2}, \\
\mathcal{A}_{Z^3} &= \mathcal{A}(Z_N^3) = N_g[-6 - 3(1+m)^3 - 3(1+n-m)^3 \\
&\quad - 2(1+n+p)^3 + (-1+m+p)^3] \\
&\quad + 2N_{2h}[(1+n-m)^3 + (1+m)^3] + (-1+\mathcal{S})^3 \\
&= rN + \eta s\frac{N^3}{8}.
\end{aligned} \tag{A7}$$

The selection rule, $(-1+\mathcal{S}) \equiv (-1+mx+ny+pz) = -(n+3)$, may be used to remove the explicit dependence of the anomaly coefficients on the singlet field charges. The case of anomalous GS symmetries is analyzed in the same way as for the ordinary symmetries by introducing the shifted anomaly coefficients. The search of generalized parity solutions can also follow a similar two-stage procedure as described earlier. One solves in a first stage the anomaly cancellation equations at fixed N for the integers (m, n, p), and in a second stage the selection rules $n+2+\mathcal{S}=0$, $3\mathcal{S}-2=0$, $\mathcal{S}-2\neq 0$, $2\mathcal{S}-2\neq 0$ for the S field charges (x, y, z). To conclude, we note that the approach discussed here appears more systematic than the alternative one where one solves the anomaly cancellation equations after assigning charges to the various particles $\alpha_Q, \alpha_{U^c}, \cdots$, subject to the selection rules.

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