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Estimate of the charmed 0^{--} hybrid meson spectrum from quenched lattice QCD

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We compute from quenched lattice QCD the ground state masses of the charmed hybrid mesons $\bar{c}cg$, with exotic quantum numbers $J^{PC} = 1^{-+}, 0^{+-}$ and 0^{--}. The 0^{--} hybrid meson spectrum has never been provided by lattice simulations due to the difficulties to extract high gluonic excitations from noise. We employ improved gauge and fermion actions on the anisotropic lattice, which reduce greatly the lattice artifacts, and lead to very good signals. The data are extrapolated to the continuum limit, with finite size effects under well control. For 1^{-+} and 0^{+-} hybrid mesons, the ground state masses are 4.405(38) GeV and 4.714(52) GeV. We predict for the first time from lattice QCD, the ground state mass of 0^{--} to be 5.883(146) GeV.

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A hybrid (exotic) meson $\bar{q}qg$ is a bound state of quark q, antiquark $\bar{q}$ and excited gluon g. The excited gluon makes quantum number of the bound state to be $1^{-+}, 0^{+-}$ or 0^{--}, ..., inaccessible to $\bar{q}q$ mesons in the quark model. The existence of hybrid mesons is one of the most important predictions of quantum chromodynamics (QCD).

So far no signal for heavy exotic hybrid mesons has been experimentally observed, though a number of potential candidates for light hybrid mesons were suggested [1,2]. Fortunately, this situation may change, due to rapid development of new experiments, for example, PEP-∐(*BABAR*), KEKB(Belle), 12 GeV Jefferson Lab [3,4], upgraded CLEO-c detector [5], and new BES3 detector [6]. Especially, 12 GeV Jefferson Lab, CLEO-c and BES3 will present well needed and more reliable data for the charmonium spectrum, including hybrid mesons.

The most reliable technique for computing hadron spectroscopy is lattice gauge theory. It is a nonperturbative approach based on first principle QCD. Of course, the lattice approach is not free of systematic errors. The discretization errors in the Wilson gluon and quark actions are the most serious ones. These errors are smaller only at very small bare coupling, and very large lattice volume is required to get rid of finite size effects. The idea of Symanzik improvement [7] is to add new terms to the Wilson actions to reduce the lattice spacing errors. In combination with tadpole improvement [8], the Symanzik program has recently led to great success in approaching the continuum physics on very coarse and small lattices. Simulations on anisotropic lattices help getting very good signal in spectrum computations.

There have been many quenched lattice calculations [9–19] of the 1^{-+} or 0^{+-} hybrid meson masses, in either light quark or heavy quark sector. The mass estimates for the light hybrid mesons might still have some uncertainties, because those simulations are still far from the chiral regime. The inclusion of dynamical quarks is still very preliminary [18], due to limited computing resources. A recent review can be found in Ref. [20]. It has been a long standing puzzle for the 0^{--} hybrid mesons [11]: no clear signal has ever been found, which might be due to the fact that the gluon is highly excited.

As for heavy quarks, special considerations have to be taken. Currently, nonrelativistic lattice QCD (NRQCD), and relativistic heavy quark (Fermilab), and anisotropic relativistic approaches are the leading methods. Let a denote the lattice spacing. The NRQCD method [21,22] is applicable for $am_q > 1$ and works well for very heavy quarks, especially for the spin-independent $\bar{b}b$ system; however the continuum limit is problematic because of the condition $am_q > 1$; it is difficult to include relativistic corrections and radiative corrections, leading to breaking down of this method for the $\bar{c}c$ system [23]. The relativistic (Fermilab) approach to quarks [24] works for both light quarks and heavy quarks; Up to $O(a^2)$, the fermionic action is equivalent to the standard Sheikholeslami-Wohlert(SW) action [25] on an isotropic lattice; however, to get rid of the $O(a)$ error all coefficients in the fermionic action are required to be mass-dependent. The anisotropic relativistic approach to quarks [26,27], which is used in this paper, generalizes the Fermilab approach to anisotropic lattice. This improved quark action has been successfully applied to the computation of the charmonium spectrum [27,28], which agree very well with experiments.

To investigate gluonic excitations in hadrons, additional improvement of the gluon action would certainly help getting better signals. The first attempt was made in

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TABLE I. Simulation parameters at largest volume. We employed the method in Ref. [27] to tune these parameters, κ_t and κ_s for the quark action. The last two columns are about the spatial lattice spacing and the lattice extent in physical units, determined from the $1P - 1S$ charmonium splitting.

β	$\xi = a_s/a_t$	$L^3 \times T$	u_s	u_t	$a_t m_{q0}$	c_s	c_t	$a_s(1^1P_1 - 1S)$ [fm]	La_s [fm]
2.6	3	$16^3 \times 48$	0.81921	1	0.229 0.260	1.8189	2.4414	0.1885(82)	3.016
2.8	3	$16^3 \times 48$	0.83099	1	0.150 0.220	1.7427	2.4068	0.1584(103)	2.534
3.0	3	$20^3 \times 60$	0.84098	1	0.020 0.100	1.6813	2.3782	0.1147(98)	2.294

Ref. [17], where the ground state masses of 1^{-+} hybrid mesons in the light quark and charm quark sectors were computed, by combining the improved gluon action [29] and a simplified relativistic fermionic action [17] on the anisotropic lattice. However, the quark masses were far away either from the chiral limit or from the charm quark regime. The statistics were low, and finite size effects and lattice spacing errors were not analyzed.

In this letter, we estimate the ground state masses of 1^{-+}, 0^{+-}, and 0^{--} exotic mesons in the charm quark sector, employing lattice QCD with tadpole improved gluon [29] and quark [26,27] actions on the anisotropic lattice. We get significantly improved signals for these particles, in particular, for the 0^{--} particle for the first time.

Our simulation parameters are listed in Table I. At each $\beta = 6/g^2$, three hundred independent configurations were generated with the improved gluonic action [29]. Two hundred configurations are the minimum for obtaining stable results. We input two values of bare quark mass m_{q0} and then compute quark propagators using the improved quark action [27], and the hybrid meson correlation function using the operators $1^{-+} = \rho \otimes B$, $0_P^{+-} = a_1(P) \otimes B$, $0_P^{--} = a_1(P) \otimes E$ and $O_S^{--} = a_1 \otimes E$ in Ref. [11], as they give the best signals.

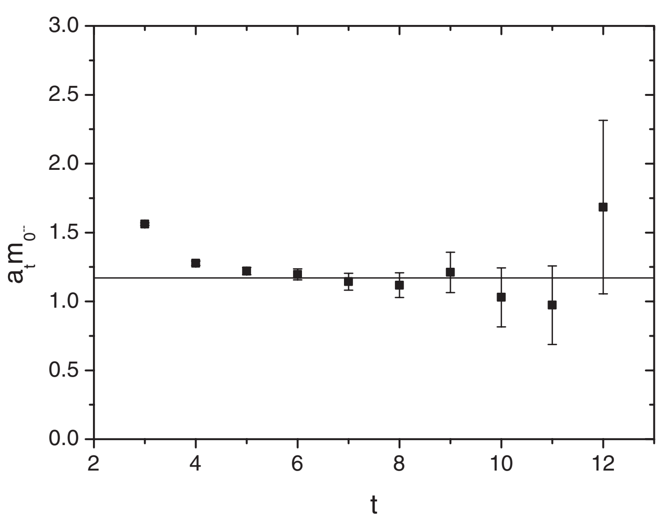

FIG. 1. Effective mass of the 0^{--} hybrid meson for $\beta = 3.0$ and $a_t m_{q0} = 0.100$. The solid line is the fitted result, ranging from $t_i = 6$ to $t_f = 12$ with $\chi^2/\mathrm{d.o.f.} = 0.4326$ and confidence level $= 0.7620$.

Figure 1 shows an example of the effective mass plot $\ln(C(t)/(C(t+1))$ of the 0^{--} hybrid, where $C(t)$ is the correlation function between the hybrid operators. We obtained the effective mass $a_t m$, with the fit range chosen according to optimal confidence level and reasonable $\chi^2/\mathrm{d.o.f.}$

We then interpolated the data to the charm quark regime using $(m_\pi(\kappa_t \rightarrow \kappa_t^{\mathrm{charm}}) + 3m_\rho(\kappa_t \rightarrow \kappa_t^{\mathrm{charm}}))/4 \rightarrow M(1S)_{\mathrm{exp}} = (m(\eta_c)_{\mathrm{exp}} + 3m(J/\psi)_{\mathrm{exp}})/4 = 3067.6$ MeV, where the right hand side is the experimental value for the $1S$ charmonium. The results for the charmed hybrid meson masses are listed in Table II for the ground state. It is also important to check whether these lattice volumes are large enough. We also did simulations on $8^3 \times 48$ and $12^3 \times 48$ at $\beta = 2.6$, $12^3 \times 36$ at $\beta = 2.8$, and $16^3 \times 48$ at $\beta = 3.0$, but here we just list the results from the largest volume. When the spatial extent is greater than 2.2 fm, the finite volume effect on the 0^{--} mass is less than 0.1% for the ground state.

The $1^1P_1 - 1S$ charmonium splitting was chosen to determine the lattice spacing, because it is roughly independent of quark mass for charm and bottom sectors, and the experimental value $\Delta M(1^1P_1 - 1S)_{\mathrm{exp}} = 457$ MeV was well measured. Here we used χ_{c_1} meson (1^{++}) mass for the P-wave and $m(\eta_c)/4 + 3m(J/\psi)/4$ for the S-wave. The results for the spacial lattice spacing a_s at different β are listed in Table I. They are consistent with those from the heavy quark potential [29].

Figure 2 shows the charmed 1^{-+}, 0^{+-}, 0^{--} hybrid meson masses as a function of a_s^2. Other hybrids have a similar behavior, indicating the linear dependence of the

TABLE II. Charmed hybrid meson spectrum for the ground state. The results in the continuum limit ($\beta = \infty$) were obtained by: (i) directly extrapolating the data to $a_s^2 \rightarrow 0$; and (ii) using the ratio of splittings R_H, as described in the text.

β	$a_s^2(fm^2)$	1^{-+}	0^{+-}	0^{--}	
2.6	0.0355	4.423(62)	4.530(63)	5.478(76)	
2.8	0.0251	4.429(78)	4.536(79)	5.533(97)	
3.0	0.0132	4.398(73)	4.670(77)	5.745(95)	
∞	0	4.390(118)	4.732(124)	5.876(152)	
∞	0	4.405(38)	4.714(52)	5.883(146)	$\leftarrow$ from R_H

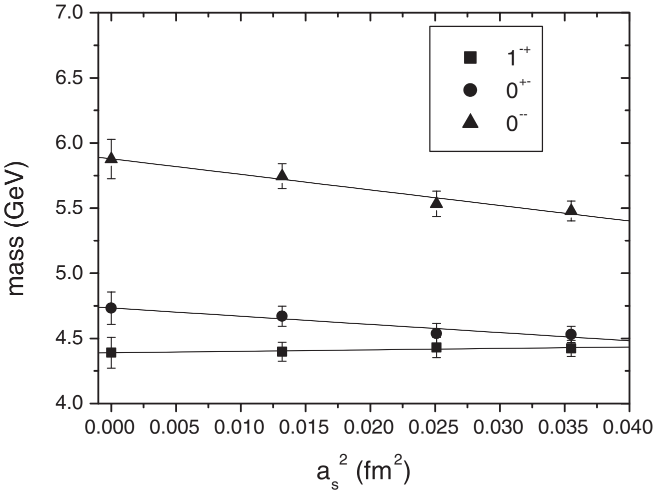

FIG. 2. Extrapolation of the charmed 1^{-+}, 0^{+-}, and 0^{--} hybrid meson masses to the continue limit.

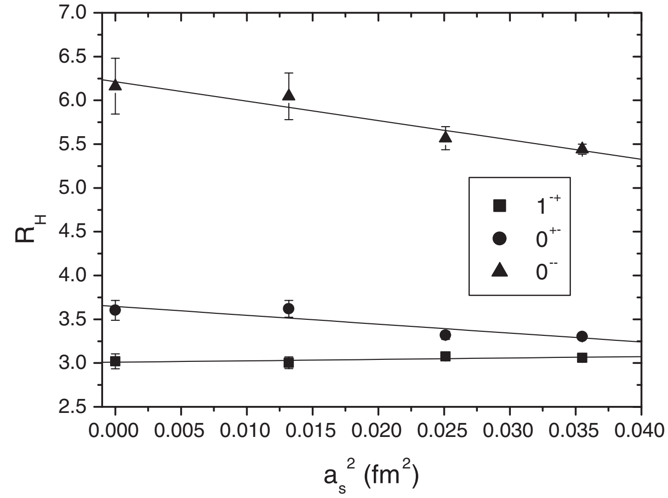

FIG. 3. Ratio of splittings $R_H = \Delta M(1H - 1S)/\Delta M(1^1P_1 - 1S)$ against a_s^2. The straight line is the extrapolation to continuum limit.

mass on a_s^2. The spectrum in the continuum limit is obtained by linearly extrapolating the data to $a_s^2 \to 0$, as also listed in Table II.

We also computed the splitting of $1H - 1S$ and the ratio $R_H = \Delta M(1H - 1S)/\Delta M(1^1P_1 - 1S)$, where $1H$ stands for the ground state of a charmed hybrid meson. Its dependence on a_s^2 and extrapolation to the continuum limit are shown in Fig. 3. The last line of Table II also lists the hybrid meson masses for the ground state, using the following equation:

$$\lim_{a_s^2 \to 0} M(1H) = M(1S)_{\rm exp} + \Delta M(1^1P_1 - 1S)_{\rm exp} \times \lim_{a_s^2 \to 0} R_H.$$

This method was claimed to be better [14], because the splitting between a hybrid and the $1S$ state is rather insensitive to the imperfect tuning of $\Delta M(1^1P_1 - 1S)$ and $M(1S)$. However, as seen in Table II, the results from two different methods agree very well.

One source of systematic errors in our calculation is the quenched approximation. Although full QCD simulations will remove this unknown error, quenched approximation in some areas [30,31], including the hybrids [32], continues to play an important role. The findings in Refs. [20,33,34] indicate that the effects of dynamical quarks on light hybrids and bottomed hybrids are very small. To have full relevance of the charmed hybrids to experiment, simulations with dynamical quarks, although extremely expensive to achieve high statistics, might be helpful to see whether the quenching error is under control. Nevertheless, our results are a very important step for comparison with future dynamical simulations.

Simulations on the anisotropic lattice with both gluon action and improved quark action improved lead to the first observation of the clear signal for the 0^{--} hybrids. We believe that our findings are useful to experimental search for these new particles, predicted by QCD.

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