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Properties of the light scalar mesons face the experimental data on the $\phi \to \pi^0\pi^0\gamma$ decay and the $\pi\pi$ scattering

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The high-statistical KLOE data on the $\phi \to \pi^0\pi^0\gamma$ decay are described simultaneously with the data on the $\pi\pi$ scattering and the $\pi\pi \to K\bar{K}$ reaction. The description is carried out taking into account the chiral shielding of the putative $\sigma(600)$ meson and its mixing with the well-established $f_0(980)$ meson. It is shown that the data do not contradict the existence of the $\sigma(600)$ meson and yield evidence in favor of the four-quark nature of the $\sigma(600)$ and $f_0(980)$ mesons.

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I. INTRODUCTION

The study of the nature of light scalar resonances is one of the central problems of nonperturbative QCD. The point is that the elucidation of their nature is important for understanding both the confinement physics and the chiral symmetry realization way in the low energy region, i.e., the main consequences of QCD in the hadron world. Actually, what kind of interaction at low energy is the result of the confinement in the chiral limit? Is QCD equivalent to the nonlinear σ-model or the linear one at low energy?

The experimental nonet of the light scalar mesons [1], the putative $f_0(600)$ [or $\sigma(600)$] and $\kappa(700–900)$ mesons, and the well-established $f_0(980)$ and $a_0(980)$ mesons, suggests the $U_L(3) \times U_R(3)$ linear σ-model [2]. The history of the linear σ-model is rather long, so that the list of its participants, quoted in Ref. [7], is far from complete.

Hunting the light σ and κ mesons had begun in the 1960s already, and preliminary information on the light scalar mesons in Particle Data Group Reviews had appeared at that time. But long-standing unsuccessful attempts to prove their existence in a conclusive way entailed general disappointment, and information on these states disappeared from Particle Data Group Reviews. One of the principal reasons against the σ and κ mesons was the fact that both $\pi\pi$ and $\pi\kappa$ scattering phase shifts do not pass over 90° at putative resonance masses. The situation changes when it was shown in Ref. [8] that in the linear σ-model there is a negative background phase which hides the σ meson [9]. It has been made clear that shielding of wide lightest scalar mesons in chiral dynamics is very natural. This idea was picked up (see, for example, Ref. [10]) and triggered a new wave of theoretical and experimental searches for the σ and κ mesons (see Particle Data Group Review [1]).

In theory the principal problem is an impossibility to use the linear σ-model in the tree-level approximation inserting widths into σ meson propagators [8] because such an approach breaks both the unitarity and the Adler self-consistency conditions [11]. Strictly speaking, the comparison with the experiment requires the nonperturbative calculation of the process amplitudes [12].

Nevertheless, now there are the possibilities to estimate odds of the $U_L(3) \times U_R(3)$ linear σ-model to the underlying physics of light scalar mesons in phenomenology. Really, even now there is a huge body of information about the S-waves of different two-particle pseudoscalar states. What is more, the relevant information goes to press almost continuously from BES, BNL, CERN, CESR, DAΦNE, FNAL, KEK, SLAC, and others. As for theory, we know quite a lot about the scenario under discussion: the nine scalar mesons, the putative chiral masking [8] of the $\sigma(600)$ and $\kappa(700–900)$ mesons, the unitarity, the analyticity, and the Adler self-consistency conditions. In addition, there is the light scalar meson treatment motivated by field theory. The foundations of this approach were formulated in Refs. [13–17] (see also Refs. [18–20]). In particular, the propagators of scalar mesons were introduced in this approach. As was shown in Ref. [19], these propagators satisfy the Källen-Lehmann representation in the wide domain of coupling constants of the light scalar mesons with two-particle states. The present paper is the first step in the realization of this plan.

The $a_0(980)$ and $f_0(980)$ scalar mesons, discovered more than 30 years ago, became the hard problem for the naive quark-antiquark ($q\bar{q}$) model from the outset. Really, on the one hand, the almost exact mass degeneration of the isovector $a_0(980)$ and isoscalar $f_0(980)$ states revealed seemingly the structure similar to the structure of the vector ρ and ω mesons, and on the other hand, the coupling of $f_0(980)$ with the $K\bar{K}$ channel pointed unambiguously to a considerable part of the strange quark pair $s\bar{s}$ in the wave function of $f_0(980)$. It was noted in the late 1970s that, in the MIT bag model (which incorporates confinement phenomenologically), there are light four-quark scalar states and it was suggested that $a_0(980)$ and $f_0(980)$ might be these states [5], containing the $s\bar{s}$ pair additionally to the nonstrange one. From that time $a_0(980)$ and $f_0(980)$ resonances became the subject of intensive investigations (see, for example, Refs. [3,4,6,10,13–19,21–45]).

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Ten years later it was shown [23] that the study of the radiative decays $\phi \to a_0\gamma \to \eta\pi^0\gamma$ and $\phi \to f_0\gamma \to \pi^0\pi^0\gamma$ can shed light on the puzzle of the light scalar mesons. Over the next ten years before the experiments (1998), this question was examined from different points of view [24–28].

Now these decays have been studied not only theoretically but also experimentally. The first measurements have been reported by the SND [29–32] and CMD-2 [33] Collaborations which obtain the following branching ratios:

$$Br(\phi \to \gamma\pi^0\eta) = (8.8 \pm 1.4 \pm 0.9) \times 10^{-5} \text{ [31]},$$

$$Br(\phi \to \gamma\pi^0\pi^0) = (12.21 \pm 0.98 \pm 0.61) \times 10^{-5} \text{ [32]},$$

$$Br(\phi \to \gamma\pi^0\eta) = (9.0 \pm 2.4 \pm 1.0) \times 10^{-5} \text{ [33]},$$

$$Br(\phi \to \gamma\pi^0\pi^0) = (9.2 \pm 0.8 \pm 0.6) \times 10^{-5} \text{ [33]}.$$

In Refs. [30,32,33] the data on the $\phi \to \pi^0\pi^0\gamma$ decay were analyzed in the K^+K^- loop model of the single $f_0(980)$ resonance production, suggested in Ref. [23]. In Ref. [37] the data on the $\phi \to \pi^0\pi^0\gamma$ decay and on the δ_0^0 phase of the $\pi\pi$ scattering were analyzed simultaneously for the first time, which allowed one to determine the relative phase between the signal amplitude $\phi \to S\gamma \to \pi^0\pi^0\gamma$ and the background one $\phi \to \rho\pi^0 \to \pi^0\pi^0\gamma$ properly. The consideration was carried out in the model suggested in Ref. [25]. This model is based on the K^+K^- loop model [23] of the $\pi\pi$ production and involves constructing the $K\bar{K} \to \pi\pi$ amplitude both above and under the $K\bar{K}$ threshold. Recall that the phase of the $K\bar{K} \to \pi\pi$ amplitude under the $K\bar{K}$ threshold equals the phase of the $\pi\pi$ scattering.

Then the data came from the KLOE experiment [34,35]:

$$Br(\phi \to \gamma\pi^0\eta) = (8.51 \pm 0.51 \pm 0.57) \times 10^{-5} \text{ in } \eta \to \gamma\gamma \text{ [34]},$$

$$Br(\phi \to \gamma\pi^0\eta) = (7.96 \pm 0.60 \pm 0.40) \times 10^{-5} \text{ in } \eta \to \pi^+\pi^-\pi^0 \text{ [34]},$$

$$Br(\phi \to \gamma\pi^0\pi^0) = (10.9 \pm 0.3 \pm 0.5) \times 10^{-5} \text{ [35]},$$

in agreement with the Novosibirsk data [29–33] but with a considerably smaller error. Note that the reanalysis of the KLOE data on the $\phi \to \gamma\pi^0\eta$ can be found in Ref. [44].

Unfortunately, in Ref. [35] interference of the signal reaction $e^+e^- \to \phi \to \pi^0\pi^0\gamma$ with the coherent background $e^+e^- \to \omega\pi^0 \to \pi^0\pi^0\gamma$ was not taken into account. As a consequence, the data in the region of low invariant $\pi\pi$ masses, $m_{\pi\pi}$, are not correct even by the order of magnitude [46]. As for the data in the high-mass region, $m > 660$ MeV, they can be treated as the correct ones.

In this paper we show that the KLOE data on the $\phi \to \pi^0\pi^0\gamma$ decay and the data on the $\pi\pi$ scattering and the $\pi\pi \to K\bar{K}$ reaction up to 1.1 GeV can be described in the upgraded model of Ref. [25], taking into account the chiral shielding of the $\sigma(600)$ meson and its mixing with the $f_0(980)$ meson.

All formulas for the $\phi \to (S\gamma + \rho^0\pi^0) \to \pi^0\pi^0\gamma$ reaction [$S = f_0(980) + \sigma(600)$] are shown in Sec. II. The results of the data analysis are presented in Sec. III. A brief summary is given in Sec. IV.

II. THE FORMALISM OF THE $\phi \to (f_0(980) + \sigma(600))\gamma \to \gamma\pi^0\pi^0$ AND $\phi \to \rho^0\pi^0 \to \gamma\pi^0\pi^0$ REACTIONS

In Refs. [23,25] it was shown that the dominant background process is $\phi \to \pi^0\rho \to \gamma\pi^0\pi^0$, while the reactions $e^+e^- \to \rho \to \pi^0\omega \to \gamma\pi^0\pi^0$ and $e^+e^- \to \omega \to \pi^0\rho \to \gamma\pi^0\pi^0$ have a small effect on $e^+e^- \to \phi \to \gamma\pi^0\pi^0$ in the region $m_{\pi^0\pi^0} \equiv m > 900$ MeV. In Ref. [37] it was shown that the $\phi \to \pi^0\rho \to \gamma\pi^0\pi^0$ background is small in comparison with the signal $\phi \to \gamma f_0(980) \to \gamma\pi^0\pi^0$ at $m > 700$ MeV.

The amplitude of the background decay $\phi(p) \to \pi^0\rho \to \gamma(q)\pi^0(k_1)\pi^0(k_2)$ has the following forms:

$$M_{\text{back}} = e^{-i\delta} g_{\rho\pi^0\phi} g_{\rho\pi^0\gamma} \phi_\alpha p_\nu \epsilon_\delta q_\epsilon \epsilon_{\alpha\beta\mu\nu}\epsilon_{\beta\delta\omega\epsilon} \times \left(\frac{k_{1\mu}k_{2\omega}}{D_\rho(q+k_2)} + \frac{k_{2\mu}k_{1\omega}}{D_\rho(q+k_1)}\right). \quad (1)$$

Here δ is the additional phase (in this work we treat it as a constant) taking into account $\rho\pi$ rescattering effects [47].

In the K^+K^- loop model, $\phi \to K^+K^- \to \gamma(f_0 + \sigma)$ [23,25], above the $K\bar{K}$ threshold the amplitude of the signal $\phi \to \gamma(f_0 + \sigma) \to \gamma\pi^0\pi^0$ is

$$M_{\text{sig}} = g(m)\left((\phi\epsilon) - \frac{(\phi q)(\epsilon p)}{(pq)}\right) T_0^0(K^+K^- \to \pi^0\pi^0) \times 16\pi, \quad (2)$$

where the $K^+K^- \to \pi^0\pi^0$ amplitude, taking into account the mixing of f_0 and σ mesons,

$$T_0^0(K^+K^- \to \pi^0\pi^0) = e^{i\delta_B}\left(\sum_{R,R'} \frac{g_{RK^+K^-} G^{-1}_{RR'} g_{R'\pi^0\pi^0}}{16\pi}\right), \quad (3)$$

where $R, R' = f_0, \sigma$,

$$\delta_B = \delta_B^{\pi\pi} + \delta_B^{K\bar{K}}, \quad (4)$$

where $\delta_B^{\pi\pi}$ and $\delta_B^{K\bar{K}}$ are phases of the elastic background of the $\pi\pi$ and $K\bar{K}$ scattering, respectively (see Refs. [15–17]).

Note that the additional phase $\delta_B^{K\bar{K}}$ changes the modulus of the amplitude at $m < 2m_K$. Let us define

$$P_K = \begin{cases} e^{i\delta_B^{K\bar{K}}} & m \geq 2m_K \\ \text{anlytical continuation of } e^{i\delta_B^{K\bar{K}}} & m < 2m_K. \end{cases} \quad (5)$$

Note also that the phase $\delta_B^{\pi\pi}$ was defined as δ_B in Refs. [25,37].

The matrix of inverse propagators [25] is

$$G_{RR'} \equiv G_{RR'}(m) = \begin{pmatrix} D_{f_0}(m) & -\Pi_{f_0\sigma}(m) \\ -\Pi_{f_0\sigma}(m) & D_\sigma(m) \end{pmatrix},$$

$$\Pi_{f_0\sigma}(m) = \sum_{a,b} \frac{g_{\sigma ab}}{g_{f_0ab}} \Pi_{f_0}^{ab}(m) + C_{f_0\sigma},$$

where the constant $C_{f_0\sigma}$ incorporates the subtraction constant for the transition $f_0(980) \to (0^-0^-) \to \sigma(600)$ and effectively takes into account the contribution of multiparticle intermediate states to $f_0 \leftrightarrow \sigma$ transition (see Ref. [25]). The inverse propagator of the R scalar meson is presented in Refs. [13–17,19,23,25,40]:

$$D_R(m) = m_R^2 - m^2 + \sum_{ab}[\text{Re}\Pi_R^{ab}(m_R^2) - \Pi_R^{ab}(m^2)], \quad (6)$$

where $\sum_{ab}[\text{Re}\Pi_R^{ab}(m_R^2) - \Pi_R^{ab}(m^2)] = \text{Re}\Pi_R(m_R^2) - \Pi_R(m^2)$ takes into account the finite width corrections of the resonance which are the one loop contribution to the self-energy of the R resonance from the two-particle intermediate ab states.

For pseudoscalar ab mesons and $m_a \geq m_b, m \geq m_+$, one has

$$\Pi_R^{ab}(m^2) = \frac{g_{Rab}^2}{16\pi}\left[\frac{m_+m_-}{\pi m^2}\ln\frac{m_b}{m_a} + \rho_{ab}\left(i + \frac{1}{\pi} \times \ln\frac{\sqrt{m^2-m_-^2} - \sqrt{m^2-m_+^2}}{\sqrt{m^2-m_-^2} + \sqrt{m^2-m_+^2}}\right)\right] \quad (7)$$

$m_- \leq m < m_+$

$$\Pi_R^{ab}(m^2) = \frac{g_{Rab}^2}{16\pi}\left[\frac{m_+m_-}{\pi m^2}\ln\frac{m_b}{m_a} - |\rho_{ab}(m)| + \frac{2}{\pi}|\rho_{ab}(m)|\arctan\frac{\sqrt{m_+^2-m^2}}{\sqrt{m^2-m_-^2}}\right]. \quad (8)$$

$m < m_-$

$$\Pi_R^{ab}(m^2) = \frac{g_{Rab}^2}{16\pi}\left[\frac{m_+m_-}{\pi m^2}\ln\frac{m_b}{m_a} - \frac{1}{\pi}\rho_{ab}(m) \times \ln\frac{\sqrt{m_+^2-m^2} - \sqrt{m_-^2-m^2}}{\sqrt{m_+^2-m^2} + \sqrt{m_-^2-m^2}}\right]. \quad (9)$$

$$\rho_{ab}(m) = \sqrt{\left(1 - \frac{m_+^2}{m^2}\right)\left(1 - \frac{m_-^2}{m^2}\right)}, \qquad m_\pm = m_a \pm m_b. \quad (10)$$

The constants g_{Rab} are related to the width

$$\Gamma(R \to ab, m) = \frac{g_{Rab}^2}{16\pi m}\rho_{ab}(m). \quad (11)$$

Note that we take into account intermediate states $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta'\eta$, $\eta'\eta'$ in the $f_0(980)$ and $\sigma(600)$ propagators:

$$\Pi_{f_0} = \Pi_{f_0}^{\pi^+\pi^-} + \Pi_{f_0}^{\pi^0\pi^0} + \Pi_{f_0}^{K^+K^-} + \Pi_{f_0}^{K^0\bar{K}^0} + \Pi_{f_0}^{\eta\eta} + \Pi_{f_0}^{\eta'\eta} + \Pi_{f_0}^{\eta'\eta'}, \quad (12)$$

and also for the $\sigma(600)$. We use $g_{f_0K^0\bar{K}^0} = g_{f_0K^+K^-}$, $g_{f_0\pi^0\pi^0} = g_{f_0\pi^+\pi^-}/\sqrt{2}$, the same for the $\sigma(600)$, too.

For other coupling constants we use the four-quark model prediction [22,23]:

$$g_{f_0\eta\eta} = -g_{f_0\eta'\eta'} = \frac{2\sqrt{2}}{3}g_{f_0K^+K^-},$$

$$g_{f_0\eta'\eta} = -\frac{\sqrt{2}}{3}g_{f_0K^+K^-};$$

$$g_{\sigma\eta\eta} = g_{\sigma\eta\eta'} = \frac{\sqrt{2}}{3}g_{\sigma\pi^+\pi^-}, \qquad g_{\sigma\eta'\eta'} = \frac{1}{3\sqrt{2}}g_{\sigma\pi^+\pi^-}.$$

In the K^+K^- loop model $g(m)$ has the following forms (see Refs. [23,38,40,44]).

For $m < 2m_{K^+}$,

$$g(m) = \frac{e}{2(2\pi)^2}g_{\phi K^+K^-}\left\{1 + \frac{1-\rho^2(m^2)}{\rho^2(m_\phi^2) - \rho^2(m^2)} \times \left[2|\rho(m^2)|\arctan\frac{1}{|\rho(m^2)|} - \rho(m_\phi^2)\lambda(m_\phi^2) + i\pi\rho(m_\phi^2) - (1-\rho^2(m_\phi^2))\left(\frac{1}{4}(\pi + i\lambda(m_\phi^2))^2 - \left(\arctan\frac{1}{|\rho(m^2)|}\right)^2\right)\right]\right\}, \quad (13)$$

where

$$\rho(m^2) = \sqrt{1 - \frac{4m_{K^+}^2}{m^2}};$$
$$\lambda(m^2) = \ln\frac{1+\rho(m^2)}{1-\rho(m^2)}; \quad (14)$$
$$\frac{e^2}{4\pi} = \alpha = \frac{1}{137}.$$

For $m \geq 2m_{K^+}$,

$$g(m) = \frac{e}{2(2\pi)^2} g_{\phi K^+K^-}\bigg\{1 + \frac{1-\rho^2(m^2)}{\rho^2(m_\phi^2) - \rho^2(m^2)} \times \bigg[\rho(m^2)(\lambda(m^2) - i\pi) - \rho(m_\phi^2)(\lambda(m_\phi^2) - i\pi) - \frac{1}{4}(1-\rho^2(m_\phi^2))((\pi + i\lambda(m_\phi^2))^2 - (\pi + i\lambda(m^2))^2)\bigg]\bigg\}. \quad (15)$$

The mass spectrum of the reaction is

$$\frac{\Gamma(\phi \to \gamma\pi^0\pi^0)}{dm} = \frac{d\Gamma_S}{dm} + \frac{d\Gamma_{\rm back}(m)}{dm} + \frac{d\Gamma_{\rm int}(m)}{dm}, \quad (16)$$

where the signal contribution $\phi \to S\gamma \to \pi^0\pi^0\gamma$

$$\frac{d\Gamma_S}{dm} = \frac{|P_K|^2|g(m)|^2\sqrt{m^2 - 4m_\pi^2}(m_\phi^2 - m^2)}{3(4\pi)^3 m_\phi^3} \times \bigg|\sum_{R,R'} g_{RK^+K^-} G^{-1}_{RR'} g_{R'\pi^0\pi^0}\bigg|^2. \quad (17)$$

The mass spectrum of the background process $\phi \to \rho\pi^0 \to \pi^0\pi^0\gamma$,

$$\frac{d\Gamma_{\rm back}(m)}{dm} = \frac{1}{2}\frac{(m_\phi^2 - m^2)\sqrt{m^2 - 4m_\pi^2}}{256\pi^3 m_\phi^3} \times \int_{-1}^{1} dx A_{\rm back}(m, x), \quad (18)$$

where

$$\begin{aligned} A_{\rm back}(m, x) &= \frac{1}{3}\sum |M_{\rm back}|^2 \\ &= \frac{1}{24} g^2_{\phi\rho\pi} g^2_{\rho\pi\gamma}\bigg\{(m_\pi^8 + 2m^2m_\pi^4\tilde{m}_\rho^2 - 4m_\pi^6\tilde{m}_\rho^2 + 2m^4\tilde{m}_\rho^4 - 4m^2m_\pi^2\tilde{m}_\rho^4 + 6m_\pi^4\tilde{m}_\rho^4 + 2m^2\tilde{m}_\rho^6 - 4m_\pi^2\tilde{m}_\rho^6 \\ &\quad + \tilde{m}_\rho^8 - 2m_\pi^6 m_\phi^2 - 2m^2m_\pi^2\tilde{m}_\rho^2 m_\phi^2 + 2m_\pi^4\tilde{m}_\rho^2 m_\phi^2 - 2m^2\tilde{m}_\rho^4 m_\phi^2 + 2m_\pi^2\tilde{m}_\rho^4 m_\phi^2 - 2\tilde{m}_\rho^6 m_\phi^2 + m_\pi^4 m_\phi^4 + \tilde{m}_\rho^4 m_\phi^4) \\ &\quad \times \bigg(\frac{1}{|D_\rho(m_\rho)|^2} + \frac{1}{|D_\rho(m_\rho^*)|^2}\bigg) + (m_\phi^2 - m^2)(m^2 - 2m_\pi^2 + 2\tilde{m}_\rho^2 - m_\phi^2)(2m^2m_\pi^2 + 2m_\pi^2 m_\phi^2 - m^4)\frac{1}{|D_\rho(m_\rho^*)|^2} \\ &\quad + 2\,{\rm Re}\bigg(\frac{1}{D_\rho(m_\rho)D_\rho^*(m_\rho^*)}\bigg)(m_\pi^8 - m^6\tilde{m}_\rho^2 + 2m^4m_\pi^2\tilde{m}_\rho^2 + 2m^2m_\pi^4\tilde{m}_\rho^2 - 4m_\pi^6\tilde{m}_\rho^2 - 4m^2m_\pi^2\tilde{m}_\rho^4 + 6m_\pi^4\tilde{m}_\rho^4 \\ &\quad + 2m^2\tilde{m}_\rho^6 - 4m_\pi^2\tilde{m}_\rho^6 + \tilde{m}_\rho^8 + m^2m_\pi^4 m_\phi^2 - 2m_\pi^6 m_\phi^2 + 2m^4\tilde{m}_\rho^2 m_\phi^2 - 4m^2m_\pi^2\tilde{m}_\rho^2 m_\phi^2 + 2m_\pi^4\tilde{m}_\rho^2 m_\phi^2 \\ &\quad - m^2\tilde{m}_\rho^4 m_\phi^2 + 2m_\pi^2\tilde{m}_\rho^4 m_\phi^2 - 2\tilde{m}_\rho^6 m_\phi^2 - m_\pi^4 m_\phi^4 - m^2\tilde{m}_\rho^2 m_\phi^4 + 2m_\pi^2\tilde{m}_\rho^2 m_\phi^4 + \tilde{m}_\rho^4 m_\phi^4)\bigg\}, \end{aligned} \quad (19)$$

$$\tilde{m}_\rho{}^2 = m_\pi^2 + \frac{(m_\phi^2 - m^2)}{2}\bigg(1 - x\sqrt{1 - \frac{4m_\pi^2}{m^2}}\bigg) \qquad \tilde{m}_\rho{}^{*2} = m_\phi^2 + 2m_\pi^2 - m^2 - \tilde{m}_\rho{}^2. \quad (20)$$

Note that in Ref. [37] there are typos, m_ρ should be replaced by $\tilde{m}_\rho$ everywhere in Eq. (18) of Ref. [37], as in Eq. (19) of the given paper; see also Ref. [39]. Note also that all calculations in Ref. [37] were done with the correct expression.

The interference between signal and background processes accounts for

$$\frac{d\Gamma_{\rm int}(m)}{dm} = \frac{1}{\sqrt{2}}\frac{\sqrt{m^2 - 4m_\pi^2}}{256\pi^3 m_\phi^3}\int_{-1}^{1} dx A_{\rm int}(m, x), \quad (21)$$

where

$$\begin{aligned} A_{\rm int}(m, x) &= \frac{2}{3}(m_\phi^2 - m^2)\,{\rm Re}\sum M_f M^*_{\rm back} \\ &= \frac{1}{3}\,{\rm Re}\bigg\{P_K e^{i\delta_B^{\pi\pi}} e^{i\delta} g(m) g_{\phi\rho\pi} g_{\rho\pi^0\gamma}\bigg(\sum_{R,R'} g_{RK^+K^-} G^{-1}_{RR'} g_{R'\pi^0\pi^0}\bigg)\bigg(\frac{((\tilde{m}_\rho^2 - m_\pi^2)^2 m_\phi^2 - (m_\phi^2 - m^2)^2\tilde{m}_\rho^2)}{D_\rho^*(\tilde{m}_\rho)} \\ &\quad + \frac{((\tilde{m}_\rho^{*2} - m_\pi^2)^2 m_\phi^2 - (m_\phi^2 - m^2)^2\tilde{m}_\rho^{*2})}{D_\rho^*(\tilde{m}_\rho{}^*)}\bigg)\bigg\}. \end{aligned} \quad (22)$$

The factor 1/2 in Eq. (18) and the factor $1/\sqrt{2}$ in Eq. (21) take into account the identity of pions, the same reason for definition $g_{R\pi^0\pi^0} = g_{R\pi^+\pi^-}/\sqrt{2}$ in Eq. (17).

The S-wave amplitude T_0^0 of the $\pi\pi$ scattering with $I = 0$ [15–17,25] is

$$T_0^0 = \frac{\eta_0^0 e^{2i\delta_0^0} - 1}{2i\rho_{\pi\pi}(m)} = \frac{e^{2i\delta_B^{\pi\pi}} - 1}{2i\rho_{\pi\pi}(m)} + e^{2i\delta_B^{\pi\pi}} \sum_{R,R'} \frac{g_{R\pi\pi} G_{RR'}^{-1} g_{R'\pi\pi}}{16\pi}. \quad (23)$$

Here $\eta_0^0 \equiv \eta_0^0(m)$ is the inelasticity, $\eta_0^0 = 1$ for $m \leq 2m_{K^+}$, and

$$\delta_0^0 \equiv \delta_0^0(m) = \delta_B^{\pi\pi}(m) + \delta_{\text{res}}(m), \quad (24)$$

where $\delta_B^{\pi\pi} \equiv \delta_B^{\pi\pi}(m)$ (δ_B in Ref. [25]) is the phase of the elastic background [see Eq. (4)], and $\delta_{\text{res}}(m)$ is half of the phase of

$$S_0^{0\,\text{res}} = \eta_0^0(m) e^{2i\delta_{\text{res}}(m)} = 1 + 2i\rho_{\pi\pi}(m) \sum_{R,R'} \frac{g_{R\pi\pi} G_{RR'}^{-1} g_{R'\pi\pi}}{16\pi}, \quad (25)$$

$$\eta_0^0 = |S_0^{0\,\text{res}}|,$$

$g_{R\pi\pi} = \sqrt{3/2}\, g_{R\pi^+\pi^-}$. The chiral shielding phase $\delta_B^{\pi\pi}(m)$, motivated by the σ-model [8], is taken in the form

$$\tan(\delta_B^{\pi\pi}) = -\frac{p_\pi}{m_\pi}\left(b_0 - b_1 \frac{p_\pi^2}{m_\pi^2} + b_2 \frac{p_\pi^4}{m_\pi^4}\right) \times \frac{1}{1 + (2p_\pi)^2/\Lambda^2}, \quad (26)$$

and

$$e^{2i\delta_B^{\pi\pi}} = \frac{1 - i\frac{p_\pi}{m_\pi}(b_0 - b_1 \frac{p_\pi^2}{m_\pi^2} + b_2 \frac{p_\pi^4}{m_\pi^4}) \frac{1}{1+(2p_\pi)^2/\Lambda^2}}{1 + i\frac{p_\pi}{m_\pi}(b_0 - b_1 \frac{p_\pi^2}{m_\pi^2} + b_2 \frac{p_\pi^4}{m_\pi^4}) \frac{1}{1+(2p_\pi)^2/\Lambda^2}}. \quad (27)$$

Here $2p_\pi = \sqrt{m^2 - 4m_\pi^2}$, and $(1 + (2p_\pi)^2/\Lambda^2)^{-1}$ is a cutoff factor. The phase $\delta_B^{K\bar{K}} = \delta_B^{K\bar{K}}(m)$ is parametrized in the following way:

TABLE I. Results of the analysis, Fits 1–4.

Fit	1	2	3	4
m_{f_0}, MeV	984.1	985.2	984.8	987.6
$g_{f_0K^+K^-}$, GeV	4.3	4.2	5.1	4.7
$\frac{g^2_{f_0K^+K^-}}{4\pi}$, GeV^2	1.44	1.39	2.09	1.79
$g_{f_0\pi^+\pi^-}$, GeV	−1.8	−2.0	−1.9	−2.1
$\frac{g^2_{f_0\pi^+\pi^-}}{4\pi}$, GeV^2	0.25	0.32	0.28	0.36
m_σ, MeV	461.9	485.0	472.0	542.6
$g_{\sigma\pi^+\pi^-}$, GeV	2.4	−2.2	2.5	−2.5
$\frac{g^2_{\sigma\pi^+\pi^-}}{4\pi}$, GeV^2	0.44	0.38	0.50	0.49
Γ_σ, MeV	286.0	240.2	319.7	289.9
$g_{\sigma K^+K^-}$, GeV	0.55	−0.93	0.43	−1.1
$\frac{g^2_{\sigma K^+K^-}}{4\pi}$, GeV^2	0.024	0.07	0.015	0.10
C, GeV^2	0.047	0.12	−0.008	0.13
δ, °	−11.4	−11.5	−24.7	−12.0
b_0	4.9	4.8	5.3	5.2
b_1	1.1	1.1	0.90	0.55
b_2	1.36	1.32	1.18	0.50
Λ, MeV	172.2	172.8	160.0	150.3
m_1, MeV	765.4	766.4	795.6	784.1
m_2, MeV	368.9	368.6	375.7	362.2
Λ_K, GeV	1.24	1.24	1.25	1.26
a_0^0, m_π^{-1}	0.209	0.209	0.22	0.22
Adler zero in $\pi\pi \to \pi\pi$	(178 MeV)^2	(207 MeV)^2	(199 MeV)^2	(185 MeV)^2
Adler zero in $\pi\pi \to K\bar{K}$	m_π^2	m_π^2	m_π^2	$m_\pi^2/2$
η_0^0 (1010 MeV)	0.48	0.48	0.39	0.41
$\chi^2_{\text{tot}}/n.d.f$ (C.L.)	40.0/48 (79%)	40.0/48 (79%)	44.2/49 (67%)	45.8/49 (60%)
χ^2_{sp} (18 points)	11.7	11.6	10.2	11.8

$$\tan\delta_B^{K\bar{K}} = f_K(m^2)\sqrt{m^2 - 4m_{K^+}^2} \equiv 2p_K f_K(m^2) \quad (28)$$

and

$$e^{2i\delta_B^{K\bar{K}}} = \frac{1 + i2p_K f_K(m^2)}{1 - i2p_K f_K(m^2)}. \quad (29)$$

Actually, $e^{2i\delta_B^{\pi\pi}(m)}$ has a pole at $m^2 = m_0^2$, $0 < m_0^2 < 4m_\pi^2$ (details are below), which is compensated by the zero in $e^{2i\delta_B^{K\bar{K}}(m)}$ to ensure a regular $K\bar{K} \to \pi\pi$ amplitude and, consequently, the $\phi \to K^+K^- \to \pi\pi\gamma$ amplitude at $0 < m^2 < 4m_\pi^2$. This requirement leads to

$$f_K(m_0^2) = \frac{1}{\sqrt{4m_{K^+}^2 - m_0^2}} \approx \frac{1}{2m_{K^+}}. \quad (30)$$

The inverse propagator of the ρ meson has the following expression:

$$D_\rho(m) = m_\rho^2 - m^2 - im^2 \frac{g_{\rho\pi\pi}^2}{48\pi}\left(1 - \frac{4m_\pi^2}{m^2}\right)^{3/2}. \quad (31)$$

The coupling constants $g_{\phi K^+K^-} = 4.376 \pm 0.074$ and $g_{\phi\rho\pi} = 0.814 \pm 0.018$ GeV^{-1} are taken from the most precise measurement, Ref. [48]. Note that in Refs. [34,37] the value $g_{\phi K^+K^-} = 4.59$ was obtained using the [49] data. To obtain the coupling constant $g_{\rho\pi^0\gamma}$, we used the data of the experiments [50,51] on the $\rho \to \pi^0\gamma$ decay and the expression

$$\Gamma(\rho \to \pi^0\gamma) = \frac{g_{\rho\pi^0\gamma}^2}{96\pi m_\rho^3}(m_\rho^2 - m_\pi^2)^3, \quad (32)$$

the result $g_{\rho\pi^0\gamma} = 0.26 \pm 0.02$ GeV^{-1} is the weighed average of these experiments.

III. DATA ANALYSIS

A. Restrictions

To analyze the data, we construct a function to minimize:

$$\tilde{\chi}_{\rm tot}^2 = \chi_{sp}^2 + \chi_{ph}^2. \quad (33)$$

Here χ_{sp}^2 is an usual χ^2 function to fit the $\pi^0\pi^0$ spectrum in the $\phi \to \pi^0\pi^0\gamma$ decay, see Appendix A for details, while χ_{ph}^2 is the δ_0^0 contribution. For the $\pi\pi$ scattering phase δ_0^0 we use the data [52–56].

TABLE II. Results of the analysis, Fits 5–8.

Fit	5	6	7	8
m_{f_0}, MeV	987.1	984.2	982.1	982.1
$g_{f_0K^+K^-}$, GeV	2.9	2.8	4.2	4.2
$\frac{g_{f_0K^+K^-}^2}{4\pi}$, GeV^2	0.67	0.62	1.44	1.42
$g_{f_0\pi^+\pi^-}$, GeV	−0.9	−0.8	−1.7	−1.7
$\frac{g_{f_0\pi^+\pi^-}^2}{4\pi}$, GeV^2	0.07	0.05	0.23	0.23
m_σ, MeV	709.0	692.5	400	415
$g_{\sigma\pi^+\pi^-}$, GeV	3.6	−3.3	−2.1	−2.2
$\frac{g_{\sigma\pi^+\pi^-}^2}{4\pi}$, GeV^2	1.01	0.89	0.36	0.39
Γ_σ, MeV	492.5	442.4	241.9	259.6
$g_{\sigma K^+K^-}$, GeV	0.13	−0.035	0.38	−0.37
$\frac{g_{\sigma K^+K^-}^2}{4\pi}$, GeV^2	0.001	≈ 0	0.01	0.01
C, GeV^2	−0.01	−0.02	−0.015	0.015
δ, °	−9.8	−6.4	−28.0	−20.5
b_0	4.6	3.0	5.6	5.4
b_1	1.2	0.86	6.8	3.7
b_2	0.18	0.16	9.0	5.0
Λ, MeV	149.4	181.0	247.4	200.4
m_1, MeV	577.5	585.4	753.4	757.0
m_2, MeV	581.7	680.4	361.5	362.9
Λ_K, GeV	0.62	0.50	1.24	1.24
a_0^0, m_π^{-1}	0.213	0.207	0.215	0.209
Adler zero in $\pi\pi \to \pi\pi$	(190 MeV)^2	(194 MeV)^2	(232 MeV)^2	(226 MeV)^2
Adler zero in $\pi\pi \to K\bar{K}$	m_π^2	m_π^2	m_π^2	m_π^2
η_0^0 (1010 MeV)	0.56	0.57	0.50	0.49
$\chi_{\rm tot}^2/n.d.f.$ (C.L.)	48.1/48 (47%)	47.1/48 (51%)	54.4/49 (28%)	43.9/49 (68%)
χ_{sp}^2 (18 points)	17.8	15.6	15.6	13.4

Some parameters are fixed by the requirement of the proper analytical continuation of amplitudes. The phase factor $e^{2i\delta_B^{\pi\pi}}$ has a singularity at m_0^2, $0 < m_0^2 < 4m_\pi^2$, when

$$1 - \frac{4m_\pi^2 - m_0^2}{\Lambda^2} - \frac{\sqrt{4m_\pi^2 - m_0^2}}{2m_\pi}\left(b_0 + b_1\frac{4m_\pi^2 - m_0^2}{(2m_\pi)^2} + b_2\frac{(4m_\pi^2 - m_0^2)^2}{(2m_\pi)^4}\right) = 0, \quad (34)$$

see Eq. (27). Inasmuch as the amplitude (23) should have no poles at $0 < m_0^2 < 4m_\pi^2$, Eq. (25) should be equal to zero at the m_0^2 [57]. This condition fixes one free parameter. Another free parameter may be removed by fixing the $\pi\pi$ scattering length a_0^0:

$$\sum_{R,R'} g_{R\pi\pi}G_{RR'}^{-1}g_{R'\pi\pi}\bigg|_{m=2m_{\pi^+}} - b_0 = a_0^0. \quad (35)$$

In two variants of fitting the data (see Fits 3 and 4 in Table I), we take $a_0^0 = 0.22m_\pi^{-1}$ from the recent calculation [58] based on the chiral perturbation theory and Roy equations. This number excellently consists with all other results, for example, with the BNL experimental value $a_0^0 = (0.228 \pm 0.012)m_\pi^{-1}$ [59]. In other variants of the data analysis, we treat a_0^0 as a free parameter but all obtained values of a_0^0 are close to the BNL one.

The Adler zeros is the separate question. The zero in the amplitude of the $\pi\pi$ scattering appears automatically near 150 MeV in all obtained analyzes, see Tables I and II, while the Adler zero existence is a rather strict constraint for the $K^+K^- \to \pi^0\pi^0$ amplitude, Eq. (3).

TABLE III. Results of the analysis, Fits 9 and 10.

Fit	9	10
m_{f_0}, MeV	983.2	987.1
$g_{f_0K^+K^-}$, GeV	4.0	3.7
$\frac{g^2_{f_0K^+K^-}}{4\pi}$, GeV^2	1.25	1.06
$g_{f_0\pi^+\pi^-}$, GeV	−1.3	−1.9
$\frac{g^2_{f_0\pi^+\pi^-}}{4\pi}$, GeV^2	0.15	0.29
m_σ, MeV	528.6	566.3
$g_{\sigma\pi^+\pi^-}$, GeV	2.8	−2.4
$\frac{g^2_{\sigma\pi^+\pi^-}}{4\pi}$, GeV^2	0.61	0.46
Γ_σ, MeV	365.8	263.8
$g_{\sigma K^+K^-}$, GeV	1.1	−1.8
$\frac{g^2_{\sigma K^+K^-}}{4\pi}$, GeV^2	0.09	0.25
C, GeV^2	0.01	0.14
δ, °	−38.8	−37.2
b_0	5.2	5.22
b_1	0.48	0.43
b_2	0.43	0.46
Λ, MeV	149.0	149.5
m_1, MeV	803.0	801.3
m_2, MeV	328.9	330.6
Λ_K, GeV	1.31	1.31
a_0^0, m_π^{-1}	0.228	0.229
Adler zero in $\pi\pi \to \pi\pi$	(179 MeV)^2	(179 MeV)^2
Adler zero in $\pi\pi \to K\bar{K}$	−(1 GeV)^2	−(0.9 GeV)^2
η_0^0 (1010 MeV)	0.54	0.54
$\chi^2_{\rm tot}/n.d.f.$ (C.L.)	38.5/47 (81%)	38.6/47 (80%)
χ^2_{sp} (18 points)	10.4	10.5

B. Investigating the general scenario and around

Analyzing data, we imply a scenario motivated by the four-quark model [5], that is, the $\sigma(600)$ coupling with the $K\bar{K}$ channel $g_{\sigma K^+K^-}$ is suppressed relative to the one with the $\pi^+\pi^-$ channel $g_{\sigma\pi^+\pi^-}$, the mass of the σ meson is in the 500–700 MeV range. In addition, we have in mind the Adler self-consistency conditions for $T_0^0(\pi\pi \to \pi\pi)$ and $T(\phi \to \pi\pi\gamma)$ [i.e., in $T_0^0(K^+K^- \to \pi\pi)$] amplitudes near the $\pi\pi$ threshold. The general aim of this subsection is to demonstrate that the data are in excellent agreement with this general scenario.

As for the $\pi\pi$ scattering amplitude, the Adler zero appears below the $\pi\pi$ threshold automatically in all variants, see Tables I, II, and III.

The ϕ decay amplitude is another matter. The analysis shows that the data prefer to have a zero in the amplitude of the ϕ decay at the negative values $m^2 \sim -1$ GeV^2, i.e., in the region of the left cut, see Table III. That is why we require the ϕ-decay amplitude to have the Adler zero in the interval $0 < m^2 < 4m_\pi^2$, see Tables I and II.

The data favor negative $f_K(m^2)$ in the resonance region, at $m > 700$ MeV. A fit with the "effective" constant $f_K(m^2) = \Delta_K$ gives $\Delta_K \approx -0.6$/GeV. Equation (30) shows that near the $\pi\pi$ threshold the function $f_K(m^2)$ should be about 1/GeV, i.e., positive. We choose $f_K(m^2)$ in the form

$$f_K(m^2) = -\frac{\arctan(\frac{m^2 - m_1^2}{m_2^2})}{\Lambda_K} \quad (36)$$

to get the desirable change of the sign at $m = m_1 \approx$ 500–800 MeV in a rather simple way [only two new parameters are introduced for Eq. (30)]. Unfortunately, now we have not enough data and theory to determine $f_K(m^2)$ more accurately.

The inelasticity $\eta_0^0(m)$ and the phase $\delta^{\pi K}(m)$ of the amplitude $T_0^0(\pi\pi \to K\bar{K})$ are essential in the fit region, $2m_{K^+} < m < 1.1$ GeV. As for the inelasticity, the experimental data of Ref. [52] give evidence in favor of low values of $\eta_0^0(m)$ near the $K\bar{K}$ threshold. The situation with the experimental data on $\delta^{\pi K}(m)$ is controversial and experimental points have large errors. We consider these data as a guide, whose main role is to fix the sign between signal and background amplitudes (2) and (1), and hold only two points of the experiment [60], see Fig. 7.

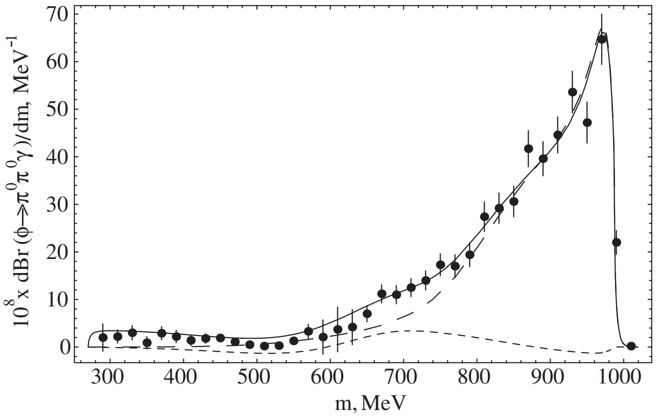

FIG. 1. The $\pi^0\pi^0$ spectrum, theoretical curve (solid line), and the KLOE data (points). The signal contribution and the interference term are shown with the dashed line and the dotted line. All figures are for Fit 1 in Table I.

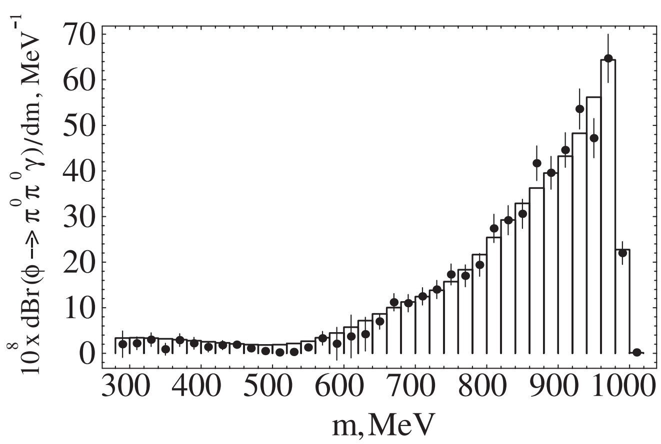

FIG. 2. The comparison of the Fit 1 and the KLOE data. Histograms show the Fit 1 curve averaged around each bin (see Appendix A).

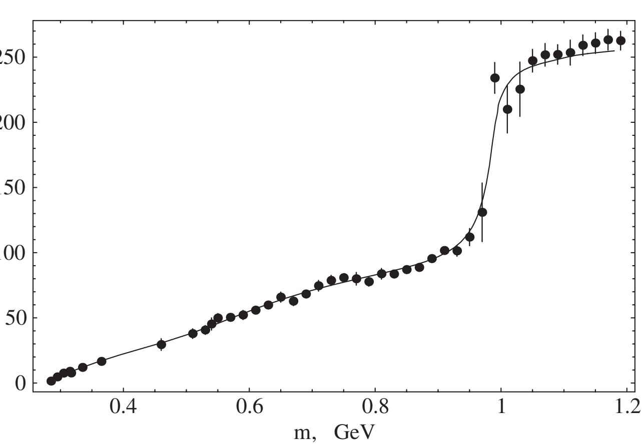

FIG. 3. The phase δ_0^0 of the $\pi\pi$ scattering (degrees).

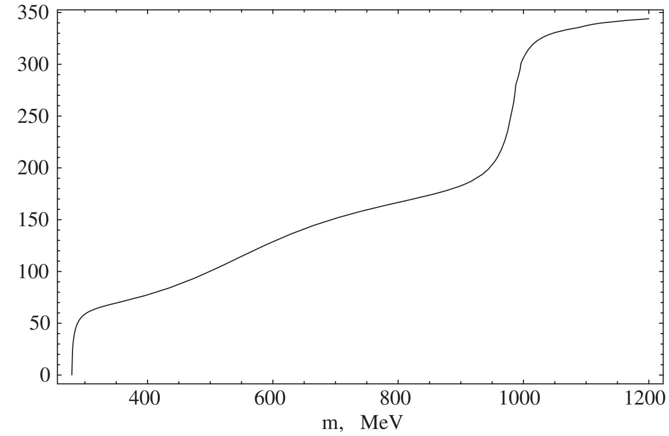

FIG. 4. The resonant phase δ_{res}.

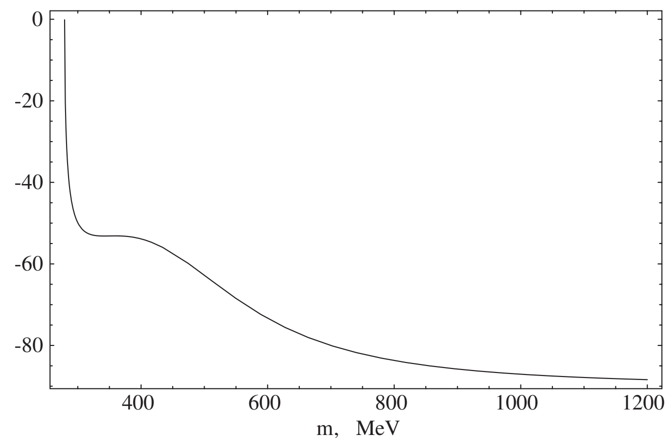

FIG. 5. The background phase $\delta_B^{\pi\pi}$.

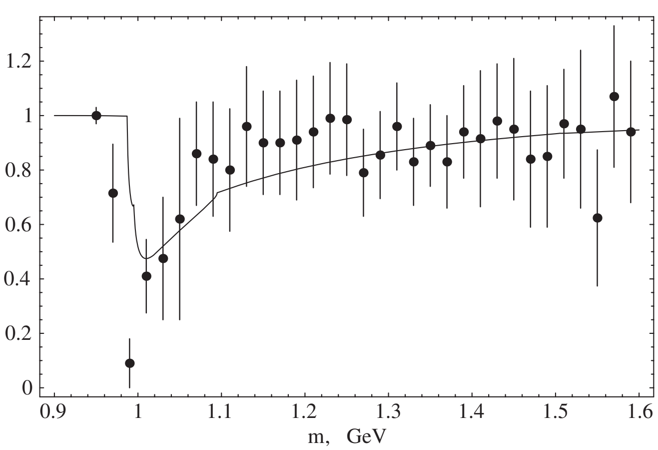

FIG. 6. The inelasticity η_0^0.

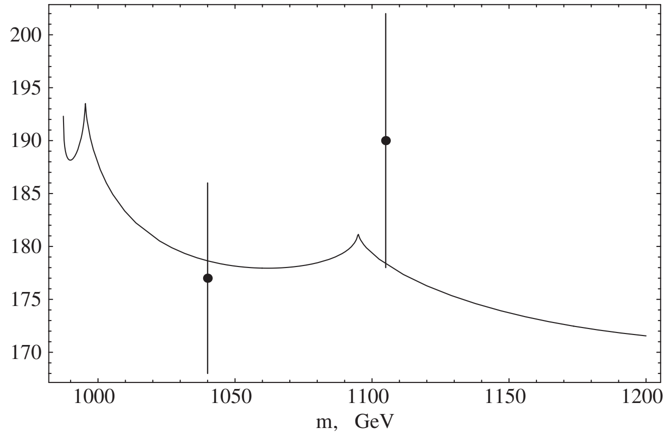

FIG. 7. The phase $\delta^{\pi K}$ of the $\pi\pi \to K\bar{K}$ scattering.

Providing all the conditions cited above, we set the Adler zero position in $\pi\pi \to K\bar{K}$ to $m^2 = m_\pi^2$ in Fits 1–3 and 5–8, listed in Tables I and II. All figures, Figs. 1–7, correspond to Fit 1 in Table I. We emphasize that Fits 1, 3, 5, 7, 9 and 2, 4, 6, 8, 10 correspond to the positive and negative $g_{\sigma\pi^+\pi^-}/g_{f_0K^+K^-}$ ratio, respectively. The sign of this ratio reveals itself only in the $f_0 - \sigma$ mixing, which is reasonably small. That is why we obtain a kind of symmetry by a slight change of the rest parameters, especially the constant $C_{f_0\sigma}$. This consideration is supported by the data, see Tables I, II, and III, and by the fact that the resonance phase $\delta_{\text{res}}(m)$ reaches 90 and 270 degrees close to m_σ and m_{f_0}, respectively, see Fig. 4.

As seen from the tables, the obtained variants do not leave any doubt that the data are in perfect agreement with the general scenario, which we discuss.

A crucial aspect of the data description is a low inelasticity. The key experimental point is $\eta_0^0(m = 1.01 \text{ GeV}) = 0.41 \pm 0.14$, see Fig. 4. To demonstrate the possibility of the low inelasticity in our model we list two variants in Table I with the fixed inelasticities: Fit 3 with $\eta_0^0(m = 1.01 \text{ GeV}) = 0.39$ and Fit 4 with $\eta_0^0(m = 1.01 \text{ GeV}) = 0.41$ for the different signs of $g_{\sigma\pi^+\pi^-}/g_{f_0K^+K^-}$.

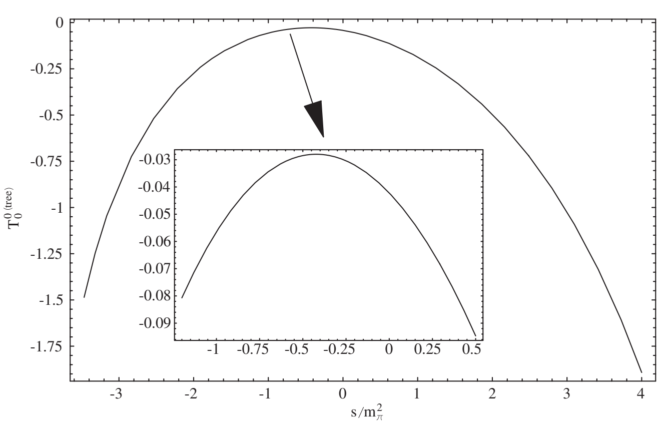

FIG. 8. Plot of the amplitude $T_0^{0(\text{tree})}$ (m), for $m_\sigma = 400$ MeV.

In Fit 4 we set the Adler zero position in $\pi\pi \to K\bar{K}$ to $m^2 = m_\pi^2/2$ for variety. Note that varying the Adler zero position in $\pi\pi \to K\bar{K}$ amplitude at $0 < m^2 < 4m_\pi^2$ does not give a noticeable effect.

The excellent description of the data gives the possibility to study different physical situations and to find crucial points for further investigations.

In particular, in Fits 5 and 6, Table II, we have the picture close to the naive four-quark model [5]: $f_0(980)$ weakly couples to the $\pi\pi$ channel, the $\sigma(600)$ practically does not couple to the $K\bar{K}$ channel, and $g^2_{\sigma\pi^+\pi^-} \approx 2g^2_{f_0K^+K^-}$, as predicted by the naive four-quark model, see Refs. [5,41].

The data may be perfectly described with low $m_\sigma \approx 400$ MeV also, see Fits 7 and 8 in Table II. We remind that low mass of $\sigma(600)$ is interesting for the sigma term problem.

Removing the requirement of the Adler zero in the $\pi\pi \to K\bar{K}$ amplitude at $0 < m^2 < 4m_\pi^2$ leads to a small improvement of χ^2 and inessential changes in the parameters. As for Adler zero, it goes to $m^2 \approx -(1 \text{ GeV})^2$, see Fits 9 and 10 in Table III.

IV. CONCLUSION

So the experimental data on the $\phi \to \pi^0\pi^0\gamma$ decay, the $\pi\pi$ scattering, and the $\pi\pi \to K\bar{K}$ reaction up to 1.1 GeV are perfectly described in the model of the coupled $\sigma(600)$ and $f_0(980)$ resonances under the general scenario (unitarity, analyticity of the amplitudes, the Adler zeros, the chiral shielding of $\sigma(600)$, $a_0^0 \approx 0.22m_\pi^{-1}$).

To reduce (if not avoid) an effect of heavier isosinglet scalars, we restrict ourselves to the analysis of the mass region $m < 1.1 \text{ GeV}^2$, where, as one may expect, an effect of heavier scalars could not be essential. As to mixing light and heavier isosinglet scalars, this question could not be resolved once and for all at present, in particular, because their properties are not well-established up to now. A preliminary consideration was carried out in Ref. [61], where, in particular, it was shown that the mixing could affect the mass difference of the isospinor and isovector.

Needless to say, analyticity considerations are essential for our analysis. It is well known that an analytical function can be restored if its values at an interval are known. One can hope that a description with good analytical characteristics is close to the exact one. So, one can consider the automatic formation of the Adler zero in the $\pi\pi \to \pi\pi$ amplitude at $0 < m^2 < 4m_\pi^2$ as a wink that we follow the right path. As for the $\pi\pi \to K\bar{K}$ amplitude, we have to put the Adler zero at $0 < m^2 < 4m_\pi^2$ as a constraint. To all appearances, it is a prompt that this process is more complicated.

The weak coupling of $\sigma(600)$ with the $K\bar{K}$ channel and $f_0(980)$ with the $\pi\pi$ channel practically in all variants agrees qualitatively with the four-quark model [5]. Certainly, there is also a suppression of the coupling of $\sigma(600)$ with the $K\bar{K}$ channel and a strong suppression of the coupling $f_0(980)$ with the $\pi\pi$ channel in the $q\bar{q}$ model: $\sigma(600) = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $f_0(980) = s\bar{s}$. But the $f_0(980)$ and $a_0(980)$ mass degeneracy cannot be explained in the naive two-quark model in this case because $a_0(980) = (u\bar{u} - d\bar{d})/\sqrt{2}$. In addition, the photon spectra in $\phi \to \gamma f_0(980) \to \gamma\pi^0\pi^0$ and $\phi \to \gamma a_0(980) \to \gamma\pi^0\eta$ cannot be explained in this case also [4]. We emphasize once more that Fits 5 and 6 in Table II are especially interesting from the four-quark model standpoint: $f_0(980)$ weakly couples to the $\pi\pi$ channel, $\sigma(600)$ practically does not couple to the $K\bar{K}$ channel, and $g^2_{\sigma\pi^+\pi^-} \approx 2g^2_{f_0K^+K^-}$. The practical absence of the $\sigma(600)$ coupling with the $K\bar{K}$ channel is excluded in the $q\bar{q}$ model, while it is the characteristic of the lightest isoscalar scalar primary states in the four-quark model [5].

Note that in all variants $g^2_{f_0K^+K^-}/4\pi$ has the same order as $g^2_{a_0K^+K^-}/4\pi$ obtained in Ref. [44] which agrees reasonably with the four-quark model.

Our investigation confirms in full the K^+K^- loop mechanism of the $f_0(980)$ production which means the radiative four-quark transition from $\phi(1020)$ to $f_0(980)$ and testifies to the four-quark nature of $f_0(980)$ [4].

The elucidation of the situation, a contraction of the possible variants, or even the selection of the unique variant, requires considerable efforts. The new precise experiment on $\pi\pi \to K\bar{K}$ would give the crucial information about the inelasticity η_0^0 and about the phase $\delta_B^{K\bar{K}}(m)$ near the $K\bar{K}$ threshold. The forthcoming precise experiment in KLOE on the $\phi \to \pi^0\pi^0\gamma$ decay will also help to judge about this phase in an indirect way. The precise measurement of the inelasticity η_0^0 near 1 GeV in $\pi\pi \to \pi\pi$ would be also very important. We hope also to find answers for some troubles in consideration of heavy quarkonia decays, which are our next aim. The new precise experiment on $\gamma\gamma \to \pi\pi$ up to 1 GeV is urgent for an understanding of the mechanism of the $\sigma(600)$ production and hence for an understanding of its nature.

ACKNOWLEDGMENTS

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APPENDIX A: χ^2 FUNCTION FOR THE DATA ON THE $\phi \to \pi^0\pi^0\gamma$ DECAY

In the experiment the whole mass region $(2m_{\pi^0}, m_\phi)$ is divided into some number of bins. Experimenters measure the average value $\bar{B}_i$ (i is the number of bin) of $dBr(\phi \to \pi^0\pi^0\gamma)/dm$ around each ith bin:

$$\bar{B}_i = \frac{1}{m_{i+1} - m_i} \int_{m_i}^{m_{i+1}} dBr(\phi \to \pi^0\pi^0\gamma)/dm,$$

In this case one should define the χ^2 function as

$$\chi^2_{sp} = \sum_i \frac{(\bar{B}_i^{\rm th} - \bar{B}_i^{\rm exp})^2}{\sigma_i^2},$$

where $\bar{B}_i^{\rm exp}$ are the experimental results, σ_i are the experimental errors, and

$$\bar{B}_i^{\rm th} = \frac{1}{m_{i+1} - m_i} \int_{m_i}^{m_{i+1}} dBr^{\rm th}(\phi \to \pi^0\pi^0\gamma)/dm$$

[$dBr^{\rm th}(\phi \to \eta\pi^0\gamma)/dm$ is the theoretical curve].

APPENDIX B: ABOUT ADLER ZEROS

The well-known Adler condition [11] means that the amplitude $T^I(\pi\pi \to \pi\pi) \equiv A^I(s, t, u)$ with isospin I satisfies

$$A^I(m_\pi^2, m_\pi^2, m_\pi^2) = 0,$$

though it does not mean, in general, that partial amplitudes $T_l^I(\pi\pi \to \pi\pi) \equiv A_l^I(s, t, u)$ must have zeros. To illustrate this idea, let us consider an example. In the linear sigma model the amplitudes $A^I(s, t, u)$, calculated in the first order in tree-level approximation, satisfy the Adler condition for all values of m_σ, while the amplitude [62]

$$T_0^{0({\rm tree})} = \frac{m_\pi^2 - m_\sigma^2}{F_\pi^2}\left[5 - 3\frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - s} - 2\frac{m_\sigma^2 - m_\pi^2}{s - 4m_\pi^2} \times \ln\left(1 + \frac{s - 4m_\pi^2}{m_\sigma^2}\right)\right]$$

has no zeros for small m_σ. For example, for $m_\sigma =$ 400 MeV the amplitude $T_0^{0({\rm tree})}$ has no zeros up to $s = 4m_\pi^2 - m_\sigma^2 \approx -4m_\pi^2$, where this amplitude has a cut, see Fig. 8. Note that, for $m_\sigma = 500$ MeV, $T_0^{0({\rm tree})}$ has a zero near $s = m_\pi^2/4$.

Hence a criticism and even a denial of some works based on the absence of the Adler zeros is, in general, incorrect. However, establishing the Adler zero near the threshold is very convenient because it guarantees weakness of $\pi\pi$ interactions near the threshold, which is the main physical consequence of the Adler condition of self-consistency. We can see it in the example mentioned above: though the amplitude does not reach zero, its absolute value is small near the threshold. That is why we believe that both T_0^0 [Eq. (23)] and $M_{\rm sig}$ [Eq. (2)] have the Adler zeros not far from the $s = 4m_\pi^2$ threshold.

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