

Exclusive semileptonic and nonleptonic decays of the B_c meson

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We study exclusive nonleptonic and semileptonic decays of the B_c -meson within a relativistic constituent quark model previously developed by us. For the nonleptonic decays we use the factorizing approximation. We update our model parameters by using new experimental data for the mass and the lifetime of the B_c meson and the leptonic decay constants of the D -meson. We calculate the branching ratios for a large set of exclusive nonleptonic and semileptonic decays of the B_c meson and compare our results with the results of other studies. As a guide for further experimental exploration we provide explicit formulas for the full angular decay distributions in the cascade decays $B_c^- \rightarrow J/\psi(\rightarrow l^+ l^-) + \rho^- (\rightarrow \pi^- \pi^0)$ and $B_c^- \rightarrow J/\psi(\rightarrow l^+ l^-) + W_{\text{off-shell}}^- (\rightarrow l^- + \bar{\nu}_l)$.

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I. INTRODUCTION

In 1998 the CDF Collaboration reported on the observation of the bottom-charm B_c meson at Fermilab [1] in the semileptonic decay mode $B_c \rightarrow J/\psi + l + \nu$ with the J/ψ decaying into muon pairs. Values for the mass and the lifetime of the B_c meson were given as $M(B_c) = 6.40 \pm 0.39 \pm 0.13$ GeV and $\tau(B_c) = 0.46_{-0.16}^{+0.18}(\text{stat}) \pm 0.03(\text{syst})$ ps. Recently, CDF reported first Run II evidence for the B_c -meson in the fully reconstructed decay channel $B_c \rightarrow J/\psi + \pi$ with $J/\psi \rightarrow \mu^+ \mu^-$ [2]. The mass value quoted for this decay channel is $6.2857 \pm 0.0053(\text{stat}) \pm 0.0012(\text{syst})$ GeV with errors significantly smaller than in the first measurement. Also D0 has observed the B_c in the semileptonic mode $B_c \rightarrow J/\psi + \mu + X$ and reported preliminary evidence that $M(B_c) = 5.95_{-0.13}^{+0.14} \pm 0.34$ GeV and $\tau(B_c) = 0.45_{-0.10}^{+0.12} \pm 0.12$ ps [3].

The B_c -meson is the lowest bound state of two heavy quarks (charm and bottom) with open flavor. The B_c -meson therefore decays weakly. It can decay via (i) b -quark decay, (ii) c -quark decay, and (iii) the annihilation channel. The modern state of art, starting from the pioneering paper [4], in the spectroscopy, production and decays of the B_c -meson can be found in the review [5] and the published talk [6].

In this article we complete the analysis of almost all accessible low-lying exclusive nonleptonic two-body and semileptonic three-body modes of the B_c -decays within our relativistic constituent quark model [7–10]. We update the model parameters by using the latest experimental data on the B_c -mass [2] and the weak decay constant f_D [11]. We give a set of numerical values for the leptonic, semileptonic and nonleptonic partial decay widths of the

B_c -meson and compare them with the results of other approaches. We provide explicit formulas for the angular decay distributions of the cascade decays $B_c^- \rightarrow J/\psi(\rightarrow l^+ l^-) + \rho^- (\rightarrow \pi^- \pi^0)$ and $B_c^- \rightarrow J/\psi(\rightarrow l^+ l^-) + W_{\text{off-shell}}^- (\rightarrow l^- + \bar{\nu}_l)$ by using the methods described in [12] and subsequently applied to various other cascade decay processes (see [8,13–16]). For the nonleptonic decay $B_c^- \rightarrow J/\psi + \rho^-$ we also include lepton mass and T -odd effects in our analysis. These angular decay distributions may be of help in analyzing the cascade decay data. Also, by analyzing the cascade angular decay distributions, one can learn more details about the spin dynamics of the decay process than from the rate analysis alone.

II. MODEL

The coupling of a meson $H(q_1 \bar{q}_2)$ to its constituent quarks q_1 and \bar{q}_2 is described by the Lagrangian [17,18]

$$\mathcal{L}_{\text{Hqq}}^{\text{int}}(x) = g_H H(x) \int dx_1 \int dx_2 F_H(x, x_1, x_2) \bar{q}_2(x_2) \times \Gamma_H q_1(x_1) + \text{h.c.} \quad (1)$$

Here, Γ_H is a Dirac matrix or a string of Dirac matrices which projects onto the spin quantum number of the meson field $H(x)$. The function F_H is related to the scalar part of the Bethe-Salpeter amplitude and characterizes the finite size of the meson. To satisfy translational invariance the function F_H has to fulfil the identity $F_H(x+a, x_1+a, x_2+a) = F_H(x, x_1, x_2)$ for any four-vector a . In the following we use a specific form for the scalar vertex function

$$F_H(x, x_1, x_2) = \delta(x - c_{12}^1 x_1 - c_{12}^2 x_2) \Phi_H((x_1 - x_2)^2), \quad (2)$$

where Φ_H is the correlation function of the two constituent quarks with masses m_{q_1} , m_{q_2} and the mass ratios $c_{ij}^i = m_{q_i}/(m_{q_i} + m_{q_j})$.

The coupling constant g_H in Eq. (1) is determined by the so-called *compositeness condition* originally proposed in [19], and extensively used in [7,8,10,17,18]. The compositeness condition requires that the renormalization constant of the elementary meson field $H(x)$ is set to zero

$$Z_H = 1 - \frac{3g_H^2}{4\pi^2} \tilde{\Pi}'_H(m_H^2) = 0, \quad (3)$$

where $\tilde{\Pi}'_H$ is the derivative of the meson mass operator. To clarify the physical meaning of the compositeness condition in Eq. (3), we first want to remind the reader that the renormalization constant $Z_H^{1/2}$ can also be interpreted as the matrix element between the physical and the corresponding bare state. The condition $Z_H = 0$ implies that the physical state does not contain the bare state and is appropriately described as a bound state. The interaction Lagrangian of Eq. (1) and the corresponding free parts of

the Lagrangian describe both the constituents (quarks) and the physical particles (hadrons) which are viewed as the bound states of the quarks. As a result of the interaction, the physical particle is dressed, i.e. its mass and wave function have to be renormalized. The condition $Z_H = 0$ also effectively excludes the constituent degrees of freedom from the space of physical states. It thereby guarantees that there is no double counting for the physical observable under consideration. The constituents exist only in virtual states. One of the corollaries of the compositeness condition is the absence of a direct interaction of the dressed charged particle with the electromagnetic field. Taking into account both the tree-level diagram and the diagrams with the self-energy insertions into the external legs (i.e. the tree-level diagram times $Z_H - 1$) yields a common factor Z_H which is equal to zero. We refer the interested reader to our previous papers [7,8,10,17,18] where these points are discussed in more detail.

In the case of the pseudoscalar and vector mesons the derivative of the meson mass operator appearing in Eq. (3) can be calculated in the following way:

$$\begin{aligned} \tilde{\Pi}'_P(p^2) &= \frac{p^\alpha}{2p^2} \frac{d}{dp^\alpha} \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_P^2(-k^2) \text{tr}[\gamma^5 \tilde{S}_1(k + c_{12}^1 p) \gamma^5 \tilde{S}_2(k - c_{12}^2 p)], \\ \tilde{\Pi}'_V(p^2) &= \frac{1}{3} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{p^\alpha}{2p^2} \frac{d}{dp^\alpha} \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_V^2(-k^2) \text{tr}[\gamma^\mu \tilde{S}_1(k + c_{12}^1 p) \gamma^\nu \tilde{S}_2(k - c_{12}^2 p)], \end{aligned} \quad (4)$$

where $\Phi_H((x_1 - x_2)^2)$ is a correlation function and $\tilde{S}_i(k)$ is the quark propagator. We have used free fermion propagators for the valence quarks given by

$$\tilde{S}_i(k) = \frac{1}{m_{q_i} - \not{k}} \quad (5)$$

with an effective constituent quark mass m_i . As discussed in [18] we have assumed that the meson mass m_H lies below the constituent quark threshold, i.e. we have

$$m_H < m_{q_1} + m_{q_2}. \quad (6)$$

Since the transitions in our approach are described by one-loop quark diagrams the condition (6) guarantees that there are no imaginary parts in our physical transition amplitudes. For the constituent quark masses that we use this is satisfied for the low-lying pseudoscalar mesons π , K , D , D_s , B , B_s , B_c , and η_c and also for the J/ψ but is no longer true for the light vector mesons (ρ , K^*), the heavy flavored vector mesons (D^* and B^*) and for the p -wave and excited charmonium states considered in this paper. We have therefore employed [7,8] identical masses for all heavy pseudoscalar and vector flavored mesons ($m_{B^*} =$

m_B , $m_{D^*} = m_D$) and for all p -wave and excited charmonium states [10] in our matrix element calculations but have used physical masses in the phase space calculation. This is quite a reliable approximation for the heavy mesons because the corresponding mass splittings are relatively small. For the light vector mesons (ρ , K^*) this approximation is not very good. However, in the present application the light vector mesons do not explicitly enter into the decay dynamics described by the transition matrix elements. They contribute only in the form of the leptonic decay constants $f_\rho = 210$ MeV and $f_{K^*} = 217$ MeV for which we use the experimental values. We emphasize that the quark mass function appearing in the Dyson-Schwinger-Equations (DSE) studies [20] is almost constant in the case of the b -quark. This is true to a lesser extent for the c -quark. However, in the case of light u , d and s quarks the momentum-dependent dressing is essential.

III. NONLEPTONIC DECAYS OF THE B_c -MESON

The effective Hamiltonian describing the B_c -nonleptonic decays is given by

$$\begin{aligned}
\mathcal{H}_{\text{eff}} = & -\frac{G_F}{\sqrt{2}} \{V_{cb}V_{ud}^\dagger [c_1(\bar{c}b)_{V-A}(\bar{d}u)_{V-A} + c_2(\bar{d}b)_{V-A}(\bar{c}u)_{V-A}] + V_{cb}V_{us}^\dagger [c_1(\bar{c}b)_{V-A}(\bar{s}u)_{V-A} + c_2(\bar{s}b)_{V-A}(\bar{c}u)_{V-A}] \\
& + V_{cb}V_{cd}^\dagger [c_1(\bar{c}b)_{V-A}(\bar{d}c)_{V-A} + c_2(\bar{d}b)_{V-A}(\bar{c}c)_{V-A}] + V_{cb}V_{cs}^\dagger [c_1(\bar{c}b)_{V-A}(\bar{s}c)_{V-A} + c_2(\bar{s}b)_{V-A}(\bar{c}c)_{V-A}] \\
& + V_{ub}V_{ud}^\dagger [c_1(\bar{u}b)_{V-A}(\bar{d}u)_{V-A} + c_2(\bar{d}b)_{V-A}(\bar{u}u)_{V-A}] + V_{ub}V_{us}^\dagger [c_1(\bar{u}b)_{V-A}(\bar{s}u)_{V-A} + c_2(\bar{s}b)_{V-A}(\bar{u}u)_{V-A}] \\
& + V_{ub}V_{cd}^\dagger [c_1(\bar{u}b)_{V-A}(\bar{d}c)_{V-A} + c_2(\bar{d}b)_{V-A}(\bar{u}c)_{V-A}] + V_{ub}V_{cs}^\dagger [c_1(\bar{u}b)_{V-A}(\bar{s}c)_{V-A} + c_2(\bar{s}b)_{V-A}(\bar{u}c)_{V-A}] \\
& + V_{cs}V_{ud}^\dagger [c_1(\bar{c}s)_{V-A}(\bar{d}u)_{V-A} + c_2(\bar{c}u)_{V-A}(\bar{d}s)_{V-A}] + V_{cs}V_{us}^\dagger [c_1(\bar{c}s)_{V-A}(\bar{s}u)_{V-A} + c_2(\bar{c}u)_{V-A}(\bar{s}s)_{V-A}] \\
& + V_{cd}V_{ud}^\dagger [c_1(\bar{c}d)_{V-A}(\bar{d}u)_{V-A} + c_2(\bar{c}u)_{V-A}(\bar{d}d)_{V-A}] + V_{cd}V_{us}^\dagger [c_1(\bar{c}d)_{V-A}(\bar{s}u)_{V-A} + c_2(\bar{c}u)_{V-A}(\bar{s}d)_{V-A}] \} + \text{h.c.},
\end{aligned} \tag{7}$$

where the subscript $V - A$ refers to the usual left-chiral current $O^\mu = \gamma^\mu(1 - \gamma^5)$. We calculate the nonleptonic B_c -decay widths by using *naive* factorization. First, we give the necessary definitions of the leptonic decay constants, invariant form factors and helicity amplitudes as they were introduced in our paper [10].

The leptonic decay constants are defined by

$$\begin{aligned}
M(H_{12} \rightarrow \bar{l}\nu) &= \frac{G_F}{\sqrt{2}} V_{q_1 q_2} \mathcal{M}_H^\mu(p) \bar{u}_l(k_l) O^\mu u_\nu(k_\nu), \\
\mathcal{M}_H^\mu(p) &= -3g_{12} \int \frac{d^4 k}{(2\pi)^4 i} \tilde{\Phi}_{12}(-k^2) \\
&\quad \times \text{tr}[\Gamma_H \tilde{S}_2(k - c_{12}^2 p) O^\mu \tilde{S}_1(k + c_{12}^1 p)], \\
\Gamma_P &= i\gamma^5, \quad \Gamma_V = \varepsilon_V \cdot \gamma, \\
\mathcal{M}_P^\mu(p) &= -if_P p^\mu, \quad \mathcal{M}_V^\mu(p) = f_V m_V \varepsilon_V^\mu. \tag{8}
\end{aligned}$$

The semileptonic decays of the B_c -meson may be induced by either a b -quark or a c -quark transition. For the sake of brevity, we use a notation where $q_1 \equiv b$ and $q_3 \equiv c$ whereas q_2 denotes either of c, u, d, s .

$$M(H_{13} \rightarrow H_{23} + \bar{l}\nu) = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} \mathcal{M}_{12}^\mu(p_1, p_2) \bar{u}_l(k_l) O^\mu u_\nu(k_\nu),$$

b -decay,

$$M(H_{13} \rightarrow H_{12} + \bar{l}\nu) = \frac{G_F}{\sqrt{2}} V_{q_2 q_3} \mathcal{M}_{23}^\mu(p_1, p_2) \bar{u}_l(k_l) O^\mu u_\nu(k_\nu),$$

c -decay,

$$\begin{aligned}
\mathcal{M}_{12}^\mu &= -3g_{13}g_{23} \int \frac{d^4 k}{(2\pi)^4 i} \tilde{\Phi}_{13}(-(k + c_{13}^3 p_1)^2) \\
&\quad \times \tilde{\Phi}_{23}(-(k + c_{23}^3 p_2)^2) \text{tr}[i\gamma^5 \tilde{S}_3(k) \\
&\quad \times \Gamma_{32} \tilde{S}_2(k + p_2) O^\mu \tilde{S}_1(k + p_1)], \tag{9}
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{23}^\mu &= -3g_{13}g_{12} \int \frac{d^4 k}{(2\pi)^4 i} \tilde{\Phi}_{13}(-(k - c_{13}^1 p_1)^2) \\
&\quad \times \tilde{\Phi}_{12}(-(k - c_{12}^1 p_2)^2) \text{tr}[i\gamma^5 \tilde{S}_3(k - p_1) \\
&\quad \times O^\mu \tilde{S}_2(k - p_2) \Gamma_{21} \tilde{S}_1(k)]. \tag{10}
\end{aligned}$$

We mention that we have checked in [7] that, in the heavy mass limit, our form factors satisfy the HQET relations written down in [21].

The invariant form factors for the semileptonic B_c -decay into the hadron with spin $S = 0, 1, 2$ are defined by

$$\mathcal{M}_{S=0}^\mu = P^\mu F_+(q^2) + q^\mu F_-(q^2), \tag{11}$$

$$\begin{aligned}
\mathcal{M}_{S=1}^\mu &= \frac{1}{m_1 + m_2} \epsilon_\nu^\dagger \{ -g^{\mu\nu} P q A_0(q^2) + P^\mu P^\nu A_+(q^2) \\
&\quad + q^\mu P^\nu A_-(q^2) + i\varepsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta V(q^2) \}, \tag{12}
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{S=2}^\mu &= \epsilon_\nu^\dagger \{ g^{\mu\alpha} P^\nu T_1(q^2) + P^\nu P^\alpha [P^\mu T_2(q^2) \\
&\quad + q^\mu T_3(q^2)] + i\varepsilon^{\mu\nu\delta\beta} P^\alpha P_\delta q_\beta T_4(q^2) \}, \tag{13}
\end{aligned}$$

$$P = p_1 + p_2, \quad q = p_1 - p_2.$$

The form factor expansions cover both the $J^P \rightarrow (J^P)'$ cases $0^- \rightarrow (0^-, 1^-, 2^-)$ and $0^- \rightarrow (0^+, 1^+, 2^+)$ needed in this paper.

One has to note that the form factors for the c -decay can be obtained from the form factors for the b -decay by simply exchanging the bottom and charm masses:

$$F_\pm^{c\text{-decay}}(m_{q_1}, m_{q_2}, m_{q_3}; q^2) = -F_\pm^{b\text{-decay}}(m_{q_3}, m_{q_2}, m_{q_1}; q^2), \tag{14}$$

$$A_i^{c\text{-decay}}(m_{q_1}, m_{q_2}, m_{q_3}; q^2) = -A_i^{b\text{-decay}}(m_{q_3}, m_{q_2}, m_{q_1}; q^2), \tag{15}$$

$$(i = 0, \pm),$$

$$V^{c\text{-decay}}(m_{q_1}, m_{q_2}, m_{q_3}; q^2) = V^{b\text{-decay}}(m_{q_3}, m_{q_2}, m_{q_1}; q^2), \tag{16}$$

where we omit the explicit dependence on the ingoing and outgoing masses and size parameters.

It is convenient to express all physical observables through the helicity form factors H_m . The helicity form factors H_m can be expressed in terms of the invariant form factors in the following way [7]:

(a) Spin $S = 0$:

$$H_t = \frac{1}{\sqrt{q^2}} \{(m_1^2 - m_2^2)F_+ + q^2 F_-\}, \quad H_\pm = 0,$$

$$H_0 = \frac{2m_1 |\mathbf{p}_2|}{\sqrt{q^2}} F_+. \quad (17)$$

 (b) Spin $S = 1$:

$$H_t = \frac{1}{m_1 + m_2} \frac{m_1 |\mathbf{p}_2|}{m_2 \sqrt{q^2}} \{(m_1^2 - m_2^2)(A_+ - A_0) + q^2 A_-\},$$

$$H_\pm = \frac{1}{m_1 + m_2} \{-(m_1^2 - m_2^2)A_0 \pm 2m_1 |\mathbf{p}_2| V\},$$

$$H_0 = \frac{1}{m_1 + m_2} \frac{1}{2m_2 \sqrt{q^2}} \{-(m_1^2 - m_2^2) \times (m_1^2 - m_2^2 - q^2)A_0 + 4m_1^2 |\mathbf{p}_2|^2 A_+\}. \quad (18)$$

 (c) Spin $S = 2$:

$$H_t = \sqrt{\frac{2}{3}} \frac{m_1^2 |\mathbf{p}_2|^2}{m_2^2 \sqrt{q^2}} \{T_1 + [|\mathbf{p}_2|^2 + E_2 q_0 + m_1 q_0] T_2 + q^2 T_3\},$$

$$H_\pm = \sqrt{\frac{1}{2}} \frac{m_1 |\mathbf{p}_2|}{m_2} \{T_1 \pm 2m_1 |\mathbf{p}_2| T_4\}, \quad (19)$$

$$H_0 = \sqrt{\frac{1}{6}} \frac{m_1 |\mathbf{p}_2|}{m_2^2 \sqrt{q^2}} \{(m_1^2 - m_2^2 - q^2) T_1 + 4m_1^2 |\mathbf{p}_2|^2 T_2\},$$

where $|\mathbf{p}_2| = \lambda^{1/2}(m_1^2, m_2^2, q^2)/(2m_1)$, $E_2 = (m_1^2 + m_2^2 - q^2)/(2m_1)$ and $q_0 = (m_1^2 - m_2^2 + q^2)/(2m_1)$ are the momentum and energies of the outgoing particles in the B_c rest frame. The widths of the semileptonic and nonleptonic decays of the B_c -meson can be conveniently expressed in terms of the helicity form factors¹. The relevant width formulas are given in the appendix.

IV. NUMERICAL RESULTS

In this paper we update the model parameters by using the new values for the B_c -mass reported by the CDF Collaboration [2] the new value of the leptonic decay constant f_D reported by the CLEO Collaboration [11] and lattice simulations [22–27]. The updated values of the quark masses and size parameters are given by

¹As regards the transverse helicity amplitudes H_\pm in Eqs. (18) and (19) we have corrected a sign error in our previous paper [10].

Eq. (20) and (21), respectively.

$$m_u = m_d \quad m_s \quad m_c \quad m_b \quad (20)$$

$$0.223 \quad 0.344 \quad 1.71 \quad 5.09 \text{ GeV}$$

$$\Lambda_\pi \quad \Lambda_K \quad \Lambda_D \quad \Lambda_{D^*} \quad \Lambda_{D_s} \quad \Lambda_B \quad \Lambda_{B^*} \quad \Lambda_{B_s} \quad \Lambda_{B_c} \quad \Lambda_{cc} \quad (21)$$

$$1.08 \quad 1.60 \quad 2.01 \quad 1.46 \quad 2.01 \quad 2.14 \quad 1.90 \quad 2.14 \quad 2.14 \quad 2.53 \text{ GeV}$$

The quality of the fit may be assessed from the entries in Table I.

The calculation of the semileptonic and nonleptonic decay widths is straightforward. For the CKM-matrix elements we use

$$|V_{ud}| \quad |V_{us}| \quad |V_{cd}| \quad |V_{cs}| \quad |V_{cb}| \quad |V_{ub}| \quad (22)$$

$$0.975 \quad 0.224 \quad 0.224 \quad 0.974 \quad 0.0413 \quad 0.0037$$

TABLE I. Leptonic decay constants f_H (MeV) used in the least-squares fit for our model parameters.

	This work	Other	Ref.
f_D	227	$222.6 \pm 16.7^{+2.8}_{-3.4}$	CLEO [11]
		$201 \pm 3 \pm 17$	MILC LAT [22]
		$235 \pm 8 \pm 14$	LAT [23]
		$210 \pm 10^{+17}_{-16}$	UKQCD LAT [26]
		$211 \pm 14^{+2}_{-12}$	LAT [27]
f_{D^*}	249	$245 \pm 20^{+3}_{-2}$	LAT [27]
f_{D_s}	255	266 ± 32	[28]
		$249 \pm 3 \pm 16$	MILC LAT [22]
		$266 \pm 10 \pm 18$	LAT [23]
		$290 \pm 20 \pm 29 \pm 29 \pm 6$	LAT [25]
		$236 \pm 8^{+17}_{-14}$	UKQCD LAT [26]
$\frac{f_{D_s}}{f_D}$	1.12	$1.24 \pm 0.01 \pm 0.07$	MILC LAT [22]
		$1.13 \pm 0.03 \pm 0.05$	LAT [23]
		$1.13 \pm 0.02^{+0.04}_{-0.02}$	UKQCD LAT [26]
		1.10 ± 0.02	LAT [27]
		1.10 ± 0.02	LAT [22]
$f_{D_s^*}$	266	$272 \pm 16^{+3}_{-20}$	LAT [27]
f_{η_c}	484	420 ± 52	[29]
	This work	Other	Ref.
f_B	187	$216 \pm 9 \pm 19 \pm 4 \pm 6$	HPQCD LAT [24]
		$177 \pm 17^{+22}_{-22}$	UKQCD LAT [26]
		$179 \pm 18^{+34}_{-9}$	LAT [27]
f_{B^*}	196	$196 \pm 24^{+39}_{-2}$	LAT [27]
f_{B_s}	218	259 ± 32	HPQCD LAT [24]
		$260 \pm 7 \pm 26 \pm 8 \pm 5$	LAT [25]
		$204 \pm 12^{+24}_{-23}$	UKQCD LAT [26]
		$204 \pm 16^{+36}_{-0}$	LAT [27]
$\frac{f_{B_s}}{f_B}$	1.16	$1.20 \pm 0.03 \pm 0.01$	HPQCD LAT [24]
		$1.15 \pm 0.02^{+0.04}_{-0.02}$	UKQCD LAT [26]
		$1.14 \pm 0.03^{+0.01}_{-0.01}$	LAT [27]
$f_{B_s^*}$	229	$229 \pm 20^{+41}_{-16}$	LAT [27]
f_{B_c}	399	395 ± 15	[30]

The results of our evaluation of the branching ratios of the exclusive semileptonic and nonleptonic B_c decays appear in Tables II, III, IV, V, VI, VII, and VIII.

In the presentation of our results we shall closely follow the format of the reviews of Kiselev [5,44]. Table II contains our predictions for the exclusive semileptonic B_c decays into ground state charmonium states, and into ground state charm and bottom meson states. Table III contains our predictions for the exclusive semileptonic B_c^- decays into p -wave charmonium states, and into the orbital excitation of the charmonium state $\psi(3836)$. In Table IV we list our predictions for exclusive nonleptonic decay widths of the B_c meson using the factorization hypothesis. In order to facilitate a comparison with other dynamical models we list our results for general values of the effective Wilson coefficients of the operator product expansion a_1 and a_2 .

We then specify the values of the effective Wilson coefficients. We take $a_1^c = 1.20$, $a_2^c = -0.317$, $a_1^b = 1.14$ and $a_2^b = -0.20$ as in [5,44]. In Table V we give our results

TABLE II. Branching ratios (in %) of exclusive semileptonic B_c decays into ground state charmonium states, and into ground state charm and bottom meson states. For the lifetime of the B_c we take $\tau(B_c) = 0.45$ ps.

Mode	This work	[31,32]	[33]	[34]	[35]	[36]	[37]	[38]
$B_c^- \rightarrow \eta_c e \nu$	0.81	0.75	0.97	0.59	0.15	0.40	0.76	0.51
$B_c^- \rightarrow \eta_c \tau \nu$	0.22	0.23	...	0.20
$B_c^- \rightarrow J/\psi e \nu$	2.07	1.9	2.35	1.20	1.47	1.21	2.01	1.44
$B_c^- \rightarrow J/\psi \tau \nu$	0.49	0.48	...	0.34
$B_c^- \rightarrow \bar{D}^0 e \nu$	0.0035	0.004	0.006	...	0.0003	0.001	0.003	0.0014
$B_c^- \rightarrow \bar{D}^0 \tau \nu$	0.0021	0.002
$B_c^- \rightarrow \bar{D}^{*0} e \nu$	0.0038	0.018	0.018	...	0.008	0.008	0.013	0.0023
$B_c^- \rightarrow \bar{D}^{*0} \tau \nu$	0.0022	0.008
$B_c^- \rightarrow \bar{B}_s^0 e \nu$	1.10	4.03	1.82	0.99	0.8	0.82	0.98	0.92
$B_c^- \rightarrow \bar{B}_s^{*0} e \nu$	2.37	5.06	3.01	2.30	2.3	1.71	3.45	1.41
$B_c^- \rightarrow \bar{B}^0 e \nu$	0.071	0.34	0.16	...	0.06	0.04	0.078	0.048
$B_c^- \rightarrow \bar{B}^{*0} e \nu$	0.063	0.58	0.23	...	0.19	0.12	0.24	0.051

TABLE III. The branching ratios (in %) of exclusive semileptonic B_c^- decays into p -wave charmonium states, and into the 3D_2 orbital excitation of the charmonium state $\psi(3836)$. For the lifetime of the B_c we take $\tau(B_c) = 0.45$ ps.

Mode	This work	[39]
$B_c^- \rightarrow \chi_{c0} e \nu$	0.17	0.12
$B_c^- \rightarrow \chi_{c0} \tau \nu$	0.013	0.017
$B_c^- \rightarrow \chi_{c1} e \nu$	0.092	0.15
$B_c^- \rightarrow \chi_{c1} \tau \nu$	0.0089	0.024
$B_c^- \rightarrow h_c e \nu$	0.27	0.17
$B_c^- \rightarrow h_c \tau \nu$	0.017	0.024
$B_c^- \rightarrow \chi_{c2} e \nu$	0.17	0.19
$B_c^- \rightarrow \chi_{c2} \tau \nu$	0.0082	0.029
$B_c^- \rightarrow \psi(3836) e \nu$	0.0066	...
$B_c^- \rightarrow \psi(3836) \tau \nu$	0.000 099	...

TABLE IV. Exclusive nonleptonic decay widths of the B_c meson in units of 10^{-15} GeV for general values of the Wilson coefficients a_1 and a_2 .

$B_c^- \rightarrow \eta_c \pi^-$	$2.11a_1^2$
$B_c^- \rightarrow \eta_c \rho^-$	$5.10a_1^2$
$B_c^- \rightarrow \eta_c K^-$	$0.166a_1^2$
$B_c^- \rightarrow \eta_c K^{*-}$	$0.276a_1^2$
$B_c^- \rightarrow J/\psi \pi^-$	$1.93a_1^2$
$B_c^- \rightarrow J/\psi \rho^-$	$5.49a_1^2$
$B_c^- \rightarrow J/\psi K^-$	$0.150a_1^2$
$B_c^- \rightarrow J/\psi K^{*-}$	$0.313a_1^2$
$B_c^- \rightarrow \chi_{c0} \pi^-$	$0.622a_1^2$
$B_c^- \rightarrow \chi_{c0} \rho^-$	$1.47a_1^2$
$B_c^- \rightarrow \chi_{c0} K^-$	$0.0472a_1^2$
$B_c^- \rightarrow \chi_{c0} K^{*-}$	$0.0787a_1^2$
$B_c^- \rightarrow \chi_{c1} \pi^-$	$0.0768a_1^2$
$B_c^- \rightarrow \chi_{c1} \rho^-$	$0.326a_1^2$
$B_c^- \rightarrow \chi_{c1} K^-$	$0.00574a_1^2$
$B_c^- \rightarrow \chi_{c1} K^{*-}$	$0.0201a_1^2$
$B_c^- \rightarrow h_c \pi^-$	$1.24a_1^2$
$B_c^- \rightarrow h_c \rho^-$	$2.78a_1^2$
$B_c^- \rightarrow h_c K^-$	$0.0939a_1^2$
$B_c^- \rightarrow h_c K^{*-}$	$0.146a_1^2$
$B_c^- \rightarrow \chi_{c2} \pi^-$	$0.518a_1^2$
$B_c^- \rightarrow \chi_{c2} \rho^-$	$1.33a_1^2$
$B_c^- \rightarrow \chi_{c2} K^-$	$0.0384a_1^2$
$B_c^- \rightarrow \chi_{c2} K^{*-}$	$0.0732a_1^2$
$B_c^- \rightarrow \psi(3836) \pi^-$	$0.0193a_1^2$
$B_c^- \rightarrow \psi(3836) \rho^-$	$0.0621a_1^2$
$B_c^- \rightarrow \psi(3836) K^-$	$0.00137a_1^2$
$B_c^- \rightarrow \psi(3836) K^{*-}$	$0.00355a_1^2$
$B_c^- \rightarrow \eta_c D_s^-$	$(2.73a_1 + 2.82a_2)^2$
$B_c^- \rightarrow \eta_c D_s^{*-}$	$(2.29a_1 + 1.51a_2)^2$
$B_c^- \rightarrow J/\psi D_s^-$	$(2.19a_1 + 1.32a_2)^2$
$B_c^- \rightarrow J/\psi D_s^{*-}$	$(3.69a_1 + 2.35a_2)^2$
$B_c^- \rightarrow \eta_c D^-$	$(0.562a_1 + 0.582a_2)^2$
$B_c^- \rightarrow \eta_c D^{*-}$	$(0.511a_1 + 0.310a_2)^2$
$B_c^- \rightarrow J/\psi D^-$	$(0.462a_1 + 0.277a_2)^2$
$B_c^- \rightarrow J/\psi D^{*-}$	$(0.785a_1 + 0.460a_2)^2$
$B_c^- \rightarrow \bar{B}_s^0 \pi^-$	$39.7a_1^2$
$B_c^- \rightarrow \bar{B}_s^0 \rho^-$	$23.6a_1^2$
$B_c^- \rightarrow \bar{B}_s^{*0} \pi^-$	$21.8a_1^2$
$B_c^- \rightarrow \bar{B}_s^{*0} \rho^-$	$115a_1^2$
$B_c^- \rightarrow \bar{B}_s^0 K^-$	$2.93a_1^2$
$B_c^- \rightarrow \bar{B}_s^{*0} K^-$	$1.34a_1^2$
$B_c^- \rightarrow \bar{B}_s^0 K^{*-}$	$0.115a_1^2$
$B_c^- \rightarrow \bar{B}_s^{*0} K^{*-}$	$5.12a_1^2$
$B_c^- \rightarrow \bar{B}^0 \pi^-$	$2.04a_1^2$
$B_c^- \rightarrow \bar{B}^0 \rho^-$	$2.05a_1^2$
$B_c^- \rightarrow \bar{B}^{*0} \pi^-$	$0.578a_1^2$
$B_c^- \rightarrow \bar{B}^{*0} \rho^-$	$3.07a_1^2$
$B_c^- \rightarrow \bar{B}^0 K^-$	$0.153a_1^2$
$B_c^- \rightarrow \bar{B}^0 K^{*-}$	$0.0490a_1^2$
$B_c^- \rightarrow \bar{B}^{*0} K^-$	$0.0361a_1^2$
$B_c^- \rightarrow \bar{B}^{*0} K^{*-}$	$0.133a_1^2$
$B_c^- \rightarrow B^- K^0$	$54.7a_2^2$
$B_c^- \rightarrow B^- K^{*0}$	$16.7a_2^2$
$B_c^- \rightarrow B^{*-} K^0$	$12.8a_2^2$
$B_c^- \rightarrow B^{*-} K^{*0}$	$46.3a_2^2$
$B_c^- \rightarrow B^- \pi^0$	$1.02a_2^2$
$B_c^- \rightarrow B^- \rho^0$	$1.03a_2^2$
$B_c^- \rightarrow B^{*-} \pi^0$	$0.289a_2^2$
$B_c^- \rightarrow B^{*-} \rho^0$	$1.54a_2^2$

for the nonleptonic decays of the B_c meson into two ground state mesons and compare our results with the results of other model calculations.

For the $b \rightarrow c$ induced decays our results are generally close to the QCD sum rule results of [31,32] and the constituent quark model results of [33–37]. In exception are the ($b \rightarrow c$; $c \rightarrow (s, d)$) results of [33] which are considerably smaller than our results, and smaller than the results of the other model calculations. Summing up the exclusive contributions one obtains a branching fraction of

8.8%. Considering the fact that the $b \rightarrow c$ contribution to the total rate is expected to be about 20% [5] this leaves plenty of room for nonresonant multibody decays

For the $c \rightarrow s$ induced decays our branching ratios are considerably smaller than those predicted by QCD sum rules [31,32] but are generally close to the other constituent quark model results. When we sum up our exclusive branching fractions we obtain a total branching ratio of 27.6% which has to be compared with the 70% expected for the $c \rightarrow s$ contribution to the total rate [5]. The sum rule

TABLE V. Branching ratios (in %) of exclusive nonleptonic B_c decays with the choice of Wilson coefficient: $a_1^c = 1.20$ and $a_2^c = -0.317$ for c -decay, and $a_1^b = 1.14$ and $a_2^b = -0.20$ for b -decay. For the lifetime of the B_c we take $\tau(B_c) = 0.45$ ps.

Mode	This work	[31,32]	[33]	[34]	[35]	[36]	[37]
$B_c^- \rightarrow \eta_c \pi^-$	0.19	0.20	0.18	0.13	0.025	0.083	0.14
$B_c^- \rightarrow \eta_c \rho^-$	0.45	0.42	0.49	0.30	0.067	0.20	0.33
$B_c^- \rightarrow \eta_c K^-$	0.015	0.013	0.014	0.013	0.002	0.006	0.011
$B_c^- \rightarrow \eta_c K^{*-}$	0.025	0.020	0.025	0.021	0.004	0.011	0.018
$B_c^- \rightarrow J/\psi \pi^-$	0.17	0.13	0.18	0.073	0.13	0.060	0.11
$B_c^- \rightarrow J/\psi \rho^-$	0.49	0.40	0.53	0.21	0.37	0.16	0.31
$B_c^- \rightarrow J/\psi K^-$	0.013	0.011	0.014	0.007	0.007	0.005	0.008
$B_c^- \rightarrow J/\psi K^{*-}$	0.028	0.022	0.029	0.016	0.020	0.010	0.018
$B_c^- \rightarrow \eta_c D_s^-$	0.44	0.28	0.054	0.35	0.50	...	0.26
$B_c^- \rightarrow \eta_c D_s^{*-}$	0.37	0.27	0.044	0.36	0.057	...	0.24
$B_c^- \rightarrow J/\psi D_s^-$	0.34	0.17	0.041	0.12	0.35	...	0.15
$B_c^- \rightarrow J/\psi D_s^{*-}$	0.97	0.67	...	0.62	0.75	...	0.55
$B_c^- \rightarrow \eta_c D^-$	0.019	0.015	0.0012	0.010	0.005	...	0.014
$B_c^- \rightarrow \eta_c D^{*-}$	0.019	0.010	0.0010	0.0055	0.003	...	0.013
$B_c^- \rightarrow J/\psi D^-$	0.015	0.009	0.0009	0.0044	0.013	...	0.009
$B_c^- \rightarrow J/\psi D^{*-}$	0.045	0.028	...	0.010	0.023	...	0.028
$B_c^- \rightarrow \bar{B}_s^0 \pi^-$	3.9	16.4	5.75	3.42	3.01	2.46	1.56
$B_c^- \rightarrow \bar{B}_s^0 \rho^-$	2.3	7.2	4.41	2.33	1.34	1.38	3.86
$B_c^- \rightarrow \bar{B}_s^{*0} \pi^-$	2.1	6.5	5.08	1.95	3.50	1.58	1.23
$B_c^- \rightarrow \bar{B}_s^{*0} \rho^-$	11	20.2	14.8	12.1	10.8	10.8	16.8
$B_c^- \rightarrow \bar{B}_s^0 K^-$	0.29	1.06	0.41	...	0.21	0.21	0.17
$B_c^- \rightarrow \bar{B}_s^{*0} K^-$	0.13	0.37	0.29	...	0.16	0.11	0.13
$B_c^- \rightarrow \bar{B}_s^0 K^{*-}$	0.011	0.0043	0.0030	0.10
$B_c^- \rightarrow \bar{B}_s^{*0} K^{*-}$	0.50	1.14
$B_c^- \rightarrow \bar{B}^0 \pi^-$	0.20	1.06	0.32	0.15	0.19	0.10	0.10
$B_c^- \rightarrow \bar{B}^0 \rho^-$	0.20	0.96	0.59	0.19	0.15	0.13	0.28
$B_c^- \rightarrow \bar{B}^{*0} \pi^-$	0.057	0.95	0.29	0.077	0.24	0.026	0.076
$B_c^- \rightarrow \bar{B}^{*0} \rho^-$	0.30	2.57	1.17	0.67	0.85	0.67	0.89
$B_c^- \rightarrow \bar{B}^0 K^-$	0.015	0.07	0.025	...	0.014	0.009	0.010
$B_c^- \rightarrow \bar{B}^0 K^{*-}$	0.0048	0.015	0.018	...	0.003	0.004	0.012
$B_c^- \rightarrow \bar{B}^{*0} K^-$	0.0036	0.055	0.019	...	0.012	0.004	0.006
$B_c^- \rightarrow \bar{B}^{*0} K^{*-}$	0.013	0.058	0.037	...	0.033	0.032	0.065
$B_c^- \rightarrow B^- K^0$	0.38	1.98	0.66	0.17	...	0.23	0.27
$B_c^- \rightarrow B^- K^{*0}$	0.11	0.43	0.47	0.095	...	0.09	0.32
$B_c^- \rightarrow B^{*-} K^0$	0.088	1.60	0.50	0.061	...	0.10	0.16
$B_c^- \rightarrow B^{*-} K^{*0}$	0.32	1.67	0.97	0.57	...	0.82	1.70
$B_c^- \rightarrow B^- \pi^0$	0.0070	0.037	0.011	0.007	...	0.003	0.004
$B_c^- \rightarrow B^- \rho^0$	0.0071	0.034	0.020	0.009	...	0.005	0.010
$B_c^- \rightarrow B^{*-} \pi^0$	0.0020	0.033	0.010	0.004	...	0.001	0.003
$B_c^- \rightarrow B^{*-} \rho^0$	0.011	0.09	0.041	0.031	...	0.023	0.031

TABLE VI. Exclusive nonleptonic decay widths of the B_c meson into DD -mesons in units of 10^{-15} GeV.

$B_c^- \rightarrow D^- D^0$	$1.19a_2^2$	$B_c^- \rightarrow D^- \bar{D}^0$	$(0.0225a_1 + 0.0225a_2)^2$
$B_c^- \rightarrow D^- D^{*0}$	$1.38a_2^2$	$B_c^- \rightarrow D^- \bar{D}^{*0}$	$(0.0118a_1 + 0.0234a_2)^2$
$B_c^- \rightarrow D^{*-} D^0$	$0.323a_2^2$	$B_c^- \rightarrow D^{*-} \bar{D}^0$	$(0.0243a_1 + 0.0117a_2)^2$
$B_c^- \rightarrow D^{*-} D^{*0}$	$0.770a_2^2$	$B_c^- \rightarrow D^{*-} \bar{D}^{*0}$	$(0.0181a_1 + 0.0181a_2)^2$
$B_c^- \rightarrow D_s^- D^0$	$0.0779a_2^2$	$B_c^- \rightarrow D_s^- \bar{D}^0$	$(0.111a_1 + 0.109a_2)^2$
$B_c^- \rightarrow D_s^- D^{*0}$	$0.0881a_2^2$	$B_c^- \rightarrow D_s^- \bar{D}^{*0}$	$(0.0580a_1 + 0.114a_2)^2$
$B_c^- \rightarrow D_s^{*-} D^0$	$0.0236a_2^2$	$B_c^- \rightarrow D_s^{*-} \bar{D}^0$	$(0.113a_1 + 0.0598a_2)^2$
$B_c^- \rightarrow D_s^{*-} D^{*0}$	$0.0574a_2^2$	$B_c^- \rightarrow D_s^{*-} \bar{D}^{*0}$	$(0.0871a_1 + 0.0932a_2)^2$

model of [31,32] gives a summed branching fraction of 73.4% for the $c \rightarrow s$ contribution, i.e. the model of [31,32] predicts that the exclusive channels pretty well saturate the $c \rightarrow s$ part of the total rate.

Of interest are the ratios of branching ratios of the pairs of modes $B_c \rightarrow VV$ and $B_c \rightarrow VP$, and $B_c \rightarrow VP$ and $B_c \rightarrow PP$ where one expects from naive spin counting that the rate ratios VP/PP and VV/VP are ≈ 3 . In many of the pairs of decay modes naive spin counting can be seen to hold. However, for some of the pairs one finds approximate equality or even an inversion of the naive spin counting ratio. The deviation from naive spin counting can be seen to be a common feature of all model results.

As was pointed out in [45] and further elaborated in [9,40,41] the decays $B_c^- \rightarrow D_s^- D^0 (\bar{D}^0)$ are well suited for an extraction of the CKM angle γ through amplitude relations. These decays are better suited for the extraction of γ than the corresponding decays of the B_u and B_d

TABLE VII. Branching ratios in units of 10^{-6} of the exclusive nonleptonic B_c decays into DD -mesons. For the Wilson coefficients we choose $a_1^b = 1.14$ and $a_2^b = -0.20$ relevant for the nonleptonic decays of the \bar{b} quark. For the lifetime of the B_c we take $\tau(B_c) = 0.45$ ps.

Mode	This work	[40]	[33]	[41]	[35]	[37]
$B_c^- \rightarrow D^- D^0$	33	53	18	86	4.1	17
$B_c^- \rightarrow D^- D^{*0}$	38	75	19	75	3.6	21
$B_c^- \rightarrow D^{*-} D^0$	8.8	49	18	30	40	7.9
$B_c^- \rightarrow D^{*-} D^{*0}$	21	330	30	55	66	23
$B_c^- \rightarrow D_s^- D^0$	2.1	4.8	0.93	4.6	0.27	1.13
$B_c^- \rightarrow D_s^- D^{*0}$	2.4	7.1	0.97	3.9	0.25	1.35
$B_c^- \rightarrow D_s^{*-} D^0$	0.65	4.5	0.91	1.8	2.38	0.55
$B_c^- \rightarrow D_s^{*-} D^{*0}$	1.6	26	1.54	3.5	4.1	1.63

Mode	This work	[40]
$B_c^- \rightarrow D^- \bar{D}^0$	0.31	0.32
$B_c^- \rightarrow D^- \bar{D}^{*0}$	0.052	0.28
$B_c^- \rightarrow D^{*-} \bar{D}^0$	0.44	0.40
$B_c^- \rightarrow D^{*-} \bar{D}^{*0}$	0.20	1.59
$B_c^- \rightarrow D_s^- \bar{D}^0$	7.4	6.6
$B_c^- \rightarrow D_s^- \bar{D}^{*0}$	1.3	6.3
$B_c^- \rightarrow D_s^{*-} \bar{D}^0$	9.3	8.5
$B_c^- \rightarrow D_s^{*-} \bar{D}^{*0}$	4.5	40.4

mesons because the unitarity triangles in the latter decays are rather squashed. We list our updated results for these decays in Tables VI and VII, where Table VI contains our results for general values of the Wilson coefficients while Table VII list values of the branching ratios for specified values of the Wilson coefficients. The branching ratios in Table VII are quite small $\mathcal{O}(10^{-5})$ but these modes should still be accessible at high luminosity hadron colliders.

Finally, in Table VIII, we compute the branching ratios of the exclusive nonleptonic B_c^- decays into p -wave charmonium states, and into the orbital excitation of the charmonium state $\psi(3836)$. We compare our results with the results of other studies when they are available.

TABLE VIII. Branching ratios [in (%)] of the exclusive nonleptonic B_c^- decays into p -wave charmonium states, and into the 3D_2 orbital excitation of the charmonium state $\psi(3836)$. The choice of Wilson coefficient is: $a_1^c = 1.20$ and $a_2^c = -0.317$ for c -decays, and $a_1^b = 1.14$ and $a_2^b = -0.20$ for b -decays. For the lifetime of the B_c we take $\tau(B_c) = 0.45$ ps.

Mode	This work	[39]	[42]	[43]
$B_c^- \rightarrow \chi_{c0} \pi^-$	0.055	0.028	0.98	...
$B_c^- \rightarrow \chi_{c1} \pi^-$	0.0068	0.007	0.0089	...
$B_c^- \rightarrow h_c \pi^-$	0.11	0.05	1.60	...
$B_c^- \rightarrow \chi_{c2} \pi^-$	0.046	0.025	0.79	0.0076
$B_c^- \rightarrow \psi(3836) \pi^-$	0.0017	...	0.030	...
$B_c^- \rightarrow \chi_{c0} K^-$	0.0042	0.00021
$B_c^- \rightarrow \chi_{c1} K^-$	0.00051	0.000052
$B_c^- \rightarrow h_c K^-$	0.0083	0.00038
$B_c^- \rightarrow \chi_{c2} K^-$	0.0034	0.00018	...	0.00056
$B_c^- \rightarrow \psi(3836) K^-$	0.00012

Mode	This work	[39]	[42]	[43]
$B_c^- \rightarrow \chi_{c0} \rho^-$	0.13	0.072	3.29	...
$B_c^- \rightarrow \chi_{c1} \rho^-$	0.029	0.029	0.46	...
$B_c^- \rightarrow h_c \rho^-$	0.25	0.12	5.33	...
$B_c^- \rightarrow \chi_{c2} \rho^-$	0.12	0.051	3.20	0.023
$B_c^- \rightarrow \psi(3836) \rho^-$	0.0055	...	0.98	...
$B_c^- \rightarrow \chi_{c0} K^{*-}$	0.0070	0.00039
$B_c^- \rightarrow \chi_{c1} K^{*-}$	0.0018	0.00018
$B_c^- \rightarrow h_c K^{*-}$	0.013	0.00068
$B_c^- \rightarrow \chi_{c2} K^{*-}$	0.0065	0.00031	...	0.0013
$B_c^- \rightarrow \psi(3836) K^{*-}$	0.00032

V. ANGULAR DECAY DISTRIBUTIONS FOR THE DECAYS OF THE B_c MESON INTO J/ψ MODES

The exclusive decays of the B_c -meson involving a J/ψ meson have an excellent experimental signature since the J/ψ can be readily reconstructed from its leptonic decay channels $J/\psi \rightarrow \mu^+ \mu^-$, $e^+ e^-$. In fact, decays of the B_c into J/ψ modes have been among the discovery channels of the B_c . In this section we write down the complete angular decay distributions for the nonleptonic decays $B_c^- \rightarrow J/\psi(\rightarrow l^+ l^-) + \rho^- (\rightarrow \pi^- \pi^0)$, $B_c^- \rightarrow J/\psi(\rightarrow l^+ l^-) + \pi^-$ and $B_c^- \rightarrow \eta_c + \rho^- (\rightarrow \pi^- \pi^0)$, and the semi-leptonic decay $B_c^- \rightarrow J/\psi + l^- + \bar{\nu}_l$.

The experimental analysis of the angular decay distributions allows one to learn more about the decay dynamics of the B_c decays. The decay dynamics is encapsulated in the helicity structure functions that multiply the angular factors in the decay distribution. Vice versa, the explicit form of the decay distributions may be a useful input for writing event generators for the decay process where one now has to make use of some theoretical input to determine explicit values for the helicity structure functions.

We first discuss the nonleptonic cascade decay $B_c^- \rightarrow J/\psi(\rightarrow l^+ l^-) + \rho^- (\rightarrow \pi^- \pi^0)$. The branching ratio of this mode is predicted to be approximately 3 times the branching ratio of the decay $B_c^- \rightarrow J/\psi(\rightarrow l^+ l^-) + \pi^-$ which has already been seen [2]. It is not difficult to anticipate that the decay mode $B_c^- \rightarrow J/\psi + \rho^-$ will be one of the next exclusive decay modes to be seen in the very near future. The decay mode $B_c^- \rightarrow J/\psi + \rho^-$ will afford an excellent opportunity to take a more detailed look at the spin dynamics of the primary weak decay process through an analysis of the joint angular decay distributions of the second stage decays $J/\psi \rightarrow l^+ l^-$ and $\rho^- \rightarrow \pi^- \pi^0$.

The angular decay distribution of the cascade decay $B_c^- \rightarrow J/\psi(\rightarrow l^+ l^-) + \rho^- (\rightarrow \pi^- \pi^0)$ has been discussed before in [15,46–48] including also lepton mass effects [15]. We rederive the angular decay distribution using the methods described in [8,12]. The angular decay distribution can be cast into the form

$$W(\theta, \chi, \theta^*) \propto \sum_{\substack{\lambda=m, \lambda'=m' \\ \lambda_l, \lambda_{l'}}} |h_{\lambda_l \lambda_{l'}}|^2 e^{i(m-m')(\pi-\chi)} \\ \times d_{m, \lambda_l - \lambda_{l'}}^1(\theta) d_{m', \lambda_l - \lambda_{l'}}^1(\theta) H_{\lambda m} H_{\lambda' m'}^\dagger \\ \times d_{\lambda 0}^1(\theta^*) d_{\lambda' 0}^1(\theta^*), \quad (23)$$

where the $d_{mm'}^1$ are Wigner's d -functions in the convention of Rose. The summation in (23) runs over $\lambda = m = 0, \pm 1$, $\lambda' = m' = 0, \pm 1$ and $\lambda_l, \lambda_{l'} = \pm 1/2$. The helicity amplitudes $h_{\lambda_l \lambda_{l'}}$ describe the decay $J/\psi(\lambda_l - \lambda_{l'}) \rightarrow l^+(\lambda_l) + l^-(\lambda_{l'})$ with lepton helicities λ_l and $\lambda_{l'}$ where l and l' denote the positively and negatively charged leptons, respectively. Similarly the helicity amplitudes $H_{\lambda m}$ describe the decay $B_c^- \rightarrow J/\psi(m) + \rho^-(\lambda)$ where the helicities of the J/ψ and the ρ^- are denoted by m and λ , respectively.

Since $\lambda = m$ from angular momentum conservation we shall drop one of the helicity labels in the helicity amplitudes, i.e. we write H_m for $H_{\lambda m}$ ($m = 0, \pm 1$). The angles θ , χ and θ^* are defined in Fig. 1.

We begin with by neglecting helicity flip effects in the decay $J/\psi \rightarrow l^+ l^-$, i.e. we take $\lambda_l = -\lambda_{l'}$ in Eq. (23). Further we assume that the helicity amplitudes are relatively real neglecting possible T -odd effects. One then obtains

$$\frac{d\Gamma}{d \cos \theta d \chi d \cos \theta^*} = \frac{1}{2\pi} \left[\frac{3}{8} (1 + \cos^2 \theta) \frac{3}{4} \sin^2 \theta^* \Gamma_U \right. \\ \left. + \frac{3}{4} \sin^2 \theta \frac{3}{2} \cos^2 \theta^* \Gamma_L \right. \\ \left. + \frac{9}{32} \sin 2\theta \cos \chi \sin 2\theta^* \Gamma_I \right. \\ \left. - \frac{1}{2} \cdot \frac{3}{4} \sin^2 \theta \cos 2\chi \frac{3}{4} \sin^2 \theta^* \Gamma_T \right]. \quad (24)$$

Integrating (24) over χ and θ^* one obtains

$$\frac{d\Gamma}{d \cos \theta} = \frac{3}{8} (1 + \cos^2 \theta) \Gamma_U + \frac{3}{4} \sin^2 \theta \Gamma_L \\ := \frac{3}{8} (\Gamma_U + 2\Gamma_L) (1 + \alpha_{L/T} \cos^2 \theta), \quad (25)$$

where the asymmetry parameter

$$\alpha_{L/T} = \frac{\Gamma_U - 2\Gamma_L}{\Gamma_U + 2\Gamma_L} \quad (26)$$

is a measure of the transverse/longitudinal composition of the produced J/ψ .

Upon full angular integration one has $\Gamma = \Gamma_U + \Gamma_L$. Note that we have taken the freedom to omit the branching ratio factors $\text{Br}(J/\psi \rightarrow l^+ l^-)$ and $\text{Br}(\rho^- \rightarrow \pi^- \pi^0)$ on the right hand side of Eq. (24). The reason is that, upon angular integration, we want to obtain the total rate $\Gamma(B_c^- \rightarrow J/\psi + \rho^-)$. The partial rates Γ_i ($i = U, P, L, T, I$) in Eq. (24) are related to bilinear products of the helicity amplitudes via

$$\Gamma_i = \frac{G_F^2}{16\pi} |V_{cb} V_{ud} a_1 f_\rho m_\rho|^2 \frac{|\mathbf{p}_2|}{m_{B_c}^2} \mathcal{H}_i, \quad (27)$$

where $|\mathbf{p}_2|$ is the magnitude of the three-momentum of the

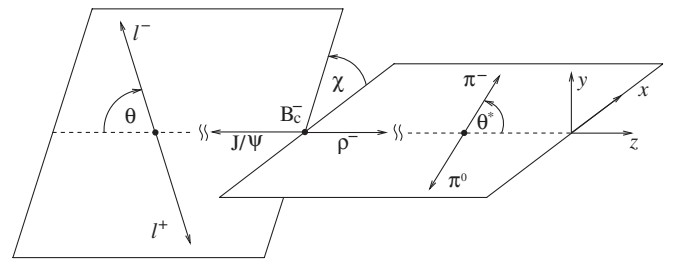


FIG. 1. Definition of the polar angles θ and θ^* and the azimuthal angle χ in the cascade decay $B_c^- \rightarrow J/\psi(\rightarrow l^+ l^-) + \rho^- (\rightarrow \pi^- \pi^0)$.

ρ^- (or the J/ψ) in the rest frame of the B_c -meson. The helicity structure functions \mathcal{H}_i ($i = U, P, L, T, I$) are given by

$$\begin{aligned}\mathcal{H}_U &= |H_+|^2 + |H_-|^2, & \mathcal{H}_P &= |H_+|^2 - |H_-|^2, \\ \mathcal{H}_L &= |H_0|^2, & \mathcal{H}_T &= \text{Re}H_+H_+^\dagger, \\ \mathcal{H}_I &= \frac{1}{2} \text{Re}(H_+H_0^\dagger + H_-H_0^\dagger).\end{aligned}\quad (28)$$

Using our constituent quark model results the partial rates Γ_i take the following values

$$\begin{aligned}\Gamma_U &= +0.826 \cdot 10^{-15} \text{ GeV}, \\ \Gamma_P &= -0.644 \cdot 10^{-15} \text{ GeV}, \\ \Gamma_L &= +6.30 \cdot 10^{-15} \text{ GeV}, \\ \Gamma_T &= +0.259 \cdot 10^{-15} \text{ GeV}, \\ \Gamma_I &= +1.46 \cdot 10^{-15} \text{ GeV}.\end{aligned}\quad (29)$$

Using the inverse lifetime $\tau(B_c)^{-1} = 1.463 \cdot 10^{-12} \text{ GeV}$ and the sum $\Gamma_U + \Gamma_L$ in Eq. (29) one numerically reproduces the branching ratio listed in Table V.

Even though \mathcal{H}_P cannot be measured in the cascade decay $B_c^- \rightarrow J/\psi(\rightarrow l^+l^-) + \rho^- (\rightarrow \pi^- \pi^0)$ we have included the parity-odd helicity structure function $\mathcal{H}_P = |H_+|^2 - |H_-|^2$ in the results for illustrative reasons in order to exemplify the hierarchy of helicity rates. The reason that \mathcal{H}_P cannot be measured is that the analyzing decays $J/\psi \rightarrow l^+l^-$ and $\rho^- \rightarrow \pi^- \pi^0$ are both parity-conserving. From the numbers in Eq. (29) one finds the hierarchy $\Gamma_L/\Gamma:\Gamma_-/\Gamma:\Gamma_+/\Gamma = 88\%:10.3\%:1.3\%$ where $\Gamma_\pm = (\Gamma_U \pm \Gamma_P)/2$. The longitudinal rate strongly dominates over the transverse rates. In terms of the asymmetry parameter $\alpha_{L/T}$ defined in Eq. (25) we find $\alpha_{L/T} = -0.88$ as compared to $\alpha_{L/T} = -0.85$ in [49]. Among the transverse rates the transverse-minus rate Γ_- dominates over the transverse-plus rate Γ_+ (for $\Gamma(B_c^+ \rightarrow J/\psi + \rho^+)$ one has $\Gamma_+ > \Gamma_-$). The predicted hierarchy of rates can be easily understood in terms of simple spin arguments as were given some time ago in [50,51] and rediscovered in [52]. In the so-called $B \rightarrow VV$ -decays the dominance of the longitudinal mode has been experimentally confirmed

in the decay $B \rightarrow \rho\rho$ but not in the decay $B \rightarrow \phi K^*$ where enhanced penguin effects may play an important role (see the discussion in [52–54]).

For the charge conjugate mode $B_c^+ \rightarrow J/\psi + \rho^+$ the angular decay distributions in Eqs. (24) and (25) will remain unchanged since from CP invariance one has $\bar{H}_\lambda(B_c^+) = H_{-\lambda}(B_c^-)$ for real helicity amplitudes. The partial helicity rates $\Gamma_{U,L,T,I}$ remain unchanged going from the B_c^- to the B_c^+ mode ($\bar{\Gamma}_{U,L,T,I} = \Gamma_{U,L,T,I}$) except for the partial helicity rate Γ_P which is not measurable in the decay.

For the sake of completeness we also list the corresponding angular decay distribution when lepton mass and T -odd effects are included. For nonvanishing lepton masses one now has to also include helicity flip effects ($\lambda_l = \lambda_{l'}$) in the decay $J/\psi \rightarrow l^+l^-$ which are nonvanishing for nonvanishing lepton masses. We also drop the assumption that the helicity amplitudes are relatively real thus including possible T -odd effects.

Although the helicity flip effects are expected to be quite small for the decay $J/\psi \rightarrow \mu^+ \mu^-$ we shall include them for completeness. Lepton mass effects have to be taken into account e.g. in the decay $J/\psi(2S) \rightarrow \tau^+ + \tau^-$ (not discussed in this paper) since the $J/\psi(2S)$ has a mass of 3686 MeV which lies above the $(\tau^+ + \tau^-)$ -threshold ($2m_\tau = 3.554 \text{ GeV}$). When helicity flip effects are included one needs to know the ratio of the squared flip and nonflip helicity amplitudes of the decay $J/\psi \rightarrow \mu^+ \mu^-$ which are given by

$$\frac{|h_{(1/2)(1/2)}|^2}{|h_{(1/2)-(1/2)}|^2} = \frac{2m_l^2}{m_{J/\psi}^2} := 4\epsilon. \quad (30)$$

The remaining two flip and nonflip amplitudes can be obtained from the parity relations $h_{-(1/2)-(1/2)} = h_{11/22}$ and $h_{-(1/2)(1/2)} = h_{(1/2)-(1/2)}$.

Using Eqs. (23) and (30) and putting in the correct normalization one obtains (we now reinstitute the branching ratio factors)

$$\begin{aligned}\frac{d\Gamma}{d\cos\theta d\chi d\cos\theta^*} &= \text{Br}(J/\psi \rightarrow l^+l^-)\text{Br}(\rho^- \rightarrow \pi^- \pi^0) \frac{G_F^2}{16\pi} |V_{cb}V_{ud}a_1 f_\rho m_\rho|^2 \frac{|\mathbf{p}_2|}{m_{B_c}^2} \frac{1}{1+4\epsilon} \frac{9}{64\pi} [(|H_+|^2 + |H_-|^2) \\ &\times (1 + \cos^2\theta)\sin^2\theta^* + 4|H_0|^2\sin^2\theta\cos^2\theta^* + (\text{Re}(H_0H_+^\dagger) + \text{Re}(H_0H_-^\dagger))\sin 2\theta \sin 2\theta^* \cos\chi \\ &- 2\text{Re}(H_-H_+^\dagger)\sin^2\theta\sin^2\theta^* \cos 2\chi + (\text{Im}(H_0H_+^\dagger) - \text{Im}(H_0H_-^\dagger))\sin 2\theta \sin 2\theta^* \sin\chi \\ &+ 2\text{Im}(H_-H_+^\dagger)\sin^2\theta\sin^2\theta^* \sin 2\chi + \frac{m_l^2}{2m_{J/\psi}^2} \{8(|H_+|^2 + |H_-|^2)\sin^2\theta\sin^2\theta^* + 32|H_0|^2\sin^2\theta\cos^2\theta^* \\ &- 8(\text{Re}(H_0H_+^\dagger) + \text{Re}(H_0H_-^\dagger))\sin 2\theta \sin 2\theta^* \cos\chi + 16\text{Re}(H_-H_+^\dagger)\cos^2\theta\sin^2\theta^* \cos 2\chi - 8(\text{Im}(H_0H_+^\dagger) \\ &- \text{Im}(H_0H_-^\dagger))\sin 2\theta \sin 2\theta^* \sin\chi - 16\text{Im}(H_-H_+^\dagger)\sin^2\theta\sin^2\theta^* \sin 2\chi\}].\end{aligned}\quad (31)$$

We have checked that the angular decay distribution in Eq. (31) agrees with the corresponding angular decay distribution written down in [15]. In addition, we have checked the correctness of the signs of the nonflip azimuthal correlations by going through a fully covariant calculation. Note that the angular decay distribution is invariant under $\theta \rightarrow \pi - \theta$, $\chi \rightarrow \chi + \pi$ and $\theta^* \rightarrow \pi - \theta^*$, $\chi \rightarrow \chi + \pi$ showing that the polar and azimuthal angles in Fig. 1 could have also been defined by changing the labels $l^+ \leftrightarrow l^-$ and/or $\pi^- \leftrightarrow \pi^0$.

We have also included so-called T -odd contributions in the decay distribution (31) which can have their origin in possible imaginary parts of the helicity amplitudes. These could arise from strong interaction phases generated from final state interaction effects or from weak phases occurring in extensions of the standard model (see e.g. [13,14]). In the standard model and in the factorization approximation these T -odd contributions vanish, i.e. the angular decay distribution (31) would be reduced to that part given by the real contributions listed in (24). It would nevertheless be interesting to experimentally check on the possible presence of T -odd contributions in the angular decay distribution Eq. (31).

The angular decay distribution for the charge conjugate mode $B_c^+ \rightarrow J/\psi + \rho^+$ can be obtained from Eq. (31) by the replacement $H_i(B_c^-) \rightarrow \bar{H}_i(B_c^+)$ in Eq. (31). The charge conjugate helicity amplitudes \bar{H}_i and the helicity amplitudes H_i are related by (see e.g. [14,46])

$$\begin{aligned} H_{\pm} &= |H_{\mp}| e^{i(\delta_{\pm} + \phi_{\pm})}, & H_0 &= |H_0| e^{i(\delta_0 + \phi_0)}, \\ \bar{H}_{\pm} &= |H_{\mp}| e^{i(\delta_{\pm} - \phi_{\pm})}, & \bar{H}_0 &= |H_0| e^{i(\delta_0 - \phi_0)}, \end{aligned} \quad (32)$$

where the δ_i and ϕ_i denote the strong and weak phases of the helicity amplitudes, respectively. A discussion of CP violating observables in this process can be found in [46–48].

$$\begin{aligned} \frac{d\Gamma}{dq^2 d\cos\theta d\chi d\cos\theta_l} &= \frac{1}{2\pi} \left[\frac{3}{8} (1 + \cos^2\theta) \frac{3}{8} (1 + \cos^2\theta_l) \frac{d\Gamma_U}{dq^2} + \frac{3}{4} \sin^2\theta \frac{3}{4} \sin^2\theta_l \frac{d\Gamma_L}{dq^2} \pm \frac{3}{8} (1 + \cos^2\theta) \frac{3}{4} \cos\theta_l \frac{d\Gamma_P}{dq^2} \right. \\ &\quad \left. + \frac{3}{4} \sin^2\theta \frac{3}{4} \sin^2\theta_l \cos 2\chi \frac{d\Gamma_T}{dq^2} + \frac{3}{4} \sin 2\theta \frac{3}{8} \sin 2\theta_l \cos\chi \frac{d\Gamma_I}{dq^2} \pm \frac{3}{4} \sin 2\theta \frac{3}{4} \sin\theta_l \cos\chi \frac{d\Gamma_A}{dq^2} \right]. \end{aligned} \quad (36)$$

The contributions proportional to Γ_P and Γ_A in Eq. (36) change signs when going from the $(l^-, \bar{\nu}_l)$ to the (l^+, ν_l) case, i.e. when going from the decay $B_c^- \rightarrow J/\psi + l^- + \bar{\nu}_l$ to the decay $B_c^+ \rightarrow J/\psi + l^+ + \nu_l$. For the terms proportional to Γ_P and Γ_A the upper and lower signs holds for the $(l^-, \bar{\nu}_l)$ and (l^+, ν_l) cases, respectively. However, since Γ_P and Γ_A also change signs when going from $B_c^- \rightarrow J/\psi + l^- + \bar{\nu}_l$ to $B_c^+ \rightarrow J/\psi + l^+ + \nu_l$ the form of the effective decay distribution will be the same in both cases. Similar to the nonleptonic decay $B_c^- \rightarrow J/\psi + \rho^-$ discussed earlier in this section the angular decay distribution (36) is invari-

ant under $\theta \rightarrow \pi - \theta$, $\chi \rightarrow \chi + \pi$ showing that the polar angle θ and the azimuthal angle χ in Fig. 2 could have also been defined by changing the labels $l^+ \leftrightarrow l^-$ in the decay $J/\psi \rightarrow l^+ l^-$.

$$\begin{aligned} \frac{d\Gamma}{d\cos\theta} &= \text{Br}(J/\psi \rightarrow l^+ l^-) \frac{G_F^2}{16\pi} |V_{cb} V_{ud} a_1 f_{\pi} m_{\pi}|^2 \\ &\quad \times \frac{|\mathbf{p}_2|}{m_{B_c}^2} \frac{1}{1 + 4\epsilon} |H_l|^2 \frac{3}{4} (\sin^2\theta + 8\epsilon \cos^2\theta). \end{aligned} \quad (33)$$

In a similar way one obtains the angular decay distribution for the decay $B_c^- \rightarrow \eta_c + \rho^- (\rightarrow \pi^- \pi^0)$ where one finds

$$\begin{aligned} \frac{d\Gamma}{d\cos\theta^*} &= \text{Br}(\rho^- \rightarrow \pi^- \pi^0) \frac{G_F^2}{16\pi} |V_{cb} V_{ud} a_1 f_{\rho} m_{\rho}|^2 \\ &\quad \times \frac{|\mathbf{p}_2|}{m_{B_c}^2} |H_0|^2 \frac{3}{2} \cos^2\theta^*, \end{aligned} \quad (34)$$

and where now H_0 is the helicity amplitude of the decay $B_c^- \rightarrow \eta_c + \rho^-$.

Finally we analyze the angular decay distribution in the semileptonic decay $B_c^- \rightarrow J/\psi (\rightarrow l^+ l^-) + l^- + \bar{\nu}_l$. We shall now neglect lepton mass effects altogether and assume that the helicity amplitudes are relatively real thus neglecting T -odd effects in the decay. Using again the methods described in [8,12] the angular decay distribution can be cast into the form

$$\begin{aligned} W(\theta, \chi, \theta_l) &\propto \sum_{\substack{\lambda=m, \lambda'=m' \\ \lambda_l = -\lambda_{\nu_l}}} e^{i(m-m')(\pi-\chi)} \times d_{m, \lambda_l - \lambda_{\nu_l}}^1(\theta) d_{m', \lambda_l - \lambda_{\nu_l}}^1(\theta) \\ &\quad \times H_{\lambda m} H_{\lambda' m'}^{\dagger} d_{\lambda, \mp 1}^1(\theta_l) d_{\lambda', \mp 1}^1(\theta_l), \end{aligned} \quad (35)$$

where the angles of the decay process are defined in Fig. 2. Putting in the correct normalization one obtains

ant under $\theta \rightarrow \pi - \theta$, $\chi \rightarrow \chi + \pi$ showing that the polar angle θ and the azimuthal angle χ in Fig. 2 could have also been defined by changing the labels $l^+ \leftrightarrow l^-$ in the decay $J/\psi \rightarrow l^+ l^-$.

Upon angular integration one has $d\Gamma/dq^2 = d\Gamma_U/dq^2 + d\Gamma_L/dq^2$. Note that we have again taken the freedom to omit the branching ratio factor $\text{Br}(J/\psi \rightarrow l^+ l^-)$. The reason is again, when integrating Eq. (36) over q^2 and doing the angular integrations, we want to obtain the total rate $\Gamma(B_c^- \rightarrow J/\psi + l + \nu_l)$. After χ and $\cos\theta$ integration one recovers the corresponding single

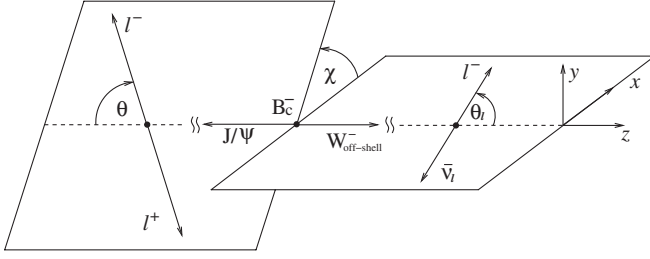


FIG. 2. Definition of the polar angles θ and θ_l and the azimuthal angle χ in the cascade decay $B_c^- \rightarrow J/\psi(\rightarrow l^+ l^-) + W_{\text{off-shell}}^- (\rightarrow l^- + \bar{\nu}_l)$.

angle decay distribution written down in [10]. The differential partial helicity rates $d\Gamma_i/dq^2$ ($i = U, L, P, T, I, A$) are defined by

$$\frac{d\Gamma_i}{dq^2} = \frac{G_F^2}{(2\pi)^3} |V_{bc}|^2 \frac{q^2 |\mathbf{p}_2|}{12m_1^2} \mathcal{H}_i, \quad (37)$$

where

$$\begin{aligned} \mathcal{H}_U &= |H_+|^2 + |H_-|^2, & \mathcal{H}_P &= |H_+|^2 - |H_-|^2, \\ \mathcal{H}_L &= |H_0|^2, & \mathcal{H}_T &= \text{Re} H_+ H_0^\dagger, \\ \mathcal{H}_I &= \frac{1}{2} \text{Re}(H_+ H_0^\dagger + H_- H_0^\dagger), & & \\ \mathcal{H}_A &= \frac{1}{2} \text{Re}(H_+ H_0^\dagger - H_- H_0^\dagger). & & \end{aligned} \quad (38)$$

Numerically, we obtain the following values for the integrated partial rates Γ_i

$$\begin{aligned} \Gamma_U &= +14.49 \cdot 10^{-15} \text{ GeV}, \\ \Gamma_P &= -8.182 \cdot 10^{-15} \text{ GeV}, \\ \Gamma_L &= +15.80 \cdot 10^{-15} \text{ GeV}, \\ \Gamma_T &= +5.850 \cdot 10^{-15} \text{ GeV}, \\ \Gamma_I &= +9.494 \cdot 10^{-15} \text{ GeV}, \\ \Gamma_A &= -3.208 \cdot 10^{-15} \text{ GeV}. \end{aligned} \quad (39)$$

The longitudinal rate Γ_L and the (unpolarized) transverse rate Γ_U are of approximately equal size where the longitudinal rate dominates at small q^2 (e.g. at $q^2 = m_\rho^2$ as discussed earlier for the decay $B_c^- \rightarrow J/\psi + \rho^-$) and the transverse rate dominates at large q^2 . This implies that one no longer has a pronounced longitudinal dominance in $B_c \rightarrow VV$ decays when the form factors are probed at higher momentum transfers as e.g. in the decay $B_c^- \rightarrow J/\psi + D^{*-}$. Reexpressing Γ_U and Γ_P in terms of the transverse-minus and transverse-plus rates one finds $\Gamma_- = 11.34 \times 10^{-15} \text{ GeV}$ and $\Gamma_+ = 3.155 \times 10^{-15} \text{ GeV}$. The dominance of Γ_- over Γ_+ reflects the basic left-chiral current structure of the $b \rightarrow c$ current transition. The interference contributions Γ_T, Γ_I , and Γ_A are large enough to

provide significant azimuthal correlations in the semileptonic decay process.

VI. SUMMARY AND CONCLUSIONS

We have performed a comprehensive analysis of the exclusive semileptonic and nonleptonic decays of the B_c -meson. The predicted branching ratios range from very small numbers of $\mathcal{O}(10^{-6})$ up to the largest branching fraction of 11% for the nonleptonic decay $B_c^- \rightarrow \bar{B}^{*0} \rho^-$. We have compared our results with the results of other studies. In general the results of the various model calculations are of the same order of magnitude while they can differ by factors of ten for specific decay modes. As a curious by-note we mention that a first attempt at estimating exclusive nonleptonic B_c decays can be found in [50]. Using the present value of V_{bc} (which was not known in 1978) and the present $\tau(B_c) = 0.45 \text{ ps}$ the authors of [50] calculated branching ratios of 0.29% and 0.69% for $B_c \rightarrow J/\psi + \rho$ and $B_c \rightarrow J/\psi + \pi$, respectively, with a $L:T_-:T_+$ ratio of 88%:10.4%:2% for $B_c \rightarrow J/\psi + \rho$. The branching ratios are $\approx 50\%$ above the branching ratios of the present calculation, whereas the helicity rate composition of the decay $B_c \rightarrow J/\psi + \rho$ is very close to that of the present model given in Sec. V. At any rate, we are looking forward to a detailed experimental study of the many exclusive decay modes of the B_c meson described in this paper.

We have taken a more detailed look at the spin dynamics of the decay modes $B_c^- \rightarrow J/\psi(\rightarrow l^+ l^-) + \rho^- (\rightarrow \pi^- \pi^0)$ and $B_c^- \rightarrow J/\psi(\rightarrow l^+ l^-) + W_{\text{off-shell}}^- (\rightarrow l^- + \bar{\nu}_l)$ involving a J/ψ in the final state for which we have presented explicit formulas for their joint angular decay distributions. We have discussed the changes in the decay distributions for the corresponding B_c^+ decay modes. It should be possible to test the joint angular decay distributions and extract values for the helicity structure functions with data samples of the $\mathcal{O}(100)$.

With our model assumptions the total exclusive rates calculated in this paper for the B_c^- decays are identical to the corresponding rates for the B_c^+ rates. As concerns the partial transverse helicity rates in the $B_c \rightarrow VV$ modes one has to change $T_- \leftrightarrow T_+$ when going from $B_c^- \rightarrow VV$ to $B_c^+ \rightarrow VV$ as discussed in Sec. V.

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APPENDIX A: WIDTHS FORMULAS FOR WEAK B_c -DECAYS

The leptonic decay widths are given by

$$\Gamma(P \rightarrow l\bar{\nu}) = \frac{G_F^2}{8\pi} |V_{q_1 q_2}|^2 f_P^2 m_P m_l^2 \left[1 - \frac{m_l^2}{m_P^2} \right]^2, \quad \Gamma(V \rightarrow l\bar{\nu}) = \frac{G_F^2}{4\pi} |V_{q_1 q_2}|^2 f_V^2 m_V^3 \left[1 - \frac{m_l^2}{m_V^2} \right]^2 \left[1 + \frac{m_l^2}{2m_V^2} \right].$$

For the semileptonic B_c -decay widths one finds

$$\begin{aligned} \Gamma(B_c^- \rightarrow M_{\bar{c}c} l \bar{\nu}) &= \frac{G_F^2}{(2\pi)^3} |V_{cb}|^2 \int_{m_l^2}^{q^2} dq^2 \frac{(q^2 - m_l^2)^2 |\mathbf{p}_2|}{12m_1^2 q^2} \left\{ \left(1 + \frac{m_l^2}{2q^2} \right) \sum_{i=\pm,0} (H_i^{B_c \rightarrow M_{\bar{c}c}}(q^2))^2 + \frac{3m_l^2}{2q^2} (H_i^{B_c \rightarrow M_{\bar{c}c}}(q^2))^2 \right\}, \\ \Gamma(B_c^- \rightarrow \bar{D}^0 l \bar{\nu}) &= \frac{G_F^2}{(2\pi)^3} |V_{ub}|^2 \int_{m_l^2}^{q^2} dq^2 \frac{(q^2 - m_l^2)^2 |\mathbf{p}_2|}{12m_1^2 q^2} \left\{ \left(1 + \frac{m_l^2}{2q^2} \right) \sum_{i=\pm,0} (H_i^{B_c \rightarrow \bar{D}^0}(q^2))^2 + \frac{3m_l^2}{2q^2} (H_i^{B_c \rightarrow \bar{D}^0}(q^2))^2 \right\}, \\ \Gamma(B_c^- \rightarrow \bar{B}_q^0 l \bar{\nu}) &= \frac{G_F^2}{(2\pi)^3} |V_{cd}|^2 \int_{m_l^2}^{q^2} dq^2 \frac{(q^2 - m_l^2)^2 |\mathbf{p}_2|}{12m_1^2 q^2} \left\{ \left(1 + \frac{m_l^2}{2q^2} \right) \sum_{i=\pm,0} (H_i^{B_c \rightarrow \bar{B}_q^0}(q^2))^2 + \frac{3m_l^2}{2q^2} (H_i^{B_c \rightarrow \bar{B}_q^0}(q^2))^2 \right\}, \quad (q = s, d), \end{aligned}$$

where $q_\pm^2 = (m_1 \pm m_2)^2$, $m_1 \equiv m_{B_c}$, and $m_2 \equiv m_f$. Note that \bar{D}^0 and \bar{B}_q^0 denote both the pseudoscalar and vector cases.

A. Nonleptonic B_c -decay widths

Finally, the nonleptonic B_c -decay widths are given by the expressions in the following two sections.

1. Transitions due to b -decays

$$\Gamma(B_c^- \rightarrow P^- M_{\bar{c}c}) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cb} V_{uq}^\dagger a_1 f_P m_P|^2 (H_t^{B_c \rightarrow M_{\bar{c}c}}(m_P^2))^2, \quad (P^- = \pi^-, K^+, \quad \text{and } q = d, s, \quad \text{respectively}),$$

$$\begin{aligned} \Gamma(B_c^- \rightarrow V^- M_{\bar{c}c}) &= \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cb} V_{uq}^\dagger a_1 f_V m_V|^2 \sum_{i=0,\pm} (H_i^{B_c \rightarrow M_{\bar{c}c}}(m_V^2))^2, \\ (V^- = \rho^-, K^{*+}, \quad \text{and } q = d, s, \quad \text{respectively}), \end{aligned}$$

$$\Gamma(B_c^- \rightarrow D_q^- \bar{D}^0) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{ub} V_{cq}^\dagger|^2 \{ a_1 f_{D_q^-} m_{D_q^-} H_t^{B_c \rightarrow \bar{D}^0}(m_{D_q^-}^2) + a_2 f_{\bar{D}^0} m_{\bar{D}^0} H_t^{B_c \rightarrow D_q^-}(m_{\bar{D}^0}^2) \}^2,$$

$$\Gamma(B_c^- \rightarrow D_q^{*-} \bar{D}^0) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{ub} V_{cq}^\dagger|^2 \{ a_1 f_{D_q^{*-}} m_{D_q^{*-}} H_0^{B_c \rightarrow \bar{D}^0}(m_{D_q^{*-}}^2) + a_2 f_{\bar{D}^0} m_{\bar{D}^0} H_t^{B_c \rightarrow D_q^{*-}}(m_{\bar{D}^0}^2) \}^2,$$

$$\Gamma(B_c^- \rightarrow D_q^- \bar{D}^{*0}) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{ub} V_{cq}^\dagger|^2 \{ a_1 f_{D_q^-} m_{D_q^-} H_t^{B_c \rightarrow \bar{D}^{*0}}(m_{D_q^-}^2) + a_2 f_{\bar{D}^{*0}} m_{\bar{D}^{*0}} H_0^{B_c \rightarrow D_q^-}(m_{\bar{D}^{*0}}^2) \}^2,$$

$$\Gamma(B_c^- \rightarrow D_q^{*-} \bar{D}^{*0}) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{ub} V_{cq}^\dagger|^2 \sum_{i=0,\pm} \{ a_1 f_{D_q^{*-}} m_{D_q^{*-}} H_i^{B_c \rightarrow \bar{D}^{*0}}(m_{D_q^{*-}}^2) + a_2 f_{\bar{D}^{*0}} m_{\bar{D}^{*0}} H_i^{B_c \rightarrow D_q^{*-}}(m_{\bar{D}^{*0}}^2) \}^2,$$

$$\Gamma(B_c^- \rightarrow D_q^- D^0) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cb} V_{uq}^\dagger a_2 f_{D^0} m_{D^0}|^2 (H_t^{B_c \rightarrow D_q^-}(m_{D^0}^2))^2,$$

$$\Gamma(B_c^- \rightarrow D_q^{*-} D^0) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cb} V_{uq}^\dagger a_2 f_{D^0} m_{D^0}|^2 (H_t^{B_c \rightarrow D_q^{*-}}(m_{D^0}^2))^2,$$

$$\Gamma(B_c^- \rightarrow D_q^- D^{*0}) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cb} V_{uq}^\dagger a_2 f_{D^{*0}} m_{D^{*0}}|^2 (H_0^{B_c \rightarrow D_q^-}(m_{D^{*0}}^2))^2,$$

$$\Gamma(B_c^- \rightarrow D_q^{*-} D^{*0}) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cb} V_{uq}^\dagger a_2 f_{D^{*0}} m_{D^{*0}}|^2 \sum_{i=0,\pm} (H_i^{B_c^- \rightarrow D_q^{*-}}(m_{D^{*0}}^2))^2,$$

$$\Gamma(B_c^- \rightarrow D_q^- \eta_c) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cb} V_{cq}^\dagger|^2 \{a_1 f_{D_q^-} m_{D_q^-} H_t^{B_c^- \rightarrow \eta_c}(m_{D_q^-}^2) + a_2 f_{\eta_c} m_{\eta_c} H_t^{B_c^- \rightarrow D_q^-}(m_{\eta_c}^2)\}^2,$$

$$\Gamma(B_c^- \rightarrow D_q^{*-} \eta_c) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cb} V_{cq}^\dagger|^2 \{a_1 f_{D_q^{*-}} m_{D_q^{*-}} H_0^{B_c^- \rightarrow \eta_c}(m_{D_q^{*-}}^2) + a_2 f_{\eta_c} m_{\eta_c} H_t^{B_c^- \rightarrow D_q^{*-}}(m_{\eta_c}^2)\}^2,$$

$$\Gamma(B_c^- \rightarrow D_q^- J/\psi) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cb} V_{cq}^\dagger|^2 \{a_1 f_{D_q^-} m_{D_q^-} H_t^{B_c^- \rightarrow J/\psi}(m_{D_q^-}^2) + a_2 f_{J/\psi} m_{J/\psi} H_t^{B_c^- \rightarrow D_q^-}(m_{J/\psi}^2)\}^2,$$

$$\Gamma(B_c^- \rightarrow D_q^{*-} J/\psi) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cb} V_{cq}^\dagger|^2 \sum_{i=0,\pm} \{a_1 f_{D_q^{*-}} m_{D_q^{*-}} H_i^{B_c^- \rightarrow J/\psi}(m_{D_q^{*-}}^2) + a_2 f_{J/\psi} m_{J/\psi} H_i^{B_c^- \rightarrow D_q^{*-}}(m_{J/\psi}^2)\}^2.$$

2. Transitions due to *c*-decays

$$\Gamma(B_c^- \rightarrow \bar{B}^0 P^-) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cd} V_{uq}^\dagger a_1 f_P m_P|^2 (H_t^{B_c^- \rightarrow \bar{B}^0}(m_P^2))^2,$$

$$\Gamma(B_c^- \rightarrow \bar{B}^0 V^-) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cd} V_{uq}^\dagger a_1 f_V m_V|^2 (H_0^{B_c^- \rightarrow \bar{B}^0}(m_V^2))^2,$$

$$\Gamma(B_c^- \rightarrow \bar{B}^{*0} P^-) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cd} V_{uq}^\dagger a_1 f_P m_P|^2 (H_t^{B_c^- \rightarrow \bar{B}^{*0}}(m_P^2))^2,$$

$$\Gamma(B_c^- \rightarrow \bar{B}^{*0} V^-) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cd} V_{uq}^\dagger a_1 f_V m_V|^2 \sum_{i=0,\pm} (H_i^{B_c^- \rightarrow \bar{B}^{*0}}(m_V^2))^2,$$

$$\Gamma(B_c^- \rightarrow \bar{B}_s^0 P^-) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cs} V_{uq}^\dagger a_1 f_P m_P|^2 (H_t^{B_c^- \rightarrow \bar{B}_s^0}(m_P^2))^2,$$

$$\Gamma(B_c^- \rightarrow \bar{B}_s^0 V^-) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cs} V_{uq}^\dagger a_1 f_V m_V|^2 (H_0^{B_c^- \rightarrow \bar{B}_s^0}(m_V^2))^2,$$

$$\Gamma(B_c^- \rightarrow \bar{B}_s^{*0} P^-) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cs} V_{uq}^\dagger a_1 f_P m_P|^2 (H_t^{B_c^- \rightarrow \bar{B}_s^{*0}}(m_P^2))^2,$$

$$\Gamma(B_c^- \rightarrow \bar{B}_s^{*0} V^-) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cs} V_{uq}^\dagger a_1 f_V m_V|^2 \sum_{i=0,\pm} (H_i^{B_c^- \rightarrow \bar{B}_s^{*0}}(m_V^2))^2,$$

$$\Gamma(B_c^- \rightarrow B^- \pi^0) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cd} V_{ud}^\dagger \frac{a_2}{\sqrt{2}} f_\pi m_\pi|^2 (H_t^{B_c^- \rightarrow B^-}(m_{\pi^0}^2))^2,$$

$$\Gamma(B_c^- \rightarrow B^- \rho^0) = \frac{G_F^2}{16\pi} \frac{|\mathbf{p}_2|}{m_1^2} |V_{cd} V_{ud}^\dagger \frac{a_2}{\sqrt{2}} f_\rho m_\rho|^2 (H_0^{B_c^- \rightarrow B^-}(m_{\rho^0}^2))^2,$$

$$\Gamma(B_c^- \rightarrow B^- K^0) = \frac{G_F^2 |\mathbf{p}_2|}{16\pi m_1^2} |V_{cs} V_{ud}^\dagger a_2 f_K m_K|^2 (H_t^{B_c^- \rightarrow B^-}(m_{K^0}^2))^2,$$

$$\Gamma(B_c^- \rightarrow B^- K^{*0}) = \frac{G_F^2 |\mathbf{p}_2|}{16\pi m_1^2} |V_{cs} V_{ud}^\dagger a_2 f_{K^*} m_{K^*}|^2 (H_0^{B_c^- \rightarrow B^-}(m_{K^{*0}}^2))^2,$$

$$\Gamma(B_c^- \rightarrow B^- \bar{K}^0) = \frac{G_F^2 |\mathbf{p}_2|}{16\pi m_1^2} |V_{cd} V_{us}^\dagger a_2 f_{\bar{K}} m_{\bar{K}}|^2 (H_t^{B_c^- \rightarrow B^-}(m_{\bar{K}^0}^2))^2,$$

$$\Gamma(B_c^- \rightarrow B^- \bar{K}^{*0}) = \frac{G_F^2 |\mathbf{p}_2|}{16\pi m_1^2} |V_{cd} V_{us}^\dagger a_2 f_{\bar{K}^*} m_{\bar{K}^*}|^2 (H_0^{B_c^- \rightarrow B^-}(m_{\bar{K}^{*0}}^2))^2.$$

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