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Quark-hadron duality and hadron properties from correlators of pseudoscalar and axial currents

Wolfgang Lucha[1] and Dmitri Melikhov[1,2]

[1]*Institute for High Energy Physics, Austrian Academy of Sciences, Nikolsdorfergasse 18, A-1050, Vienna, Austria*
[2]*Institute of Nuclear Physics, Moscow State University, 119992, Moscow, Russia*
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We study the operator product expansion (OPE) and quark-hadron duality for 2- and 3-point correlators of the axial (A) and pseudoscalar (P) currents of the light quarks. In the chiral limit these correlators are often dominated by nonperturbative power corrections leading to subtleties of quark-hadron duality relations and of the extraction of properties of light pseudoscalars. For the 2-point correlators, we show the sum rule for $\langle PP\rangle$ to be sensitive to the excited light pseudoscalar. For the 3-point correlators, we derive the Ward identities which provide the normalization of the pion electromagnetic form factor at zero momentum transfer. For large momentum transfer, we demonstrate the way the correct behavior of the pion form factor in agreement with perturbative QCD emerges from condensate terms in the OPE for the $\langle PVP\rangle$ and $\langle AVP\rangle$ correlators. The local-duality sum rule for $\langle AVA\rangle$ is shown to lead to the pion form factor with the required properties for all values of the momentum transfer.

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I. INTRODUCTION

Correlators of the axial and pseudoscalar currents are the basic objects for studying properties of light pseudoscalars within QCD sum rules [1–3], bound-state equations [4], and lattice QCD [5]. The axial current $j^5_\alpha(x) = \bar{u}(x)\gamma_\alpha\gamma_5 d(x)$ and the pseudoscalar current $j^5(x) = i\bar{u}(x)\gamma_5 d(x)$ satisfy the equation

$$\partial^\alpha j^5_\alpha(x) = (m_u + m_d) j^5(x). \qquad (1.1)$$

The axial current is conserved in the limit of massless quarks (the chiral limit). Therefore, the correlators of the axial current have the following property: perturbative contributions to the longitudinal structures of these correlators are suppressed by the light quark mass, and the operator product expansion (OPE) [6] for the longitudinal structures is dominated by nonperturbative power corrections. This leads to specific features of the quark-hadron duality [7] in these cases.

We shall be interested in extracting contributions of light pseudoscalars from the correlators of axial and pseudoscalar currents of light quarks. The coupling of pseudoscalar mesons P to these currents is governed by the decay constants f_P defined according to

$$\langle 0|\bar{u}\gamma_\alpha\gamma_5 d|P(q)\rangle = i f_P q_\alpha,$$
$$\langle P(q)|\bar{d}\gamma_\alpha\gamma_5 u|0\rangle = -i f_P q_\alpha, \qquad (1.2)$$

and

$$\langle 0|i\bar{u}\gamma_5 d|P(q)\rangle = \frac{f_P m_P^2}{m_u + m_d},$$
$$\langle P(q)|i\bar{d}\gamma_5 u|0\rangle = \frac{f_P m_P^2}{m_u + m_d}. \qquad (1.3)$$

The divergence equation requires

$$f_P m_P^2 \propto m, \qquad (1.4)$$

implying that at least one of the quantities on the left-hand side vanishes in the chiral limit. If chiral symmetry is spontaneously broken (as in QCD at zero temperature), Eq. (1.4) leads to the following alternatives [1][1]:

$$m_\pi^2 = O(m), \qquad f_\pi = O(1), \qquad \text{ground-state pion},$$
$$m_P^2 = O(1), \qquad f_P = O(m), \qquad \text{excited pseudoscalars}. \qquad (1.5)$$

Therefore, the vanishing of the decay constants of the excited pseudoscalars in the limit of massless quarks is a direct consequence and signature of chiral symmetry. (Hereafter, the subscript P denotes the excited massive pseudoscalar, and π stands for the pion.) An important relation between the pion observables and the quark condensate is given by the Gell-Mann–Oakes–Renner (GMOR) formula [9]

$$f_\pi^2 m_\pi^2 = -(m_u + m_d)\langle \bar{u}u + \bar{d}d\rangle + O(m_\pi^4). \qquad (1.6)$$

In this paper we study the consequences of spontaneously broken chiral symmetry for the 2- and 3-point correlators of the axial and pseudoscalar currents, and for the extraction of properties of pseudoscalar mesons from these correlators.

In Sec. II we discuss OPE and duality for the $\langle PP\rangle$, $\langle AP\rangle$, and $\langle AA\rangle$ correlators, and the possibility to extract from these correlators the decay constants of the excited pseudoscalars. (For simplicity, we denote the vacuum average of the T product of axial and pseudoscalar currents as $\langle AP\rangle$, etc.)

In Sec. III we consider the 3-point correlators $\langle PVP\rangle$, $\langle AVP\rangle$, and $\langle AVA\rangle$, V being the electromagnetic current.

[1]If no spontaneous breaking of chiral symmetry occurs, then in the chiral limit the pion behaves the same way as the excited pseudoscalars: it stays massive and its decay constant vanishes [8].

We derive Ward identities for these correlators and show the way the normalization of the pion form factor at zero momentum transfer arises due to these identities. We then analyze the region of large momentum transfers and study the way quark-hadron duality works for this case. We discuss a recent statement in the literature that the pion form factor as extracted from the $\langle PVP\rangle$ correlator does not have the right asymptotics according to perturbative QCD (pQCD). We show that the OPE for $\langle PVP\rangle$ and $\langle AVP\rangle$ at large q^2 is dominated by nonperturbative corrections. In particular, the four-quark condensate $\langle\bar\psi\psi\bar\psi\psi\rangle$ in the case of $\langle PVP\rangle$ and the mixed condensate $\langle\bar\psi\sigma^{\mu\nu}G_{\mu\nu}\psi\rangle \equiv \langle\bar\psi\sigma G\psi\rangle$ in the case of $\langle AVP\rangle$ are demonstrated to be crucial for extracting the correct large-q^2 behavior of the pion form factor. We then discuss $\langle AVA\rangle$ and point out that the recent calculation of radiative corrections to this correlator opens the possibility to obtain the pion electromagnetic form factor for all spacelike momentum transfers making use of the local-duality sum rule.

Section IV summarizes our results.

II. TWO-POINT FUNCTIONS, f_π AND $f_{\pi'}$

In this section, we consider all possible 2-point correlators of the axial current $j^5_\alpha = \bar u\gamma_\alpha\gamma_5 d$, $j^{5\dagger}_\alpha = \bar d\gamma_\alpha\gamma_5 u$ and the pseudoscalar current $j^5 = i\bar u\gamma_5 d$, $j^{5\dagger} = i\bar d\gamma_5 u$.

A. $\langle PP\rangle$

We start with the correlator of two pseudoscalar currents

$$\Pi^5(q) \equiv i\int d^4x e^{iqx}\langle T(j^5(x)j^{5\dagger}(0))\rangle = \Pi_5(q^2), \quad (2.1)$$

where $\langle\cdot\rangle$ stands for $\langle 0|\cdot|0\rangle$. At small q^2, the correlator is dominated by the Goldstone boson, leading in the chiral limit to the expression [10]

$$\Pi^5(q) = -\frac{\langle\bar u u + \bar d d\rangle^2}{f_\pi^2}\frac{1}{q^2}. \quad (2.2)$$

At large q^2, the OPE for this correlator reads [1]

$$\Pi_5(q^2) = \Pi_5^{\rm pert}(q^2) + \frac{(m_u+m_d)\langle\bar u u + \bar d d\rangle}{4q^2} - \frac{\langle\alpha_s GG\rangle}{8\pi q^2} + O\left(\frac{1}{q^4}\right), \quad (2.3)$$

where ${\rm Im}\Pi_5^{\rm pert}(s) = 3s/8\pi$ (for radiative corrections and higher condensates, see Ref. [11] and references therein).

B. $\langle AP\rangle$

The correlator of a pseudoscalar and an axial current has the form

$$\Pi^5_\alpha(q) \equiv i\int d^4x e^{iqx}\langle T(j^5_\alpha(x)j^{5\dagger}(0))\rangle = iq_\alpha\Pi(q^2). \quad (2.4)$$

To determine $\Pi(q^2)$, we calculate

$$\begin{aligned} q^\alpha\Pi^5_\alpha(q) &= -\int d^4x e^{iqx}\frac{\partial}{\partial x_\alpha}\langle T(j^5_\alpha(x)j^{5\dagger}(0))\rangle \\ &= -\int d^3x e^{-i\vec q\vec x}\langle[j^5_0(0,\vec x), j^{5\dagger}(0)]\rangle \\ &\quad -\int d^4x e^{iqx}\langle T(\partial^\alpha j^5_\alpha(x)j^{5\dagger}(0))\rangle. \end{aligned} \quad (2.5)$$

Making use of the commutation relation

$$[\bar u\gamma_0\gamma_5 d(0,\vec x), \bar d\gamma_5 u(0)] = -(\bar u u + \bar d d)\delta(\vec x), \quad (2.6)$$

and the divergence equation (1.1), we obtain

$$\Pi(q^2) = \frac{\langle\bar u u + \bar d d\rangle}{q^2} + (m_u+m_d)\frac{\Pi_5(q^2)}{q^2}, \quad (2.7)$$

with $\Pi_5(q^2)$ given by Eq. (2.1). In the chiral limit $m_u = m_d = 0$, the OPE for Π^5_α contains only one operator and takes the simple form

$$\Pi^5_\alpha(q) = iq_\alpha\frac{\langle\bar u u + \bar d d\rangle}{q^2}. \quad (2.8)$$

Let us now saturate the correlator Π^5_α with intermediate hadron states. Here we have contributions of single-state pseudoscalars (ground-state π and excitations P) and the hadron continuum (e.g., multipion states with the relevant quantum numbers). The contribution of a single pseudoscalar state P to the invariant amplitude $\Pi(q^2)$ has the form

$$\frac{f_P^2 m_P^2}{m_u+m_d}\frac{1}{(m_P^2-q^2)}. \quad (2.9)$$

In the chiral limit, neither excited pseudoscalars (since $f_P \propto m$) nor continuum states contribute to the correlator and it is fully saturated by the intermediate Goldstone boson for all values of q^2,

$$\Pi^5_\alpha(q) = -i\frac{q_\alpha}{q^2}\frac{f_\pi^2 m_\pi^2}{m_u+m_d}, \quad (2.10)$$

and both representations (2.8) and (2.10) are equal to each other by virtue of the GMOR relation. (Notice that the analysis of this correlator also provides a derivation of the GMOR relation.) It is interesting to note that in the chiral limit the duality between the OPE and the Goldstone contribution is locally fulfilled for all values of q^2.

C. $\langle AA\rangle$

The correlator of two axial currents contains two Lorentz structures:

$$\begin{aligned} \Pi^5_{\alpha\beta}(q) &\equiv i\int d^4x e^{iqx}\langle T(j^5_\alpha(x)j^{5\dagger}_\beta(0))\rangle \\ &= \left(g_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}\right)\Pi_T(q^2) + \frac{q_\alpha q_\beta}{q^2}\Pi_L(q^2). \end{aligned} \quad (2.11)$$

To determine the longitudinal part, let us calculate $q^\alpha \Pi^5_{\alpha\beta}$:

$$\begin{aligned} q^\alpha \Pi^5_{\alpha\beta}(q) &= -\int d^4x e^{iqx}\frac{\partial}{\partial x_\alpha}\langle T(j^5_\alpha(x) j^{5\dagger}_\beta(0))\rangle \\ &= -\int d^3x e^{-i\vec{q}\,\vec{x}}\langle [j^5_0(0,\vec{x}), j^{5\dagger}_\beta(0)]\rangle \\ &\quad -\int d^4x e^{iqx}\langle T(\partial^\alpha j^5_\alpha(x) j^{5\dagger}_\beta(0))\rangle \\ &= -(m_u+m_d)\int d^4x e^{iqx}\langle T(j^5(x) j^{5\dagger}_\beta(0))\rangle \\ &= i(m_u+m_d)\Pi^5_\beta(-q) = (m_u+m_d)q_\beta \Pi(q^2). \end{aligned} \tag{2.12}$$

The contribution of the commutator term in Eq. (2.12) vanishes since

$$[j^5_0(0,\vec{x}), j^{5\dagger}_\beta(0)] = (\bar{u}\gamma_\beta u(0) - \bar{d}\gamma_\beta d(0))\delta(\vec{x}) \tag{2.13}$$

and $\langle \bar{u}\gamma_\beta u\rangle = \langle \bar{d}\gamma_\beta d\rangle = 0$ because of the Lorentz invariance of the vacuum. Making use of Eq. (2.7) we arrive at

$$\Pi^5_{\alpha\beta}(q) = \left(g_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}\right)\Pi_T(q^2) + \frac{q_\alpha q_\beta}{q^4}((m_u+m_d) \times \langle \bar{u}u + \bar{d}d\rangle + (m_u+m_d)^2\Pi_5(q^2)). \tag{2.14}$$

The OPE for the transverse function $\Pi_T(q^2)$ is known from Ref. [1].

In the chiral limit, the axial current is conserved and the correlator is transverse. At small q^2, the correlator is dominated by the Goldstone pole

$$\Pi^5_{\alpha\beta}(q) = \left(g_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}\right)[f_\pi^2 + O(q^2)]. \tag{2.15}$$

If we switch on a small quark mass, the pion pole is shifted to its physical value and we have [10]

$$\Pi^5_{\alpha\beta}(q) = \left(g_{\alpha\beta} + \frac{q_\alpha q_\beta}{m_\pi^2 - q^2}\right)[f_\pi^2 + O(q^2)] + O(m^2). \tag{2.16}$$

The shift of the pion pole gives the only effect linear in the quark masses, all other corrections are quadratic in m. The correlator may be split into transverse and longitudinal structures as follows:

$$\Pi^5_{\alpha\beta}(q) = \left(g_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}\right)[f_\pi^2 + O(q^2) + O(m^2)] + q_\alpha q_\beta\left[\frac{m_\pi^2 f_\pi^2}{q^2(m_\pi^2 - q^2)} + O(m^2)\right]. \tag{2.17}$$

As soon as we are not in the precise chiral limit, the pion pole appears in the longitudinal structure of the correlator, corresponding to $J^P = 0^-$. Nevertheless, the transverse part of the correlator "remembers" the Goldstone nature of the pion and contains f_π^2. Therefore, f_π^2 can be related to the OPE for the transverse function $\Pi_T(q^2)$ by a "transverse" sum rule, as done in Ref. [1].[2]

Notice that the pion contribution to the longitudinal part has a different structure than the contribution of the excited pseudoscalar states, which has the form

$$q_\alpha q_\beta \frac{f_P^2}{m_P^2 - q^2}. \tag{2.18}$$

In the longitudinal structure of the expression (2.17), the pion contribution is the only hadron contribution of order $O(m)$, and, by virtue of the GMOR relation, it is locally dual to the quark-condensate term of the OPE (2.14) [1].

D. QCD sum rules for two-point correlators

Let us now summarize the OPE results for the 2-point correlators discussed above:

$$\begin{aligned} \Pi^5(q) &= \Pi_5(q^2), \\ \Pi^5_\alpha(q) &= \frac{iq_\alpha}{q^2}(\langle \bar{u}u + \bar{d}d\rangle + (m_u+m_d)\Pi_5(q^2)), \\ \Pi^5_{\alpha\beta,L}(q) &= \frac{q_\alpha q_\beta}{q^4}((m_u+m_d)\langle \bar{u}u + \bar{d}d\rangle + (m_u+m_d)^2\Pi_5(q^2)), \end{aligned} \tag{2.19}$$

where $\Pi^5_{\alpha\beta,L}$ is the longitudinal part of the correlator $\Pi^5_{\alpha\beta}$. The corresponding hadron saturation for these correlators reads

$$\begin{aligned} \Pi^5(q) &= \left(\frac{f_\pi m_\pi^2}{m_u+m_d}\right)^2\frac{1}{m_\pi^2 - q^2} + \left(\frac{f_P m_P^2}{m_u+m_d}\right)^2\frac{1}{m_P^2 - q^2} + \cdots, \\ \Pi^5_\alpha(q) &= iq_\alpha\left(\frac{f_\pi^2 m_\pi^2}{m_u+m_d}\frac{1}{m_\pi^2-q^2} + \frac{f_P^2 m_P^2}{m_u+m_d}\frac{1}{m_P^2-q^2} + \cdots\right), \\ \Pi^5_{\alpha\beta,L}(q) &= q_\alpha q_\beta\left(\frac{f_\pi^2 m_\pi^2}{q^2(m_\pi^2-q^2)} + \frac{f_P^2}{m_P^2-q^2} + \cdots\right), \end{aligned} \tag{2.20}$$

where dots indicate the contribution of the hadron continuum states.

The representations (2.19) and (2.20) for Π^5, Π^5_α, and $\Pi^5_{\alpha\beta,L}$ lead to a single relation between the imaginary parts:

$$\frac{f_\pi^2 m_\pi^4}{(m_u+m_d)^2}\pi\delta(s - m_\pi^2) + \frac{f_P^2 m_P^4}{(m_u+m_d)^2}\pi\delta(s - m_P^2) + \cdots \simeq \mathrm{Im}\Pi_5(s). \tag{2.21}$$

The sign $\simeq$ means that both sides are equal to each other

[2] We note that the excited pseudoscalars do not contribute to the transverse structure, and therefore decay constants of excited pseudoscalars have no relationship to Π_T.

after application of an appropriate smearing, which gives a ''longitudinal'' sum rule. Taking into account the GMOR relation and performing a Borel transform of the equation above (with the Borel parameter M^2), we find

$$\frac{\langle\bar{u}u+\bar{d}d\rangle^2}{f_\pi^2}\exp\left(-\frac{m_\pi^2}{M^2}\right)+\frac{f_P^2 m_P^4}{(m_u+m_d)^2}\exp\left(-\frac{m_P^2}{M^2}\right) = \frac{1}{\pi}\int_{(m_u+m_d)^2}^{s_0} ds\exp\left(-\frac{s}{M^2}\right)\mathrm{Im}\Pi_5(s), \quad (2.22)$$

where s_0 denotes the continuum subtraction point.

It is not our goal to perform here a detailed analysis of this sum rule. We would rather like to point out the sensitivity of this sum rule to the contribution of the excited pseudoscalar state. If we keep, on the hadron side, only the pion pole and neglect, on the QCD side, higher-order (both radiative and power) corrections and terms which vanish in the chiral limit, we obtain, after Borel transform,

$$\frac{4\langle\bar{\psi}\psi\rangle^2}{f_\pi^2}\exp\left(-\frac{m_\pi^2}{M^2}\right)=\frac{3}{8\pi^2}\int_0^{s_0} ds\, s\exp\left(-\frac{s}{M^2}\right) + \frac{1}{8\pi}\langle\alpha_s GG\rangle, \quad (2.23)$$

where we assume that $\langle\bar{u}u\rangle=\langle\bar{d}d\rangle$ and denote this quantity as $\langle\bar{\psi}\psi\rangle$. Taking into account the known values of the condensates and of f_π, we can neglect the $\langle GG\rangle$ term on the right-hand side and arrive at the sum rule

$$\frac{4\langle\bar{\psi}\psi\rangle^2}{f_\pi^2}=\frac{3}{8\pi^2}\int_0^{s_0} ds\, s\exp\left(\frac{m_\pi^2-s}{M^2}\right). \quad (2.24)$$

For $s_0 \simeq 1$–2 GeV we have an approximate relation:

$$\frac{4\langle\bar{\psi}\psi\rangle^2}{f_\pi^2}\simeq\frac{3}{8\pi^2}M^4. \quad (2.25)$$

To obtain $f_\pi = 0.13$ GeV for $\langle\bar{\psi}\psi\rangle = -(0.22\text{–}0.25\ \text{GeV})^3$ requires $M = 0.9$–1.1 GeV. On the other hand, the contribution of the excited pseudoscalar meson $\pi'(1300)$ to the sum rule (2.22) is sizable for a Borel mass M around $M \simeq 1$ GeV. Thus the sum rule (2.22) provides an interesting possibility to study the decay constant $f_{\pi'}$ of the excited pseudoscalar meson $\pi'(1300)$.

III. THREE-POINT FUNCTIONS AND THE PION FORM FACTOR

In this section, we discuss properties and duality relations for 3-point correlators, laying our main emphasis on $\langle PVP\rangle$ and $\langle AVP\rangle$. Nonperturbative power corrections play a crucial role in the OPE for these correlators: for $\langle PVP\rangle$ power corrections dominate the correlator at large momentum transfers, and for $\langle AVP\rangle$ only nonperturbative terms contribute in the chiral limit. This leads to subtleties in extracting the single-hadron contribution, in particular, the pion form factor, from these correlators.

A. $\langle PVP\rangle$

Let us start with the correlator

$$\Gamma_\mu(q,p,p') \equiv i^2\int d^4x d^4y e^{iqx-ipy}\langle T(j^5(y)j_\mu(x)j^{5\dagger}(0))\rangle, \quad p'=p-q, \quad (3.1)$$

where j_μ is the electromagnetic current

$$j_\mu = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d. \quad (3.2)$$

The conservation of the electromagnetic current and the commutation relation

$$[j_0(0,\vec{x}), \bar{d}\gamma_5 u(0)] = -\bar{d}\gamma_5 u(0)\delta(\vec{x}) \quad (3.3)$$

lead to the Ward identity

$$q^\mu\Gamma_\mu(q,p,p') = \Pi^5(p') - \Pi^5(p). \quad (3.4)$$

This relation determines the longitudinal structure of Γ_μ, so we can write

$$\Gamma_\mu(q,p,p') = 2\left(p_\mu - \frac{qp}{q^2}q_\mu\right)\Gamma(q^2,p^2,p'^2) + \frac{q_\mu}{q^2}(\Pi^5(p') - \Pi^5(p)). \quad (3.5)$$

Since there is no massless hadron state in the vector channel, the function Γ_μ is regular for $q^2 \to 0$. This leads to the relation, valid for all values of p^2 and p'^2,

$$\Gamma(0,p^2,p'^2) = -\frac{\Pi^5(p') - \Pi^5(p)}{p'^2 - p^2}. \quad (3.6)$$

Let us set the quark masses equal to zero. At small p^2 and p'^2, the Goldstones dominate the correlators, leading to a single pole in Π^5,

$$\Pi^5(p) = -\frac{4\langle\bar{\psi}\psi\rangle^2}{f_\pi^2 p^2}, \quad (3.7)$$

and to a double pole in $\Gamma(q^2,p^2,p'^2)$,

$$\Gamma(0,p^2,p'^2) = -\frac{F_\pi(0)}{p^2p'^2}\frac{4\langle\bar{\psi}\psi\rangle^2}{f_\pi^2}, \quad (3.8)$$

with the pion electromagnetic form factor $F_\pi(q^2)$ defined by

$$\langle\pi(p')|j_\mu(0)|\pi(p)\rangle = (p+p')_\mu F_\pi(q^2). \quad (3.9)$$

The Ward identity (3.6) then implies the normalization of the pion form factor at $q^2 = 0$:

$$F_\pi(q^2=0) = 1. \quad (3.10)$$

At large values of p^2, p'^2, and q^2, we have the following two representations for $\Gamma(q^2,p^2,p'^2)$, namely, a hadronic one and a QCD one:

$$\Gamma^{\rm hadr}(q^2, p^2, p'^2) = -\frac{4\langle\bar\psi\psi\rangle^2}{f_\pi^2}\frac{1}{p^2p'^2}F_\pi(q^2) + \cdots,$$
$$\Gamma^{\rm QCD}(q^2, p^2, p'^2) = \Gamma_{\rm pert}(q^2, p^2, p'^2) + \Gamma_{\rm inst}(q^2, p^2, p'^2) + \Gamma_{\rm cond}(q^2, p^2, p'^2), \tag{3.11}$$

where the dots in $\Gamma^{\rm hadr}$ represent double-pole contributions of pseudoscalars (π-P and P-P), mixed pole-continuum and double-continuum contributions. On the QCD side, one has an instanton contribution $\Gamma_{\rm inst}$ [12] in addition to the perturbative part $\Gamma_{\rm pert}$ and the condensate part $\Gamma_{\rm cond}$. Quark-hadron duality states that $\Gamma^{\rm hadr}$ and $\Gamma^{\rm QCD}$ should be equal to each other after a proper smearing is applied. For large values of q^2 the pion double pole behaves like

$$-\frac{4\langle\bar\psi\psi\rangle^2}{f_\pi^2}\frac{F_\pi(q^2)}{p^2p'^2} \propto \frac{\alpha_s\langle\bar\psi\psi\rangle^2}{q^2p^2p'^2}. \tag{3.12}$$

On the QCD side the double spectral density of $\Gamma_{\rm pert}$, including α_s corrections, was found to have the leading behavior $1/q^4$ at large q^2 [13]. The absence of the term α_s/q^2 in the perturbative diagrams led the authors of Ref. [13] to the conclusion that the behavior of the pion form factor as extracted from the $\langle PVP\rangle$ correlator should not reproduce the correct pQCD asymptotics because of the "wrong" twist of the pseudoscalar current. However, this argument cannot be correct: the correlator $\langle PVP\rangle$ has a nonzero overlap with the pion double pole, and the residue in this double pole should reproduce the full pion form factor, behaving according to pQCD at large q^2. As we shall demonstrate, the solution to this puzzle is simple: the condensates dominate the OPE for $\Gamma^{\rm QCD}(q^2, p^2, p'^2)$ at large q^2 and provide the necessary α_s/q^2 terms. The odd-dimensional operators $\bar\psi\psi$ and $\bar\psi\sigma G\psi$ are irrelevant for the effect under discussion as their contribution vanishes in the chiral limit:

$$\Gamma_{\rm cond}^{\langle\bar\psi\psi\rangle} \propto m\langle\bar\psi\psi\rangle, \qquad \Gamma_{\rm cond}^{\langle\bar\psi\sigma G\psi\rangle} \propto m\langle\bar\psi\sigma G\psi\rangle. \tag{3.13}$$

The contribution which is essential in the chiral limit comes from the even-dimensional operators GG and $\bar\psi\psi\bar\psi\psi$. Since the pion double pole (3.12) contains the factor $\langle\bar\psi\psi\rangle^2$, one can expect the operator $\bar\psi\psi\bar\psi\psi$ to provide the contribution of interest. For dimensional reasons, its contribution may contain the structures

$$\Gamma_{\rm cond}^{\langle\bar\psi\psi\rangle^2} \propto \alpha_s\langle\bar\psi\psi\rangle^2\left\{\frac{1}{p^2p'^2q^2}, \frac{1}{p^2q^4} + \frac{1}{p'^2q^4}, \frac{1}{p^4p'^2} + \frac{1}{p'^4p^2}, \frac{q^2}{p^4p'^4}, \cdots\right\}. \tag{3.14}$$

The explicit calculation of all coefficients is a cumbersome task [2]. Of particular interest for us is, however, the term $1/p^2p'^2q^2$. We find that only the two diagrams with hard gluon exchange shown in Fig. 1 give rise to this structure. Using the factorization formula for the four-quark condensate [11], this contribution is easily calculable, leading to

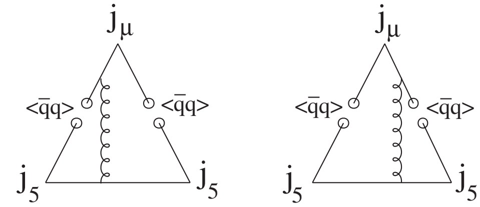

FIG. 1. $\Gamma_{\rm cond}^{\langle\bar\psi\psi\rangle^2}$: diagrams leading to the structure $\propto \alpha_s\langle\bar\psi\psi\rangle^2/(q^2p^2p'^2)$.

$$\Gamma_{\rm cond}^{\langle\bar\psi\psi\rangle^2} = \frac{8}{9}\alpha_s\langle\bar\psi\psi\rangle^2\frac{1}{p^2p'^2q^2}. \tag{3.15}$$

This result is sufficient for our argument and we do not need the explicit GG contribution: because of its dimension, the operator GG leads to a constant term, whereas we are interested in $1/q^2$ terms.

Now, notice that the perturbative term is power suppressed at large q^2 compared to the nonperturbative contribution, which dominates the correlator. In order to use the sum rule for the analysis of the form factor at large q^2, precisely following Ref. [3] we doubly Borelize the function $\Gamma(q^2, p^2, p'^2)$ in the variables p^2 and p'^2, choose the same Borel parameter M^2 for both channels, and send $M^2 \to \infty$.

The contribution of order $1/q^2$ which survives on the QCD side is the one given by Eq. (3.15). Other structures in $\Gamma_{\rm cond}^{\langle\bar\psi\psi\rangle^2}$ are either killed by the double Borel transform or, similar to $\Gamma_{\rm pert}$, have a $1/q^4$ behavior.

At order $1/q^2$ the sum rule takes the form

$$-4\frac{\langle\bar\psi\psi\rangle^2}{f_\pi^2}F_\pi(q^2) + \cdots = \frac{8}{9}\pi\alpha_s\langle\bar\psi\psi\rangle^2\frac{1}{q^2}. \tag{3.16}$$

Clearly, we have the necessary analytic structure on the QCD side, but the coefficient is different from the known pQCD result [14]

$$F_\pi(q^2) = 8\pi\alpha_s f_\pi^2/(-q^2). \tag{3.17}$$

This does not seem to signal an error in our calculation: on the hadronic side there are contributions coming from other states which are not suppressed (since we have set $M^2 \to \infty$). So we do not see a possibility to isolate the pion contribution and thus Eq. (3.16) is a genuine sum rule which relates the sum of several hadron contributions to a single term of the OPE.

To summarize, we do not find any disagreement between the large-q^2 behavior of the pion form factor and the OPE for $\langle PVP\rangle$. Of interest for us is that, unlike the case of the 2-point correlators, in the chiral limit there is no local duality between the pion double pole and separate double-pole terms in the OPE.

We would like to add one more comment on the interesting finding of Ref. [13] on the absence of the α_s/q^2 term in the double spectral density of the quark triangle diagram. This result may pose difficulties for the analysis of the large-q^2 behavior of the pion form factor within the constituent quark model [15] since this model makes use of the $\bar{q}\gamma_5 q$ structure for the description of the pseudoscalar meson.

B. $\langle AVP\rangle$

Next, we discuss the correlator

$$\Gamma_{\mu\alpha}(q,p,p') \equiv i^2 \int d^4x d^4y e^{iqx-ipy}\langle T(j^5_\alpha(y) j_\mu(x) j^{5\dagger}(0))\rangle, \quad p' = p - q. \tag{3.18}$$

Let us calculate $q^\mu \Gamma^5_{\mu\alpha}(q,p,p')$ and $p^\alpha \Gamma^5_{\mu\alpha}(q,p,p')$. We make use of the divergence equations for j^5_α and j_μ, and the commutation relations

$$\begin{aligned} [j_0(0,\vec{x}), \bar{u}\gamma_\alpha\gamma_5 d(0)] &= \bar{u}\gamma_\alpha\gamma_5 d(0)\delta(\vec{x}), \\ [j_0(0,\vec{x}), \bar{d}\gamma_5 u(0)] &= -\bar{d}\gamma_5 u(0)\delta(\vec{x}), \\ [\bar{u}\gamma_0\gamma_5 d(0,\vec{x}), \bar{d}\gamma_5 u(0)] &= -(\bar{u}u + \bar{d}d)\delta(\vec{x}). \end{aligned} \tag{3.19}$$

Taking into account the relation

$$\int d^4x e^{iqx}\langle T(j_\mu(x)(\bar{u}u + \bar{d}d)\rangle = 0 \tag{3.20}$$

in the evaluation of $p^\alpha \Gamma^5_{\mu\alpha}$, we find the following Ward identities:

$$\begin{aligned} q^\mu \Gamma_{\mu\alpha}(q,p,p') &= \Pi^5_\alpha(p') - \Pi^5_\alpha(p), \\ p^\alpha \Gamma_{\mu\alpha}(q,p,p') &= \Pi^5_\mu(p') + (m_u + m_d)\Gamma_\mu(p,p',q). \end{aligned} \tag{3.21}$$

In the chiral limit, by virtue of Eq. (2.8), these Ward identities take the form

$$\begin{aligned} q^\mu \Gamma_{\mu\alpha}(q,p,p') &= 2i\langle\bar{\psi}\psi\rangle\left(\frac{p'_\alpha}{p'^2} - \frac{p_\alpha}{p^2}\right), \\ p^\alpha \Gamma_{\mu\alpha}(q,p,p') &= 2i\langle\bar{\psi}\psi\rangle\frac{p'_\mu}{p'^2}. \end{aligned} \tag{3.22}$$

We may split $\Gamma_{\mu\alpha}$ into transverse and longitudinal parts,

$$\Gamma_{\mu\alpha}(q,p,p') = \Gamma^\perp_{\mu\alpha}(q,p,p') + \Gamma^L_{\mu\alpha}(q,p,p'), \tag{3.23}$$

where $q^\mu \Gamma^\perp_{\mu\alpha} = 0$, $p^\alpha \Gamma^\perp_{\mu\alpha} = 0$. Making use of the Ward identities (3.22), we determine the longitudinal part $\Gamma^L_{\mu\alpha}$ in the form

$$\Gamma^L_{\mu\alpha}(q,p,p') = 2i\langle\bar{\psi}\psi\rangle\left[\frac{q_\mu}{q^2}\left(\frac{p'_\alpha}{p'^2} - \frac{p_\alpha}{p^2}\right) + \left(g_{\mu\alpha} - \frac{q_\mu q_\alpha}{q^2}\right)\frac{1}{p'^2}\right]. \tag{3.24}$$

The transverse part $\Gamma^\perp_{\mu\alpha}$ may be parametrized by two invariant amplitudes:

$$\begin{aligned} \Gamma^\perp_{\mu\alpha}(q,p,p') = {}& \left(g_{\mu\alpha} - \frac{q_\mu p_\alpha}{qp}\right) iF_1(q^2,p^2,p'^2) \\ &+ \left(p_\mu - \frac{qp}{q^2}q_\mu\right)\left(q_\alpha - \frac{qp}{p^2}p_\alpha\right) \\ &\times iF_2(q^2,p^2,p'^2). \end{aligned} \tag{3.25}$$

The terms in $\Gamma^5_{\mu\alpha}$ singular at $q^2 = 0$ should cancel each other since there is no massless hadron state in the q^2 channel. This leads to the relation

$$F_2(0,p^2,p'^2) = -\frac{8\langle\bar{\psi}\psi\rangle}{p'^2(p^2 - p'^2)}, \tag{3.26}$$

valid for all values of p^2 and p'^2.

Let us look at the structure $p_\mu p_\alpha$ in $\Gamma_{\mu\alpha}$ at $q^2 = 0$. Because of the Ward identity (3.26), Eqs. (3.24) and (3.25) lead to the following expression for this structure:

$$\frac{4\langle\bar{\psi}\psi\rangle}{p^2 p'^2} p_\mu p_\alpha. \tag{3.27}$$

At small values of p^2 and p'^2, and for all values of q^2, the Goldstone pole dominates the correlator, leading to the following $p_\mu p_\alpha$ term:

$$\frac{4\langle\bar{\psi}\psi\rangle}{p^2 p'^2} F_\pi(q^2) p_\mu p_\alpha. \tag{3.28}$$

Comparing Eq. (3.28) at $q^2 = 0$ with Eq. (3.27), we see that the Ward identity guarantees the correct normalization of the pion form factor $F_\pi(0) = 1$.

The other region where rigorous relations for the correlator may be obtained is the region of large q^2, p^2, p'^2. Here the OPE may be applied. Perturbative diagrams, being $O(m)$, do not contribute to the correlator in the chiral limit. So the leading contribution to the OPE series for $\Gamma_{\mu\alpha}$ comes from the $\bar{\psi}\psi$ operator. This contribution is given by the triangle diagrams with quark-condensate insertions in one of the quark lines in the loop. The result reads

$$\begin{aligned} \Gamma^{\langle\bar{\psi}\psi\rangle}_{\mu\alpha}(q,p,p') = i\langle\bar{\psi}\psi\rangle\Bigg(& \frac{p_\mu p'_\alpha + p'_\mu p_\alpha - g_{\alpha\mu} pp'}{p^2 p'^2} \\ &+ \frac{q_\mu p'_\alpha + p'_\mu q_\alpha - g_{\alpha\mu} qp'}{q^2 p'^2} \\ &+ \frac{p_\mu q_\alpha - q_\mu p_\alpha - g_{\alpha\mu} qp}{p^2 q^2}\Bigg). \end{aligned} \tag{3.29}$$

This expression fully saturates the chiral Ward identities (3.22).

Let us now study which operator in the OPE corresponds to the pion contribution at large q^2. To this end, we look at the Lorentz structure $p_\mu p_\alpha$, perform the double Borel transform of $\Gamma_{\mu\alpha}$, and take the limit $M^2 \to \infty$. After this procedure, the pion double pole takes the form

$$4F_\pi(q^2)\langle\bar{\psi}\psi\rangle p_\mu p_\alpha. \tag{3.30}$$

Taking into account the α_s/q^2 behavior of the pion form factor, we now look for the term on the OPE side which, after the transformations described above, behaves as $1/q^2$. Only the dimension-5 operator $\bar{\psi}\sigma G\psi$ can lead to such a structure; the operator $\bar{\psi}\psi$ leads to the constant term

$$2\langle\bar{\psi}\psi\rangle p_\mu p_\alpha, \tag{3.31}$$

which is relevant for continuum states, and not for the pion. The contribution of the $\bar{\psi}\sigma G\psi$ operator to the correlator has been found in Ref. [16]:

$$\frac{1}{3}\langle\bar{\psi}\sigma G\psi\rangle\left(\frac{1}{q^2p'^4}-\frac{q^2}{p^4p'^4}\right)p_\mu p_\alpha. \tag{3.32}$$

This structure does not contribute after performing the double Borel transform and sending $M^2\to\infty$. The radiative corrections to the Wilson coefficient have not been calculated. Nevertheless, the term in the OPE series

$$\frac{C\alpha_s}{q^2p^2p'^2}\langle\bar{\psi}\sigma G\psi\rangle p_\mu p_\alpha \tag{3.33}$$

leads to

$$\frac{C\alpha_s}{q^2}\langle\bar{\psi}\sigma G\psi\rangle p_\mu p_\alpha. \tag{3.34}$$

All other possible kinematic structures of the relevant dimension vanish in the limit $M^2\to\infty$. Therefore, the duality relation takes the form

$$4F_\pi(q^2)\langle\bar{\psi}\psi\rangle+\cdots=C\frac{\alpha_s}{q^2}\langle\bar{\psi}\sigma G\psi\rangle, \tag{3.35}$$

with the dots standing for contributions of other hadronic states (such as pseudoscalar-axial meson transition form factors, etc.). We do not see a possibility to isolate the pion term. Nevertheless, it is interesting that, while at small values of q^2 the pion pole is due to Ward identities "dual" to $\langle\bar{\psi}\psi\rangle$, at large values of q^2 it contributes to the sum rule for a different operator, viz., $\langle\bar{\psi}\sigma G\psi\rangle$.

C. $\langle AVA\rangle$

The extraction of the pion form factor from the correlator

$$\Gamma_{\mu\alpha\beta}(q,p,p')\equiv i^2\int d^4xd^4ye^{ip'z-ipy}\langle T(j^5_\alpha(y)j_\mu(0)j^{5\dagger}_\beta(z))\rangle,$$
$$q=p-p', \tag{3.36}$$

is known quite well [2,3,17]. We would only like to point out the following features of this case. To consider the pion form factor at large q^2 one has to proceed along the lines of Ref. [3] which we followed above: namely, to perform the double Borel transform and to take $M^2\to\infty$. Then the condensate contributions vanish and one obtains the "local-duality" representation for the form factor [3]

$$f_\pi^2F_\pi(q^2)=\int_0^{s_0}ds_1\int_0^{s_0}ds_2\Delta_{\rm pert}(s_1,s_2,q^2), \tag{3.37}$$

where s_0 is the pion duality interval and $\Delta_{\rm pert}$ is the double spectral density of the $p_\mu p_\alpha p_\beta$ structure of $\Gamma_{\mu\alpha\beta}$. Similarly, one has the local-duality representation for the decay constant

$$f_\pi^2=\int_0^{s_0}ds\rho_{\rm pert}(s), \tag{3.38}$$

where $\rho_{\rm pert}(s)=(1+\alpha_s/\pi)/4\pi^2$ is the spectral density of the transverse invariant amplitude $\Pi_T(q^2)$ (2.11). Equation (3.38) yields $s_0=4\pi^2f_\pi^2/(1+\alpha_s/\pi)$. Because of the vector Ward identity for $\Gamma_{\mu\alpha\beta}$, one has the relation [18]

$$\lim_{q^2\to 0}\Delta_{\rm pert}(s_1,s_2,q^2)=\delta(s_1-s_2)\rho_{\rm pert}(s). \tag{3.39}$$

Clearly, in $\Delta_{\rm pert}$ and $\rho_{\rm pert}$ radiative corrections up to the same order should be included. Recently, the calculation of the $O(\alpha_s)$ contributions to $\Delta_{\rm pert}$ has been reported [19]. For large negative q^2 and at fixed s_1 and s_2, the spectral density was found to behave like

$$\Delta_{\rm pert}(s_1,s_2,q^2)\to-\frac{\alpha_s}{2\pi^3q^2}+O(1/q^4). \tag{3.40}$$

Interestingly, substituting this expression into the representation (3.37) one precisely reproduces the asymptotic pQCD result (3.17).

Thus the local-duality representation for the form factor has several attractive features: (i) It is applicable for all spacelike momentum transfers. (ii) At $q^2=0$ the form factor is properly normalized due to the Ward identity. (iii) At large q^2 the form factor behaves in accordance with pQCD. As a result, the local-duality representation with the spectral density which includes $O(\alpha_s)$ corrections [19] is expected to give reliable predictions for the form factor for all values of q^2.

To summarize, in the $\langle VAV\rangle$ case it is possible to relate the pion form factor for all spacelike q to a definite part of the perturbative contribution.

IV. SUMMARY AND RESULTS

We analyzed 2- and 3-point correlators of the pseudoscalar and axial currents. Because of the partial conservation of the axial current, the OPE for correlators of this current exhibits a rather specific feature: namely, in the chiral limit the OPE is dominated in many cases by nonperturbative power corrections.

Our results are as follows:

(i) A detailed study of the OPE and duality for the $\langle PP\rangle$, $\langle AA\rangle$, and $\langle AP\rangle$ correlators was presented. We pointed out that quark-hadron duality for $\langle AP\rangle$, similar to the longitudinal structure of $\langle AA\rangle$, has an interesting feature: in the chiral limit the pion contribution turns out to be dual to a single $\bar{\psi}\psi$ term in the OPE. We discussed sum rules for $\langle PP \rangle$, $\langle AA \rangle$, and $\langle AP \rangle$, which are governed by the quark condensates $\langle \bar{u}u \rangle$ and $\langle \bar{d}d \rangle$, and the two independent functions $\mathrm{Im}\Pi_T(q^2)$ and $\mathrm{Im}\Pi_5(q^2)$. Accordingly, one can obtain two independent QCD sum rules: The first sum rule, for the transverse structure of the $\langle AA \rangle$ correlator, involves $\mathrm{Im}\Pi_T(q^2)$. On the hadronic side, it includes the contributions of axial-meson states, but contains also f_π^2 as the reminder of the Goldstone nature of the pion. This sum rule was used for the extraction of f_π^2 in [1]. The second sum rule, for the $\langle PP \rangle$ correlator, involves $\mathrm{Im}\Pi_5(q^2)$. On the hadronic side, it contains contributions of the ground-state and excited light pseudoscalars. According to our estimates, this sum rule receives a sizable contribution from the excited $\pi'(1300)$ and provides a promising possibility to extract its decay constant $f_{\pi'}$.

(ii) We studied properties of the 3-point correlators $\langle PVP \rangle$ and $\langle AVP \rangle$. We derived the Ward identities for these correlators and demonstrated the way the normalization of the pion form factor at zero momentum transfer arises due to these Ward identities. We then analyzed the region of large momentum transfers and the way quark-hadron duality works in these cases. We proved that the OPE for $\langle PVP \rangle$ and $\langle AVP \rangle$ at large q^2 is dominated by nonperturbative corrections, and identified the operators responsible for providing the correct large-q^2 asymptotics of the pion form factor in accordance with pQCD: the four-quark condensate $\langle \bar{\psi}\psi\bar{\psi}\psi \rangle$ in the case of $\langle PVP \rangle$, and the mixed condensate $\langle \bar{\psi}\sigma G\psi \rangle$ in the case of $\langle AVP \rangle$. We have thus disproved the recent statement in the literature that the pion form factor as extracted from the $\langle PVP \rangle$ correlator does not have the right asymptotics required by pQCD.

(iii) For the $\langle AVA \rangle$ correlator, we pointed out that the local-duality representation for the pion form factor with the double spectral density, which includes the radiative corrections, is applicable for all spacelike momentum transfers and has the following interesting features: At $q^2 = 0$ the form factor is properly normalized due to the vector Ward identity, and at large $q^2 < 0$ it reproduces the pQCD asymptotic behavior. Therefore, this parameter-free representation is expected to give reliable predictions for all spacelike momentum transfers.

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