Behavior of single-scale hard small-*x* processes in QCD near the black disk limit

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We argue that at sufficiently small Bjorken *x* where pQCD amplitudes rapidly increase with energy and violate probability conservation the shadowing effects in the single-scale small *x* hard QCD processes can be described by an effective quantum field theory of interacting quasiparticles—perturbative QCD ladders. We find, within the WKB approximation, that the smallness of the QCD coupling constant ensures the hierarchy among many-quasiparticle interactions evaluated within the physical vacuum and, in particular, the dominance in the Lagrangian of the triple quasiparticle interaction. It is explained that the effective field theory considered near the perturbative QCD vacuum contains a tachyon relevant for the divergency of the perturbative QCD series at sufficiently small *x*. We solve the equations of motion of the effective field theory within the WKB approximation and find the physical vacuum and the transitions between the false (perturbative) and physical vacua. Classical solutions which dominate transitions between the false and physical vacua are kinks that cannot be decomposed into perturbative series over the powers of α_s. These kinks lead to color inflation and the Bose-Einstein condensation of quasiparticles. The account of the quantum fluctuations around the WKB solution reveals the appearance of the ''massless'' particles—phonons. It is explained that phonons are relevant for the black disk behavior of cross sections of small *x* processes. The Bose-Einstein condensation of the ladders produces a color network occupying a ''macroscopic'' longitudinal volume. We discuss briefly the possible detection of new QCD effects. We outline albeit briefly the relationship between the small *x* hard QCD processes and the coherent critical phenomena.

I. INTRODUCTION

One of the challenging properties of QCD is the rapid increase with energy of the cross sections of the hard processes. Initially the increase has been predicted within the leading order DGLAP approximation [1,2]. The rapid increase of the structure functions of the proton with the energy has been observed in ep scattering, for the review and proper references see Ref. [3]. The deep-inelastic structure functions of a proton, calculated in the perturbative QCD (pQCD) within the leading-twist (LT) approximation, can be fitted at small *x* as $F_{2p}(x, Q^2)$, $xG_p(x, Q^2) \propto x^{-\mu(Q^2)}$, where $\mu \approx 0.2$ at $Q^2 \approx 10$ GeV^2. The generalization of the QCD factorization theorems to the amplitudes of hard diffractive processes shows that these cross sections (that are higher-twist effects) should increase with energy even faster than the structure functions, calculated in the leading-twist approximation [4,5]. Predicted by the perturbative QCD increase $\propto x^{-2\mu(Q^2)}$ of cross sections of hard diffractive processes, has been recently observed at HERA (DESY) in the diffractive photo-production of the heavy flavor mesons J/ψ, Y, in the high Q^2 diffractive electroproduction of ρ^0 mesons, for the review and references see Ref. [3]. The rapid increase of amplitudes of hard processes with energy predicted in perturbative QCD within the DGLAP approximation can not continue forever since it violates at sufficiently small *x*

the strict inequality: $\sigma(\text{tot}) \geq \sigma(\text{diff})$. Here $\sigma(\text{tot})$ is the full, and $\sigma(\text{diff})$ is the differential cross-section, and the cross-section $\sigma(\text{tot})$ is calculated within the LT approximation, see Ref. [6].

To quantify the theoretical challenge it is useful to introduce an auxiliary theoretical object—the amplitude describing the scattering of colorless and spatially small dipole of the transverse size $\approx 1/Q$ of a hadron target or of another small size dipole. It follows from the probability conservation that properly normalized such amplitude for the scattering at the fixed impact parameter *b* should be restricted from above: $|f(b, Q^2, s)| \leq 1$, see Refs. [7,8] and references therein. This inequality is violated within the DGLAP approximation beyond the value of Bjorken *x* equal to $x_{\text{cr}}(Q^2)$. Probability conservation restricts region of applicability of the DGLAP approximation but it does not precludes the fast increase of the dipole cross section $\sigma_{\text{dipole}T} \sim \log^2(s/s_0)$ corresponding to black disk limit, cf. Ref. [9]. The structure functions should increase with energy even faster: $F_{2p}, xG_p \propto Q^2 \ln^3(x_o/x)$ resulting from the increase with energy of essential impact parameters and from the ultraviolet divergence of hadronic contribution into renormalization of the electric charge, see Ref. [9].

The applicability of the competing leading order (LO) $\alpha_s \ln(x_o/x)$ approximation requires rather large energies, because the convergency of the perturbative QCD series at small *x* is rather slow, (c.f. Eq. (2.5) below). The evaluation

of the next-to-leading order effects found huge correction and significant reduction of the increase of the amplitudes with energy obtained within LO approximation [10] due to the necessity to restore the energy-momentum conservation [11,12]. The further reduction of the energy dependence of the amplitude has been found in Ref. [12] where the separation of scales characteristic for hard and soft QCD processes has been performed and the running of the coupling constant and the energy-momentum conservation were taken into account. The practical conclusion is that almost in the whole range of LHC energies the LO + NLO DGLAP and BFKL approximations predict rather similar energy dependence of the structure functions, cf. Ref. [13]. It seems now that the challenge of violation of probability conservation becomes important at lesser energies than the difference between the different pQCD calculations reveals itself at small x.

Small x physics problem for the two-scale processes has been discussed recently in [8,14–18] within the color glass condensate (CGC) approach. The unitarity of scattering matrix has not been achieved within the made approximations [19]. (See however the Refs. [20] whose approach is similar to that used in Ref. [9] but different from the mechanism discussed in this paper.)

The perturbative QCD calculation of the scattering of a color-neutral two-gluon dipole of the size $\approx 1/Q$ off a proton target violates probability conservation for $Q^2 \approx 10$ GeV^2 for the scattering at central impact parameters at $x \lesssim x_{\rm cr}(Q^2) \sim 10^{-5}$–$10^{-4}$, i.e. in the kinematics typical for the large hadron collider (LHC) [7]. The puzzle arises in the kinematics when few gluons are produced only. It reveals itself in the gluon structure function of a proton as the consequence of the large gluon density in the initial condition for QCD evolution and the rapid increase with energy of the perturbative QCD amplitudes [7]. The violation of the leading-twist approximation should become important at the values of x where the perturbative QCD calculations in the leading-twist approximation are still more or less unambiguous. The scale of the hard processes $Q^2 \sim 10$ GeV^2 is chosen to guarantee the smallness of the running invariant charge. Note that the existence of the discussed above puzzle in the amplitude for the scattering of two spatially small dipoles investigated in this paper is unrelated to the poorly understood physics of the quark confinement and the spontaneous broken chiral symmetry.

With the increase of the collision energy the higher-twist effects blow up: higher is the twist of the term—more rapidly it increases with the energy. At sufficiently small x the perturbative QCD series have the form:

$$\sum_{n=0}^{n=\infty} c_n (1/x)^{n\mu}$$

where coefficients c_n are rapidly increasing with n. The rapid increase of c_n with n is known for a long time from the study of the topology of the iterations of the ladder

diagrams [21]. The challenging problem is to develop the unambiguous method of summing the divergent series. One of the aims of this paper is the reconstruction of the nonperturbative terms related to this divergence, the terms that can not be decomposed into powers of α_s.

At sufficiently large energies, the hadron scattering at central impact parameters becomes completely absorptive, as the consequence of the compositeness of the projectile and the increase of the interaction with the energy, see Ref. [22] and references therein. The FNAL data on the differential cross sections of elastic pp collisions shows that the pp scattering at central impact parameters is black (cf. discussion in Refs. [23,24]). This fact can be considered as an experimental hint of expected limiting behavior of hard processes also at small x where the interactions become strong.

Another distinctive property of small x hard processes is the rapid increase with energy of the longitudinal distances important in the scattering process. The coherence length l_c in the total cross section of the deep-inelastic scattering in the kinematics near the black disk limit is increasing with energy as $l_c \propto x^{\mu(Q^2)-1}$. This increase is somewhat less rapid than the one familiar from the analysis of the leading-twist approximation, see Ref. [25] and references therein. Nevertheless, at current and future accelerators l_c significantly exceeds the static radius of a hadron or nucleus. In fact, l_c becomes comparable with the electromagnetic radius of the hydrogen atom in the kinematic region to be studied at the Large Hadron Collider. As a result, a variety of observable new coherent phenomena are expected to appear in the small-x processes [6]. In addition it has been shown that in the coordinate space the correlators of the currents evaluated within both the DGLAP and BFKL approximations increase rapidly with the distance [25,26]. Such increase is a necessary condition for the onset of the critical phenomena [27]. The perturbative QCD produces branch points in the angular-momentum plane located at $j \gtrsim 1$—i.e. in the region forbidden by the causality and conservation of probability. Thus the theory has a tachyon. In this respect, certain similarity may exist between the theory of small x phenomena and the theory of strings in 26 dimensions where tachyon is also present in the perturbative (nonphysical) vacuum, cf. discussion in Ref. [28]. In these theories the account of the nonperturbative phenomenona is necessary to find a true ground state.

The well understood property of QCD is the strong dependence on Bjorken x of the piece of the quark-gluon component of the wave function of the virtual photon that dominates the structure function of a hadron target. At moderately small x, both soft components in the wave function of the virtual photon and the quark-gluon configurations, where the constituents have large relative transverse momenta, give comparable contributions, in the spirit of the aligned jet model [29,30]. With the increase of the

collision energy this conspiracy disappears: the hard configurations that occupy most of the phase volume in the photon wave function begin to dominate due to the rapid increase of the hard cross sections with the energy. This corresponds to the serious change of the Q^2 dependence of the structure functions to the regime where $F_2(x, Q^2) \propto Q^2 \ln(x_0/x)^3$, for the review and references see Ref. [31]. In the new regime hard pQCD phenomena should dominate in the structure functions. The cross section of the diffractive production of quark-gluon system with large mass determines the triple-ladder vertex which enters our calculations. In order to simplify our task we restrict ourselves to the kinematics where the wave function of the longitudinally polarized photon $\gamma_L^*(Q^2)$ is dominated by the configurations of the constituents with large transverse momenta and large invariant mass—the hard perturbative QCD analogue of the triple-reggeon limit. The account of this phenomenon helps to evaluate the triple-reggeon vertex near the black disk limit where the leading logarithmic approximations are violated.

A necessary kinematical condition of the applicability of our method is $x \lesssim x_{cr}(Q^2) \cdot 10^{-2}$. Here $x_{cr}(Q^2)$ can be determined from the condition that the contribution of one ladder in dipole-dipole scattering at central impact parameters is near the black disk limit in the leading-twist approximation. This inequality is just the condition for the existence of the triple-Regge limit and it is the main kinematical limitation of our approach.

The aim of this paper is to show that some of the challenges discussed above can be met for one-scale hard small x processes, such as $\gamma_L^*(Q^2) + \gamma_L^*(Q^2) \rightarrow$ hadrons, $\gamma + \gamma \rightarrow Y + Y$, etc. We restrict ourselves by the consideration of the one-scale hard processes where the contribution into the amplitude of the QCD evolution with scale is suppressed. Consequently the factorization of the information on the wave functions of the projectile and target from the interaction, similar to the one that occurs in the Regge-pole exchanges, becomes a reasonable approximation.

The rapid increase of the perturbative QCD amplitudes with energy leads to even more rapid increase of the role of the shadowing effects. It was found long ago that the generalization of the technology of calculations of shadowing effects, encoded in the Reggeon Field Theory, may appear useful for the description of the small x processes [32]. The generalization of the latter approach, which accounts for any diffractive processes, leads to the effective field theory (EFT). The evident advantage of the EFT, as compared to models based on elastic eikonal approximation (see i.e. Ref. [33]) is in the possibility to account for the whole variety of rescattering effects, and to ensure the energy—momentum conservation. In addition, we will show in this paper, that the transition to the black disk limit (BDL) in hard processes has a certain resemblance to a critical phenomena.

Note that the soft QCD contribution to the scattering processes is almost absent in the chosen processes. This suppression is achieved by the choice of the longitudinally polarized highly virtual photons as the projectile and the target. Although the method developed in the paper is inapplicable beyond the one-scale hard processes, the new QCD phenomena we found in the paper may appear important for the many-scale hard processes as well.

To account for the coherence of the high-energy processes and the rapid increase with energy of the amplitudes of the hard small-x processes, we construct an effective field theory (EFT) of the interacting perturbative colorless ladders. The evident advantage of this approach is the significant reduction of the number of variables in the problem and the mapping of the physics of the coherent processes into the framework resembling the statistical models. (This approach is in the spirit of the statistical models of critical phenomena that account for the interactions between the major modes only. The specifics of the physics of the large longitudinal distances is included in the concept of the quasiparticle.) The interaction between the quasiparticles, when the amplitudes of the hard processes are near the unitarity limit, can be easily evaluated within the WKB approximation. The use of the WKB approximation is justified by the smallness of the running coupling constant in the perturbative QCD. We show that account of the running coupling constant helps to establish the dominance of the triple-ladder vertex (see Appendix B). The smallness of the multiladder vertices in pQCD implies that the basic phenomena that characterizes the black disk limit should be insensitive to the restriction by the triple-ladder vertex. This observation helps to fix the form of the Lagrangian of the EFT near the unitarity limit.

Since $\mu \gtrsim 0.2$ in pQCD, the quasiparticle of the EFT is a tachyon: the singularities in the complex angular-momentum plane are located at $j - 1 = n\mu$ where $n = 1, 2, \ldots$. Such a behavior is in the evident conflict with the bound for the total cross section of a small dipole-dipole scattering, that follows from the causality and conservation of probability in QCD. The causality and unitarity of the S matrix, however, are not necessarily valid in the perturbative QCD vacuum. The rapid increase of the amplitude with energy predicted by the perturbative QCD approximations (i.e. the presence of the tachyon in the EFT) reflects a rapid transition from the false, perturbative vacuum to the nonperturbative, physical vacuum. To visualize this phenomenon it is useful to consider physical processes in coordinate space [25]. So these approximations should fail to predict actual energy dependence of amplitudes of physical processes which is nonperturbative phenomenon.

We find in the paper that account of nonlinear phenomena leads to serious restructuring of produced QCD states besides the onset of black disk limit. With the Lagrangian of the EFT at hand, we may identify the order

parameter relevant for the critical phenomena in small-x processes: the order parameter is the condensate of the quasiparticles, that are the pQCD ladders. We find the new nonperturbative phenomena in the regime of the strong interaction with a small coupling constant, including the new nonperturbative classical fields, whose effects are $\propto \exp(1/\alpha_s)$, the color inflation due to the tunneling transitions, the zero modes in the dominant classical fields and related massless particles—"phonons" in the EFT, and the formation of a color network where the overlapping quasiparticles (ladders) do exchange the colored constituents.

The approach developed in this paper differs from the EFT suggested in Ref. [34]. The major practical differences are that we restrict ourselves by one-scale hard processes only, in the dominance in the WKB approximation of the tunneling transitions and the quantum fluctuations around them, while the contribution of the quantum loops calculated near the vacuum of the pQCD vacuum is negligible. Another fundamental difference from the approach of [34] as well as from the Color Glass Condensate approaches (cf. reviews [8,16,17]) is in the account of the important role played by the diffusion to large impact parameters. Near the unitarity limit this diffusion leads to the fast rise of structure functions with energy [9]. In our paper we take into account the universal Gribov diffusion related to randomness of radiation [35], but neglect the diffusion to large distances due to the running coupling constant and the diffusion to small distances related to the increase of the final state phase volume. The latter two types of diffusion were shown in Ref. [12] to be strongly suppressed after an account of the next-to-leading order BFKL type approximation. There is no systematic evaluation of these effects because of the sensitivity to the restriction of the phase volume of the produced particles and related sensitivity to NLO approximations.

In this paper we heavily use results of analysis of particular preQCD model of Reggeon field theory obtained in [36–40]. The major difference from these papers is that dominance of triple-ladder vertex is justified in QCD. Besides we found that EFT predicts significant overlap between ladders near BDL. In the Appendix A we account for the exchange of the constituents between the overlapping ladders. At sufficiently small x this leads to the formation of the color network instead of a system of ladders (see discussion in Appendix A).

The paper is organized in the following way. In Sec. II we introduce the concept of the pQCD ladder as the quasiparticle of the EFT. We explain the hierarchy among the multiladder vertices in pQCD (see also Appendix B). We show that a restriction to the triple-ladder vertex evaluated within the physical vacuum is sufficient for the theoretical description of the phenomena near the unitarity transition, and we construct the Lagrangian of the EFT. In Secs. III and IV we discover and analyze the analogies with

the critical phenomena. In Sec. V we briefly discuss observable phenomena that follow from the solution of the effective field theory. Our results are summarized in the conclusion. In Appendix A we show that color is confined within ladders in the tube of the small transverse size. We argue that a color network appears once the ladders overlap significantly in space. In Appendix B we present the estimates of the multiladder vertices within the physical vacuum.

II. EFFECTIVE FIELD THEORY IN QCD

The construction of an effective field theory requires the knowledge of the dominant degrees of freedom—the quasiparticles—and their interactions. We assume, based on the linear pQCD calculations, that the major degrees of freedom are the pQCD color-singlet ladders with the two-gluon state in the t channel, that we will denote the "*Pomeron*". (The four gluon exchange, where each pair of gluons forms an 8_F representation of the $SU(3)_c$, is included in this Pomeron.) Such a definition is of common use, although the pQCD "Pomeron" has no direct relation to the Regge-pole Pomeron exchange relevant for the successful description of the phenomena dominated by the nonperturbative soft QCD. In principle, the intercept of Pomeron depends on Q^2 in pQCD. However at very small x the Q^2 dependence of Pomeron intercept becomes weak [12,41], and the factorization of information on projectile and target is valid for a ladder because of the disappearance of the color transparency phenomenon.

The contribution of the pQCD ladders that have 4, 6, ... gluons in the t channel where pairs of gluons form the color representations 8_D, 10, $\overline{10}$, or 27 cannot be topologically and analytically reduced to the contribution of the two-gluon exchange ladder. In principle it is necessary to introduce the new varieties of quasiparticles into the Hamiltonian of the EFT and to account for their interactions. However, the smallness of the running coupling constant leads to the hierarchy of the interactions especially within large N_c limit evaluated within the physical vacuum. The account of this hierarchy justifies the neglect of this kind of quasiparticles. In particular, it is well known that the two-gluon colorless ladder has maximal μ as compared to the colored ladders. The contributions of all quasiparticles except the ladders with the two-gluon color-singlet intermediate states in t channel can be neglected even near the unitarity limit, where the amplitudes of the high-energy processes are close to the maximum permitted by the probability conservation.

When the contribution of one ladder to the amplitude becomes large it is necessary to take into account the shadowing effects. The technology of the evaluation of the rescattering effects due to the iterations of the pQCD ladder is the adaptation to pQCD of methods of the Reggeon Field Theory(RFT) [42]. The account of the shadowing effects leads to the $2 + 1$ dimensional field

theory where variables are the position of the leading singularity in the angular-momentum plane—$j - 1$ and the transverse momentum k_t. The equivalent but more convenient approach to the problem considered in the paper is to use the description in terms of the variables y-rapidity and $\vec{b}$-impact parameter. The effective Field Theory includes the Pomeron loops and all variety of multi-"Pomeron" vertices. Technologically it is more convenient to find the Lagrangian of the EFT rather than to evaluate the particular set of diagrams, cf. Ref. [21]. Consequently, we may restrict ourselves to the brief description of the Lagrangian of the EFT near the unitarity limit.

The equations of the EFT can be derived as the Lagrange equations of motion from an effective Lagrangian that contains five terms,

$$L = L_0 - L_1 - L_2 - L_3 - L_4. \qquad (2.1)$$

The first three terms have a straightforward interpretation in the case of noninteracting quasiparticles, while the other terms describe interactions between quasiparticles and with the virtual photon as the source. The L_0, L_1 and L_2 follow from the Mellin transformation of the Green function of the free quasiparticle, $G = [j - 1 - \mu(Q^2) - \alpha' k^2]^{-1}$, in the plane of the complex angular momentum j in the crossed channel. Thus

$$L_0 = \tfrac{1}{2}(q\partial_y p - p\partial_y q), \qquad (2.2)$$

where $p(y, b) = \psi^+$ and $q(y, b) = \psi$ are the quasiparticle fields. We denote $\partial_y = \partial_{\log(x_0/x)}$ where y is rapidity and $x = Q^2/(2PQ)$, P is the 4-momentum of the target. The quantity $x_0 \approx 0.1$ denotes the length of the fragmentation region where there are no $\log(x_0/x)$ factors. L_1 is the "mass" term,

$$L_1 = -\mu(Q^2)pq. \qquad (2.3)$$

This term accounts for the rise of the structure functions with energy as derived for the scattering of two dipoles within the LT approximation,

$$F_{2p}(x, Q^2), \qquad xG_p(x, Q^2) \sim (x_0/x)^{\mu(Q^2)}, \qquad (2.4)$$

with $\mu > 0$. The intercept $\mu(Q^2)$ has been evaluated in the pQCD in the next-to-leading order (for a review and references see Ref. [13]),

$$\mu \approx 4\log 2\alpha_s(Q^2)N_c/\pi(1 - 4(\alpha_s N_c/\pi)^{2/3}$$
$$- 6.5(\alpha_s N_c/\pi) + O(\alpha_s)^{4/3} + \cdots), \qquad (2.5)$$

where α_s is the effective coupling constant and N_c is the number of colors. The dependence of $\mu(Q^2)$ on Q^2 is rather weak at sufficiently small x [13,41]. Note also slow convergence of pQCD effects.

The term L_2 describes the dependence on the collision energy of the essential impact parameters. We assume that it has the form:

$$L_2 = \alpha'_P p \triangle_{\vec{b}} q. \qquad (2.6)$$

Here $\vec{b}$ is a two-dimensional impact parameter, and α'_P is the Pomeron slope, $\triangle_{\vec{b}}$ is Laplace operator in "b" space. The slope α'_P is small within the perturbative QCD. Within the approximation, that takes into account the Q^2 evolution, the running of the coupling constant, and to some extent the energy-momentum conservation, the fast QCD evolution to small and large distances (that exists beyond Gribov diffusion) is suppressed [12], and we can neglect it. The Gribov diffusion to large impact parameters due to randomness of gluon radiation [35] is still there. The parametric estimate gives: $\alpha'_P \propto N_c \alpha_s(Q^2)/Q^2$. At the same time, near the unitarity limit the additional mechanism (complementary to Gribov diffusion) of increase with energy of essential impact parameters, due to the fast increase of the amplitudes with energy, begins to play an important role [7,9,43]. There the effective α'_P cannot be negligible but it is difficult to evaluate it because of the sensitivity to the uncalculated nonleading order terms in the running coupling constant, etc. Since the exact form of L_2 is unknown at present it is chosen in the paper in the simplest form.

The evaluation of the multi-Pomeron vertices near the unitarity limit (see Appendix B) shows that within the WKB approximation, the relative contribution of the fourth and higher multi-Pomeron vertices is suppressed by the powers of α_s compared to the triple-ladder term. Thus for the description of hard QCD phenomena it should be sufficient to restrict ourselves to the triple-ladder interaction.

Near the unitarity limit the dominant contribution into the triple-"Pomeron" vertex κ:

$$L_3 = \kappa pq(p + q), \qquad (2.7)$$

is given by the component of the virtual photon wave function having transverse size $\leq 1/Q$. Really at moderately small x soft QCD and pQCD components of the photon wave function give comparable contributions into the total cross section and cross section of the diffraction. This is due to the conspiracy of small probability of the soft component $\propto 1/Q^2$ and the large cross section of the soft component interaction with a target. The probability of a hard component in the virtual photon wave function is ≈ 1 but the cross section of its interaction with a hadron target is $\propto 1/Q^2$—the aligned jet model [29,30]. This conspiracy disappears at sufficiently small x as the consequence of the rapid increase with the energy of the cross sections of the hard processes. So small x, and therefore small κ processes are dominated by hard QCD, for the review and references see Ref. [31]. Then at moderately small x the vertex κ is determined by the interplay of the soft and hard QCD dynamics. However with the decrease of x in the kinematics near the black disk limit the vertex κ should be dominated by the hard QCD.

In the absence of a detailed calculation, we limit our-selves to the dimensional estimate of κ and neglect any possible weak dependence of the coupling κ on y and on t. In the lowest order in the coupling constant, the triple-''Pomeron'' vertex, is due to the interaction of ladders via one gluon loop (see Fig. 1), where near the black disk limit hard QCD dominates, as explained in the above discussion. In addition the account of the running coupling constant, and Sudakov form factors suppresses the contribution of almost on shell effects. Our estimate gives

$$\kappa \propto \frac{\alpha_s^2 N_c}{\lambda}. \tag{2.8}$$

Here $\lambda \propto Q$ and it is somewhat increasing with the energy increase. This estimate of λ differs from the one obtained within the leading order BFKL approximation far from the black disk limit. The singularity in the dependence of the triple-''Pomeron'' vertex on the momentum transfer found in Ref. [44] reveals an important role of the soft QCD, the appearance of the effects encoded into aligned jet model and represents another challenging problem for the BFKL approximation at moderately small x. We follow the method of calculations of Ref. [12] where the BFKL approximation arises as the small x limit of pQCD series where separation of scales is made before summing them. In this case for single-scale hard processes κ is dominated by hard pQCD effects. Besides the nonperturbative con-tribution into the three—Pomeron vertex is suppressed by Sudakov form factors, and by the dominance of the hard regime near the black disk limit, cf. the above discussion. The existence (but not the properties) of the new QCD phenomena is not sensitive to the actual value of λ which is effectively the characteristic transverse momentum of the

FIG. 1.   Three Reggeon vertex

constituents of the pQCD ladder where it splits into the two new ones.

A tricky point in the evaluation of the ''multi-Pomeron'' vertices within the pQCD is the necessity to account for causality and energy-momentum conservation. Diagrams where singularities are located on the same side of the contour of integration in the energy plane (in particular eikonal diagrams) are suppressed by the powers of energy [42,45]). So we neglect the eikonal-type inelastic rescatter-ings since a bare particle may have one inelastic collision and any number of elastic collisions. For the interactions that rapidly increase with the energy, the requirements of the causality, the positivity of the probability for the physi-cal processes, and the energy-momentum conservation can be hardly satisfied within such a set of diagrams [31]. In contrast, the contribution of rescatterings due to an inelas-tic diffraction into the final state with the invariant mass M^2, where $\beta = Q^2/(Q^2 + M^2)$, is not too small. This contribution dominates in the two-scale hard small-x phe-nomena at $x \approx x_{cr}$. We include this contribution in the scale factor of the source. (This contribution can be inter-preted as the coupling of the Pomeron to secondary re-ggeon trajectories.)

Within the approximations made in this paper the cou-pling of the pQCD ladder to a hadron can be treated as the interaction with a source. The actual form of the $L_4 \equiv L_\gamma$ term is unimportant for the most of the results obtained in this paper.

Nonetheless, let us discuss the interaction of the ladders with a virtual photon as an external source in the center-of-mass frame of the reaction. There is the coupling with the projectile and target virtual photons:

$$L_\gamma = \int_0^1 \frac{dz}{z} d^2 b \Phi^2(z, \vec{b}, Q^2) q(z, \vec{B} - \vec{b}) c_{dipole} \delta(Y + y). \tag{2.9}$$

Here Y is the total rapidity, and $\vec{B}$ is the total impact parameter. In this formula we neglect the Q^2 dependence of the interaction in the vicinity of the black disk limit because, near the black disk limit, the color transparency phenomena and the related decomposition over twists dis-appear. In the above equation $\Phi^2(z, \vec{b}, Q^2)$ is the square of the dipole wave function of the virtual photon in the transverse parameter space, z is the fraction of the photon momentum carried by the constituent, and $dz/z = dy$, where y is the rapidity. The variable $\vec{b}$ is the transverse distance between the constituents in the photon wave function. The dipole wave function is given by (for sim-plicity we restrict ourselves to the collisions of longitudi-nally polarized photons only)

$$\Phi_L(b, z, Q^2)^2 = c(N_c \alpha_{em})(Qz(1-z))^2)$$
$$\times K_0^2(bQ\sqrt{z(1-z)}). \tag{2.10}$$

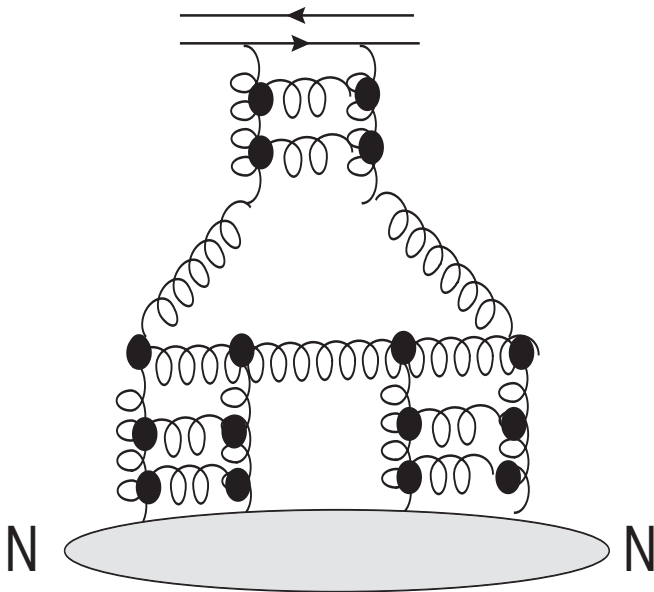The numerical factor c follows from a convention on the

normalization of this wave function. Note that this function is localized in the space of relative transverse distances between the constituents as $\exp(-bQ\sqrt{z(1-z)})$ and in the space of rapidities y. Since the distribution over z is centered at $z = 1/2$ in the case of the wave function of the longitudinally polarized photon, we can safely assume that this contribution into the source corresponds to the finite y negligible as compared to the full rapidity span. So we assume for certainty that $Y - y \sim 0$. The contribution of inelastic diffraction that dominates at not extremely small x [31] corresponds to $-Y \leq y \leq -Y + y_{\text{dif}}$. Since our interest is in $y \gg y_{\text{dif}}$ we may skip in the analysis of EFT the difference between $-Y + y_{\text{dif}}$ and $-Y$, although this approximation restricts region of applicability of approximations made in the paper.

Altogether, we obtain the Lagrangian:

$$L = 1/2(q\partial_y p - p\partial_y q) - \alpha' p \triangle_b q + \mu pq - \kappa pq(p+q)$$

$$- c_{\text{dipole}} \int \exp(-bQ/2) q(y, \vec{B} - \vec{b}) d^2 b \, \delta(y + Y)$$

$$- c_{\text{dipole}} \int \exp(-bQ/2) p(y, \vec{B} - \vec{b}) d^2 b \, \delta(y - Y),$$

$$(2.11)$$

where $\kappa \propto (\alpha_s^2 N_c / \lambda)$, and c_{dipole} accounts for the normalization of the virtual photon wave function.

The interaction between overlapping ladders due to the exchange by the constituents will be considered in the end of the paper.

III. CRITICAL PHENOMENA IN HARD PERTURBATIVE QCD REGIME NEAR THE UNITARITY LIMIT

The form of the effective Lagrangian (2.11) relevant for the single-scale hard QCD phenomena near the unitarity limit is rather close to preQCD model analyzed in Refs. [36–40] (except for the form of the source term). In the further analysis we use the WKB solution of the Lagrangian equations of motion and quantum fluctuations around them found in Refs. [36–40] and adjust it to describe the hard QCD phenomena.

A. Classical solutions of EFT

The distinctive property of the Lagrangian (2.11) is the existence, in addition to the usual perturbative vacua

$$p = 0, \qquad q = 0, \qquad (3.1)$$

of the two new vacua:

$$p = \mu/\kappa, \qquad q = 0 \qquad (3.2)$$

and

$$p = 0, \qquad q = \mu/\kappa. \qquad (3.3)$$

Since Lagrangian of EFT is nonhermitean it has no vacuum

in the usual sense and the term "vacuum" really means the critical points of the action (2.11) of the EFT. The action (2.11) has actually four critical points: the three critical points (3.1) and (3.2) and an additional critical point $(\mu/(3\kappa), \mu/(3\kappa))$. However the contribution of the last point to the S-matrix (see below) is suppressed by $\exp(-HY) \sim \exp(-\mu^3/(27\kappa^2)Y)$ relative to three critical points (3.1) and (3.2). Consequently the fourth critical point can be neglected.

Having Lagrangian (2.11) we may deduce the Lagrangian equations of motion that will be analyzed first in the classical approximation. The system of nonlinear partial differential equations in $2 + 1$ dimensions is

$$-dq/dy = \alpha' \triangle q - \mu q + \kappa(2pq + q^2)$$
$$dp/dy = \alpha' \triangle p - \mu p + \kappa(2pq + p^2). \qquad (3.4)$$

This system has kink solutions. The boundary conditions in the system (3.4) are at $y = \pm\infty$ and correspond to the vacua (3.1), (3.2), and (3.3).

The detailed analysis of the kink solution is possible in $1 + 1$ dimensions only, where the above equations can be reduced to the ordinary differential equation. In Refs. [36–38] the family of kinks, characterized by a 2d velocity parameter v has been found. These kinks interpolate between two vacua, say (3.2) and (3.3). The action of the kink is finite $S \sim (\mu/\kappa)^2(2 - v)\phi_0^2$, where ϕ_0 is the field value at the impact parameter value $b = vY$, and v is the kink velocity. It is proportional to $1/\alpha_s^2$, where we used the dependence of μ and κ on N_c discussed in Sec. II. For the value of parameter $v = 2$ we obtain critical kinks with zero action. Quantum fluctuations around these kinks are described by a positive quadratic form, cf. [38].

The characteristic property of the kinks, both in unphysical $1 + 1$, and in physical $2 + 1$ dimensions, is their step-function form. One of the functions p or q behaves like a step function

$$p(q) \sim \theta(v\sqrt{\alpha'\mu}(y - y_0) - |\vec{b} - \vec{b}_0|), \qquad (3.5)$$

where v is the kink velocity (a free parameter, $v = 2$ for critical kink). The solution contains an arbitrary parameter y_0 that helps to understand why the physics related to the fragmentation can be hidden into the properties of the source. The arbitrary solution depends also on $\vec{b} - \vec{b}_0$. The value of $\vec{b}_0$ is not fixed by the equations. This is a zero mode relevant for the appearance of the phonons in quantum fluctuations.

The calculations in $2 + 1$ dimensions are significantly more difficult. However one can easily prove the existence of the kinks with the finite action that tunnel between the vacua (3.1) and (3.2). We need to find a solution of Eq. (3.4) with boundary conditions

$$y \to \infty, p \to \mu/\kappa, q \to 0 \qquad y \to -\infty, q, p \to 0. \quad (3.6)$$

Then for $y \to -\infty$ it is legitimate to neglect by nonlinear

interaction and obtain the usual diffusion equations:

$$-dq/dy = \alpha' \triangle q - \mu q \qquad dp/dy = \alpha' \triangle p - \mu p. \tag{3.7}$$

The general solution, say for q, is given by

$$q(y, b) = \exp(\mu y) \int \exp(-((\vec{b} - \vec{b}')^2/4\alpha'_P y)) f(\vec{b}') d^2 b'. \tag{3.8}$$

Here $f(b) = cq(y = 0, b)$ where c is a numerical factor.

The boundary condition for p will be $p \to \mu/\kappa$ for $y \to \infty$. Then near $y \to \infty$, we may write $p = \mu/\kappa - z$, where z is small. The linearized equation for z is

$$dz/dy = \alpha' \triangle z - \mu z + 2\mu q \tag{3.9}$$

The system of Eqs. (3.4) imposes the condition $q \to 0$ for $y \to \infty$, since $\alpha' \triangle p \sim 2\mu q$, and $p \to$ constant. Thus for each value of the impact parameter $\vec{b}$ the function q has a maximum. It follows from Eq. (3.9) that when z is small (and y is approaching Y), the field p decreases sharply near Y, so it can be approximated by the step function, while q reaches its maximum at $y \sim Y$ and then goes to zero.

It is easy to see that the action of the kink is finite and proportional to (μ/κ) in some power.

This solution is similar to the one found in the $1 + 1$ dimensional theory. However there is no analytical expression for the solution giving critical kinks with zero action as well as a full classification of kinks. So in $2 + 1$ dimensions we rely on the results of the numerical simulations made in Refs. [36,37], who found the same properties of kinks for the $2 + 1$ dimensional theory as for $1 + 1$ dimensional one.

The existence of the step like solutions made it possible to calculate S-matrix for the case of the sources given by Eq. (2.10). Indeed, the classical equations of motion for this case have the form

$$-dp/dy = \alpha' \triangle p - \mu p + \kappa(2pq + qp^2)$$
$$+ c_{\text{dipole}} \int d^2 B \exp(-BQ/2) q(y, b - B) \delta(y - Y)$$
$$dq/dy = \alpha' \triangle q - \mu q + \kappa(2pq + q^2)$$
$$+ c_{\text{dipole}} \int d^2 B \exp(-BQ/2) p(y, b - B) \delta(y + Y). \tag{3.10}$$

Since coupling to dipole for the field q is much smaller than $\mu/\kappa \sim 1/N_R^3 \alpha_s^2$ these equations are close to that analyzed in Ref. [36]. The field p is small for $y \le 0$, and it is the solution of the usual diffusion equation with the boundary condition that follows from the first of Eqs. (3.10), times $\exp(\mu y)$. The field p starts to rise exponentially for $y \sim 1/\mu$. At this point one cannot however neglect the nonlinear terms. Since for $y \to \infty$ $p \to \mu/\kappa$, we can linearize Eqs. (3.4) as $p = \mu/\kappa - z$. Then for z one

gets equation

$$dz/dY = \alpha' \triangle z + \mu z. \tag{3.11}$$

The equation for z is the same equation as above, except that the mass term changes sign, and it is small up to $y \sim 1/\mu$, when it blows up, leading to a step like decrease of p.

The point where solutions for p blows up can be estimated from the equation

$$q(b, y) \propto \exp(\mu y - b^2/(R^2 + 4\alpha'_P y)). \tag{3.12}$$

The transition occurs near the point where $q \sim 1$, i.e.

$$(R^2 + 4\alpha'y)\mu y \sim b^2. \tag{3.13}$$

Here $R \sim 1/\lambda$ is the transverse scale in the problem. Asymptotically

$$p(b, y) = (\mu/\kappa)\theta(2\sqrt{\alpha'\mu y} + \delta - b), \tag{3.14}$$

where δ is a phase shift determined by boundary conditions.

We see that in the classical approximation the equations for p and q effectively decouple. Moreover it can be proved that in the classical limit it is enough to solve the decoupled equations with suitably chosen boundary conditions.

The classical solution leads to the black disk limit, since in the classical theory the S-matrix is given by

$$S \sim \exp\left(-\int d^2 b g(b) p(b, Y)\right), \tag{3.15}$$

cf. refs. [36,37]. Here $g(b)$ characterizes the photon and it is concentrated near $b \sim B$. Since $p(b, Y)$ has a step-function form, and $g(b)$ is concentrated near B, in the classical theory S-matrix is 1 outside the black disk and zero inside (total absorption).

B. Quantum fluctuations around the WKB solutions

The knowledge of the family of the kink solutions permits the semiclassical quantization of the theory and the calculation of the S-matrix.

The distinctive feature of the semiclassical approximation is the existence of the "critical" kinks, with zero action, in particular, the kinks with both the energy and the classical momentum being zero. Classical contributions of these kinks into wave function are not exponentially suppressed. (Energy and momentum of kinks are defined as components of $\int d^2 b T^{00}$ and $\int d^2 b T^{0i}$, where the energy-momentum tensor of the EFT is found via the Nether theorem).

The remarkable property of the quantum fluctuations around critical kinks is the existence of the zero modes-"phonons" in the EFT, that are characterized by the linear dispersion:

$$E = i2\sqrt{\alpha'\mu}k = 2\sqrt{\alpha'\mu}P_{\text{cl}}, \tag{3.16}$$

where $P_{cl} = \int d^2b\, p \frac{dq}{db}$ is the total classical momentum derived from the EFT action. We present here plausible reasoning for the existence of such modes, following Refs. [39,40]. Let us begin from the analysis of $1 + 1$ dimensional model. In this case kinks are $f(x - vY)$ solutions of the ordinary differential equations. Quantum fluctuations around kinks can be calculated using the standard procedure [46]. The zero mode (3.16) leads to the gapless spectrum of excitations (''phonons''). The ''phonon'' wave function satisfies the equation:

$$H_R|k\rangle_\pm = \pm i2\sqrt{\alpha'\mu}k|k\rangle_\pm \qquad (3.17)$$

and

$$P_R|k\rangle_\pm = k|k\rangle_\pm, \qquad (3.18)$$

where H_R and P_R are the total Hamiltonian and momentum of the quantized EFT ($P_R = \int db T^{01}$ and $H_R = \int db T^{00}$, where T^{00}, T^{01} were calculated via the Nether theorem). The corresponding full set of wave functions can be found explicitly:

$$|k\rangle_\pm = \int da \exp(-ika)(\exp(-(\mu/\kappa)$$
$$\times \phi(\pm\rho - a)q(\rho))|\Phi_0\rangle. \qquad (3.19)$$

Here $\rho = x - v_0 Y$, and $\phi(\rho)$ is a solution of a classical equation of motion for a kink, Φ_0 is wave function of a perturbative vacuum. These states propagate in one direction in time—to $p \to \mu/\kappa$ at $Y \to \infty$ and in the opposite directions in impact parameter space. It is straightforward to calculate the quasiparticle correlation function:

$$D(B, Y) = \langle\Phi_0|p(0, \vec{0}) \exp(i\vec{P}\vec{B}) \exp(-HY)q(0, \vec{0})|\Phi_0\rangle, \qquad (3.20)$$

using the set of states (3.19) together with a perturbative and nonperturbative vacua, and get a black disk limit as the asymptotic answer.

Let us emphasize the key role of a linear spectrum. Indeed, for the linear spectrum, as it was shown in Ref. [38],

$$D(B, Y) = \frac{\mu^2}{\kappa^2} \int \frac{dk}{2\pi} \exp(ikB) \exp(iv_0 kY)\langle\Phi_0|p(0)|k\rangle$$
$$\times \langle k|q(0)|\Phi_0\rangle. \qquad (3.21)$$

It is possible to prove that the above matrix elements are equal to

$$\langle\Phi_0|p(0)|k\rangle = -\frac{\mu}{\kappa}\frac{1}{\epsilon + ik}, \qquad (3.22)$$

where ϵ defines contour of integration around singularities. Then we obtain:

$$D(B, Y) = (\mu/\kappa)^2 \theta(-B + 2\sqrt{\alpha'\mu}Y). \qquad (3.23)$$

In order to connect this correlator with the amplitudes we have to normalize it properly [38,40]:

$$G(B, Y) = \frac{\kappa^2}{\mu^2} D(B, Y). \qquad (3.24)$$

Then

$$G(k, J) = \frac{2}{(J - 1)^2 + v_0^2 k^2} \qquad (3.25)$$

Here $v_0 = 2\sqrt{\alpha'\mu}$ is a critical kink speed. The scattering amplitude is ([42], Chapter 16)

$$A(s, t) = s \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi^2} \exp(\omega\xi)G(\omega, k^2)\left(i + \tanh\left(\frac{\pi}{2}\omega\right)\right) \qquad (3.26)$$

Here $\xi = \log(s)$, $\omega = J - 1$, $t = -k_t^2$.

In addition, there exists a band of low lying states with a dispersion relation $E \sim k^2$, and a band of states with a gap $\sim\mu$. It can be argued that the first set of states corresponds to the unshifted quasiparticle fluctuations around the perturbative vacuum $|\Phi_0\rangle$ in the presence of a kink and viewed from the reference frame moving with a critical speed v_0. The higher modes can be interpreted as the collective fluctuations of a condensate of ladders, i.e. the quasiparticles interacting with the kinks. It is easy to prove that these modes do not influence the expanding disk solution asymptotically, although can be important outside the disk, and near the transition to a black disk regime.

The numerical analysis of the spectrum of the quantum fluctuations around the kinks has been performed in Refs. [36,37]. Moreover, it was proved on the discrete lattice that the Field Theory can be continuously connected with the Ising model in transverse magnetic field. Two types of collective excitations were found—the zero modes with a spectrum $E \sim k$ and solutions with a gap and spectrum $E \sim a + bk^2$, similar to the ones found in the $1 + 1$ dimensional model in the continuum. Evidently, only solutions with linear spectrum are relevant for the asymptotic behavior of the high-energy processes.

To summarize, the quasiclassical solution of EFT has following distinctive features:

(A) EFT has three degenerated ''vacua'' (3.1), (3.2), and (3.3), (The word vacua is in the brackets, since EFT is the theory with nonhermitian Hamiltonian, and actually means critical points of the action with zero value of the Hamiltonian). The true wave function of the physical ground state is a linear combination of these three vacua, and the Hamiltonian is diagonalized by critical kinks with zero action. If the initial state is a perturbative one interacting with a source, it evolves in rapidity and becomes a condensate of quasiparticles (ladders). (The term ''ground state'' means here that all states in the relevant physical sector the theory are the excitations of this state).

(B) The *S*-matrix is given by the functional integral

$$S(B, Y, g, f) = \int dp \int dq \exp(-L(B, Y, g, f)),$$

$$(3.27)$$

where the corresponding action is calculated via the classical kink solution described above, but includes now the source terms—the couplings with the virtual photons. The main contribution to the *S*-matrix comes from the quantum fluctuations around the critical kinks with the zero action. The spectrum of the excitations begins from the massless excitations—phonons. The contribution of the kinks into the two—quasiparticle Green functions that determines the cross-section is:

$$G(B, Y) = \theta(B^2 - 4\alpha'_P\mu Y^2) \qquad (3.28)$$

for large *Y*, $\mu Y \gg 1$. This Green function defined by Eq. (3.24) is related in a usual way to a scattering amplitude by the Mellin transformation, (see e.g. Ref. [42], chapter 16 and Eq. (3.26)). We denote the perturbative vacuum (3.1) as Φ_0.

(C) The obtained state is described by the asymptotic wave function

$$\Psi(y) = \exp\left(-\frac{\mu}{\kappa}\int d^2bq(b,y)\theta(b^2 - 4\alpha'\mu Y^2)\right)|\Phi_0\rangle. \qquad (3.29)$$

Here $|\Phi_0\rangle$ is a perturbative vacuum. To the extent that the correlations may be neglected the asymptotic vector (3.29) is a coherent state [38] and the *S*-matrix is given by

$$S(B, Y) = \exp\left(-\frac{\mu}{\kappa}\theta(2\sqrt{\alpha'\mu}Y - B)\right), \qquad (3.30)$$

where we explore that the target is localized near the impact parameter $b \sim B$. The *S*-matrix as given by Eq. (3.30) is 1 inside the black disk and suppressed exponentially as $\mu/\kappa \sim \sim 1/\alpha_s$, outside. This is just the BDL behavior. Note the nonperturbative structure of Eq. (3.28), and that the wave function $\Psi(y)$ can not be obtained by decomposition over powers of α_s.

(D) In other words in the limit of the infinite energies $Y \to \infty$ the produced state corresponds to a Bose-Einstein condensate of ladders in the entire space. However, for finite energies the solution is the black disk of radius $R^2 = R_0^2 + 4\alpha'_P\mu Y^2$. The Eq. (3.29) gives the exact form of the wave function of the Bose-Einstein condensate of ladders as a function of rapidity *Y*.

(E) The crucial reason why both quantum and classical approximations lead to the black disk behavior is the existence of the phonon zero mode in the quantum kink Hamiltonian, that has been proved on the lattice, and in the $1 + 1$ dimensional case. Moreover,the analysis of Refs. [36–38] shows that the contribution of the excited states that are different from the phonons is relevant only in the vicinity of the transition point. There are two types of states: collective fluctuations near the perturbative vacuum interacting with quantum kinks and the quantum fluctuations of the kink condensate. None of them influences the asymptotic behavior of the black disk.

IV. KINKS AND QCD

Some properties of the classical solutions of the effective theory can be understood directly in QCD. A kink produces action proportional to $(\mu/\kappa) \sim (1/\alpha_s)$. The dependence of the *S*-matrix (3.30), of the critical kink action (understood as the limit of a family of kinks with nonzero action) on the coupling constant α_s and existence of phonon show that this is a novel nonperturbative QCD phenomenon.

Let us note here that the 2d translational invariance is broken in the theory at finite rapidity *Y* due to the existence of black disk behavior. Mathematically it reveals itself in the existence of a translational zero mode for the kink solution. This phenomenon resembles a spontaneous symmetry breaking since the system, after being at moderate rapidities in the perturbative vacuum chooses in the process of space-time evolution the state which is a definite combination of the vacuum states of the system. Whether this phenomenon corresponds to the conventional spontaneous symmetry breaking is unclear at present but the existence of phonons looks suggestive. Moreover it is easy to show that in the approximation where $\alpha' = 0$ EFT can be mapped by substitution of variables into the quantum mechanics with hermitian double well potential with the two nonperturbative critical points of the action (2.11) corresponding to two minima of Higgs type potential. In this case two nonperturbative vacuums of Sec. III are real minima of the action and tunneling transitions are relevant for the change of symmetry.

The estimate Y_c where transition to the regime of Bose-Einstein condensation of ladders follows from the inequality $Y_c \gtrsim \ln(10^2/x_{cr})$ where x_{cr} can be found from the requirement of conservation of probability, of unitarity of the *S* matrix cf. Ref. [9]. (Additional factor $\leq 10^2$ accounts larger energies needed for the applicability of triple "reggeon" = ladders limit and all ladders should be near unitarity limit.) Assuming $x_{cr} \sim 10^{-4}–10^{-5}$ cf. [9] we obtain $Y_c \gtrsim 14–16$. At these rapidities the Bose-Einstein condensation of the ladders starts.

The characteristic form of the kink is the step-function in the *Y* space, with the width of the order $\delta Y \approx \log(\delta E/Q) = 1/\mu$. The coherent length relevant for the evolution of the kink is enhanced due to the large Lorentz

slowing down gamma-factor as

$$T_I \sim \delta E/Q^2 \approx \exp(1/\mu)/Q \approx 10^2/Q \qquad (4.1)$$

In other words, for sufficiently low x we have $T_I \ll T_c$, where T_c is coherence length $T_c \sim 1/(Qx^{1-\mu})$. This rapid transition to the black disk regime can be called the "color inflation": one ladder due to the tunneling transition blows up during time T_I and creates an entire region of space filled with the gluon ladders. During time T_I approximately $(\mu^2/\kappa^2)R(Y)^2$ ladders are created, where $R(Y)$ is a black disk radius for a given rapidity Y.

It is easy to evaluate the density of ladders in the coordinate space by solving discussed above diffusion equations (cf. similar analysis in Ref. [40]):

$$|\Psi(y)\rangle = \exp\left(-(\mu/\kappa) \int_0^{R(Y)} d^2 b q(b, Y)\right). \qquad (4.2)$$

The conjugate state is

$$\langle\Psi(y)| = \exp\left(+(\mu/\kappa) \int_0^{R(Y)} d^2 b p(b, Y)\right). \qquad (4.3)$$

Expanding these states into series of powers over q we obtain the multiladder wave functions $\Psi(b_1, \dots b_N)$,

$$\Psi = \sum_{N=0}^{N=\infty} \int \Pi_{i=0}^{i=N} \int d^2 b_1 \dots d^2 b_i \dots d^2 b_N \Psi(b_1, \dots b_N)$$
$$\times q(b_1)\dots\dots q(b_N)/(N!), \qquad (4.4)$$

where $\Psi(b_1, \dots b_N)$ are products of theta functions, limiting all the integration in Eq. (4.4) inside the black disk. We neglect the correlations (see Sec. III).

The average distance between the ladders l_t can be estimated from the wave function of the condensate. Indeed,

$$l_t^2 = \frac{\int_0^{R(Y)}(\vec{b}_1 - \vec{b}_2)^2 \Psi(b_1, b_2, \dots)\Psi^*(b_1, b_2, \dots)d^2 b_1 \dots d^2 b_N}{(\int_0^{R(Y)} \Psi^* \Psi d^2 b_1 \dots d^2 b_N)} \qquad (4.5)$$

The integration is over the impact parameters of the ladders. The condensate wave function is homogeneous inside the black disk. Therefore

$$\langle p(b_1) \dots p(b_n) q(b_1) \dots q(b_n)\rangle = 1 \qquad (4.6)$$

Consequently, the transverse distance between the ladders is

$$l_t \sim \kappa/\mu \sim \alpha_s/\lambda \qquad (4.7)$$

On the other hand, the characteristic scale d_t of a ladder in the transverse parameter space is determined by the coefficient in the effective Lagrangian in front of the kinetic term, that is equal to $\sim N_c \alpha_s/\lambda^2$, hence

$$d_t^2 \sim N_c \alpha_s/\lambda^2 \qquad (4.8)$$

Consequently,

$$l_t^2/d_t^2 \approx \alpha_s/N_c \ll 1. \qquad (4.9)$$

It follows from the above estimate that the pQCD ladders overlap significantly. But overlapping ladders can exchange the quarks and the gluons because in the perturbative QCD there are no barriers between the ladders. The resulting color network, as we shall call this object, is macroscopical in the longitudinal direction. The distinctive features of the color network resemble the quark-gluon plasma.

Understanding the actual longitudinal structure of the system is not possible within the effective field theory. Indeed, even after the phase transition, the ladders continue to grow till the color network achieves the longitudinal length $1/Qx^{1-\mu}$.

In our analysis we used the fact the QCD ladders can be considered as the effective degrees of freedom up to the scales when they significantly overlap. This overlap is controlled by the density of ladders since in the perturbative QCD (see Appendix A) there are no long range forces operating between the ladders.

V. OBSERVABLE PHENOMENA

We predict a variety of the new nonperturbative QCD hard phenomena for the case of the collision of the two small dipoles near the black disk limit. Their very existence shows that transition to the black disk limit is a kind of a critical phenomenon.

The phenomenon of color inflation may significantly change the physics of hard processes at sufficiently small x by softening the parton distribution over the longitudinal momenta and will reveal itself as the threshold like increase of multiplicity. (We postpone quantitative analysis of such phenomena to the next publications).

Comparatively clean way to identify the onset of the new regime would be to measure Mueller-Navelet process [47]: $p + p \rightarrow \text{jet} + X + \text{jet}$ where the distance in the rapidity between the high p_t jets is large. The expected behavior is the following: initial fast increase of cross section with y predicted by the pQCD should change to the fast decrease at larger y because of the color inflation, i.e. disappearance of the long range correlations in the rapidity (coordinate) space near the black disk limit due to the creation of the color network.

VI. CONCLUSION

We argue that the absence of the long range forces between the colorless ladders as the consequence of the color screening phenomenon (see Appendix A) justifies the neglect of the exchanges of the constituents between the colorless ladders in the first approximation. The colorless ladders should be a dominant degree of freedom even near the black disk limit. As a result it is possible to build the effective field theory where ladders are quasiparticles. We showed that at sufficiently small x the form of the Hamiltonian of this EFT is dictated for the single-scale hard processes in QCD by the smallness of running coupling constant. Moreover smallness of the effective triple-ladder vertex: $\lambda^2 \kappa \ll 1$ justifies the applicability of the semiclassical approximation. We show that the effective field theory is solvable within the WKB approximation and leads to the black disk behavior and other QCD phenomena.

The transition to the black disk regime within the effective field theory in one-scale hard processes is a kind of a critical phenomena. There exist classical hard QCD fields relevant for the transition from false to physical vacuum— kinks in the rapidity-impact parameter space. The transition due to general EFT kink is suppressed as $\sim \exp(-\mu\lambda/\kappa) \sim \exp(-1/\alpha_s)$, because $\kappa \sim \alpha_s^2 N_c$ (see Appendix B). The critical kinks that correspond to actual transitions between vacua are the limiting cases of the families of these noncritical kinks.

The transition to the black disk limit is of the inflationary type: in the case of collisions of two small dipoles the time scale $T_I \sim \exp(1/\mu)/Q$ of the transition is significantly smaller than the time needed for the formation of the perturbative ladder $\propto 1/Qx^{1-\mu(Q^2)}$.

The nonperturbative transition produces ladders that strongly overlap in the impact parameter space. Because of the exchange of the constituents between the overlapping ladders the system of ladders becomes the color network that resembles the lengthy but narrow pencil. The difference between the pencil and the jet is the softer distribution of hadrons over transverse and longitudinal momenta. However to study this color network, we need to be able to describe the BDL transitions (the kinks) directly in terms of the QCD language, the goal that is yet not achieved.

To summarize, we were able to show that QCD leads to solvable in quasiclassical approximation effective effective field theory, and this effective field theory has a transition to the black disk limit, that in the QCD language is a color network.

The most challenging problems for the future work will be to calculate both the kinks and the phonons of the EFT and to study the condensation of ladders directly in terms of QCD (quark,gluon) degrees of freedom and to understand the role of the found QCD phenomena in two-scale processes.

APPENDIX A: FORCES BETWEEN THE PERTURBATIVE LADDERS

It has been known long ago [48] that ladders are the dominant degrees of freedom in the theoretical description of the high-energy processes. In the leading-twist approximation this assumption has been proved in pQCD. At the same time near the black disk limit due to the existence of the triple-ladder coupling the multiladder configurations become important and may overlap. We explain here that there are no long range forces between the pQCD ladders and quarks and gluons are confined within the ladders, till the latter start to overlap inside the space-time.

Indeed, consider the DGLAP ladder. The vector potential created by this ladder can be calculated as

$$A_\mu^a(x) = \int d^4y D_{\mathrm{ret}\,\mu\nu}^{ab}(x - y)J_\nu^b(y) \qquad (A1)$$

Here D_{ret} is the retarded gluon propagator in a light-cone gauge, while J is the matrix element of a current for gluon emission by a ladder. The Eq. (A1) can be rewritten in the momentum space as

$$A_\mu^a(x) = \int d^4s \exp(ixs)D_{\mathrm{ret}\,\mu,\nu}^{ab}(s)J_\nu^b(s) \qquad (A2)$$

The retarded propagator in the light-cone gauge has the form:

$$D_{\mathrm{ret}}(s) = \frac{1}{s + i\epsilon s_0}(g_{\mu\nu} - (s^\mu\eta^\nu + s^\nu\eta^\mu)/(s\eta)). \quad (A3)$$

As usual we take into account for the cancellation of the most singular term in the propagator that follows from the Ward identities. $J_\nu^b(s)$ is the current of a soft gluon emission by the ladder and η is a light-cone vector.

$$J_\mu(s) = \sum_{i=1}^{i=N} q_i^\mu/(sq_i) \qquad (A4)$$

where the sum is taken over all external lines of a cut ladder. The contributions from the internal lines cancel at least in the leading order [49]. Consider integral over s, i.e. $\int ds_- ds_+ d^2s_t$. Calculating the integral over the residues, we see that the integral is controlled by $1/s^2$ pole in the propagator. Potentially dangerous terms like $1/(q_i - s)^2$ are far from the mass shell and have no singularity. Thus integral for the vector potential is given by the pole $s^2 = 0$. The emitted gluon is always on the mass shell and transverse. In other words no long range forces exist within the DGLAP approximation.

The same arguments can be applied to BFKL ladder. The integral over s is also determined by $1/s^2$ pole, and leads to transverse gluons (Ref. [50]).

We conclude that the leading order ladders do not create perturbatively long-range fields in the leading-twist approximation. Generalization of this statement to the next-to-leading order should be straightforward.

APPENDIX B: MANY POMERON COUPLINGS IN PQCD

The conventional strategy can be used to evaluate parameters of effective Hamiltonian of EFT: substitution of variables in the path integral from quarks and gluons to ladders. However such calculation is too cumbersome at present. So we restrict ourselves by the qualitative evaluation of multiladder vertices near the unitarity limit in the lowest order of pQCD. The most difficult point is to take into account for causality, i.e. location of singularities in the complex plane of energies in respect to the contour integration. It has been shown by S. Mandelstam [45] and in the more straightforward way by V. Gribov [42] that the diagrams having no third spectral function $\rho(s, u)$ in Mandelstam representation for the scattering amplitude give no contribution into the leading power of energy. The transparent interpretation of this result is that bare particle may experience only one inelastic collision within the semiclassical approximation.

Thus lowest diagram for triple-reggeon vertex is given by the triangle gluon loop with 6 gluon lines attached. So

$$\kappa = G_{3P} \propto \frac{\alpha_s^2 N_c}{\lambda}, \qquad (B1)$$

cf. recent discussion in Ref. [44]. To evaluate dependence on N_c it is useful to represent vectors in the color space in terms of color spinors and then to find the disentaglment of color contours.

The account of causality (i.e. of the fact that the bare particle may have only one inelastic collision and any number of elastic ones) shows that the lowest order diagram for the four ladder vertex corresponds to the attachments of 4 ladders to the 2 gluon loops.

$$G_{4P} \propto \frac{\alpha_s^4 N_c^2}{\lambda^3}. \qquad (B2)$$

Similarly as the consequence of the causality the n ladder vertex is given in the lowest order over α_s by the attachment of n ladders to the $n - 2$ gluon loops. Then

$$G_{nP} \propto \frac{\alpha_s^{2n-4} N_c^{n-2}}{\lambda^{n-2}}. \qquad (B3)$$

The presence of the multiladder couplings does not change the behavior of the system near the extremum. This is because their relative contribution is characterized by the parameters:

$$G_{4P}\phi^4/G_{3P}\phi^3 \approx \alpha_s N_c$$

Here ϕ is the field of the quasiparticle estimated in the WKB approximation as $\approx \mu/\kappa$. Similarly one may estimate the contribution of the $n \geq 3$ ladder vertices:

$$G_{nP}\phi^n/G_{3P}\phi^3 \approx \alpha_s^{n-3} N_c^{n-3}$$

Thus relative contribution of higher vertexes is suppressed by the powers of α_s.

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