PHYSICAL REVIEW D **73,** 054006 (2006)

Scalar glueball spectrum

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(Received 3 June 2005; revised manuscript received 24 January 2006; published 15 March 2006)

I discuss scenarios for scalar glueballs using arguments based on sum rules, spectral decomposition, the $\frac{1}{N_c}$ approximation, the scales of the strong interaction and the topology of the flux tubes. I analyze the phenomenological support of those scenarios and their observational implications. My investigations hint a rich low lying glueball spectrum.

DOI: 10.1103/PhysRevD.73.054006 PACS numbers: 12.38.−t, 12.38.Aw, 14.70.−e, 14.80.−j

I. INTRODUCTION

The glueball spectrum has attracted much attention since the formulation of the theory of the strong interactions quantum chromodynamics (QCD)[1,2]. QCD sum rules [3], QCD based models [4,5] and Lattice QCD computations, both with sea quarks [6,7] and in the pure glue theory [8–12] have been used to determine their spectra and properties. However, from the phenomenological point of view it has become clear by now that it is difficult to single out which states of the hadronic spectrum are glueballs because we lack the necessary knowledge to determine their decay properties [13]. Moreover the strong expected mixing between glueballs and quark states leads to a broadening of the possible glueball states which does not simplify their isolation [14]. The wishful sharp resonances which would confer the glueball spectra the beauty and richness of the baryonic and mesonic spectra are lacking. This confusing picture has led to a loss of theoretical and experimental interest in these hadronic states. However, it is important to stress that if they were to exist, they would be a beautiful and unique consequence of QCD.

Glueballs have not been an easy subject to study due to the lack of phenomenological support and therefore much debate has been associated with their properties[14]. The lightest glueball is the scalar 0^{++} [8–12,15]. Its properties, i.e., mass and widths still differ among the various approaches. For the pure Glue Theory the situation that arises from lattice calculations is clear and the masses of the scalar glueballs are large $m > 1$ GeV [8–12]. However, when sea quarks are considered no firm conclusion about the scalar spectrum can be drawn. The theoretical calculations based on QCD sum rules and/or low energy theorems lead to contradictory results. While Dominguez and Paver [16], Bordes, Peñarrocha and Giménez [17], and Kisslinger and Johnson [18] obtain, using low energy theorems and/or sum rule calculations with (or without) instanton contributions, a low lying and narrow 0^{++} (mass $<$700 MeV and $\Gamma_{\pi\pi} < 100$ MeV), Narison and collaborators[19], using two (substracted and unsubstracted) sum rules, prefer a broader (200–800 MeV), heavier (700–1000 MeV) scalar glueball whose properties imply a strong violation of the Okubo-Zweig-Ishimura's (OZI) rule.[1] In a recent state of the art sum rule calculation, Forkel [20], obtains the scalar glueball at 1250 ± 200 MeV with a large width (~ 300 MeV). However, he has some strength at lower masses which he is not able to ascribe to a resonance in the fits.[2] Present day interpretation of experiments[21,22] claim a heavy glueball (~ 1500 MeV). I found though illuminating the discussion of Kisslinger and Johnson [18] since using their calculation they can explain the existence of two scalar glueballs, a light one (~ 500 MeV) and heavy one (~ 1700 MeV), by studying the influence of the higher condensates in their sum rule approach.

To investigate the scalar glueball sector I develop my description initially in a world where the OZI rule is exactly obeyed, i.e., decays into quarks which require gluons are strictly forbidden. OZI dynamics (OZID) generates a glueball spectrum which is formed by towers of states disconnected from mesons, baryons and leptons. The lowest lying scalar glueball (hereafter called g) is, in this world, a bound state of strongly interacting gluons with a twisted flux tube configurations [23,24]. OZID confers this topology a super selection rule inhibiting any decays from this state into other particles. It is therefore stable and (almost) invisible. However, OZID is an idealized scenario which breaks down because a low energy theorem requires the existence of a low lying meson with the same quantum numbers of the g, which I call σ. Moreover, to lift the initial mass degeneracy of g and σ I implement an additional OZID breaking scheme which I call mixing because it reminds me of the flavor breaking patterns of the mesons. Through this breaking the interactions of the glueballs with quarks, and through them with all other standard model probes, arise. The mixing leads to scenarios, which I investigate by comparing with data.

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[1]However, a lighter glueball would be narrow since the coupling to $\pi\pi$ is proportional to the square of the mass.
[2]Private communication.

II. QCD SCALARS

To transform the OZID scenario into a Gedanken picture of reality we need the support of theory. In the extreme OZID picture gluodynamics is the theory describing g. The Lagrangian of gluodynamics does not contain dimensional parameters. As a consequence, there are an infinity set of Ward identities induced by the trace of the regularized energy momentum tensor. An effective low energy Lagrangian can be constructed which includes one scalar field and saturates the Ward identities, i.e.

$$L = \frac{1}{2}(\partial g)^2 - V(g), \tag{1}$$

with $V(g)$ completely determined [25–27]. Following the description of Ref. [27] we have,

$$V(g) = H_0\left(-\frac{1}{4} + \frac{\tilde{g}(x)}{g_0}\right)\exp\left(\frac{4\tilde{g}(x)}{g_0}\right), \tag{2}$$

where H_0 is related to the gluon condensate and in the notation of Ref. [28] is given by ,

$$H_0 = -\left\langle 0 \left| \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \right| 0 \right\rangle, \tag{3}$$

g_0 is the vacuum expectation value of g, $g_0 = \langle 0|g|0\rangle = -f_g$ and $\tilde{g}$ is defined from g by

$$g(x) = g_0 \exp\left(\frac{\tilde{g}(x)}{g_0}\right). \tag{4}$$

This scalar field describes the lowest lying 0^{++} glueball. An immediate consequence of the implementation of the anomaly in the effective Lagrangian (see Eq. (11) of Ref. [27]) is the following relation between the mass of g, m_g, and the condensate,

$$m_g^2 f_g^2 = -4\left\langle 0 \left| \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \right| 0 \right\rangle. \tag{5}$$

The coefficient in front of G^2 has been chosen in such a way as to make the right-hand side renormalization group invariant [28]. The same relation was also obtained by Novikov *et al.* [29] isolating the leading power correction in their calculation.

OZID arises naturally in the $\frac{1}{N_c}$ expansion from QCD. Equation (5) is consistent with the expected behavior

$$m_g \sim 1 \quad \text{and} \quad f_g \sim N_c. \tag{6}$$

Let us introduce the following correlator

$$\Pi(q^2) = i \int dx e^{iqx} \left\langle 0 \left| T\left(\frac{\beta(\alpha_s)}{4\alpha_s} G^2(x) \frac{\beta(\alpha_s)}{4\alpha_s} G^2(0)\right) \right| 0 \right\rangle. \tag{7}$$

A well-known low energy theorem, proven in Section 3 of Ref. [29], is

$$\Pi(0) = -4\left\langle 0 \left| \frac{\beta(\alpha_s)}{4\alpha_s} G^2(0) \right| 0 \right\rangle, \tag{8}$$

which is a consequence, if the quark masses are too small or too large to be relevant, of the fact that there is no dimensional parameter in QCD except the ultraviolet cut-off. To leading order in $\frac{1}{N_c}$,

$$\Pi(q^2) = \sum_{\text{glueballs}} \frac{N_c^2 a_n^2}{M_n^2 - q^2} + \sum_{\text{mesons}} \frac{N_c c_n^2}{m_n^2 - q^2}, \tag{9}$$

where M_n and m_n represent, respectively, the masses of the glueballs and mesons contributing to the correlator, and the numerators are related to the following transition matrix elements

$$N_c a_n = \left\langle 0 \left| \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \right| n\text{th glueball} \right\rangle \tag{10}$$

and

$$\sqrt{N_c} c_n = \left\langle 0 \left| \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \right| n\text{th meson} \right\rangle. \tag{11}$$

In the extreme $\frac{1}{N_c}$ limit at low q^2, $\Pi(q^2)$ is dominated by the lowest mass glueball m_g, and using Eqs. (8) and (9) I obtain

$$\left| \left\langle 0 \left| \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \right| g \right\rangle \right|^2 = -4m_g^2 \left\langle 0 \left| \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \right| 0 \right\rangle. \tag{12}$$

This results is in agreement with the effective theory, Eq. (1), since it establishes a relation of the g coupling strength, recall Eq. (10), with the gluon condensate. Using Eqs. (5) and (12) we obtain,

$$m_g^2 f_g = \left\langle 0 \left| \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \right| g \right\rangle. \tag{13}$$

In order to achieve consistency with the low energy theorem of Voloshin and Zakharov, Eq. (4) of Ref. [30], a consequence of which is that

$$\left\langle 0 \left| \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \right| (\pi\pi)_{J=0} \right\rangle \neq 0, \tag{14}$$

we need to incorporate in our scheme scalar mesons since in the extreme OZID limit g does not connect with the 2π state, and this matrix element vanishes. Thus, this exact result of QCD, implies that my description should go beyond the lowest order in the $\frac{1}{N_c}$ low q^2 approximation and incorporate scalar mesons. We assume for simplicity that around the g mass region only one scalar meson suffices. Thus, we are led to depart from pure gluodynamics and to extend the description by introducing one more term in the correlator in the $\frac{1}{N_c}$ low q^2 approximation, i.e., the contribution of a low lying scalar meson, which I call σ, and which behaves as

$$m_\sigma^2 f_\sigma = \left\langle 0 \left| \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \right| \sigma \right\rangle, \tag{15}$$

and from Eq. (11) we know that,

$$m_\sigma \sim 1 \quad \text{and} \quad f_\sigma \sim \sqrt{N_c}. \tag{16}$$

We have gathered all the ingredients to estimate the masses of these two states following the analysis of Shifman [28] adapting it to my description. I proceed in the broken OZID limit, i.e. g does not couple to the 2π state but σ does, and it is therefore the latter which plays the role of saturating the matrix elements.

Equation (13) leads to the following glueball spectral function,

$$\mathrm{Im}\,\Pi(q^2) = \pi m_g^2 f_g^2 \delta(q^2 - m_g^2) \tag{17}$$

Equation (15) and the discussion leading to it imply that σ is the 2π state appearing in Ref. [28] and thus we obtain

$$m_g^2 f_g^2 + \frac{s_r^2}{8\pi^2} = -4\left\langle 0 \left| \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \right| 0 \right\rangle, \tag{18}$$

where I have used $m_\sigma^2 f_\sigma^2 = \frac{s_r^2}{8\pi^2}$. The additional term appearing from the existence of the nondecaying g implies a reduction of the masses with respect to the cited analysis.

Let me use $\frac{1}{N_c}$ in here,

$$\frac{m_g^2 f_g^2}{m_\sigma^2 f_\sigma^2} = N_c.$$

Repeating the numerical estimate of Ref. [28] for massless quarks and for $N_c = 3$ we get

$$m_{\mathrm{res}} = 600 \text{ MeV}$$

and therefore

$$m_g \sim m_\sigma \sim 600 \text{ MeV}$$

and

$$f_g \sim \sqrt{3} f_\sigma.$$

For massive quarks Shifman's estimates lead to

$$m_g \sim m_\sigma \sim 750 \text{ MeV}$$

Thus g and σ are degenerate in mass in this naive scenario.

My calculation has been carried out in a theory with quarks, i.e., broken OZID. It is therefore not equivalent to a calculation in gluodynamics (quenched QCD). Nevertheless, the value obtained for the mass of the glueball is compatible with the larger value, 1.6–1.7 GeV, obtained in lattice calculations for pure glue [8–12]. In gluodynamics the absence of quarks increases the coupling constant. The right hand side of Eq. (18) is related to the energy of the vacuum which increases [12] and the left-hand side has no contribution from the quarks, i.e. σ meson, thus

$$(m_g f_g)^{\mathrm{gluodynamics}} \sim \sqrt{\frac{44}{27}} (m_g f_g)^{\mathrm{QCD}}. \tag{19}$$

Moreover one expects f_g to decrease in gluodynamics since the quarks contribute to the equation of the scale anomaly in the full theory [27].

These statement together with my previous estimate Eq. (19) leads to

$$m_g^{\mathrm{gluodynamics}} \gg 1 \text{ GeV}.$$

In the most recent calculation of Chen *et al.* [12] Eq. (12) (their Eq. (59)) is used to relate matrix elements with the gluon condensate. My development shows that it can be used in the quenched approximation and moreover it will only hold, as it does, there. In order to make it work in the unquenched, the unquenched parameters have to be used and my statement as expressed all along is that they do change considerably.

Finally to close this comment recall that the original calculation of Shifman [28], in which there was no doubling of the lowest scalar, led to a glueball mass of 1.3 GeV, closer to the quenched result. Thus the doubling mechanism proposed here states that the results of the unquenched calculation for the glueball spectrum differ considerably from those of the quenched calculation, in particular, regarding the spectrum.

III. TOPOLOGY AND DYNAMICS

Nature does not realize OZID, namely, the number of colors is not very large. We have to establish a scheme for breaking OZID. How should we incorporate corrections to the leading order in the $\frac{1}{N_c}$ expansion? In order to understand how nature departs from OZID I resort to symmetry breaking and topological arguments.

Dynamical transmutation in QCD gives rise to the confinement scale, Λ, which introduces dimensions into a dimensionless (apart from quark masses) theory. Conventional low energy physics is governed by the chiral symmetry breaking scale f_π [31], which ultimately should be a function of Λ. In this case low energy dynamics will be governed by f_g and f_σ respectively. Recalling the results of previous section we notice that $f_\pi \sim f_\sigma \sim O(\sqrt{N_c})$, Eq. (16), while $f_g \sim O(N_c)$, Eq. (6). The breaking of OZID is governed by powers of their inverses. Thus I expect the corrections to the mesons to be $O(\frac{1}{N_c})$ while that for the glueball $O(\frac{1}{N_c^2})$. OZID is better realized in the glueball sector than in the meson sector.

A second idea which guides our intuition about the breaking of OZID is the topology of the flux tubes and their relation with perturbative emission. The mesonic $q\bar{q}$ states have a nontwisted flux tube configuration and therefore only one length scale describing confinement [32–34]. The glueballs in most treatments arise from twisted flux tube configurations [24,35–37]. In particular I conjecture, based for simplicity on a the simplest possible nontrivial topology, namely, a torus like configuration [23], the behavior of particle emission.

Gluon and quark emission occur inside the flux tubes and therefore the scale of the perturbative emission is limited by the confinement size, i.e. the running coupling constant takes its maximum possible value when the particles are emitted with the lowest possible momentum, which is bounded from below by Heisenberg's principle,

(i) for the meson: $L < L_{\rm conf} \sim \frac{1}{\Lambda_{\rm QCD}} \sim 1$ fm.

(ii) for the glueball:[3] $L < \frac{L_{\rm conf}}{\sqrt{2}\pi} \sim \frac{1}{4}$ fm.

Therefore,

$$\alpha_{\rm meson} \sim \alpha(L_{\rm conf}) \gg \alpha_{\rm glueball} = \alpha\left(\frac{L_{\rm conf}}{\sqrt{2}\pi}\right), \qquad (20)$$

where α is the running coupling constant.

This argument also suggests that OZID dynamics is a better approximation in the case of glueballs than in the case of mesons since for the former the perturbative emission is weak. I expect therefore that the pure perturbative emission approximation of QCD to gluon and quark emission for g is very appropriate at any scale, while for the σ nonperturbative chiral effects will be important [38].

IV. g-σ MIXING

Since g and σ have the same quantum numbers they can easily mix in broken OZID and the observed particles are coherent superpositions of them. We consider that g and σ mix due to additional terms in the Hamiltonian which are of higher order in $\frac{1}{N_c}$. Since $f_\sigma \sim \sqrt{N_c}$ and $f_g \sim N_c$ the following is the most general Hamiltonian in this reduced Fock space,

$$\begin{pmatrix} m & \delta \\ \delta & m + \Delta m \end{pmatrix} \qquad (21)$$

where $\Delta m \sim \frac{1}{N_c}$, $\delta \sim (\frac{1}{N_c})^{3/2}$ and we exclude terms $O((\frac{1}{N_c})^2)$ and higher powers. The diagonal basis of this Hamiltonian can be presented as,

$$\tilde{g} = g\cos(\theta/2) - \sigma\sin(\theta/2), \qquad (22)$$

$$\tilde{\sigma} = g\sin(\theta/2) + \sigma\cos(\theta/2), \qquad (23)$$

where the tilde labels the physical particles and θ is the mixing angle.

The masses of the physical particles become

$$m_{\tilde{g}} = m + \frac{\Delta m}{2} - r, \qquad (24)$$

[3]The torus flux tube is basically a planar figure since the cross section radius of the tube is small compared to the other radius. Thus $\Delta p_x \sim \frac{1}{2R}$ where R is the radius of the large circle of the torus. Thus $\Delta p = \sqrt{(\Delta p_x)^2 + (\Delta p_y)^2} = \frac{1}{\sqrt{2}R}$. But $L_{\rm conf} = 2\pi R$ thus $L < \frac{L_{\rm conf}}{\sqrt{2}\pi}$.

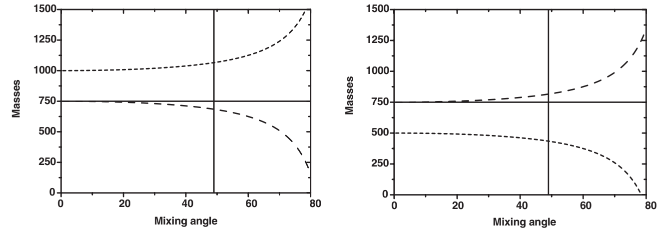

FIG. 1. The limiting values for the masses of the physical $\tilde{g}$ (solid-dashed lines) and $\tilde{\sigma}$ (solid-short-dashed lines) are shown as a function of the mixing angle for the range $0 < |\Delta m| < 250$ MeV. The left (right) figure corresponds to positive (negative) Δm. The degenerate initial mass has been taken, as discussed in the text, at $m = 750$ MeV. The vertical line defines the approximate limit of the validity of the $\frac{1}{N_c}$ expansion.

$$m_{\tilde{\sigma}} = m + \frac{\Delta m}{2} + r, \qquad (25)$$

where

$$\tan\theta = \frac{2\delta}{\Delta m}, \qquad (26)$$

and

$$r = \frac{\Delta m}{2}\sqrt{1 + \left(\frac{2\delta}{\Delta m}\right)^2} = \frac{\Delta m}{2\cos\theta}. \qquad (27)$$

In Figs. 1 I represent the masses of the physical states as a function of the mixing angle θ. The curves separate possible mass regions. The horizontal solid line represents the mass of both states for $\delta m \to 0$.[4] The dashed lines represent the value of $m_{\tilde{g}}$ for $\Delta m = \pm 250$ MeV and the short-dotted lines those for $m_{\tilde{\sigma}}$. To the left of the vertical line the values are consistent with the $\frac{1}{N_c}$ expansion, the condition that defines that line is

$$\tan\theta = \frac{2\delta}{\Delta m} \sim \frac{2}{N_c} \sim \frac{2}{3}.$$

In Figs. 1 the curves on the left show that the two state mixing scenario for positive Δm leads to a "light" glueball with a mass in the range 650 MeV $< m_{\tilde{g}} <$ 750 MeV and a scalar meson with a mass in the range 750 MeV $< m_{\tilde{\sigma}} <$ 1050 MeV. The $\frac{1}{N_c}$ expansion favors small mixings in the physical states. The curves on the right show that for negative Δm the meson becomes lighter 450 MeV $< m_{\tilde{\sigma}} <$ 750 MeV, while the glueball becomes heavier 750 MeV $< m_{\tilde{g}} <$ 850 MeV.

Let me speculate about strong OZID breaking. If we abandon the $\frac{1}{N_c}$ expansion, i.e. allow the mixing matrix elements to be larger than required by this approximation, the masses separate notoriously and, in particular, the

[4]We take a value for Δm small enough so that the deviation from this line which occurs for $\theta \to \frac{\pi}{2}$, which leads ultimately to a $\pm\delta$ splitting for δ finite, is beyond the shown values.

glueball (meson) becomes very light in the $\Delta m > 0$ ($\Delta m < 0$) scenario. Correspondingly, the associated meson (glueball) becomes heavy. In this case however the mixing is large thus it is difficult to talk about glueball or meson since both states are an almost perfect mixture, i.e. the $\tilde{g}$ state has a large a large σ component and the $\tilde{\sigma}$ state a large glueball component.

We have performed here a mathematical analysis of our theoretical scheme, in the next section we put the present analysis under the scrutiny of data.

V. DISCUSSION

The OZID glueball does not interact with quarks, neither with leptons nor electroweak gauge bosons, therefore in this approach it is sterile. However, the physical glueball does because of its admixture with the σ. From now on I only address the physical particles and omit their tilde in the notation. Using a σ-model interaction we get

$$\Gamma_{\sigma\to 2\pi} = \frac{3}{64\pi f_\pi^2}\left(\frac{m_\sigma^2 - m_\pi^2}{m_\sigma}\right)^2\sqrt{m_\sigma^2 - 4m_\pi^2} \sim \frac{3}{64\pi}\frac{m_\sigma^3}{f_\pi^2} \sim 1.5\left(\frac{m_\sigma(\mathrm{GeV})}{1\ \mathrm{GeV}}\right)^3\ \mathrm{GeV}, \quad (28)$$

where we have taken $f_\pi \sim 100$ MeV and neglected terms $O(m_\pi^2)$ in the last line.

Let us look at the lower spectrum of scalars shown in Table I. Below 750 MeV the only existing resonance is the broad $f_0(600)$, whose mass and width are still quite undetermined. Using the data on the width and using Eq. (28) we obtain

$$737\ \mathrm{MeV} < m_\sigma < 874\ \mathrm{MeV}. \quad (29)$$

Thus the $\Delta m < 0$ scenario is discarded by the data. Therefore the glueball is lighter than the meson, i.e. within the limits of the $\frac{1}{N_c}$ expansion

$$650\ \mathrm{MeV} < m_g < 750\ \mathrm{MeV}. \quad (30)$$

Note that in this approximation the mixing angle is small and therefore

$$\Gamma_{g\to 2\pi} \sim 1.5\sin^2(\theta/2)\left(\frac{m_g(\mathrm{GeV})}{1\ \mathrm{GeV}}\right)^3\ \mathrm{GeV} < 100\ \mathrm{MeV}, \quad (31)$$

$$\Gamma_{\sigma\to 2\pi} \sim 1.5\cos^2(\theta/2)\left(\frac{m_\sigma(\mathrm{GeV})}{1\ \mathrm{GeV}}\right)^3\ \mathrm{GeV} > 500\ \mathrm{MeV}. \quad (32)$$

My analysis supports that the broad $f_0(600)$ hides, within its experimental indetermination the two states, the conventional σ meson and the lightest glueball g.

If we relax the broken OZID hypothesis we could arrive to an exotic scenario in which for large mixings one of the states has a small mass close to the 2π threshold and an extremely small width due to the kinematical threshold factor appearing in Eq. (28). This exotic scenario is characterized by a quasi stable state close to the observed lower mass limit (~ 400 MeV) and a broad width state in the upper mass limit (~ 1200 MeV).

The $f_0(980)$, which belongs to the meson nonet, is too narrow to correspond to our sigma-model state. The $f_0(980)$ survives the large N_c limit [38], therefore I ascribe it to the first excited meson. Since its width is low, it does not seem to arise from a mixing with the lower lying states and therefore it sets the upper bound for the mass of the σ. Thus the existence of the $f_0(980)$ excludes, in my view, the extreme exotic scenario and validates a broken OZID scheme.

The $f_0(1370)$ region is again ill-determined experimentally. In this case new channels, like 2η, open up. A similar mixing scheme would predict two excited states, a glueball and a meson. We cannot apply here the naive sigma model width and therefore the discussion for the widths to separate the two states is absent. However, the recent analysis of Forkel [20] comes up with a broad glueball at 1250, which leads me to conjecture that in this region of the spectrum the glueball is the heavier particle of the pair and therefore a $\Delta m < 0$ scenario takes place. The proximity of the $f_0(980)$, and a minimal population hypothesis, leads me to propose the $f_0(980)$ as the required mesonic com-

TABLE I. The scalar spectrum according to the Particle Data Group [39]

	$f_0(600)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$
mass (MeV)	$400-1200$	980 ± 10	$120-1500$	1507 ± 5	1714 ± 5
width (MeV)	$600-1000$	$40-100$	$200-500$	109 ± 7	140 ± 10
Decay	$\pi\pi$ dominant	$\pi\pi$ dominant	$\pi\pi$ seen	$\pi\pi$ 35%	$\pi\pi$ seen
modes	$\gamma\gamma$ seen	$K\bar{K}$ seen	4π	4π 50%	$K\bar{K}$ seen
		$\gamma\gamma$ seen	... $\rho\rho$ dominant	$\eta\eta$ 5%	$\eta\eta$ seen
			... other 4π seen	$\eta\eta'$ 2%	
			$\eta\eta$ seen	$K\bar{K}$ 9%	
			$K\bar{K}$ seen	$\gamma\gamma$ not seen	
			$\gamma\gamma$ seen		

panion. The fact that the lower mass particle, a meson in this case, is narrow confirms the breaking scheme.

Many authors claim that the $f_0(1500)$ is a glueball [21,22], then by assuming the same mixing scheme, I propose a $\Delta m > 0$ scenario in the higher mass region, which ascribes the $f_0(1700)$ as its companion meson. The $f_0(1500)$ could be also the higher lying glueball of Ref. [18] after mixing.

Thus, my analysis leads to the existence of three glueball states in the low lying scalar spectrum with three companion mesons. The precise dynamical mechanisms by which they arise are as of yet unknown, however more precise studies within the large N_c approximation might shed light to my proposal. The duality of the Δm mechanism which leads to an ordering of the spectrum in the form

$$m_g < m_\sigma < m_{\sigma_1} < m_{g_1} < m_{g_2} < m_{\sigma_2} \ldots,$$

has been guided by observation and physical intuition as explained above.

The analysis could be completed by studying other decay modes. In particular 2γ decays also hint about the mass orderings. Using the trace anomaly [27]

$$\Gamma_{\sigma\to 2\gamma} = \frac{\alpha^2}{16\pi^3}\frac{m_\sigma^3}{f_\sigma^2} \sim 10.5\left(\frac{m_\sigma}{1\ \mathrm{GeV}}\right)^3 \mathrm{eV} \qquad (33)$$

where we have used $N_c = 3$ and $f_\sigma \sim f_\pi \sim 100$ MeV. We obtain therefore

$$\Gamma_{g\to 2\gamma} \sim 10.5 \sin^2(\theta/2)\left(\frac{m_g(\mathrm{GeV})}{1\ \mathrm{GeV}}\right)^3 \mathrm{eV} < 1\ \mathrm{eV}, \qquad (34)$$

$$\Gamma_{\sigma\to 2\gamma} \sim 10.5 \cos^2(\theta/2)\left(\frac{m_\sigma(\mathrm{GeV})}{1\ \mathrm{GeV}}\right)^3 \mathrm{eV} > 3\ \mathrm{eV}. \qquad (35)$$

Thus, in my weak mixing scenario, the lightest glueball state is narrower than the lightest meson state.

VI. CONCLUSIONS

I have analyzed the possible existence of a 0^{++} glueball low lying state from different perspectives. The analysis has been modeled by $\frac{1}{N_c}$ physics on which I have also based the estimates. I am led to a scenario of broken OZID and a low mass glueball. This glueball is narrow since only its σ state component is allowed to decay and the small mixing angle inhibits decays. It represents a beautiful example of approximate OZID in the decays, certainly not in the spectrum.

The discussion and mechanisms can be repeated for the higher lying scalars and a spectrum arises in which glueballs and scalar mesons appear in pairs, with masses ordered according to the sign of the $1/N_c$ breaking parameter Δm.

The lowest lying states, g and σ, appear within the $f_0(600)$ peak in agreement with previous estimates [17–19], and therefore they might be difficult to isolate [40], although their widths are vastly different for strong and electromagnetic decays. Maybe more precise experiments could manage to see the two peaks.

The existence of low lying glueballs might strongly influence the transition towards the quark gluon plasma [41,42], and it might be in this physical regime where it might appear unquestioned.

The present investigation lies at the foundations for the understanding of the scalar spectrum. My reasonings can be made more quantitative by lattice studies and more sophisticated model studies. It opens up the possibility of understanding glueballs and their dynamics.

ACKNOWLEDGMENTS

Correspondence with C. A. Dominguez, M. Pennington and M. Teper is gratefully acknowledged. The author has benefitted from long and fruitful discussions with H. Forkel. This work was supported by MCYT-FIS2004-05616-C02-01 and GV-GRUPOS03/094. The author acknowledges the hospitality of CERN-TH where this work was completed.

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