PHYSICAL REVIEW D **73,** 054003 (2006)

Chiral Lagrangian with heavy quark-diquark symmetry

Jie Hu[1,*] and Thomas Mehen[1,2,†]

[1]*Department of Physics, Duke University, Durham, North Carolina 27708, USA*
[2]*Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*
(Received 5 December 2005; published 13 March 2006)

We construct a chiral Lagrangian for doubly heavy baryons and heavy mesons that is invariant under heavy quark-diquark symmetry at leading order and includes the leading $O(1/m_Q)$ symmetry violating operators. The theory is used to predict the electromagnetic decay width of the $J = \frac{3}{2}$ member of the ground state doubly heavy baryon doublet. Numerical estimates are provided for doubly charm baryons. We also calculate chiral corrections to doubly heavy baryon masses and strong decay widths of low lying excited doubly heavy baryons.

DOI: 10.1103/PhysRevD.73.054003 PACS numbers: 12.39.Fe, 12.39.Hg, 14.20.Lq

I. INTRODUCTION

Heavy quark-diquark symmetry relates mesons with a single heavy quark to antibaryons with two heavy antiquarks. Savage and Wise [1] argued that quark-diquark symmetry was realized in the heavy quark limit of quantum chromodynamics (QCD) and studied this symmetry using the methods of heavy quark effective theory (HQET) [2]. Recently, Refs. [3,4] derived effective Lagrangians for heavy diquarks within the framework of nonrelativistic QCD (NRQCD) [5–7]. These papers obtain a prediction for the hyperfine splitting of the ground state doubly heavy baryons in terms of the ground state heavy meson hyperfine splitting.[1] Heavy quark-diquark symmetry also relates other properties of heavy mesons and doubly heavy baryons. A useful tool for studying low energy strong and electromagnetic interactions of heavy hadrons is heavy hadron chiral perturbation theory (HHχPT) [10–12]. This theory has heavy hadrons, Goldstone bosons, and photons as its elementary degrees of freedom and incorporates the approximate chiral and heavy quark symmetries of QCD. In this paper we derive a version of HHχPT that includes doubly heavy baryons and incorporates heavy quark-diquark symmetry. The theory is used to calculate chiral corrections to doubly heavy baryon masses and to obtain model-independent predictions for the electromagnetic decay of the $J = \frac{3}{2}$ member of the ground state doubly heavy baryon doublet. We also discuss the low lying excited doubly heavy baryons, show how these states can be included in the effective theory, and calculate their strong decay widths.

Hadrons with two or more heavy quarks are characterized by several scales: the mass of the heavy quarks, m_Q, their momentum $\sim m_Q v$, their kinetic energy $\sim m_Q v^2$, and $\Lambda_{\rm QCD}$. Our formalism works best in the limit $m_Q \gg m_Q v \gg m_Q v^2 \gtrsim \Lambda_{\rm QCD}$. In this limit, the spatial extent of the diquark, $\sim 1/(m_Q v)$, is small compared to the size of the hadron, $\sim 1/\Lambda_{\rm QCD}$, and the diquark can be treated as a pointlike object. Such a separation of scales is only approximately realized in doubly charmed and doubly bottom baryons. The scale $\Lambda_{\rm QCD}$ should be identified with the hadronic matrix elements typical of heavy quark systems. Using the hyperfine splittings of heavy mesons, which are $O(\Lambda_{\rm QCD}^2/m_Q)$, or the excitation energies of excited heavy mesons, which are $O(\Lambda_{\rm QCD})$, one finds $\Lambda_{\rm QCD} \sim$ 350–500 MeV. Taking $m_b \sim 5$ GeV, $m_c \sim 1.5$ GeV, $v_b^2 \sim 0.1$ and $v_c^2 \sim 0.3$ [5] one finds $m_b v_b^2 \sim m_c v_c^2 \sim \Lambda_{\rm QCD}$, while $m_b v_b \sim 1.5$ GeV and $m_c v_c \sim 800$ MeV. If one assumes that corrections to heavy quark-diquark symmetry scale as $\Lambda_{\rm QCD}/(m_Q v)$ then these naive estimates lead to the expectation that heavy quark-diquark symmetry predictions should have ~30% errors for doubly bottom baryons and ~60% errors for doubly charm baryons. Nevertheless, there is some empirical support for applying heavy quark-diquark symmetry to doubly charm baryons, since the leading heavy quark-diquark symmetry prediction for the hyperfine splitting of doubly charm baryons only differs from quenched lattice QCD calculations and preliminary experimental data by about 30% [3,4,8]. More data and theoretical predictions are needed to test whether heavy quark-diquark symmetry applies to doubly charm baryons, and an important goal of this paper is to provide new predictions based on heavy quark-diquark symmetry for this purpose. Though doubly bottom baryons would make a better testing ground for the formalism of this paper, no experimental observation of such states has been reported to date. Our formulas can be applied to doubly bottom baryons once they are discovered. In our numerical estimates, we will focus on doubly charmed baryons since there is some experimental evidence for the existence of these states [13–15].

An alternative power counting for doubly charm baryons would be to assume that $m_Q v \sim \Lambda_{\rm QCD}$. Since our previous estimates for $m_c v_c$ and $\Lambda_{\rm QCD}$ do not greatly differ this is also a plausible power counting for doubly charm baryons.

*Electronic address: hujie@phy.duke.edu
†Electronic address: mehen@phy.duke.edu

[1]The formula for the hyperfine splittings in Ref. [1] differs from the correct formula in Refs. [3,4] by a factor of 2. The result is easily derived in the quark model, see e.g. Refs. [8,9].

The assumption $m_Q v \sim \Lambda_{\rm QCD}$ underlies the construction of strongly coupled pNRQCD which has been developed for $Q\bar{Q}$ systems, see e.g., Chap. 7 in Ref. [16]. Presumably, such a theory would treat diquarks as extended objects rather than pointlike. Development of such a theory is beyond the scope of this paper. It is not clear whether quark-diquark symmetry predictions would survive in this theory and resolution of this question requires a detailed analysis of the structure of possible $O(\Lambda_{\rm QCD}/(m_Q v))$ corrections which does not exist at the present time.

Motivation for this work comes from the SELEX experiment's recent observation of states which have been tentatively interpreted as doubly charm baryons [13–15], and also the COMPASS experiment, which in its second phase run in 2006 hopes to observe doubly charm baryons [17]. Many aspects of the SELEX states are difficult to understand. States observed by SELEX include the $\Xi_{cc}^{+}(3520)$, which decays weakly into $\Lambda_c^+\pi^+K^-$ [13] as well as pD^+K^- [15], the $\Xi_{cc}^{++}(3460)$, which decays weakly into $\Lambda_c^+K^-\pi^+\pi^+$ [14], and a broader state, $\Xi_{cc}^{++}(3780)$, also seen to decay into $\Lambda_c^+K^-\pi^+\pi^+$ [14]. The ground states of the Ξ_{cc}^{+} and Ξ_{cc}^{++} are related by isospin symmetry and therefore should differ in mass by only a few MeV, so the observed difference of 60 MeV seems implausible. On the other hand, an unpublished talk [18] and conference proceedings [19] present evidence for additional states, $\Xi_{cc}^{+}(3443)$ and $\Xi_{cc}^{++}(3541)$. If these states exist the isospin splittings are closer to theoretical expectations, but still quite large. The difference between the mass of the $\Xi_{cc}^{+}(3520)$ and $\Xi_{cc}^{+}(3443)$ is 77 MeV, and the splitting between $\Xi_{cc}^{++}(3541)$ and $\Xi_{cc}^{++}(3460)$ is 81 MeV. These splittings agree remarkably well with calculations of the doubly charm hyperfine splittings in quenched lattice QCD [8,20,21] and are within $\sim$25 MeV of the heavy quark-diquark symmetry prediction obtained in Refs. [3,4,8,9], an acceptable discrepancy given the expected corrections. However, interpretation of the $\Xi_{cc}^{+}(3520)$ as the $J=\frac{3}{2}$ member of the ground state doublet is impossible to reconcile with the fact that the $\Xi_{cc}^{+}(3520)$ is observed to decay weakly because if the $\Xi_{cc}^{+}(3520)$ is not the ground state of the ccd system it should decay electromagnetically. There are also discrepancies between the weak decay lifetimes predicted by HQET [22–24]($\sim$ 100 fs) and the observed lifetimes ($<$ 33 fs) [13,15]. Production cross sections are also poorly understood within perturbative QCD [25,26]. However, the SELEX states are observed in the forward region, $\langle x_F\rangle \sim 0.3$, where nonperturbative production mechanisms such as intrinsic charm [27–29] or parton recombination [30,31] may be important.

Even if there is difficulty interpreting the SELEX data, doubly charm baryons must exist and are expected to have masses of approximately 3.5 GeV [25,32–37], where the SELEX states are. In light of existing and future experimental efforts to observe doubly charm baryons, it is desirable to have model-independent predictions for other properties besides the relation for the hyperfine splittings derived in Refs. [1,3,4]. Therefore it is important to develop theoretical tools for analyzing the properties of doubly heavy baryons systematically.

The rest of this paper is organized as follows. In Sec. II, we derive a chiral Lagrangian for the ground state doubly heavy baryons and heavy mesons with approximate heavy quark-diquark symmetry. In Sec. III, the electromagnetic decay widths of the $J=\frac{3}{2}$ members of the ground state doubly heavy baryon doublets are calculated. In Sec. IV, we compute the chiral corrections to doubly heavy baryon masses. In Sec. V, we discuss the lowest lying excitations of the doubly heavy baryon, which turn out to be excitations of the diquark. The strong decay rates of these states are calculated. In Sec. VI, a brief summary is given.

II. DERIVATION OF THE CHIRAL LAGRANGIAN

Savage and Wise [1] wrote down a version of heavy quark effective theory (HQET) which includes diquarks as elementary degrees of freedom and derived a formula relating heavy meson and doubly heavy baryon hyperfine splittings. HQET only separates the scales $\Lambda_{\rm QCD}$ and m_Q. The correct effective theory for hadrons with two heavy quarks is nonrelativistic QCD (NRQCD) [5], which properly accounts for the scales $m_Q v$ and $m_Q v^2$. Analysis of heavy diquarks within the framework of NRQCD was recently performed in Refs. [3,4]. These papers derived Lagrangians for diquark fields starting from NRQCD and obtained the correct heavy quark symmetry prediction for the hyperfine splittings of the doubly heavy baryons. For simplicity, we will consider only one flavor of heavy quark. The lowest mass diquark will consist of two heavy antiquarks in an orbital S-wave in the $\mathbf{3}$ representation of color $SU(3)$. Then Fermi statistics demands that they have total spin one. In the rest frame of the heavy quark and lowest mass diquark, the Lagrangian to $O(1/m_Q)$ is [1,4]

$$\mathcal{L} = h^\dagger\Big(iD_0 - \frac{\vec{D}^2}{2m_Q}\Big)h + \vec{V}^\dagger\cdot\Big(iD_0 + \delta - \frac{\vec{D}^2}{4m_Q}\Big)\vec{V} + \frac{g_s}{2m_Q}h^\dagger\vec{\sigma}\cdot\vec{B}^a\frac{\lambda^a}{2}h + \frac{ig_s}{2m_Q}\vec{V}^\dagger\cdot\vec{B}^a\frac{\lambda^a}{2}\times\vec{V}. \quad (1)$$

Here h is the heavy quark field, $\vec{V}$ is the field for the diquarks, the $\lambda^a/2$ are the $SU(3)$ color generators, $\mathrm{Tr}[\lambda^a\lambda^b] = 2\delta^{ab}$, D_0 and $\vec{D}$ are the time and spatial components of the gauge covariant derivative, respectively, and $\vec{B}^a$ is the chromomagnetic field. The term proportional to δ is the residual mass of the diquark. The heavy antiquarks in the diquark experience an attractive force and therefore the mass of the diquark is not $2m_Q$ but $2m_Q - \delta$, where δ is the binding energy. This residual mass can be removed by rephasing the diquark fields. Physically, this corresponds to measuring diquark energies relative to the mass of the diquark, rather than $2m_Q$. Once this is done the

Lagrangian, at lowest order in $1/m_Q$, is invariant under a $U(5)$ symmetry which permutes the two spin states of the heavy quark and the three spin states of the heavy antiquark. The $U(5)$ symmetry is broken by the $O(1/m_Q)$ kinetic energy and chromomagnetic couplings of the heavy quark and diquark. The latter terms are responsible for the hyperfine splittings.

The ground state doublet of heavy mesons is usually represented in HHχPT as a 4×4 matrix transforming covariantly under Lorentz transformations, and transforming as a doublet under $SU(2)$ heavy quark spin symmetry,

$$H_v = \left(\frac{1+\not{v}}{2}\right)(P_v^{*\mu}\gamma_\mu - \gamma_5 P_v). \tag{2}$$

Here $P_v^{*\mu}$ is the $J^P = 1^-$ vector heavy meson field which obeys the constraint $v_\mu P_v^{*\mu} = 0$, where v^μ is the four-velocity of the heavy meson. P_v is the $J^P = 0^-$ pseudoscalar heavy meson field. The superfield H_v obeys the constraints $\not{v} H_v = -H_v \not{v} = H_v$, so H_v only has four independent degrees of freedom. These can be collected in a 2×2 matrix. For example, in the heavy meson rest frame where $v^\mu = (1, 0, 0, 0)$,

$$H_v = \begin{pmatrix} 0 & -\vec{P}_v^* \cdot \vec{\sigma} - P_v \\ 0 & 0 \end{pmatrix}, \tag{3}$$

where we have used the Bjorken-Drell conventions for γ_μ and γ_5. For a process such as the weak decay $B \to D\ell\nu$, in which the initial and final heavy hadrons have different four-velocities, the covariant representation of fields is needed to determine heavy quark symmetry constraints on heavy hadron form-factors. However, for studying low energy strong and electromagnetic interactions in which the heavy meson four-velocity is conserved (up to $O(\Lambda_{\rm QCD}/m_Q)$ corrections), it is also possible to work in the heavy meson rest frame and use 2×2 matrix fields. This makes some calculations simpler and we find it easiest to formulate the extension of HHχPT with $U(5)$ quark-diquark symmetry in this frame. We define the heavy meson field in our theory to be

$$H_a = \vec{P}_a^* \cdot \vec{\sigma} + P_a, \tag{4}$$

where a is an $SU(3)$ flavor antifundamental index and the $\vec{\sigma}$ are the Pauli matrices. Since we have chosen to work in the heavy meson rest frame, Lorentz covariance is lost and the symmetries of the theory are rotational invariance, $SU(2)$ heavy quark spin symmetry, parity, time reversal, and $SU_L(3) \times SU_R(3)$ chiral symmetry. Under these symmetries the field H_a transforms as

$$\begin{aligned} &\text{rotations} && H'_a = U H_a U^\dagger \\ &\text{heavy quark spin} && H'_a = S H_a \\ &\text{parity} && H'_a = -H_a \\ &\text{time reversal} && H'_a = -\sigma_2 H_a^* \sigma_2 \\ &SU_L(3) \times SU_R(3) && H'_a = H_b V_{ba}^\dagger. \end{aligned} \tag{5}$$

Here U and S are 2×2 rotation matrices and $V_{ba}^\dagger$ is an $SU(3)$ matrix which gives the standard nonlinear realization of $SU_L(3) \times SU_R(3)$ chiral symmetry. In the two-component notation the HHχPT Lagrangian is:

$$\begin{aligned} \mathcal{L} &= \mathrm{Tr}[H_a^\dagger (iD_0)_{ba} H_b] - g\,\mathrm{Tr}[H_a^\dagger H_b \vec{\sigma} \cdot \vec{A}_{ba}] \\ &\quad + \frac{\Delta_H}{4}\,\mathrm{Tr}[H_a^\dagger \sigma^i H_a \sigma^i]. \end{aligned} \tag{6}$$

The last term breaks heavy quark spin symmetry and Δ_H is the hyperfine splitting of the heavy mesons. The time component of the covariant chiral derivative is $(D_0)_{ba}$, $\vec{A}_{ba}$ is the spatial part of the axial vector field, and g is the heavy meson axial coupling. Our definitions for the chiral covariant derivative, the axial current, and the Lagrangian for the Goldstone boson fields are the same as Ref. [38].

We are now ready to generalize the Lagrangian to incorporate the doubly heavy baryons and the $U(5)$ quark-diquark symmetry. The field H_a transforms like the tensor product of a heavy quark spinor and a light antiquark spinor. (This is how representations of heavy hadron fields were constructed in Ref. [39].) Writing the field with explicit indices, $(H_a)_{\alpha\beta}$, the index α corresponds to the spinor index of the heavy quark and the index β is that of the light antiquark spinor. In the theory with quark-diquark symmetry, the heavy quark spinor is replaced with a five-component field, the first two components corresponding to the two heavy quark spin states and the last three components corresponding to the three spin states of the diquark:

$$\mathcal{Q}_\mu = \begin{pmatrix} h_\alpha \\ V_i \end{pmatrix}. \tag{7}$$

In terms of $\mathcal{Q}_\mu$ the kinetic terms of the Lagrangian in Eq. (1) are

$$\mathcal{L} = \mathcal{Q}_\mu^\dagger i D_0 \mathcal{Q}_\mu. \tag{8}$$

The fields in HHχPT with heavy quark-diquark symmetry transform as tensor products of the five-component field $\mathcal{Q}_\mu$ and a two-component light antiquark spinor. Thus, the 2×2 matrix field H_a is promoted to a 5×2 matrix field

$$H_{a,\alpha\beta} \to \mathcal{H}_{a,\mu\beta} = H_{a,\alpha\beta} + T_{a,i\beta}. \tag{9}$$

Here the index μ takes on values between 1 and 5, $\alpha, \beta =$ 1 or 2, and $i = 3$, 4, or 5. The doubly heavy baryon fields are contained in $T_{a,i\beta}$. Under the symmetries of the theory

$\mathcal{H}_a$ transforms as

$$\begin{aligned} \text{rotations} \quad & \mathcal{H}'_a = \mathcal{R}\mathcal{H}_a U^\dagger \\ \text{heavy quark spin} \quad & \mathcal{H}'_a = S\mathcal{H}_a \\ \text{parity} \quad & \mathcal{H}'_a = -\mathcal{H}_a \\ \text{time reversal} \quad & \mathcal{H}'_a = -\Sigma_2 \mathcal{H}^*_a \sigma_2 \\ SU_L(3)\times SU_R(3) \quad & \mathcal{H}'_a = \mathcal{H}_b V^\dagger_{ba}. \end{aligned} \tag{10}$$

The matrix S is now an element of $U(5)$ and $\mathcal{R}$ is a 5×5 reducible rotation matrix

$$\mathcal{R}_{\mu\nu} = \begin{pmatrix} U_{\alpha\beta} & 0 \\ 0 & R_{ij} \end{pmatrix}, \tag{11}$$

where $U_{\alpha\beta}$ is an $SU(2)$ rotation matrix and R_{ij} is an orthogonal 3×3 rotation matrix related to U by $U^\dagger \sigma_i U = R_{ij}\sigma_j$. The 5×5 matrix appearing in the time reversal transformation is

$$(\Sigma_2)_{\mu\nu} = \begin{pmatrix} (\sigma_2)_{\alpha\beta} & 0 \\ 0 & \delta_{ij} \end{pmatrix}. \tag{12}$$

Under rotations the field $T_{a,i\beta}$ transforms as $T'_{a,i\beta} = R_{ij} T_{a,j\gamma} U^\dagger_{\gamma\beta}$. $T_{a,i\beta}$ can be further decomposed into its spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ components,

$$T_{a,i\beta} = \sqrt{2}\left(\Xi^*_{a,i\beta} + \frac{1}{\sqrt{3}}\Xi_{a,\gamma}\sigma^i_{\gamma\beta}\right), \tag{13}$$

where $\Xi^*_{a,i\beta}$ and $\Xi_{a,\gamma}$ are the spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ fields, respectively. The factor of $\sqrt{2}$ is a convention that ensures that the kinetic terms of the doubly heavy baryon fields have the same normalization as the heavy meson fields. The field $\Xi^*_{a,i\beta}$ obeys the constraint $\Xi^*_{a,i\beta}\sigma^i_{\beta\gamma} = 0$.

The $U(5)$ invariant generalizations of the first two terms of Eq. (6) are simply obtained by making the replacement $H_a \to \mathcal{H}_a$. To determine the proper generalization of the $U(5)$ breaking term we note that the chromomagnetic couplings in Eq. (1) can be written as

$$\frac{g_s}{2m_Q} \mathcal{Q}^\dagger_\mu \vec{\Sigma}_{\mu\nu}\cdot \vec{B}^a \frac{\lambda^a}{2}\mathcal{Q}_\nu, \tag{14}$$

where the $\vec{\Sigma}_{\mu\nu}$ are the 5×5 matrices

$$\vec{\Sigma}_{\mu\nu} = \begin{pmatrix} \vec{\sigma}_{\alpha\beta} & 0 \\ 0 & \vec{\mathcal{T}}_{jk} \end{pmatrix}, \tag{15}$$

and $(\mathcal{T}^i)_{jk} = -i\epsilon_{ijk}$. It is now obvious that the correct generalization of Eq. (6) is

$$\begin{aligned} \mathcal{L} &= \mathrm{Tr}[\mathcal{H}^\dagger_a (iD_0)_{ba}\mathcal{H}_b] - g\,\mathrm{Tr}[\mathcal{H}^\dagger_a \mathcal{H}_b \vec{\sigma}\cdot\vec{A}_{ba}] \\ &\quad + \frac{\Delta_H}{4}\mathrm{Tr}[\mathcal{H}^\dagger_a \Sigma^i \mathcal{H}_a \sigma^i] \\ &= \mathrm{Tr}[H^\dagger_a (iD_0)_{ba} H_b] - g\,\mathrm{Tr}[H^\dagger_a H_b \vec{\sigma}\cdot\vec{A}_{ba}] \\ &\quad + \frac{\Delta_H}{4}\mathrm{Tr}[H^\dagger_a \sigma^i H_a \sigma^i] + \mathrm{Tr}[T^\dagger_a (iD_0)_{ba} T_b] \\ &\quad - g\,\mathrm{Tr}[T^\dagger_a T_b \vec{\sigma}\cdot\vec{A}_{ba}] + \frac{\Delta_H}{4}\mathrm{Tr}[T^\dagger_a \mathcal{T}^i T_a \sigma^i]. \end{aligned} \tag{16}$$

The last three terms of Eq. (16) are relevant for doubly heavy baryons. Heavy quark-diquark symmetry relates the couplings in the doubly heavy baryon sector to the heavy meson sector. The propagator for the spin-$\frac{1}{2}$ doubly heavy baryon is

$$\frac{i\delta_{ab}\delta_{\alpha\beta}}{2(k_0 + \Delta_H/2 + i\epsilon)},$$

while the propagator for the spin-$\frac{3}{2}$ doubly heavy baryon is

$$\frac{i\delta_{ab}\mathcal{P}_{i\alpha,j\beta}}{2(k_0 - \Delta_H/4 + i\epsilon)} = \frac{i\delta_{ab}(\delta_{ij}\delta_{\alpha\beta} - \frac{1}{3}(\sigma^i\sigma^j)_{\alpha\beta})}{2(k_0 - \Delta_H/4 + i\epsilon)}.$$

The projection operator $\mathcal{P}_{i\alpha,j\beta}$ satisfies $\sigma^i_{\gamma\alpha}\mathcal{P}_{i\alpha,j\beta} = \mathcal{P}_{i\alpha,j\beta}\sigma^j_{\beta\gamma} = 0$. Comparison of the poles of the propagators shows that the hyperfine splitting for the doubly heavy baryons is $\frac{3}{4}\Delta_H$, reproducing the heavy quark-diquark symmetry prediction

$$m_{\Xi^*} - m_{\Xi} = \tfrac{3}{4}(m_{P^*} - m_P), \tag{17}$$

obtained in Refs. [3,4].

III. ELECTROMAGNETIC DECAYS

The decay $\Xi^* \to \Xi\gamma$ is related to the decay $P^* \to P\gamma$ by heavy quark-diquark symmetry and therefore is interesting to study in our formalism. For interpreting the SELEX states, it is useful to have estimates of the electromagnetic decay widths. Even with the substantial $O(\Lambda_{\mathrm{QCD}}/(m_c v))$ corrections that could be present for doubly charm baryons, such estimates should be helpful for deciding whether an observed Ξ_{cc} is the $J = \frac{3}{2}$ member of the gound state doublet.

The Lagrangian for electromagnetic decays of the heavy mesons in the two-component notation is [40]

$$\mathcal{L} = \frac{e\beta}{2}\mathrm{Tr}[H^\dagger_a H_b \vec{\sigma}\cdot\vec{B} Q_{ab}] + \frac{e}{2m_Q}Q'\,\mathrm{Tr}[H^\dagger_a \vec{\sigma}\cdot\vec{B} H_a], \tag{18}$$

where $Q_{ab} = \mathrm{diag}(2/3, -1/3, -1/3)$ is the light quark charge matrix, β is the parameter introduced in Ref. [40], and Q' is the heavy quark charge. For charm, $Q' = 2/3$. The first term is the magnetic moment coupling of the light degrees of freedom and the second term is the magnetic moment coupling of the heavy quark. Both terms are needed to understand the observed electromagnetic branching fractions of the D^{*+} and D^{*0} because a cancellation between the two terms accounts for the very small width of the D^{*+} relative to the D^{*0} [40]. The magnetic couplings of the heavy quark and diquark are

$$\begin{aligned}\mathcal{L}_{em} &= \frac{e}{2m_Q} Q' h^\dagger \vec{\sigma} \cdot \vec{B} h - \frac{ie}{m_Q} Q' \vec{V}^\dagger \cdot \vec{B} \times \vec{V} \\ &= \frac{e}{2m_Q} Q' \mathcal{Q}_\mu^\dagger \vec{\Sigma}'_{\mu\nu} \cdot \vec{B} \mathcal{Q}_\nu, \end{aligned} \tag{19}$$

where the $\vec{\Sigma}'_{\mu\nu}$ are the 5×5 matrices

$$\vec{\Sigma}'_{\mu\nu} = \begin{pmatrix} \vec{\sigma}_{\alpha\beta} & 0 \\ 0 & -2\vec{\mathcal{T}}_{jk} \end{pmatrix}. \tag{20}$$

The magnetic coupling of the diquark has the opposite sign as that of the heavy quark because it is composed of two heavy antiquarks. The coefficient of the chromomagnetic coupling of the diquark in Eqs. (1) and (14) is a factor of 2 smaller than the coefficient of the electromagnetic coupling of the diquark in Eq. (19) due to a color factor. The magnetic couplings in the HHχPT Lagrangian for heavy mesons and doubly heavy baryons are

$$\begin{aligned}\mathcal{L} &= \frac{e\beta}{2} \mathrm{Tr}[\mathcal{H}_a^\dagger \mathcal{H}_b \vec{\sigma} \cdot \vec{B} Q_{ab}] \\ &\quad + \frac{e}{2m_Q} Q' \mathrm{Tr}[\mathcal{H}_a^\dagger \vec{\Sigma}' \cdot \vec{B} \mathcal{H}_b]. \end{aligned} \tag{21}$$

The part of this Lagrangian involving the doubly heavy baryon fields is

$$\mathcal{L} = \frac{e\beta}{2} \mathrm{Tr}[T_a^\dagger T_b \vec{\sigma} \cdot \vec{B} Q_{ab}] - \frac{e}{m_Q} Q' \mathrm{Tr}[T_a^\dagger \vec{\mathcal{T}} \cdot \vec{B} T_b]. \tag{22}$$

This can be used to obtain the following tree level predictions for the electromagnetic decay widths:

$$\begin{aligned}\Gamma[P_a^* \to P_a \gamma] &= \frac{\alpha}{3}\left(\beta Q_{aa} + \frac{Q'}{m_Q}\right)^2 \frac{m_P}{m_{P^*}} E_\gamma^3 \\ \Gamma[\Xi_a^* \to \Xi_a \gamma] &= \frac{4\alpha}{9}\left(\beta Q_{aa} - \frac{Q'}{m_Q}\right)^2 \frac{m_\Xi}{m_{\Xi^*}} E_\gamma^3. \end{aligned} \tag{23}$$

Here E_γ is the photon energy. These results could also be obtained in the quark model, with the parameter $\beta = 1/m_q$, where m_q is the light constituent quark mass. The effective theory allows one to improve upon this approximation by including corrections from loops with light Goldstone bosons, which give $O(\sqrt{m_q})$ corrections to the decay rates [40]. If these loop corrections are evaluated in an approximation where heavy hadron mass differences are neglected, the correction to the above formulas can be incorporated by making the following replacements [40]

$$\begin{aligned} &\beta Q_{11} \to \frac{2}{3}\beta - \frac{g^2 m_K}{4\pi f_K^2} - \frac{g^2 m_\pi}{4\pi f_\pi^2} \\ &\beta Q_{22} \to -\frac{1}{3}\beta + \frac{g^2 m_\pi}{4\pi f_\pi^2} \qquad \beta Q_{33} \to -\frac{1}{3}\beta + \frac{g^2 m_K}{4\pi f_K^2}. \end{aligned} \tag{24}$$

For charm mesons, hyperfine splittings are ≈ 140 MeV and the $SU(3)$ splitting is ≈ 100 MeV, while for bottom mesons the hyperfine splittings are ≈ 45 MeV and $SU(3)$ splitting is ≈ 90 MeV. The approximation of neglecting heavy hadron mass differences and keeping Goldstone boson masses is reasonable for kaon loops but not for loops with pions. However, the largest $O(\sqrt{m_q})$ corrections come from loops with kaons. When data on double heavy baryon electromagnetic decays is available, more accurate calculations along the lines of Ref. [38] should be performed. In this paper, we will use Eqs. (23) and (24) to obtain estimates of doubly charm baryon electromagnetic decay widths.

Currently $\Gamma[D^{*+}]$ is measured to be 96 ± 22 KeV [41], while there is only an upper limit for $\Gamma[D^{*0}]$. The branching ratios for the D^{*+} decays are $\mathrm{Br}[D^{*+} \to D^0\pi^+] = 67.7 \pm 0.5\%$, $\mathrm{Br}[D^{*+} \to D^+\pi^0] = 30.7 \pm 0.5\%$ and $\mathrm{Br}[D^{*+} \to D^+\gamma] = 1.6 \pm 0.4\%$, and the branching ratios for D^{*0} decays are $\mathrm{Br}[D^{*0} \to D^0\pi^0] = 61.9 \pm 2.9\%$ and $\mathrm{Br}[D^{*0} \to D^0\gamma] = 38.1 \pm 2.9\%$ [41]. Isospin symmetry can be used to relate the strong partial width of the D^{*0} to the known strong partial width of the D^{*+}. Then the measured branching fractions of the D^{*0} can be used to obtain the partial electromagnetic width of the D^{*0}. We find

$$\begin{aligned}\Gamma[D^{*0} \to D^0\gamma] &= 26.1 \pm 6.0 \text{ keV} \\ \Gamma[D^{*+} \to D^+\gamma] &= 1.54 \pm 0.35 \text{ keV}, \end{aligned} \tag{25}$$

where the error is dominated by the uncertainty in $\Gamma[D^{*+}]$. $\Gamma[D^{*+} \to D^+\gamma]$ is suppressed because of a partial cancellation between the magnetic moments of the light degrees of freedom and the charm quark. Using the partial widths in Eq. (25) and the formulas in Eqs. (23) and (24), we obtain predictions for doubly charm baryon electromagnetic decays in Table I.

TABLE I. Predictions for the electromagnetic widths of the Ξ_{cc}^{*+} and Ξ_{cc}^{*++}. The fits are explained in the text.

Fit	β^{-1} (MeV)	m_c (MeV)	$\Gamma[\Xi_{cc}^{*++}]$ (keV)	$\Gamma[\Xi_{cc}^{*+}]$ (keV)
QM 1	379	1863	$3.3\left(\frac{E_\gamma}{80\text{ MeV}}\right)^3$	$2.6\left(\frac{E_\gamma}{80\text{ MeV}}\right)^3$
QM 2	356	1500	$3.4\left(\frac{E_\gamma}{80\text{ MeV}}\right)^3$	$3.2\left(\frac{E_\gamma}{80\text{ MeV}}\right)^3$
χPT 1	272	1432	$2.3\left(\frac{E_\gamma}{80\text{ MeV}}\right)^3$	$3.5\left(\frac{E_\gamma}{80\text{ MeV}}\right)^3$
χPT 2	276	1500	$2.3\left(\frac{E_\gamma}{80\text{ MeV}}\right)^3$	$3.3\left(\frac{E_\gamma}{80\text{ MeV}}\right)^3$

In our calculations of the doubly charm baryon decay widths the factor m_{Ξ}/m_{Ξ^*} in Eq. (23) has been set equal to one. For the expected masses and hyperfine splittings of the doubly charm baryons, this factor changes the predictions for the widths by less than 3%. The fits are labeled in the left hand column of Table I. In the fits labeled QM we have not included the $O(\sqrt{m_q})$ corrections in Eq. (24). Therefore, these predictions for the doubly charm baryon electromagnetic decays are the same as what would be obtained in the quark model. The values of the parameters β and m_c for each fit are shown along with the predictions for the electromagnetic decay widths. In QM 1, we have treated β and m_c as free parameters and fit these to the central values in Eq. (25). In QM 2 we have set $m_c = 1500$ MeV and performed a least squared fit to β. In the fits labeled χPT, we have included the leading $O(\sqrt{m_q})$ chiral corrections in Eq. (24). We have used $f_\pi = 130$ MeV, $f_K = 159$ MeV, and $g = 0.6$ which is extracted from a tree level fit to the D^{*+} width. In χPT 1, we fixed β and m_c to reproduce the central values in Eq. (25). In χPT 2, we set $m_c = 1500$ MeV and performed a least squares fit to β.

There are corrections to the decay rates coming from counterterms we have not included in the Lagrangian. For a comprehensive listing of counterterms contributing to the decay $D^* \to D\gamma$, see Eq. (14) of Ref. [38]. Counterterms contributing to our theory can be obtained from that paper by substituting $H_a \to \mathcal{H}_a$. In addition there are operators which do not violate $SU(2)$ heavy quark spin-symmetry but do violate $U(5)$ heavy quark-diquark symmetry which cannot be obtained by the substitution $H_a \to \mathcal{H}_a$. An example is the operator $\mathrm{Tr}[\mathcal{H}_a^\dagger \mathcal{X} \mathcal{H}_a \vec{\sigma} \cdot \vec{B}]$, where $\mathcal{X}$ is the 5×5 matrix

$$\mathcal{X}_{\mu\nu} = \begin{pmatrix} \delta_{\alpha\beta} & 0 \\ 0 & -\delta_{ij} \end{pmatrix}. \qquad (26)$$

There are a large number of counterterms, and not enough data to fix all the free parameters in any calculation that includes them systematically. Still we can estimate the uncertainty due to these higher order effects. Operators with explicit factors of m_q are higher order than the nonanalytic $O(\sqrt{m_q})$ corrections we have included, and their contributions to the decay rates are suppressed by m_π/Λ_χ, which is of order 10%. ($SU(3)$ symmetry breaking counterterms give larger contributions, $\sim m_K/\Lambda_\chi$, but our predictions do not rely on $SU(3)$ symmetry, and therefore these counterterms do not affect our predictions.) There are operators that contain additional derivatives, these give corrections suppressed by $E_\gamma/\Lambda_\chi \sim 10\%$. Heavy quark spin-symmetry breaking operators should be suppressed by $\Lambda_{\mathrm{QCD}}/m_c \sim 30\%$. Operators that break $U(5)$ quark-diquark symmetry may give large $\Lambda_{\mathrm{QCD}}/(m_Q v) \sim 60\%$ corrections. Finally there is a 25% uncertainty from the experimentally measured value of $\Gamma[D^{*+}]$. Taking all these sources of error into account, we expect that there is almost $O(1)$ uncertainty in the predictions in Table I.

Chiral perturbation theory and the nonrelativistic quark model give similar size estimates for the Ξ_{cc}^* electromagnetic decay widths which are expected to be ~2–3 keV if the hyperfine splitting is 80 MeV. The electromagnetic decay should completely dominate any possible weak decay, even if the weak decay rates are an order of magnitude greater than calculated in Refs. [22–24]. Despite the large uncertainties discussed in the previous paragraph, it is safe to conclude that SELEX states observed via weak decays cannot be $J = \frac{3}{2}$ members of the ground state doublet. The quark model predicts $\Gamma[\Xi_{cc}^{*++}]$ slightly greater than $\Gamma[\Xi_{cc}^{*+}]$. This is in contrast with the charm meson sector where the magnetic moment of the light degrees of freedom and the magnetic moment of the charm quark add constructively to give a large $\Gamma[D^{*0} \to D^0\gamma]$ and destructively to give a small $\Gamma[D^{*+} \to D^+\gamma]$. In the doubly heavy baryon sector, the relative sign of the magnetic moments is reversed, and both decay rates are approximately the same. In fact from Eq. (23), we can see that for $\beta = 4/m_c$ the two rates are exactly equal in the quark model. Fits to the D^* electromagnetic decays yield values of β and m_c that are close to this point in parameter space. Including the $O(\sqrt{m_q})$ corrections from chiral perturbation theory, the most important effect is the kaon loop correction whose contribution to the Ξ_{cc}^{*++} decay has opposite sign as the contribution from β at tree level, therefore suppressing the Ξ_{cc}^{*++} decay relative to Ξ_{cc}^{*+}.

IV. MASS CORRECTIONS

The theory can also be used to compute chiral corrections to doubly heavy baryon masses. The corrections to the hadron masses from one-loop diagrams are

$$\begin{aligned}
\delta m_{\Xi_a^*} &= \sum_{i,b} C_{ab}^i \frac{g^2}{16\pi^2 f_i^2}\left(\frac{5}{9}K(m_{\Xi_b^*} - m_{\Xi_a^*}, m_i, \mu) + \frac{4}{9}K(m_{\Xi_b} - m_{\Xi_a^*}, m_i, \mu)\right) \\
\delta m_{\Xi_a} &= \sum_{i,b} C_{ab}^i \frac{g^2}{16\pi^2 f_i^2}\left(\frac{1}{9}K(m_{\Xi_b} - m_{\Xi_a}, m_i, \mu) + \frac{8}{9}K(m_{\Xi_b^*} - m_{\Xi_a}, m_i, \mu)\right) \qquad (27) \\
\delta m_{H_a} &= \sum_{i,b} C_{ab}^i \frac{g^2}{16\pi^2 f_i^2} K(m_{H_b^*} - m_{H_a}, m_i, \mu) \\
\delta m_{H_a^*} &= \sum_{i,b} C_{ab}^i \frac{g^2}{16\pi^2 f_i^2}\left(\frac{1}{3}K(m_{H_b} - m_{H_a^*}, m_i, \mu) + \frac{2}{3}K(m_{H_b^*} - m_{H_a^*}, m_i, \mu)\right).
\end{aligned}$$

Here m_i and f_i are the mass and decay constant of the

Goldstone boson in the one-loop diagram and $\mathcal{C}^i_{ab}$ is a factor which comes from $SU(3)$ Clebsch-Gordan coefficients in the couplings. For loops with charged pions we have $\mathcal{C}^{\pi^\pm}_{12} = \mathcal{C}^{\pi^\pm}_{21} = 1$, for loops with neutral pions $\mathcal{C}^{\pi^0}_{11} = \mathcal{C}^{\pi^0}_{22} = \frac{1}{2}$, for loops with kaons $\mathcal{C}^K_{3i} = \mathcal{C}^K_{i3} = 1$ ($i = 1$ or 2), and for loops with η mesons $C^\eta_{11} = C^\eta_{22} = \frac{1}{6}$ and $C^\eta_{33} = \frac{2}{3}$. The function $K(\delta, m, \mu)$ is related to the finite part of the integral

$$i \int \frac{d^D l}{(2\pi)^D} \frac{\vec{l}^2}{l^2 - m_\pi^2 + i\epsilon} \frac{1}{l_0 - \delta + i\epsilon} = \frac{1}{(4\pi)^2} K(\delta, m, \mu), \tag{28}$$

evaluated using dimensional regularization in the $\overline{MS}$ scheme. We find

$$K(\delta, m, \mu) = (-2\delta^3 + 3m^2\delta) \ln\left(\frac{m^2}{\mu^2}\right) + 2\delta(\delta^2 - m^2) F\left(\frac{\delta}{m}\right) + 4\delta^3 - 5\delta m^2, \tag{29}$$

where

$$F(x) = 2\frac{\sqrt{1-x^2}}{x}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right] \qquad |x| < 1$$
$$= -2\frac{\sqrt{x^2-1}}{x} \ln(x + \sqrt{x^2-1}) \qquad |x| > 1,$$

and μ is the renormalization scale. The μ dependence in the one-loop calculation is cancelled by counterterms that have not been included. Counterterms contributing to the self-energies of heavy mesons can be found in, e.g., Ref. [42]. Again there are many counterterms that contribute to the doubly heavy baryon self-energies, only some of which can be obtained from Ref. [42] by the substitution $H_a \to \mathcal{H}_a$. We focus on the nonanalytic dependence of the self-energies because they provide a rough estimate of the long distance corrections to the leading prediction for hyperfine splittings obtained earlier and because the nonanalytic dependence on the light quark mass could be useful for chiral extrapolations in unquenched lattice QCD calculations of doubly heavy baryon masses.

We are interested in how the one-loop corrections affect the leading order prediction for the hyperfine splittings. Unfortunately, it is impossible to give a reliable estimate without knowing the numerical value of the counterterms required to cancel the μ dependence in the nonanalytic contribution. Furthermore, to compute the contribution from kaon loops, one must know the masses of doubly charm strange baryons which have not been observed. We will assume that the ground state doubly charm strange baryons are 100 MeV higher in mass than their nonstrange counterparts, similar to the D meson system. This is consistent with theoretical estimate of the $SU(3)$ breaking in Refs. [25,32–34,36,37]. We work in the isospin limit and use $g = 0.6$, $\Delta_H = 140$ MeV, $m_\pi = 137$ MeV, $m_K = 496$ MeV, $m_\eta = 548$ MeV and the experimental values of the pseudoscalar meson decay constants: $f_\pi = 130$ MeV, $f_K = 159$ MeV, and $f_\eta = 156$ MeV. The nonanalytic part of the one-loop correction to the nonstrange doubly charm baryon hyperfine splitting is

$$\delta m_{\Xi^*_{cc}} - \delta m_{\Xi_{cc}} = \begin{cases} -7.0 \text{ MeV} & \mu = 500 \text{ MeV} \\ 8.1 \text{ MeV} & \mu = 1000 \text{ MeV}, \\ 16.9 \text{ MeV} & \mu = 1500 \text{ MeV} \end{cases} \tag{30}$$

where we have shown our results for three values of μ. For these choices of μ the nonanalytic part of the chiral correction varies between -7 MeV and $+17$ MeV. The nonanalytic part of the chiral correction to the doubly charm baryon hyperfine splitting is quite sensitive to the choice of μ, and lies within 15% of the tree level prediction. We also calculate the correction to the hyperfine splitting relationship of Eq. (17) and find for the masses in the nonstrange sector

$$\delta m_{\Xi^*_{cc}} - \delta m_{\Xi_{cc}} - \frac{3}{4}(\delta m_{D^*} - \delta m_D) = \begin{cases} 3.9 \text{ MeV} & \mu = 500 \text{ MeV} \\ 5.3 \text{ MeV} & \mu = 1000 \text{ MeV}. \\ 6.1 \text{ MeV} & \mu = 1500 \text{ MeV} \end{cases} \tag{31}$$

The nonanalytic correction to the symmetry prediction is small (< 10 MeV) and relatively insensitive to the choice of μ. Chiral perturbation theory predicts the nonanalytic dependence of the doubly heavy baryon masses on the light quark masses, and generalized to include the effects of quenching as well as other lattice artifacts, formulas such as those in Eq. (27) should be useful for chiral extrapolations of doubly heavy baryon masses and hyperfine splittings in lattice simulations.

V. EXCITED STATES

In this section, we discuss excited doubly heavy baryons. There are two types of excitations in the doubly heavy baryon system: excitations of the light degrees of freedom and excitations of the diquark. Excitations of the first type are related to analogous excitations in the heavy meson sector by heavy quark-diquark symmetry. The lowest lying excited charm mesons are in a doublet of $J^P = 0^+$ and 1^+ mesons with masses approximately 425 MeV above the ground state in the nonstrange sector [43–45] and 350 MeV above the ground state in the strange sector [46,47]. In the nonstrange sector these states decay via S-wave pion emission and have widths in the range 250–350 MeV, while in the strange sector the strong decay is via π^0 emission which violates isospin, and therefore the states are very narrow with widths expected to be of order 10 keV [42]. These states have light degrees of freedom with angular momentum and parity $j^p = \frac{1}{2}^+$. The doubly charm baryons related to the even-parity excited charm mesons by quark-diquark symmetry are a doublet with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$. The excited charm mesons and doubly charm baryons can be incorporated into HHχPT with a 5×2 matrix field $\mathcal{S}_{\mu\beta}$ which is like the field $\mathcal{H}_{\mu\beta}$ except $\mathcal{S}_{\mu\beta}$ has opposite parity. The excitation energies and strong decay widths of these excited doubly charm baryons should be similar to their counterparts in the charm meson sector. Since the excited $\Xi_{cc}^{++}(3780)$ state observed by SELEX is only 320 MeV above the $\Xi_{cc}^{++}(3460)$, the lowest mass Ξ_{cc}^{++} candidate, and its width is considerably less than 300 MeV, it does not seem likely that this excited doubly charm baryon is related to the excited charm mesons by heavy quark-diquark symmetry.

This is not unexpected as the lowest lying excited doubly charm baryons are not excitations of the light degrees of freedom but rather states in which the diquark is excited. The lowest mass excited diquark is a P-wave excitation. Because of Fermi statistics the diquark is a heavy quark spin singlet. The diquark's orbital angular momentum couples with the angular momentum of the light degrees of freedom to form baryons with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$, which we will refer to as $\Xi_{cc}^{\mathcal{P}}$ and $\Xi_{cc}^{\mathcal{P}*}$, respectively. The next lowest lying states are doubly heavy baryons with a radially excited diquark, which form a heavy quark doublet with $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ baryons, which we will refer to as Ξ'_{cc} and Ξ'^*_{cc}, respectively. If the heavy antiquarks are sufficiently heavy that the force between them is approximately Coulombic, they interact via a potential which is $1/2$ as strong as the potential between the quark and antiquark in a quarkonium bound state. Therefore we expect the excitation energies of the charm diquarks to be significantly smaller than the analogous excitation energies in charmonium. Quark model calculations of excited doubly charm baryons predict that the $\Xi_{cc}^{\mathcal{P}}$ and $\Xi_{cc}^{\mathcal{P}*}$ states are about 225 MeV above the Ξ_{cc} and Ξ_{cc}^*, respectively, and that the heavy quark doublet containing Ξ'_{cc} and Ξ'^*_{cc} is about 300 MeV above the ground state doublet [9,36,48–50]. These excitation energies are about $1/2$ the corresponding excitation energies in the charmonium system: $m_{h_c} - m_{J/\psi} = 430$ MeV and $m_{\psi'} - m_{J/\psi} = 590$ MeV. The charm diquark excitation energies are less than the expected excitation energy of the light degrees of freedom and therefore the lowest lying excited doubly charm baryons have excited diquarks. Excitation energies of a diquark made from two bottom quarks are similar to the excitation energies of a diquark made from two charm quarks, so the same conclusion holds for doubly bottom baryons.

Note that the excitation energies of the light degrees of freedom, which should scale as $\sim\Lambda_{\rm QCD}$, are as much as 2 times larger than the excitations of the diquark, which should scale as $\sim m_Q v^2$. Since the difference is only a factor of 2, either power counting $m_Q v > \Lambda_{\rm QCD} \sim m_Q v^2$ or $m_Q v \sim \Lambda_{\rm QCD} > m_Q v^2$ seems plausible for excited doubly heavy baryons. Our calculations assume the first power counting, but if the second power counting is more appropriate a formalism similar to strongly coupled pNRQCD needs to be developed instead [16]. It should be kept in mind that since excited diquarks will be less pointlike than the ground states, corrections to heavy quark-diquark symmetry could be larger for excited doubly heavy baryons. Our predictions for strong decays listed below could be used to test the validity of the assumption $m_Q v > \Lambda_{\rm QCD} \sim m_Q v^2$ for excited doubly heavy baryons.

The doubly heavy baryons with P-wave excited diquarks decay to the ground state via S-wave pion emission. These decays violate heavy quark spin symmetry because the total spin of the diquark is changed in the transition. The Lagrangian for the excited $\Xi^{\mathcal{P}}$ and $\Xi^{\mathcal{P}*}$ states, including kinetic terms, residual mass terms and terms which mediate the S-wave decays, is

$$\begin{aligned}\mathcal{L} = {}& 2(\Xi_a^{\mathcal{P}})^\dagger (i(D_0)_{ba} - \delta_{\mathcal{P}}\delta_{ab})\Xi_b^{\mathcal{P}} \\ &+ 2(\Xi_a^{\mathcal{P}*})^\dagger (i(D_0)_{ba} - \delta_{\mathcal{P}*}\delta_{ab})\Xi_b^{\mathcal{P}*} \\ &+ 2\lambda_{1/2}(\Xi_a^\dagger \Xi_b^{\mathcal{P}} A_{ba}^0 + \text{h.c.}) \\ &+ 2\lambda_{3/2}(\Xi_a^{*\dagger}\Xi_b^{\mathcal{P}*} A_{ba}^0 + \text{h.c.}). \end{aligned} \tag{32}$$

The strong decay widths of the P-wave excited nonstrange doubly charm baryons are

$$\begin{aligned}\Gamma[\Xi_{cc}^{\mathcal{P}*} \to \Xi_{cc}^*\pi] &= \frac{\lambda_{3/2}^2}{2\pi f^2}\left(\frac{1}{2}E_{\pi^0}^2 p_{\pi^0} + E_{\pi^+}^2 p_{\pi^+}\right)\frac{m_{\Xi^*}}{m_{\Xi^{\mathcal{P}*}}} \\ &= \lambda_{3/2}^2 111 \text{ MeV} \\ \Gamma[\Xi_{cc}^{\mathcal{P}} \to \Xi_{cc}\pi] &= \frac{\lambda_{1/2}^2}{2\pi f^2}\left(\frac{1}{2}E_{\pi^0}^2 p_{\pi^0} + E_{\pi^+}^2 p_{\pi^+}\right)\frac{m_{\Xi}}{m_{\Xi^{\mathcal{P}}}} \\ &= \lambda_{1/2}^2 111 \text{ MeV}.\end{aligned}$$

To obtain numerical estimates, we have assumed the masses $m_{\Xi_{cc}} = 3440$ MeV, $m_{\Xi_{cc}^*} = 3520$ MeV, $m_{\Xi_{cc}^{\mathcal{P}}} = 3665$ MeV and $m_{\Xi_{cc}^{\mathcal{P}*}} = 3745$ MeV, corresponding to a diquark excitation energy of 225 MeV. We sum over both charged and neutral pion decay modes. The coupling constants $\lambda_{1/2}$ and $\lambda_{3/2}$ are $O(\Lambda_{\rm QCD}/m_Q)$ so we should expect this suppression makes $\lambda_{1/2}$ and $\lambda_{3/2} < 1$. Therefore these states could be quite narrow despite decaying via S-wave pion emission. The small widths are due to the small excitation energy which leaves little phase space for the decay. If the excitation energy is increased to 280 MeV, the widths are twice as large. Like the isospin violating decays of the D_s^* [51] and the even-parity excited D_s mesons [42,52], the excited doubly heavy strange baryons below the kaon threshold decay through a virtual η which mixes into a π^0. Denoting the ground state doubly charm strange baryons as $\Omega_{cc}^{(*)}$ and the P-wave excited doubly charm strange baryons as $\Omega_{cc}^{\mathcal{P}(*)}$ we obtain the following formulas for the isospin violating strong decay widths

$$\Gamma[\Omega_{cc}^{\mathcal{P}*} \to \Omega_{cc}^{*}\pi^0] = \frac{\lambda_{3/2}^2}{2\pi f^2}\frac{2}{3}\theta^2 E_{\pi^0}^2 p_{\pi^0}$$

$$\Gamma[\Omega_{cc}^{\mathcal{P}} \to \Omega_{cc}\pi^0] = \frac{\lambda_{1/2}^2}{2\pi f^2}\frac{2}{3}\theta^2 E_{\pi^0}^2 p_{\pi^0}.$$

Here $\theta = 0.01$ is the $\pi^0 - \eta$ mixing angle. We expect these widths to be in the range 1–5 keV, but without knowing the masses of the $\Omega_{cc}^{(*)}$ and $\Omega_{cc}^{\mathcal{P}(*)}$ states or the couplings $\lambda_{1/2}$ and $\lambda_{3/2}$ we cannot make more precise predictions.

The $J^P = \frac{3}{2}^-$ and $J^P = \frac{1}{2}^-$ doubly heavy baryons with radially excited diquarks are members of a heavy quark doublet we will denote T'_a whose definition in terms of component fields is identical to Eq. (13). The Lagrangian describing this field, including terms which mediate its decay to the ground state, is

$$\begin{aligned}\mathcal{L} &= \mathrm{Tr}[T_a'^{\dagger}((iD_0)_{ab} - \delta_{T'}\delta_{ab})T'_b] \\ &\quad - g\,\mathrm{Tr}[T_a'^{\dagger}T'_b\vec{\sigma}\cdot\vec{A}_{ba}] + \frac{\Delta_H}{4}\,\mathrm{Tr}[T_a'^{\dagger}\mathcal{T}^i T'_b\sigma^i] \\ &\quad - \tilde{g}(\mathrm{Tr}[T_a^{\dagger}T'_b\vec{\sigma}\cdot\vec{A}_{ba}] + \text{h.c.}). \qquad (33)\end{aligned}$$

In the limit of infinite heavy quark mass, the light degrees of freedom in the radially excited doubly heavy baryons are in the same configuration as the ground state. Therefore, they are also related to the heavy meson ground state doublet by heavy quark-diquark symmetry. The axial coupling and hyperfine splitting of T'_a are the same as T_a, as long as the spatial extent of the excited diquark, which is of order $1/(m_Q v)$, is much smaller than $1/\Lambda_{\mathrm{QCD}}$. This is valid in the heavy quark limit, but could receive significant corrections in the charm sector. The last term in Eq. (33) mediates P-wave decays from the excited $J^P = \frac{3}{2}^-$ and $J^P = \frac{1}{2}^-$ doubly heavy baryons to the ground state. The partial decay widths are

$$\begin{aligned}\Gamma[\Xi_a'^{*} \to \Xi_b^{*}\pi] &= \mathcal{C}_{ab}\frac{5}{9}\frac{\tilde{g}^2}{2\pi f^2}\frac{m_{\Xi^*}}{m_{\Xi'^*}}|p_\pi|^3 \\ \Gamma[\Xi_a'^{*} \to \Xi_b\pi] &= \mathcal{C}_{ab}\frac{4}{9}\frac{\tilde{g}^2}{2\pi f^2}\frac{m_{\Xi}}{m_{\Xi'^*}}|p_\pi|^3 \\ \Gamma[\Xi'_a \to \Xi_b^{*}\pi] &= \mathcal{C}_{ab}\frac{8}{9}\frac{\tilde{g}^2}{2\pi f^2}\frac{m_{\Xi^*}}{m_{\Xi'}}|p_\pi|^3 \\ \Gamma[\Xi'_a \to \Xi_b\pi] &= \mathcal{C}_{ab}\frac{1}{9}\frac{\tilde{g}^2}{2\pi f^2}\frac{m_{\Xi}}{m_{\Xi'}}|p_\pi|^3. \qquad (34)\end{aligned}$$

Here $\mathcal{C}_{ab}$ is an $SU(3)$ factor which is 1/2 for decays involving π^0 and one for decays involving charged pions. The radially excited doubly heavy strange baryons should also be below the threshold for decays into kaons, and therefore should be quite narrow. The formulas in Eq. (34) can be used to obtain these decay widths as well. The isospin violating strong partial decay widths are obtained by using Eq. (34) with $C_{33} = \frac{2}{3}$ then multiplying by θ^2. The expected widths of these states are of order 10 keV, but more precise estimates cannot be made until the masses of the states and the coupling $\tilde{g}$ are known. For the nonstrange doubly heavy baryons, in the limit of infinite heavy quark mass, we obtain

$$\begin{aligned}\Gamma[\Xi'] = \Gamma[\Xi'^{*}] &= \frac{3\tilde{g}^2}{4\pi f^2}p_\pi^3 \\ &= 55\ \mathrm{MeV}\left(\frac{\tilde{g}}{0.5}\right)^2\left(\frac{p_\pi}{250\ \mathrm{MeV}}\right)^3 \qquad (35)\end{aligned}$$

for the total widths, and for the branching fractions we find

$$\frac{\mathrm{Br}[\Xi'^{*} \to \Xi^{*}\pi]}{\mathrm{Br}[\Xi'^{*} \to \Xi\pi]} = \frac{5}{4} \qquad \frac{\mathrm{Br}[\Xi' \to \Xi^{*}\pi]}{\mathrm{Br}[\Xi' \to \Xi\pi]} = 8. \qquad (36)$$

These relations receive large corrections due to phase space effects. Once the hyperfine splittings are taken into account the factors of p_π^3 will differ greatly for the four decays. To get a feeling for these effects in the doubly charm sector we choose $m_{\Xi_{cc}} = 3440$ MeV, $m_{\Xi_{cc}^*} = 3520$ MeV, $m_{\Xi'_{cc}} = 3740$ MeV, and $m_{\Xi_{cc}^{*\prime}} = 3820$ MeV, which corresponds to a diquark excitation energy of 300 MeV and hyperfine splittings of 80 MeV. We then find

$$\Gamma[\Xi'_{cc}] = \tilde{g}^2 336\ \mathrm{MeV} \qquad \Gamma[\Xi_{cc}^{*\prime}] = \tilde{g}^2 78\ \mathrm{MeV}$$

$$\frac{\Gamma[\Xi_{cc}'^{*} \to \Xi_{cc}^{*}\pi]}{\Gamma[\Xi_{cc}'^{*} \to \Xi_{cc}\pi]} = 0.56 \qquad \frac{\Gamma[\Xi'_{cc} \to \Xi_{cc}^{*}\pi]}{\Gamma[\Xi'_{cc} \to \Xi_{cc}\pi]} = 2.3. \qquad (37)$$

Note that the Ξ'_{cc} unlike the $\Xi_{cc}'^{*}$ strongly prefers to decay to Ξ_{cc}^{*} relative to Ξ_{cc} despite the phase space suppression. This may be useful for distinguishing $\Xi_{cc}'^{*}$ and Ξ'_{cc} experimentally.

The SELEX $\Xi_{cc}^{++}(3780)$ is broad relative to the other SELEX doubly charm candidates. Since it is 260 MeV heavier than the $\Xi_{cc}^{+}(3520)$, it is a natural candidate for one of the low lying excited doubly charm baryons. Unfortunately, no measurement of the width exists and the pattern of decays is also hard to understand, since Ref. [14] states that 50% of the decays to $\Lambda_c^+ K^- \pi^+ \pi^+$ are through $\Xi_{cc}^{+}(3520)\pi^+$ while the other 50% are weak decays. More information on the quantum numbers of the $\Xi_{cc}^{++}(3780)$ and the $\Xi_{cc}^{+}(3520)$ are needed before we can determine which of the excited doubly charm baryons should be identified with the $\Xi_{cc}^{++}(3780)$.

VI. SUMMARY

In this paper we have developed a generalization of HHχPT which incorporates heavy quark-diquark symmetry and includes the leading symmetry breaking corrections from the chromomagnetic couplings of the heavy quark and diquark. We also included electromagnetic interactions in the Lagrangian, and obtained an estimate of the width of the $J = \frac{3}{2}$ member of the ground state doubly charm baryon doublet. The width of this state is completely dominated by electromagnetic decays. Our theory was used to calculate chiral corrections to doubly heavy baryon masses. The nonanalytic correction to the leading heavy quark-diquark symmetry prediction for the hyperfine splittings is small. Computations of chiral corrections to doubly heavy baryon masses which include effects of quenching and other lattice artifacts will be useful for chiral extrapolations in future lattice QCD calculations of doubly heavy baryon masses.

We showed how to include the lowest lying doubly charm baryons which are expected to be excitations of the doubly charm diquark rather than the light degrees of freedom. Strong decay widths of low lying excited states were calculated and the states are expected to be rather narrow because of limited phase space available for the decays. Of particular interest is the doubly charm strange sector where we expect three pairs of excited baryons whose strong decay must violate isopsin conservation because they are below the kaon decay threshold. These states will have narrow widths of 10 keV or less. Experimental efforts to observe the narrow doubly charm strange baryons would be of great interest.

ACKNOWLEDGMENTS

This research is supported in part by DOE Grant Nos. DE-FG02-05ER41368, DE-FG02-05ER64101, and DE-AC05-84ER40150.

[1] M. J. Savage and M. B. Wise, Phys. Lett. B **248**, 177 (1990).
[2] A. V. Manohar and M. B. Wise, Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. **10**, 1 (2000).
[3] N. Brambilla, A. Vairo, and T. Rösch, Phys. Rev. D **72**, 034021 (2005).
[4] S. Fleming and T. Mehen, Phys. Rev. D **73**, 034502 (2006).
[5] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995); **55**, 5853(E) (1997).
[6] N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Nucl. Phys. **B566**, 275 (2000).
[7] M. E. Luke, A. V. Manohar, and I. Z. Rothstein, Phys. Rev. D **61**, 074025 (2000).
[8] R. Lewis, N. Mathur, and R. M. Woloshyn, Phys. Rev. D **64**, 094509 (2001).
[9] D. Ebert, R. N. Faustov, V. O. Galkin, and A. P. Martynenko, Phys. Rev. D **66**, 014008 (2002).
[10] M. B. Wise, Phys. Rev. D **45**, R2188 (1992).
[11] G. Burdman and J. F. Donoghue, Phys. Lett. B **280**, 287 (1992).
[12] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, and H. L. Yu, Phys. Rev. D **46**, 1148 (1992); **55**, 5851(E) (1997).
[13] M. Mattson *et al.* (SELEX Collaboration), Phys. Rev. Lett. **89**, 112001 (2002).
[14] M. A. Moinester *et al.* (SELEX Collaboration), Czech. J. Phys. **53**, B201 (2003).
[15] A. Ocherashvili *et al.* (SELEX Collaboration), Phys. Lett. B **628**, 18 (2005).
[16] N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Rev. Mod. Phys. **77**, 1423 (2005).
[17] L. Schmitt, S. Paul, R. Kuhn, and M. A. Moinester, hep-ex/0310049.
[18] J. S. Russ (SELEX Collaboration), "The Double Charm Baryon Family at SELEX: An Update", http://www-selex.fnal.gov/Welcome.html.
[19] J. S. Russ, AIP Conf. Proc. No. 717 (AIP, New York, 2004) p. 507.
[20] N. Mathur, R. Lewis, and R. M. Woloshyn, Phys. Rev. D **66**, 014502 (2002).
[21] J. M. Flynn, F. Mescia, and A. S. B. Tariq (UKQCD Collaboration), J. High Energy Phys. 07 (2003) 066.
[22] A. K. Likhoded and A. I. Onishchenko, hep-ph/9912425.
[23] B. Guberina, B. Melic, and H. Stefancic, Eur. Phys. J. C **9**, 213 (1999); **13**, 551 (2000).
[24] V. V. Kiselev, A. K. Likhoded, and A. I. Onishchenko, Phys. Rev. D **60**, 014007 (1999).
[25] V. V. Kiselev and A. K. Likhoded, Usp. Fiz. Nauk **172**, 497 (2002); [Phys. Usp. **45**, 455 (2002)].
[26] V. V. Kiselev and A. K. Likhoded, hep-ph/0208231.
[27] R. Vogt and S. J. Brodsky, Nucl. Phys. **B478**, 311 (1996).
[28] R. Vogt and S. J. Brodsky, Phys. Lett. B **349**, 569 (1995).
[29] R. Vogt and S. J. Brodsky, Nucl. Phys. **B438**, 261 (1995).
[30] E. Braaten, M. Kusunoki, Y. Jia, and T. Mehen, Phys. Rev. D **70**, 054021 (2004).
[31] E. Braaten, Y. Jia, and T. Mehen, Phys. Rev. Lett. **89**, 122002 (2002).
[32] J. G. Korner, M. Kramer, and D. Pirjol, Prog. Part. Nucl. Phys. **33**, 787 (1994).
[33] R. Roncaglia, D. B. Lichtenberg, and E. Predazzi, Phys. Rev. D **52**, 1722 (1995).
[34] D. Ebert, R. N. Faustov, V. O. Galkin, A. P. Martynenko, and V. A. Saleev, Z. Phys. C **76**, 111 (1997).
[35] V. V. Kiselev and A. I. Onishchenko, Nucl. Phys. **B581**, 432 (2000).
[36] S. S. Gershtein, V. V. Kiselev, A. K. Likhoded, and A. I. Onishchenko, Phys. Rev. D **62**, 054021 (2000).
[37] V. V. Kiselev and A. E. Kovalsky, Phys. Rev. D **64**, 014002 (2001).
[38] I. W. Stewart, Nucl. Phys. **B529**, 62 (1998).
[39] A. F. Falk, Nucl. Phys. **B378**, 79 (1992).
[40] J. F. Amundson *et al.*, Phys. Lett. B **296**, 415 (1992).
[41] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).
[42] T. Mehen and R. P. Springer, Phys. Rev. D **72**, 034006 (2005).

[43] S. Anderson *et al.* (CLEO Collaboration), Nucl. Phys. **A663**, 647 (2000).
[44] K. Abe *et al.* (Belle Collaboration), Phys. Rev. D **69**, 112002 (2004).
[45] J. M. Link *et al.* (FOCUS Collaboration), Phys. Lett. B **586**, 11 (2004).
[46] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **90**, 242001 (2003).
[47] D. Besson *et al.* (CLEO Collaboration), AIP Conf. Proc. No. 698 (AIP, New York, 2004) p. 497.
[48] V. V. Kiselev, A. K. Likhoded, O. N. Pakhomova, and V. A. Saleev, Phys. Rev. D **66**, 034030 (2002).
[49] S. S. Gershtein, V. V. Kiselev, A. K. Likhoded, and A. I. Onishchenko, Mod. Phys. Lett. A **14**, 135 (1999).
[50] S. S. Gershtein, V. V. Kiselev, A. K. Likhoded, and A. I. Onishchenko, Heavy Ion Phys **9**, 133 (1999); [Phys. At. Nucl. **63**, 274 (2000)]; [Yad. Fiz. **63**, 334 (2000)].
[51] P. L. Cho and M. B. Wise, Phys. Rev. D **49**, 6228 (1994).
[52] P. Colangelo and F. De Fazio, Phys. Lett. B **570**, 180 (2003).