# Landscape of string theory and the wave function of the universe

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We explore the possibility that quantum cosmology considerations could provide a selection principle in the landscape of string vacua. We propose that the universe emerged from the string era in a thermally excited state and determine, within a mini-superspace model, the probability of tunneling to different points on the landscape. We find that the potential energy of the tunneling endpoint from which the universe emerges and begins its classical evolution is determined by the primordial temperature. By taking into account some generic properties of the moduli potential we then argue that the tunneling to the tail of the moduli potentials is disfavored, that the most likely emergence point is near an extremum, and that this extremum is not likely to be in the outer region of moduli space where the compact volume is very large and the string coupling very weak. As a concrete example we discuss the application of our arguments to the Kachru, Kallosh, Linde and Trivedi model of moduli stabilization.

## I. INTRODUCTION

The existence of a multiverse of solutions to string theory makes it eminently desirable to find a dynamical principle that would select a universe, or a subset of the multiverse that has properties similar to ours. Such a principle might be provided in the framework of quantum cosmology [1–3]. The wave function of the universe yields a probability distribution for the dynamical parameters of the multiverse. Depending on the boundary condition chosen for the wave function, the probability distribution is sharply peaked at a zero value of the cosmological constant (CC) [4] or is peaked at a large value of the CC [3,5] giving rise to an inflationary universe. A related and complementary approach is based on the statistics of solutions [6–9], one of whose outcomes is that the number density of solutions is uniform as a function of the value of the CC.

In this paper we propose that the universe emerged from the string era in a thermally excited state above the Hartle-Hawking vacuum. We show that imposing this boundary condition on the wave function of the universe leads, within a mini-superspace model, to interesting restrictions on the allowed dynamics. High temperature effects were first introduced in a related context, to the best of our knowledge, by Vilenkin [10] who suggested including them in the context of the so-called tunneling boundary condition, but did not work out the consequences. We will show in detail the relevance of these effects both to the Hartle-Hawking (HH) wave function and to the tunneling wave function.

Our proposal is motivated by an interesting observation that was made recently by Sarangi and Tye [11] (see also

[12,13]). They considered the tunneling amplitude as a function of the CC $\Lambda$, and found that if rather than evaluating it from the HH wave function [2] that describes tunneling from nothing,

$$\Psi \simeq e^{3\pi/G\Lambda}, \tag{1}$$

a modified wave function is used, then the tunneling amplitude

$$\Psi \simeq e^{3\pi/G\Lambda - (a/2)(3\pi/G\Lambda)^2}, \tag{2}$$

has a critical point (a maximum for the HH choice). The additional parameter $a$ is a constant that depends on the string mass scale $M_s$, the Planck scale $M_p = G^{-1/2}$ and the number of fluctuating degrees of freedom $n_{\mathrm{dof}}$. The critical CC that Sarangi and Tye found is $\Lambda_c = 4n_{\mathrm{dof}}(\frac{M_s^2}{M_p^2})M_s^2$.

The original argument by Hawking was that since the wave function (1) is peaked at $\Lambda \to 0^+$ it explains why the observed value of the CC today is zero. Of course at that time there was just an observational upper bound on the CC, and it was assumed that the upper bound implies that the true value is zero. The problem with this argument is that it predicts a large but empty universe. Sarangi and Tye argue that their modified wave function (2) predicts inflation and hence makes the choice of the HH boundary condition more interesting.

We use a mini-superspace model for our discussion. In this model only the time-dependent scale factor of the universe, and some time-dependent but homogeneous fields are considered and all other degrees of freedom are ignored. We regard the mini-superspace calculation as a toy model which by itself is a well-defined quantum mechanical problem that can be solved self-consistently and hopefully incorporates some features of the complete quantum gravity calculation. At the current stage of development an attempt to do a more complete quantum gravity

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analysis seems to lead to the notorious problems of Euclidean quantum gravity. Thus our calculation should be regarded as an indication of the true solution of the full dynamical problem. For self-consistency we apply the quantum tunneling picture in four dimensional mini-superspace only for large scale factors and energies and temperatures that are small compared to the string scale and the Kaluza-Klein scale. The existence of the primordial temperature allows us to do this at the price of not being able to say anything about the quantum origin at zero scale factor. We regard the inclusion of primordial radiation as a parametrization of our ignorance of the physics at the quantum origin.

We start our discussion in the simpler case for which the moduli, the dilaton $S$, and the volume modulus $T$ are fixed by stringy effects. In this case the thermal boundary conditions lead to results that are similar to those of Sarangi and Tye. Then we take into account the dynamics of the moduli by assuming they are stabilized as in the recent works on moduli stabilization by fluxes and nonperturbative effects [14,15]. Moduli stabilization by fluxes was considered previously in [16–20]. We show that in order to have a finite potential barrier and therefore a large tunneling amplitude, the endpoint of the tunneling should be to a region of the moduli potential that supports accelerated expansion. Taking into account additional features of the moduli potentials we further show that the tunneling takes place to a maximum or a saddle point that is within a bounded region in moduli space. In particular, we find that the tunneling to the tail of the moduli potentials, where they runaway to decompactification, for example, is disfavored. The reason for this is that these regions do not lead to sustained accelerated expansion.

The organization of the paper is as follows. After a brief review of the wave function calculus in Sec. II we discuss in Sec. III thermal effects and their significance to the tunneling amplitude. In Sec. IV we discuss the dependence of the tunneling amplitude on the parameters and conclude that we must add the dependence on moduli. We follow our own conclusion and in Secs. V and VI where we extend the analysis to a case with dynamical moduli. In Sec. VII we discuss in detail the application of our results to the model of Kachru, Kallosh, Linde and Trivedi (KKLT). Section VIII contains our conclusions, and in the appendix we compare our work to that of Sarangi and Tye.

## II. REVIEW OF THE WAVE FUNCTION CALCULUS

Let us first briefly review the calculation of the wave function of the universe in the mini-superspace context [21]. Consider a theory with a dynamical CC $\Lambda$ whose action is

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{g}(R - 2\Lambda). \tag{3}$$

Here all other dynamical fields are ignored and it is assumed that their effects are incorporated into a single parameter $\Lambda$ that is essentially an integration constant. The problem is further simplified by considering only homogeneous and isotropic metrics that describe a closed universe. For such metrics the line element is

$$ds^2 = \sigma^2(-dt^2 + a^2(t)d\Omega_3^2), \tag{4}$$

$\sigma^2 = 2G/3\pi$. For this class of models the action simplifies considerably,

$$S = \frac{1}{2} \int dt(-a\dot{a}^2 + a - \lambda a^3). \tag{5}$$

All quantities in the action (5) are dimensionless, in particular, the dimensionless CC $\lambda = \sigma^2\Lambda/3 = 2G\Lambda/9\pi$. The canonical momentum conjugate to $a$ is $\Pi_a = -a\dot{a}$ and the classical Hamiltonian constraint is

$$H = -\frac{1}{2a}(\Pi_a^2 + U(a)) \approx 0. \tag{6}$$

The last relation means that the Hamiltonian vanishes on the space of classical solutions. The potential $U(a) = a^2 - \lambda a^4$ is positive for $0 < a < 1/\sqrt{\lambda}$ and negative for $a > 1/\sqrt{\lambda}$.

The quantum equation derived from the Hamiltonian constraint is the Wheeler-deWitt (WDW) equation which replaces the Schroedinger equation in quantum gravity. The momentum operator $\hat{\Pi}_a = -i\partial/\partial a$ replaces the classical momentum,

$$\hat{H}\Psi[a] = (\hat{\Pi}_a^2 + U(a))\Psi(a) = 0. \tag{7}$$

The WDW equation can be solved in the WKB approximation,

$$\Psi_\pm[a] \sim e^{\pm iS_c} = e^{\pm i \int_0^a \Pi_a da} = e^{\pm 1/3\lambda[1-(1-\lambda a^2)^{3/2}]}. \tag{8}$$

The last equality is valid only for $0 < a < 1/\sqrt{\lambda}$. In Eq. (8) we have ignored the prefactors because they will not be important for the rest of the discussion. The classical action $S_c$ is evaluated on a classical solution so that $\Pi_a = \partial S_c/\partial a = \sqrt{-U(a)}$. For $0 < a < 1/\sqrt{\lambda}$, $\Pi_a$ is of course imaginary and the wave function is real. Hartle and Hawking [2] have imposed the no-boundary boundary condition which in this context means that the positive sign should be taken in Eq. (8). Then the under the barrier wave function near $a = 1/\sqrt{\lambda}$ is given by the growing exponential, and the probability of the universe emerging after tunneling through the barrier is given by

$$P_{\text{HH}}[\lambda] \sim e^{2/3\lambda}. \tag{9}$$

Hawking [4] has argued that this distribution explains the observed vanishing of the CC. At that time it was assumed that the observations indicated that it was in fact zero. Now there is strong evidence that the expansion of the universe is accelerated and therefore that the CC, or a

similar form of dark energy is small and positive. However, as many have pointed out, the probability distribution (9) also predicts an empty universe—there would have been no primordial inflationary stage.

The result (9) depends very sensitively on the nature of the state and, in particular, on the boundary conditions. If one uses the Euclidean path integral to define the state, the no-boundary proposal gives the ground state wave function. The resulting distribution is rather surprising. It implies that the tunneling amplitude increases exponentially when the barrier becomes larger. This is certainly not what happens in laboratory tunneling experiments which show that the tunneling amplitude decreases exponentially with the size of the barrier.

Linde [3] and Vilenkin [1] have proposed different boundary conditions that yield results that are similar to the probability distributions in standard tunneling processes. Vilenkin has proposed that only an outgoing wave should exist in the Lorentzian region for $a > 1/\sqrt{\lambda}$. This means that the universe has only an expanding component whereas the HH wave function is a superposition of expanding and contracting universes. The probability distribution that results from Linde's and Vilenkin's proposals is

$$P_{L,V}[\lambda] \sim e^{-2/3\lambda}. \tag{10}$$

This clearly favors a large CC. In fact it favors a situation for which the barrier is as small as possible, and even no barrier at all. Since the semiclassical theory is valid only up to some cutoff scale, which we will choose to be the string scale $M_s$, this seems to imply that the universe is created in a state with string scale CC. Linde has proposed recently [23] that a flat compact universe may perhaps be more likely, since in this case there is no barrier.

To discuss inflation it is necessary to include at least one inflaton field. This is done for instance in [24] where it is shown that the formulas (9),(10) remain valid when one replaces the CC with the scalar potential $V(\phi)$, $\lambda \rightarrow 4\pi^2\sigma^2 V(\phi)$, provided that it is slowly varying, i.e. $|V^{-1}dV/d\phi| \ll 1$. Obviously, these semiclassical considerations are valid only for regions in field space where $|V(\phi)| \ll M_s^4$. In [24] various possible potentials are illustrated with the corresponding probability distributions for the two cases (9) and (10). In all cases it is clear that the HH distribution will favor tunneling into the lowest positive points in the potential, while the LV distribution will favor tunneling into the highest points of the potential allowed by the cut off.

Vilenkin [24] has argued that the above considerations lead to a "prediction" of inflation from the LV wave function, in contrast to what is obtained from the HH wave function. However the problem is that the conclusion is cutoff dominated. Besides, the natural value of a cutoff would be close to the Planck scale, perhaps one or two orders of magnitude below, and this would be too high to agree with observations. The predicted Hubble parameter during inflation is $H \sim \sqrt{M_s^4/M_p^2}$. If the string scale is taken to be an order of magnitude below the Planck scale—which is the case for string compactifications where there are no anomalously large extra dimensions, then $H \sim 10^{16}$ GeV, in conflict with the WMAP result that $H \lesssim 10^{14}$ GeV. To get agreement, the cutoff scale would have to be at least 2 orders of magnitude below the Planck scale.

We would like to make two remarks about the possible application of the wave function calculus to string theory. First, in the string theoretic context $V(\phi)$ is typically steep, so that the condition $|V^{-1}dV/d\phi| \ll 1$ is obeyed only in the vicinity of the critical points of the potential. Thus, strictly speaking the analysis can only compare the relative probability of tunneling to different critical points. The LV wave function would predict tunneling to the highest critical point while the HH wave function would favor the lowest positive one. Second, recent work has shown that in string theory the acceptable solutions that have potentials that can stabilize the moduli necessarily involve fluxes of RR and NS-NS fields. The fluxes make the parameters of the potential, and, in particular, the CC, discrete integration constants. In the string theory context one should really consider a probability distribution that depends on all the moduli as well as on the flux parameters. In contrast, the original Hawking argument was made in a model without a scalar field. The CC was treated as a dynamical quantity arising as an integration constant characterizing the flux of a four form field strength.

## III. THERMAL EFFECTS

Our boundary condition proposal for the wave function of the universe can be stated as follows: The universe emerges from the string era in a thermal state above the Hartle-Hawking vacuum. We propose that the decay of all the string excited states has created a primordial thermal gas of radiation at a temperature that is somewhat below the Hagedorn temperature. At this high temperature there would be in addition to the massless states some population of Boltzmann suppressed massive string states, and perhaps also a gas of branes [25]. These may behave as pressureless matter, or have some other behavior. We will ignore such contributions for simplicity, since we do not expect this to change the qualitative behavior that we find. Within the context of string theory the effective field theory arguments that we use make sense only at the end of the string era when the energy densities in the universe are somewhat below the string scale.

Rather than evaluating the no-boundary thermal partition function, we evaluate the Euclidean mini-superspace path integral with a modified effective potential that includes the temperature corrections. The leading temperature correction is very simple, a negative term proportional to the fourth power of the temperature is added to the

potential in the WDW equation. Equivalently, we can solve the WDW equation with a modified potential, and even though one does not expect a coherent wave function to describe a thermal state, the square of this WDW "wave function" would have the interpretation of a density matrix that measures relative probabilities.

Since the temperature scales as the inverse scale factor the energy density of the radiation will be of the form $\rho_{\text{RAD}} = K/(\sigma^4 a^4)$ where $K$ is a constant. The effective potential that goes into the WDW equation becomes

$$U(a) - K = a^2 - \lambda a^4 - K. \tag{11}$$

The potential barrier is now in the region limited by the roots of $U(a) - K = 0$, $a_- < a < a_+$,

$$a_\pm^2 = \frac{1}{2\lambda}(1 \pm \sqrt{(1 - 4K\lambda)}). \tag{12}$$

Of course to have a barrier at all the radiation term cannot be too large $4K\lambda < 1$. This condition needs to be satisfied for the semiclassical theory to remain valid as argued below.

To keep the semiclassical effective field theory approach self-consistent the highest radiation density in the region of interest should be less than string scale. i.e. $\rho(a_-) \sim K/\sigma^4 a_-^4 = CM_s^4$ with $C < 1$. Using the value $a_-^2 \sim 1/2\lambda$ from Eq. (12) we get $K \sim \frac{1}{4\lambda^2}M_s^4\sigma^4$. Expressing $C$ in terms of the number of degrees of freedom $n_{\text{dof}}$ in thermal equilibrium , $C = n_{\text{dof}}/c^4$ we have our final expression for the radiation energy density,

$$\rho = \frac{n_{\text{dof}}}{c^4}\frac{1}{4\lambda^2 a^4}M_s^4. \tag{13}$$

The constant $c$ must satisfy $c^4 > n_{\text{dof}}$ for the consistency of these arguments. This energy density corresponds to an initial temperature at $a = a_-$ of $T \sim M_s/c$ i.e. a temperature close to the Hagedorn temperature. The same estimate can also be obtained by requiring that the initial entropy is close to saturating the entropy bound as discussed in the appendix. The condition for the existence of a barrier is thus equivalent in this context to the condition that the cosmological constant be smaller than the string scale.

Including the contribution of the thermal radiation energy density into the effective action (5), it becomes,

$$S = \frac{1}{2} \int dt(-a\dot{a}^2 + a - \lambda a^3 - \frac{\nu}{a\lambda^2}), \tag{14}$$

where

$$\nu = n_{\text{dof}}\frac{1}{9\pi^2 c^4}\frac{M_s^4}{M_p^4}. \tag{15}$$

The corresponding classical Hamiltonian that replaces the one in Eq. (6) is

$$H = -\frac{1}{2a}\left(\Pi_a^2 + U(a) - \frac{\nu}{\lambda^2}\right) \approx 0. \tag{16}$$

The boundaries of the barrier can be rewritten as

$$a_\pm^2 = \frac{1}{2\lambda}\left(1 \pm \sqrt{1 - \frac{4\nu}{\lambda}}\right). \tag{17}$$

It is important that for the range $\lambda \ll 1, 0 < 4\nu/\lambda < 1$ the scale factor is large under the barrier $a_\pm \gg 1$, so the semiclassical mini-superspace calculation is self-consistent. Also since one expects $n_{\text{dof}} \sim 10^2$–$10^3$, the above restriction on $c$ means that it is about 10 but need not be much bigger. In fact it is reasonable to expect that the initial temperature at the beginning of the classical evolution $T \sim M_s/c$ is close to but not quite at the string scale. The initial volume of the universe as it emerges from under the potential barrier into the classically allowed region is $\sigma^3 a_+^3 \sim \sigma^3/\lambda^{3/2}$. The horizon volume on the other hand is approximately $H^{-3} \simeq \sigma^3/\lambda^{3/2}$. Thus the universe starts its classical evolution having a volume which is approximately one horizon volume.

Now, in addition to the Lorentzian region to the right of $a_+$, there is also a Lorentzian region to the left of $a_-$. This is depicted in Fig. 1. The solution of the WDW equation in the WKB approximation is obtained by matching the solutions on the boundaries of the three different regions,

$$\Psi_{\text{I}} = A_+ e^{+i\Phi_{\text{I}}} + A_- e^{-i\Phi_{\text{I}}}, \tag{18}$$

$$\Psi_{\text{II}} = B_+ e^{+\Phi_{\text{II}}} + B_- e^{-\Phi_{\text{II}}}, \tag{19}$$

$$\Psi_{\text{III}} = C_+ e^{+i\Phi_{\text{III}}} + C_- e^{-i\Phi_{\text{III}}}. \tag{20}$$
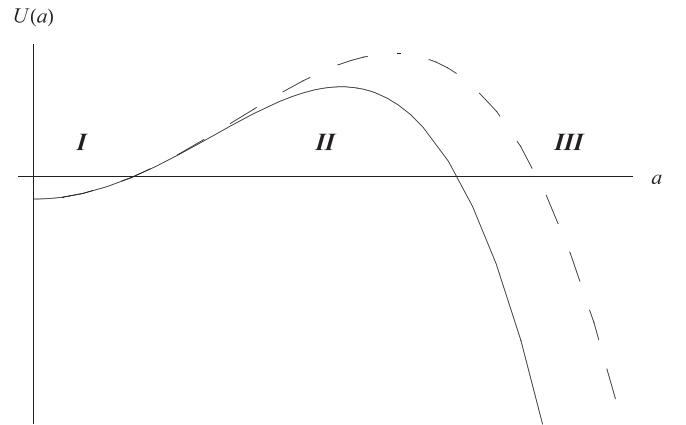
The exponents are given by



FIG. 1. The potential of the WDW equation for a closed universe with thermal radiation. The solid line corresponds to the potential for a universe with a CC, and the dashed line corresponds to a universe with dark energy with a softer EOS. Regions I, III are classically allowed regions where the wave function is oscillatory, while region II is a classically forbidden region where the wave function is exponential.

$$\Phi_{\text{I}}(a) = \sqrt{\lambda} \int_0^a \sqrt{(a_-^2 - a^2)(a_+^2 - a^2)}, \quad 0 < a < a_- \tag{21}$$

$$\Phi_{\text{II}}(a) = \sqrt{\lambda} \int_{a_-}^a \sqrt{(a^2 - a_-^2)(a_+^2 - a^2)}, \quad a_- < a < a_+ \tag{22}$$

$$\Phi_{\text{III}}(a) = \sqrt{\lambda} \int_{a_+}^a \sqrt{(a^2 - a_-^2)(a^2 - a_+^2)}, \quad a_+ < a. \tag{23}$$

In practice the matching has to be done with care since the boundaries of the three regions are turning points where $E = U$, and therefore the WKB approximation breaks down there.

The new Lorentzian region $a < a_-$ can clarify and resolve the debate about which linear combinations to take inside the forbidden region, and which boundary conditions to choose. If one puts boundary conditions in this region that correspond to "initial conditions" and not to "final conditions" about the state of the universe when it starts its classical evolution after tunneling then any generic choice will effectively be equivalent to the HH choice as we now show. An example of quantum cosmology with a Lorentzian region for small $a$ was considered in the context of brane gravity in [26].

A generic boundary condition in region I that is not too far from a "stationary state" in the sense that it has comparable incident and reflected waves, will yield some finite ratio of $B_+$ to $B_-$. Unless the coefficient of the increasing exponential $B_+$ is tuned specifically to vanish or to be much smaller than $B_-$ then the rising exponential will dominate the wave function inside the barrier and at the beginning of the Lorentzian region. Hence in practice, the HH boundary conditions can be taken, and will be a very good approximation to the generic situation. Of course, if for some reason (for instance, in analogy with $\alpha$-decay) one would like to impose, as advocated by Vilenkin, that there is only an outgoing wave function in region III, then the coefficient $B_+$ would need to be very small. In our setup, this would seem to be a very special choice.

The tunneling amplitude is of the form $e^{\pm\Phi}$ with

$$\Phi = \sqrt{\lambda} \int_{a_-}^{a_+} da \sqrt{(a^2 - a_-^2)(a_+^2 - a^2)}$$

$$= \frac{1}{\lambda}\left(1 - \frac{4\nu}{\lambda}\right)\frac{1}{8} \int_{-\pi/2\sqrt{\lambda}}^{\pi/2\sqrt{\lambda}} \frac{\sin^2(2\sqrt{\lambda}\tau)}{a(\tau)} d\tau \tag{24}$$

In order to get the second equality we have made the substitution $a^2 = a(\tau)^2 \equiv (1 + \sqrt{(1 - \frac{4\nu}{\lambda}}\cos(2\sqrt{\lambda}\tau))/(2\lambda)$. A simple estimate of the integral which is quite sufficient for our purposes is obtained by assuming a

triangular integrand $\sqrt{U(a) - \nu/\lambda^2}$ whose height is the maximal height of the potential barrier $\frac{1}{\sqrt{4\lambda}}\sqrt{1 - \frac{4\nu}{\lambda}}$, and whose width is $a_+ - a_- = \frac{1}{\sqrt{\lambda}}\sqrt{1 - \frac{4\nu}{\lambda}}$. The resulting estimate for $\Phi$ is

$$\Phi = \frac{1}{4\lambda}\left(1 - \frac{4\nu}{\lambda}\right). \tag{25}$$

A more sophisticated analysis shows that the integral can be expressed in terms of complete elliptic integrals of the first and second kind. In the limit $\nu/\lambda \to 0$ the exact result reduces, as it should, to the previous case [see Eq. (8)] and its value is 8/3, such that in this limit $\Phi = \frac{1}{3\lambda}$. The exact expression is

$$\Phi = \frac{\sqrt{2}}{\lambda}\left(1 - \frac{4\nu}{\lambda}\right)\frac{1}{\sqrt{1 + \sqrt{1 - \frac{4\nu}{\lambda}}}}\frac{1}{2m}$$

$$\times \left\{2\frac{m-1}{m}K[m] + \frac{2-m}{m}E[m]\right\}, \tag{26}$$

where $m \equiv 2\sqrt{1 - \frac{4\nu}{\lambda}}/(1 + \sqrt{1 - \frac{4\nu}{\lambda}})$.

Let us now choose the HH + sign for the logarithm of the wave function for the reasons that were explained previously (we will discuss what happens when the other sign is chosen later). In this case, the tunneling amplitude is maximized at the maximum of $\Phi$. Using the simple estimate of Eq. (25), we find that when

$$\lambda = 8\nu \tag{27}$$

$\Phi$ is maximized, and its value is

$$\Phi|_{\text{max}} = \frac{1}{64\nu}. \tag{28}$$

Using a more accurate numerical evaluation of the exact expression gives,

$$\lambda = 5.26\nu, \tag{29}$$

and

$$\Phi|_{\text{max}} = \frac{0.12}{\nu}. \tag{30}$$

The important point here is that the ratio of the radiation energy density to the CC is some finite fixed numerical constant. This means that the initial radiation energy density determines the value of the CC,

$$\Lambda \simeq \frac{5n_{\text{dof}}}{\pi c^4}\frac{M_s^2}{M_p^2}M_s^2. \tag{31}$$

It is clear from this formula that the result is sensitive to the initial temperature and that the precise estimation of the value of $\nu/\lambda$ is not particularly important. This is essentially the same result as the one obtained in [11], except that they have effectively put the constant $c = 1$. Our

derivation shows that setting $c$ to unity is inconsistent, in that it would in effect give at the barrier a radiation energy density that is greater than the string energy density.

## IV. THE TUNNELING PROBABILITY

In order to discuss in a meaningful way the relative probabilities for tunneling into different points in the landscape and the issue of whether or not inflation is favored one really needs, in addition to the CC, to introduce the set of moduli fields $\phi$ and their potential $V(\phi)$. If the moduli potential is not steep, one could (following [24]) take over the results of Sec. II with the substitution $\lambda \rightarrow 4\pi^2 \sigma^4 V(\phi)$. However, the string moduli potentials are steep except in a limited domain around their extrema. Let us ignore this for the moment and come back to this issue in Sec. VI.

Let us consider the set of dynamical parameters in the potential that are determined by the fluxes, gauge groups, etc., and denote them collectively by $\beta$. These vary from point to point in the landscape. Now the string to Planck mass ratio depends on the moduli $\frac{M_s}{M_p}(\phi)$. Using the above substitution, our previous maximization argument gives,

$$4\pi^2 \sigma^4 V_{\max}(\phi; \beta) \simeq 5\nu = \frac{5n_{\mathrm{dof}}}{9\pi^2 c^4}\left(\frac{M_s}{M_p}(\phi)\right)^4. \qquad (32)$$

It is clear from Eq. (32) that the maximization of the tunneling amplitude puts a constraint on the parameters $\beta$ and the values of the moduli fields to which the universe tunnels.

Let us now observe what happens when the LV sign for the under the barrier wave function is chosen corresponding to the usual tunneling situation, with only outgoing waves in the final Lorentzian region. Then within the class of models that we consider: a closed universe with a positive CC, and radiation whose temperature does not exceed the Hagedorn temperature, the maximum of the tunneling amplitude becomes a minimum. Now the tunneling amplitude is maximized at the edge of parameter space when $\lambda = 4\nu$, exactly at the point that the barrier disappears. In this case Eq. (32) should be replaced by

$$4\pi^2 \sigma^4 V_{\max}(\phi; \beta) = 4\nu = \frac{4n_{\mathrm{dof}}}{9\pi^2 c^4}\left(\frac{M_s}{M_p}(\phi)\right)^4. \qquad (33)$$

Clearly, given that we are ignoring order one factors, the difference between the two cases is not that significant. The thermal boundary condition switches the physical consequences of the two wave functions. In the absence of radiation the LV wave function favors a larger CC while the HH wave function favors a zero CC. With radiation the HH wave function favors a larger CC than the LV.

The maximization that we have performed was essentially with respect to $\lambda$ (or $V$) keeping $\nu$ fixed. In [11] probabilities for tunneling for different values of $\nu$ are compared. However, if we strictly follow their logic there

appears to be a problem. From Eq. (30) we have,

$$\Phi_{\max} \simeq \frac{\sqrt{2}}{12\nu} = \frac{\sqrt{2}}{12}\frac{9\pi^2 c^4}{n_{\mathrm{dof}}}\frac{M_p^4}{M_s^4}. \qquad (34)$$

The number of light degrees freedom does not change that much—being around $10^2$–$10^3$ so the tunneling probability is essentially controlled by the ratio of the Planck scale to the string scale. In the heterotic string, for example, this is given by (See for example [27] chapter 18). $\frac{M_s^2}{M_p^2} = \frac{\alpha_{\mathrm{YM}}}{8}$ where $\alpha_{\mathrm{YM}} = g_{\mathrm{YM}}^2/4\pi^2$ is the gauge field coupling strength at the string scale. In type I theory on the other hand the ratio is given by (See for example [28]). $\frac{M_s^2}{M_p^2} = g\frac{\alpha_{\mathrm{YM}}}{4}$ where $g$ is the string coupling [29]. Plugging these into Eq. (34) we get

$$\begin{aligned}\Phi_{\max} &= \frac{\sqrt{2}}{12}\frac{9\pi^2 c^4}{n_{\mathrm{dof}}}\frac{8}{\alpha_{\mathrm{YM}}} \quad \text{Heterotic,}\\ &= \frac{\sqrt{2}}{12}\frac{9\pi^2 c^4}{n_{\mathrm{dof}}}\frac{4}{g\alpha_{\mathrm{YM}}} \quad \text{Type I.}\end{aligned}$$

This seems to favor tunneling into very weakly coupled universes. If the value of $M_s/M_p$ is not fixed then it is preferable to have it vanishingly small, and then the radiation energy density also vanishes and consequently also the CC. The final result in this case is very similar to the original HH result, the universe tunnels to the smallest possible value of the CC. We believe that to come to any reliable conclusion it is really necessary to explicitly consider the dependence on the moduli scalar fields, which we do in the next sections.

## V. ACTION AND WHEELER-DE WITT EQUATION FOR GRAVITY AND A SCALAR FIELD

As we have explained, to determine the tunneling probability it is necessary to reexamine the wave function of the universe when in addition to the gravity sector, we also have scalar fields. For simplicity we will consider just one field $\phi$ with the action

$$S_\phi = -\int d^4x \sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right). \qquad (35)$$

For a closed universe the total mini-superspace action is

$$S = \frac{1}{2}\int dt\{-a\dot{a}^2 + \sigma^2 a^3\dot{\phi}^2 + a - 4\pi^2\sigma^4 a^3 V(\phi)\}. \tag{35}$$

Here we have absorbed the CC into the scalar potential $V$. The conjugate momenta are

$$\Pi_a = -a\dot{a} \qquad \Pi_\phi = a^3\sigma^2\dot{\phi}, \qquad (37)$$

and the Hamiltonian is

$$H = -\frac{1}{2a}\Pi_a^2 + \frac{1}{2a^3\sigma^2}\Pi_\phi^2 - \frac{1}{2a}(a^2 - 4\pi^2 a^4 \sigma^4 V(\phi)).$$
(38)

The term $\frac{1}{2a^3\sigma^2}\Pi_\phi^2$ determines the kinetic energy (KE) of the scalar field.

The WDW equation is $H\Psi(a, \phi) = 0$, and can be obtain by the substitution $\Pi_a \to -i\frac{\partial}{\partial a}$, and $\Pi_\phi \to -i\frac{\partial}{\partial \phi}$. The result is then the following,

$$\left[a^2\hat{\Pi}_a^2 - \frac{1}{\sigma^2}\hat{\Pi}_\phi^2 + a^2(a^2 - 4\pi^2 a^4 \sigma^4 V(\phi))\right]\Psi(a, \phi) = 0.$$
(39)

We have assumed a particular operator ordering but this is not particularly important for our arguments.

### A. A slowly rolling scalar field

In the limit that the scalar field is moving very slowly and where $2\sigma^4 V(\phi)$ can be treated as a constant, the WDW equation is separable $\Psi(a, \phi) = \chi(\phi)\psi(a)$. Then,

$$\frac{\partial^2 \chi(\phi)}{\partial \phi^2} = -\sigma^2 E\chi(\phi)$$
(40)

$$a^2\frac{\partial^2 \psi(a)}{\partial a^2} - \left(a^2 - a^4 4\pi^2 \sigma^4 V(\phi) - \frac{E}{a^2}\right)\psi(a) = 0.$$ (41)

The significance of $E$ is clear: it represents the average KE of the scalar field. The term $E/a^2$ in the WDW equation originates from a term $E/a^6$ in the energy density, which is indeed a scalar field KE term.

Positive $E$ corresponds to positive KE, and leads to an oscillating scalar field wave function while negative $E$ leads to an exponential scalar field wave function whose interpretation is unclear. The solutions of the scalar field WDW are simply linear combinations of "free particle" solutions: wave packets. For positive $E$, denoting $k = \sqrt{E}$,

$$\chi(\phi) = \int_{-\infty}^{\infty} dk e^{i\sigma k\phi}\tilde{\chi}(k).$$
(42)

The choice $E = 0$ leads to a constant wave function and a uniform probability for all values of the field. If one chooses $E = 0$, and the path integral is dominated by a single classical configuration, then this classical configuration is a constant scalar field, for consistency.

### B. A rolling scalar field

If the potential is such that the field is moving significantly, we may use the following method to solve the WDW equation. We find the classical solution for the scale factor and scalar field, and express the scalar field as a function of the scale factor. This is possible provided that the scale factor is a monotonic function of time. We then use the parametric solution to find the kinetic and potential energy of the scalar field as a function of the scale factor as

shown below. This results in a modified equation for the scale factor only.

The energy density of the scalar field $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and its pressure is $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$. The conservation equation

$$d\rho_\phi + 3(\rho_\phi + p_\phi)d\ln a = 0,$$
(43)

which is valid also for a closed universe, can be formally integrated if an additional relation between $\rho$ and $p$ is supplied,

$$\rho_\phi(a) = \rho_\phi(a_0)e^{-3\int_{a_0}^{a}(1+p_\phi/\rho_\phi)d\ln a}.$$
(44)

Let us consider as a simple instructive example the Euclidean scaling solution for which the total energy density of the field behaves as a fixed power of the scale factor: $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \sim C/a^m$, corresponding to the case that the equation of state (EOS) parameter $w_\phi(a) = p_\phi(a)/\rho_\phi(a)$ is constant. The constant $C$ replaces the CC for the general case in which $\rho_\phi$ is not constant, and the solution is the same as the one with a fluid whose EOS parameter is $w = p/\rho = m/3 - 1$. In general, the scalar field will not have a constant EOS parameter.

In the presence of space curvature it is difficult to find potentials that yield a constant EOS. For the Lorentzian spatially flat case it is well known that exponential potentials lead to scaling solutions. A potential of the form $V(\phi) = Ae^{\alpha\phi}$ leads to solutions that have $\dot{\phi}^2 \sim V(\phi) \sim 1/t^2$ and $w = -1 + 2\alpha^2/3$. For $\alpha < 1$ the potential is flat enough and the expansion is accelerated. The maximal EOS parameter for a scalar field with a positive potential is $w = 1$ in the limit that the evolution is KE dominated. This limit is reached when $\alpha$ approaches the critical value $\sqrt{3}$, and $A$ vanishes. For a scaling solution to exist for larger values of $\alpha$, the prefactor $A$ needs to be negative, and in this case values of the EOS parameter that are larger than unity are allowed. A positive potential that is steeper than the critical steepness leads very quickly to a KE dominated evolution, as discussed, for example, in [30]. This will be relevant to our discussion in Sec. VII

The Euclidean equations are in fact the same as the Lorentzian equations for the spatially flat case, and therefore the Euclidean scaling solutions lead to the same $\rho_\phi(a)$ as do the Lorentzian ones.

In the WDW Eq. (39), the terms that correspond to $\rho_\phi$ are $-\frac{1}{2a^2\sigma^2}\Pi_\phi^2 - 2a^4\sigma^4 V(\phi)$, so on a scaling solution we obtain the following effective WDW equation for the scale factor only,

$$\left[-\frac{\partial^2}{\partial a^2} + a^2 - Ca^{4-m}\right]\Psi(a) = 0.$$
(45)

One can immediately realize that for the potential barrier to exist and be finite, one needs that $m < 2$, so that the scaling behavior corresponds to a power law inflation with an EOS

$p/\rho$ that is more negative than that of spatial curvature $p/\rho < -1/3$. In general, on a solution $\rho_\phi$ can be replaced by some function of $a$. The potential is then $U(a) = a^2 - \rho_\phi(a)$. Clearly $\rho_\phi$ needs to grow at large $a$ faster than $a^2$, and therefore needs to have an EOS more negative than that of curvature.

Recall that the EOS of space curvature is $-1/3$ which is just the borderline between decelerated and accelerated expansion. Thus, our conclusion is that the universe tunnels to a state of accelerated expansion. The scalar field energy density needs to dominate over the space curvature for a while, in order to get the potential down to zero, where the Lorentzian era begins. This conclusion will be very important for the following discussion about the landscape.

An additional conclusion is that if the dominant power in $\rho_\phi$ has a less negative EOS, the larger the range of $a$ that is required for the "dark energy" to become dominant over the space curvature. This is depicted in Fig. 1. In addition, for lower dominant power in $\rho_\phi$, the faster the field $\phi$ is moving. As the evolution becomes less and less similar to that in the presence of a CC, the scale factor is moving more when under the barrier and the scalar field needs to move more. This requires that the potential of the scalar field be flat over a larger region in field space so the accelerated expansion could be supported. Another general comment is that one can compare the tunneling amplitude for a CC to that in the case of "softer" EOS (less negative than $-1$). Assuming that the EOS is a constant, leading to a power law dependence $\rho_\phi \sim C/a^{(2-m)}$, $m > 2$, we can see that the tunneling amplitude that comes from the positive exponential mode (in the case that the magnitudes of the potentials are similar) is much larger for the softer EOS, since the barrier is much larger. This is depicted in Fig. 1.

It should be stressed here that the considerations of this section are independent of the specific choice of the boundary conditions on the wave function. We have argued that generic *initial* conditions lead to the dominance of the rising mode under the barrier. If, however, one imposes final conditions such that only the outgoing wave is allowed, then the usual tunneling picture emerges. Obviously, in this case too the barrier needs to end and therefore, as argued above, the tunneling needs to take place to an accelerating universe. By altering the topology of the final state as in [23] (or even just the geometry by taking flat or negatively curved universes) one could eliminate the barrier altogether and in this case our arguments would be irrelevant.

## VI. THE TUNNELING PROBABILITY WITH MODULI

The discussion and results of Secs. III and IV about tunneling probability and its extremization did not take into account the fact that the ratio of the string scale to the Planck scale $M_s/M_p$ is dynamical and moduli dependent, and that the CC should be replaced by the moduli potential. We reevaluate them in the light of the discussion of Sec. V. We make several assumptions about the form of the moduli potentials that allow us to obtain more definite results.

In the case that the scalar field $\phi$ is a modulus, there are two possibilities. First, that the true CC is large and then the potential is dominated by the CC. This case is not very interesting because it does not lead to a universe that is similar to ours. The more interesting possibility is that the true CC is substantially less than the string scale. In the latter case we know from general arguments about Peccei-Quinn symmetries and how they break, that the potential is a sum of steep functions. Further, we know that at generic points that are not extrema of the potential and that are in the outer region of moduli space, the potential is dominated by a single steep function (see, for example, a related discussion in [31]). It follows that the only flat regions in the potential of the moduli fields where $V'/V \ll 1$ occur in the vicinity of an extremum.

We argue that consequently, when the CC is small the universe cannot tunnel to a generic point in the outer region of moduli space. We have seen that the tunneling endpoint needs to be where the potential energy dominates the energy balance in the universe, and, in particular, dominates the field's KE. However, we have just argued that the potential for a generic point in the outer region of moduli space is steep. On the other hand, we know that the cosmological solution of a scalar field on a steep potential leads to the dominance of KE over the potential energy. Our conclusion follows. Additionally, we have just argued that the only flat regions in the potential of the moduli fields are in the vicinity of an extremum. Our tentative conclusion is therefore that the universe tunnels to a region that is not far from an extremum of the moduli potential. We proceed to examine this conclusion in a more concrete setup.

A possible loophole in our argument could exist if some hitherto unknown Euclidean transient solutions that support accelerated expansion for a short period of time could be found. We expect such solutions, if they exists at all, to require some special initial conditions. We cannot discuss their possible existence in the general setup without adding more specific information on the moduli potentials. In Sec. VII we show that such solutions, even if they do exist, do not modify our conclusions.

The moduli potential can be put in the form of an $N = 1$ SUGRA potential,

$$V = e^K[D_i W D_{\bar{j}} \bar{W} K^{i\bar{j}} - 3|W|^2] \qquad (46)$$

with

$$K = -\ln(2S_R) - 3\ln(2T_R) + \cdots \qquad (47)$$

Here $T_R$ is the real part of $T$ and $S_R$ is the real part of $S$ and the ellipses represent the contribution of the other (Kaehler

and complex structure) moduli. The field $S$ is the (complex) dilaton axion field whose expectation value determines the coupling and $T$ is the so-called volume modulus whose expectation value determines the size of the internal manifold. Together, they determine the string to Planck mass ratio $M_s^4/M_p^4 = S_R^{-1}T_R^{-3}$. Let us denote the set of dimensionless moduli $(S, T, \cdots)$ by $\phi^i$. The potential is a function of these moduli as well as of the flux parameters and Casimirs of the gauge groups which we collectively denoted by $\beta$ as in the discussion above Eq. (32). Thus we may express the potential as

$$V = V(\phi^i; \beta). \tag{48}$$

The WDW equation in the presence of the radiation term becomes,

$$\left[ -a^2\left(\frac{\partial}{\partial a}\right)^2 + K^{i\bar{j}} \frac{\partial^2}{\partial\phi^i\partial\phi^{\bar{j}}} + a^2(a^2 - a^4 V(\phi; \beta) - K) \right]$$
$$\times \Psi(a, \phi) = 0. \tag{49}$$

Here $K$ is a constant not to be confused with the Kahler potential, and we have substituted $\lambda \to V$ as in Sec. V. We can reason as in Sec. V B that tunneling to an accelerated expansion phase is favored since otherwise the barrier does not end. Since an accelerated expansion phase requires that the potential is flat enough, it reasonable to expand to leading order in an expansion in $V'/V$, and as argued previously it is also reasonable to assume that the moduli are near a critical point in the potential. Thus the arguments of Sec. III that lead to Eq. (32) can be used. The new probability maximization conditions are

$$\frac{1}{V} \frac{\partial V}{\partial\phi^i} \simeq 0 \tag{50}$$

$$V(\phi; \beta) \simeq 5\nu(\phi), \tag{51}$$

with

$$\nu = \frac{n_{\text{dof}}}{9\pi^2 c^4} \frac{M_s^4}{M_p^4} = \frac{n_{\text{dof}}}{9\pi^2 c^4} (T_R)^{-3}(S_R)^{-1}. \tag{52}$$

Equations (50) (one for each modulus) determine, for any set of parameters $\beta$, a region in moduli space for which the slow roll conditions are satisfied. Of course, there may be some values of $\beta$ for which there is no solution. Equation (51) is then a further constraint that restricts the parameter values only to those that satisfy it.

Equations (50)–(52), however, leave the system underdetermined. The only exception is when there is only one parameter in the set $\beta$. The logarithm of the WKB wave function at the point of emergence from the barrier into the Lorentzian spacetime depends on the values of the moduli at this point $\{S^L, T^L, z^L\} \equiv \{\phi_i^L\}$ and on $a_+$,

$$a_+ = \frac{1}{2V(\phi)} \left[ 1 + \sqrt{1 - 4\frac{\nu(S, T)}{V(\phi)}} \right]. \tag{53}$$

Hence, it takes the following form in general

$$\Phi = \Phi(S^L, T^L, z^L, a_+, \beta). \tag{54}$$

The wave function should be extremized with respect to all the moduli as well as the parameters $\beta$. Since $\Phi$ depends on $\beta$ only through its dependence on the potential, we have

$$\frac{\partial\Phi}{\partial\beta} = \frac{\partial\Phi}{\partial V|_{\phi_i^L}} \frac{\partial V}{\partial\beta|_{\phi_i^L}} = 0, \tag{55}$$

$$\frac{\partial\Phi}{\partial\phi_L^i} = \frac{\partial\Phi}{\partial a_+|_{\phi_i^L}} \frac{\partial a_+}{\partial\phi_i^L} + \frac{\partial\Phi}{\partial\phi_L^i|_{a_+}} = 0. \tag{56}$$

In principle, these equations determine the tunneling endpoint in the moduli space that serves as the initial values for the classical evolution of all the moduli. They also determine the discrete parameters, the fluxes, gauge group parameters, etc. These equations therefore determine the particular flux configuration to which the tunneling occurs. Of course, in general there may be more than one solution to these conditions, so their solution may be a multiverse rather than a universe.

In practice we used the particular expression for $\Phi$ that was obtained in Eq. (26) for the case of a constant potential (CC) by arguing that the tunneling took place to a flat region of the potential. Thus we argued that even with the moduli, the equations of Sec. III could be used with the appropriate replacements for $\lambda$ and $\nu$. Equation (51) then follows from imposing $\partial\Phi/\partial V = 0$ which implies that the whole set of Eqs. (55), one for each parameter in the set $\beta$, are satisfied. Clearly this is a sufficient but not a necessary condition for extremization with respect to $\beta$. It would be interesting to explore the nature of the general solutions to the extremization equations.

In the next section we will investigate the nature of the restrictions that we have found for the specific potentials for moduli that have been suggested recently by KKLT.

## VII. APPLICATION OF THE THERMAL BOUNDARY CONDITION TO THE KKLT MODEL

We would like to determine more accurately the point on the moduli potential to which the universe tunnels. For this we need to input some additional information on the properties of the moduli potential. We will use here the KKLT model [15]. In this model, the potential has two contributions $V_{\text{SUGRA}}$ and $V_{\overline{D}}$. The first takes the form of an $N = 1$ SUGRA potential (using the same units as before) with a Kaehler potential given by Eq. (47) and a superpotential

$$W = A + BS + Ce^{-aT}. \tag{57}$$

The first two terms in Eq. (57) come from the fluxes [14], $A$ and $B$ are functions of the complex structure moduli, and there is only one Kaehler modulus $T$. The third term in this expression can arise from gaugino condensation in a gauge

group living on a stack of 7-branes wrapping a four cycle on the compact manifold. KKLT assume that it is possible to ignore the third term and integrate out $S$ and the complex structure moduli, assuming a flux configuration which makes their masses heavy. While this is not strictly correct (see [32]) the corrections are not important for the current discussion so we will ignore them. Then $S$ and the complex structure moduli are constants, and the effective superpotential is of the form $W = W_0 + Ce^{-aT}$ with a Kaehler potential $K = -3\ln(T + \bar{T})$ giving

$$V_{\text{SUGRA}} = \frac{aCe^{-aT_R}}{2T_R^2}\left(W_0 + \left(\frac{1}{3}T_R a + 1\right)Ce^{-aT_R}\right). \quad (58)$$

The minimum of $V_{\text{SUGRA}}$ is at $DW = \partial_T W + \partial_T K W = 0$, and for consistency with the assumption that the volume is large, and therefore that ten dimensional supergravity is valid, and for consistency with the expansion in nonperturbative terms, one needs $T_R \gg 1$, $aT > 1$, so that we need to have $W_0 < 1$. This can be achieved by fine-tuning the fluxes.

The second contribution to the KKLT potential $V_{\overline{D}}$ comes from anti-$D_3$ branes and breaks supersymmetry explicitly from the 4D perspective. It is equal to $\frac{d}{T_R^3}$ in a naive calculation of the anti-$D$-brane tension, however, a term proportional to $\frac{1}{T_R^2}$ has also been proposed in the literature. The total potential has a shallow positive minimum at a largish value of $T_R$, and a small barrier separates it from a positive tail that goes to zero as $\sim 1/T_R^3$.

Now let us check whether the endpoint of the tunneling can be on the asymptotic tail using the arguments of Sec. V B. The field $T_R$ is not canonically normalized. The canonically normalized field is $x$ where $T_R = e^{\sqrt{2x/3}}$. So the asymptotic dependence of the full potential on $x$ is $V \sim e^{-3\sqrt{2x/3}}$. Recall the discussion in Sec. V where we concluded that the (Euclidean) cosmological scaling solution of a canonically normalized scalar field $\phi$ with an exponential potential $V = Ae^{\alpha\phi}$ gives a power law dependence for the scale factor $a(t) \sim t^{p_a}$, with $p_a = 1/\alpha^2$. If $p_a > 1$, $\alpha < 1$, the expansion is accelerated. Recall also that for $|\alpha| > \sqrt{3}$ the prefactor $A$ needs to be negative for a scaling solution to exist. In the case that the potential goes like $1/T_R^3$ then $\alpha = \sqrt{6}$ or in the case that the potential goes like $1/T_R^2$ then $\alpha = \sqrt{\frac{8}{3}}$, both significantly above 1. The prefactor $d$ is positive. Hence we can conclude that these potentials do no lead to accelerated expansion.

The argument of Sec. V B indicated that for the tunneling barrier to be finite, one needed an accelerating scale factor. Our conclusion is therefore that the tunneling endpoint cannot be on the asymptotic tail of the potential in the region $T_R \gg 1$ where $V_{\overline{D}}$ dominates. A possible loophole in the argument is that perhaps it is possible to find a transient solution that includes a brief period of accelerated

expansion. However, also in this case the potential energy has to dominate the energy balance. Then, ignoring numerical factors of order unity and since $S_R$ is fixed at a number of $O(1)$, Eq. (52) leads to the condition

$$\frac{5n_{\text{dof}}}{9\pi^2 c^4} = d. \quad (59)$$

As we discuss below, this relationship cannot be satisfied since $d$ has to be very small, and the left-hand side is not particularly small. Hence it is not possible to satisfy the extremization conditions even in this case.

If the anti-$D$-brane term is not included then the tail of the potential is steeper, and if a potential of the form $\tilde{d}/T_R^2$ is added then Eq. (59) is replaced by $\frac{5n_{\text{dof}}}{9\pi^2 c^4} = \tilde{d}T_R$ which is still hard to satisfy. Our conclusion is therefore valid for these additional cases.

Let us now check whether the tunneling endpoint can be near the shallow minimum, or near the maximum of the barrier separating the minimum from the asymptotic region. The dimensionless radiation $\nu$ is given by Eq. (52) and the dimensionless potential by

$$V = V_{\text{SUGRA}} + \frac{d}{T_R^3}. \quad (60)$$

At the two extrema, the two terms are comparable. In the event that the tunneling occurs to an extremum, which we have argued is a reasonable approximation, the wave function is extremized at points where $\nu$ and $V$ are related as in Eq. (51). In our case $S_R$ fixed at a number of $O(1)$.

Equation (51) gives

$$\frac{5n_{\text{dof}}}{9\pi^2 c^4} = \frac{aAT_R e^{-aT_R}}{2}\left(W_0 + \left(\frac{1}{3}T_R a + 1\right)Ce^{-aT_R}\right) + d, \quad (61)$$

where we have ignored a factor of $O(1)$. The factor $\frac{5n_{\text{dof}}}{9\pi^2 c^4}$ is about $O(10^{-1})$, and is not expected to vary much with different flux choices. On the other hand, for models with one condensate (i.e. one nonperturbative term as in the original KKLT example) and a small CC, both extrema are at quite large values of $aT_R$. For large values of $aT_R$ the right-hand side of Eq. (61) is much smaller than $1/10$. We conclude that Eq. (61) cannot be satisfied at either extremum. For example, let us consider the choice of parameters in KKLT, where $a = 0.1$, $A = 1$, $T_R \sim 100$–150 and $d = 1 \times 10^{-9}$. Although the normalization of the potential in KKLT is somewhat different than ours, it is clear that the right-hand side of Eq. (61) is of order $10^{-9}$ for the maximum, and much smaller for the minimum where the CC was tuned to be very small.

Our conclusion is that the quantum cosmology arguments would prefer a modified KKLT model, with additional exponential terms in the superpotential to allow extrema that are closer to the central region of moduli space where Eq. (51) has a chance to be obeyed. Such

models can be constructed by having several nonperturbative terms in the superpotential (see, for example [33,34]). Clearly, it is easier to satisfy Eq. (51) at a maximum, a saddle point, or a metastable minimum rather than at a global minimum.

## VIII. CONCLUSIONS

We have seen that the probability distribution obtained from the wave function of the universe can provide an interesting and restrictive dynamical selection principle on the landscape of string solutions without reference to the anthropic principle.

We have proposed that the universe emerged from the string era in a thermally excited state above the HH vacuum, and determined, within a mini-superspace model, the probability of tunneling to different points on the landscape. We have clarified the significance of including a radiation term for the HH wave function of the universe and have shown that the radiation term switches the roles of the Hartle-Hawking and Linde-Vilenkin wave functions. We have found that the potential energy of the tunneling endpoint from which the universe emerges and begins its classical evolution, is determined by the primordial temperature, and that this starting point can be followed by some interesting dynamics.

We have found that a more accurate treatment, even within the mini-superspace approximation, requires the inclusion of the moduli fields, and we have included them. By taking into account some generic properties of the moduli potential we then argue that the tunneling to the tail of the moduli potentials is disfavored, that the most likely emergence point is near an extremum, and that this extremum is not likely to be in the outer region of moduli space where the compact volume is very large and the string coupling very weak. Combined, these considerations select a class of values of the flux parameters etc., that characterize a universe, or a multiverse, in the landscape.

We explicitly demonstrated the applicability of our arguments for the KKLT model of moduli stabilization. We have determined that for the KKLT model the tunneling to the tail of the potential or to the vicinity of the barrier that separates the minimum from the asymptotic region is disfavored. Our quantum cosmology arguments favor tunneling to an extremum of the potential that is close to the central region of moduli space as might be obtained from generalizations of the original KKLT model.

Finally, we might consider the relevance of the counting program of Douglas and collaborators [7–9] to our arguments. We have calculated a quantity that is analogous to the square of the tunneling amplitude in the calculation of a decay rate of an unstable particle. To obtain the total tunneling rate one needs also the density of final states. Perhaps the counting program can supply the latter, so that a complete calculation of the relative probabilities of finding one or another universe, or a certain subset of the

multiverse, is obtained by taking the product of the two factors. The number density of solutions does not seem to influence much the preferred value of the CC, because the tunneling amplitude is sharply peaked as a function of the CC, while the number density of solutions is uniform as a function of the CC. Perhaps it is more relevant to the preferred values of other dynamical parameters.

## APPENDIX

Here we explain the relation of our work to the calculations of Sarangi and Tye [11].

Sarangi and Tye (ST) have argued that the wave function needs to be modified due to decoherence effects. They argue that the fluctuations of the metric and of other light fields should be integrated out and traced over. Their calculation is rather involved, but the final result can be justified and related to that in Sec. III by the following simple argument.

ST are essentially computing the thermal partition function in a Friedman-Robertson-Walker (FRW) background at some undefined temperature. It is well known that this calculation gives a contribution to the effective action (free energy) which is just the energy density of radiation i.e. a term proportional to $T^4$ [35]. It is also well known that in the FRW background the temperature scales as the inverse of the scale factor $a^{-1}$. So the only question is: what is the temperature?

Let us work with the action (3). The effect of the unobserved fluctuations can be represented by the entropy $\Sigma$ within a horizon volume. An order of magnitude estimate of the upper bound on this is (a related though not identical calculation is performed by ST)

$$\Sigma \sim \int d^3 n = \left(\frac{H^{-1}}{l_s}\right)^3 \sim \left(\frac{1}{\sqrt{\Lambda}l_s}\right)^3,$$

where we have assumed as in [11] that the infrared cutoff is the horizon size and the ultraviolet one is the string scale $l_s$. Also we have used the Friedman equation to estimate $H \sim 1/\sqrt{\Lambda}$. Comparing to the entropy of a thermal state $S \sim \sigma^3 a^3 T^3$, we expect $S$ and $\Sigma$ to agree to within a time independent factor $c^3$. The factor $c$ may be numerically large but is not expected to be parametrically large. More importantly it is time independent. Equating them

$(\frac{1}{\sqrt{\Lambda}l_s})^3 = c^3\sigma^3 a^3 T^3$ we find that the effective temperature is $T = 1/(ca\sqrt{\lambda}l_s)$ where $\lambda$ is the dimensionless CC introduced after Eq. (5). The energy density associated with this radiation is then given by Eq. (13). ST effectively have $c = 1$. However this is not consistent with the effective field theory mini-superspace starting point, since as can be seen from Eq. (17), at $a = a_{\pm} \sim 1/(\lambda^{1/4})$, this would give a radiation energy density that is greater than string scale. So for consistency one should have $c^4 > n_{\mathrm{dof}}$ as discussed in Sec. III.

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