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Dyonic branes and linear dilaton background

Gérard Clément,[1,*] Dmitri Gal'tsov,[1,2,†] Cédric Leygnac,[1,‡] and Dmitri Orlov[2,§]

[1]*Laboratoire de Physique Théorique LAPTH (CNRS), B.P. 110, F-74941 Annecy-le-Vieux Cedex, France*
[2]*Department of Theoretical Physics, Moscow State University, 119899, Moscow, Russia*

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We study dyonic solutions to the gravity-dilaton-antisymmetric form equations with the goal of identifying new p-brane solutions on the fluxed linear dilaton background. Starting with the generic solutions constructed by reducing the system to decoupled Liouville equations for certain values of parameters, we identify the most general solution whose singularities are hidden behind a regular event horizon, and then explore the admissible asymptotic behaviors. In addition to known asymptotically flat dyonic branes, we find two classes of asymptotically nonflat solutions which can be interpreted as describing magnetically charged branes on the electrically charged linear dilaton background (and the S-dual configuration of electrically charged branes on the magnetically charged background), and uncharged black branes on the dyonically charged linear dilaton background. This interpretation is shown to be consistent with the first law of thermodynamics for the new solutions.

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I. INTRODUCTION

Holographic dualities which relate classical supergravities with quantum field theories in lower dimensions were first discovered in nondilatonic theories (AdS/CFT correspondence) [1] and then extended to the generic case of theories with a dilaton [2]. An important step in establishing these dualities consists in the study of the near-horizon limit of p-branes. In the case of dilatonic branes the near-horizon geometry is either anti-de Sitter (AdS) or Minkowski with a nontrivial dilaton field depending linearly on an appropriate radial coordinate which we call in what follows the linear dilaton background (LDB). Such configurations are $1/2$ supersymmetric in the context of supergravities (contrary to maximally supersymmetric ones in the case of nondilatonic branes), but the conformal symmetry is broken by the dilaton. These backgrounds are dual to nonconformal quantum field theories (QFT) with 16 supercharges living on their boundary [2]. In the particular case of the NS5 brane the corresponding dual theory is the little string theory [3], while in the general case one finds a class of theories exhibiting the domain wall (DW)/QFT correspondence [4]. By the standard argument, the *thermal* version of the dual quantum theory should have as a holographic dual the linear dilaton background endowed with an event horizon. A variety of relevant supergravity configurations were obtained both in the black hole case [5–8] and for general p-branes [9–11]. These solutions have a nontrivial electric or magnetic field which is attributed to the LDB, while the presence of the event horizon is interpreted as due to a neutral p-brane on this background.

A natural question arises whether there exist *charged* branes on the linear dilaton background. To answer this, here we study systematically the *dyonic* brane configurations supported by a unique form field with both electric and magnetic sectors nonempty. It is worth noting that branes with both electric and magnetic charges may exist in any spacetime with electric and magnetic branes having different dimensions (branes within branes of the type of Ref. [12]. In even dimensions and with an antisymmetric form of a suitable rank, electric and magnetic branes may both have the same dimension [13]. Here we will be interested by dyonic branes of this latter type, which are derivable from Liouville systems. Some dyonic branes can also be identified within the class of intersecting branes for suitable values of the parameters [14–16], though in this approach the Chern-Simons terms in the Bianchi identities (transgressions), which can also be relevant for dyonic branes, are usually not taken into account (for a more detailed discussion see [13,17]). We will restrict ourselves here to dyonic configurations which exist in even spacetime dimensions $d = 2n$ in the presence of a form field of rank $q = p + 2 = n$. Asymptotically flat (AF) branes possessing both electric and magnetic charges were discussed in a number of papers [18–21], some non-AF solutions were also mentioned in [22,23].

Our strategy consists in obtaining the generic solution of the supergravity field equations for $ISO(p)$ symmetric branes with the transverse space being the product of a homogeneous space of dimension k and a flat $(q - k)$-dimensional Euclidean space. Such a possibility exists for certain particular values of the dilaton coupling constant, when the system of equations can be reduced to decoupled Liouville equations. We then demand the absence of naked singularities without imposing any asymptotic conditions. The resulting solution possesses an (either nondegenerate or degenerate) event horizon and can be

*Electronic address: gclement@lapp.in2p3.fr
†Electronic address: gdmv04@mail.ru
‡Electronic address: leygnac@lapp.in2p3.fr
§Electronic address: orlov_d@mail.ru

interpreted as a black brane. Then we explore all possible asymptotic behaviors in the region at an infinite geodesic distance from the horizon and find three different classes. The first class consists of the usual asymptotically flat black branes possessing both electric and magnetic charges. The solutions of the second class are magnetically charged black branes on the electrically charged LDB (and the dual electrically charged black branes on the magnetically charged LDB). The last class contains uncharged branes on the dyonic LDB.

In order to test this interpretation we develop the thermodynamics of our general dyonic configurations. To calculate the brane tension and other physical characteristics of the asymptotically nonflat solutions, one needs to generalize the formalism of quasilocal charges developed, in particular, in Refs. [24–26] to the case of an arbitrary number of spacetime dimensions and to the presence of the antisymmetric form fields. This was done in Ref. [9], so we can directly apply this technique here. We find that the asymptotically flat dyons satisfy the expected first law including variations of both the electric and magnetic charges. For the second class the first law includes only the variation of the magnetic charge, while the electric field remains frozen. This fits nicely with the expectation that we deal with a magnetic brane on the electric LDB (similarly, with an electric brane on the magnetic LDB). Finally, for the third class of solutions, neither electric nor magnetic charge variations contribute to the first law. Hence both these charges must be attributed indeed to the background, not to the brane.

II. SETUP

Our starting point is the standard action for the metric, dilaton and an antisymmetric form

$$S = \int d^d x \sqrt{-g}\left(R - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2q!} e^{a\phi} F_{[q]}^2\right), \quad (2.1)$$

with the Newton constant $G = 1/16\pi$.

We consider p-brane spacetime configurations with a $p+1$-dimensional world volume and the q-dimensional transverse space $\Sigma_{k,\sigma} \times R^{q-k}$ ($p + q = d - 2$):

$$ds^2 = -e^{2B} dt^2 + e^{2D}(dx_1^2 + \cdots + dx_p^2) + e^{2A} dr^2 + e^{2C} d\Sigma_{k,\sigma}^2 + e^{2E}(dy_1^2 + \cdots + dy_{q-k}^2), \quad (2.2)$$

where the metric functions $A, \ldots, E$ depend only on the radial coordinate r, and $\Sigma_{k,\sigma}$ is a constant curvature space ($k > 1$):

$$d\Sigma_{k,\sigma}^2 = \bar{g}_{ab} dz^a dz^b = \begin{cases} d\varphi^2 + \sin^2\varphi\, d\Omega_{(k-1)}^2, & \sigma = +1, \\ d\varphi^2 + \varphi^2 d\Omega_{(k-1)}^2, & \sigma = 0, \\ d\varphi^2 + \sinh^2\varphi\, d\Omega_{(k-1)}^2, & \sigma = -1. \end{cases} \quad (2.3)$$

The Ricci tensor for the metric (2.2) has the nonvanishing components

$$R_{tt} = e^{2B-2A}[B'' + B'(-A' + B' + kC' + pD' + (q-k)E')], \quad (2.4)$$

$$R_{\alpha\beta} = -e^{2D-2A}[D'' + D'(-A' + B' + kC' + pD' + (q-k)E')]\delta_{\alpha\beta}, \quad (2.5)$$

$$R_{rr} = -B'' - B'(B' - A') - k(C'' + C'^2 - A'C') - p(D'' + D'^2 - A'D') - (q-k)(E'' + E'^2 - A'E'), \quad (2.6)$$

$$R_{ab} = -\{e^{2C-2A}[C'' + C'(-A' + B' + kC' + pD' + (q-k)E')] - \sigma(k-1)\}\bar{g}_{ab}, \quad (2.7)$$

$$R_{ij} = -e^{2E-2A}[E'' + E'(-A' + B' + kC' + pD' + (q-k)E')]\delta_{ij}, \quad (2.8)$$

Dyonic configurations for the q-form field $F_{[q]}$ are possible in an even dimensional spacetime $d = 2n$, with

$$q = p + 2 = n. \quad (2.9)$$

In this case the Maxwell equations and the Bianchi identities are solved by

$$F_{[n]} = b_1 \mathrm{vol}_n + b_2 e^{-a\phi} * \mathrm{vol}_n, \qquad \mathrm{vol}_n = \mathrm{vol}(\Sigma_{k,\sigma}) \wedge dy_1 \wedge \cdots \wedge dy_{n-k}. \quad (2.10)$$

Define the gauge function $\mathcal{F}$

$$\ln\mathcal{F} \equiv -A + B + kC + pD + (q-k)E. \quad (2.11)$$

Fixing the form of $\mathcal{F}$ we thereby choose some gauge condition. The field equations are particularly simple to solve in the gauge $\mathcal{F} = 1$. The corresponding radial coordinate ρ is related to the radial coordinate r in a generic gauge $\mathcal{F}$ by

$$dr = \mathcal{F} d\rho. \quad (2.12)$$

Denoting the derivatives with respect to ρ by a dot, and putting

$$G_1 = a\phi + 2B + 2(n-2)D, \qquad G_2 = -a\phi + 2B + 2(n-2)D, \qquad H = 2(A - C), \quad (2.13)$$

we obtain the following form for the Einstein equations and the dilaton field equation:

$$\ddot{B} = \tfrac{1}{4}\{b_1^2 e^{G_1} + b_2^2 e^{G_2}\} \quad (2.14)$$

$$\ddot{D} = \tfrac{1}{4}\{b_1^2 e^{G_1} + b_2^2 e^{G_2}\} \quad (2.15)$$

$$\ddot{A} = -\tfrac{1}{4}\{b_1^2 e^{G_1} + b_2^2 e^{G_2}\} + \sigma k(k-1) e^{2H} \quad (2.16)$$

$$\ddot{C} = -\tfrac{1}{4}\{b_1^2 e^{G_1} + b_2^2 e^{G_2}\} + \sigma(k-1)e^{2H} \quad (2.17)$$

$$\ddot{E} = -\tfrac{1}{4}\{b_1^2 e^{G_1} + b_2^2 e^{G_2}\} \quad (2.18)$$

$$\ddot{\phi} = \frac{a}{2}\{b_1^2 e^{G_1} - b_2^2 e^{G_2}\}, \quad (2.19)$$

together with the constraint

$$-\dot{A}^2 + \dot{B}^2 + k\dot{C}^2 + (n-2)\dot{D}^2 + (n-k)\dot{E}^2 + \frac{1}{2}\dot{\phi}^2 = \frac{b_1^2}{2}e^{G_1} + \frac{b_2^2}{2}e^{G_2} - \sigma k(k-1)e^{2H}. \quad (2.20)$$

From the above system, we obtain the equations for the functions G_1, G_2 and H:

$$\ddot{G}_1 = \tfrac{1}{2}\{b_1^2\Delta_1 e^{G_1} + b_2^2\Delta_2 e^{G_2}\} \quad (2.21)$$

$$\ddot{G}_2 = \tfrac{1}{2}\{b_1^2\Delta_2 e^{G_1} + b_2^2\Delta_1 e^{G_2}\} \quad (2.22)$$

$$\ddot{H} = 2\sigma(k-1)^2 e^H, \quad (2.23)$$

where

$$\Delta_1 = a^2 + (n-1), \qquad \Delta_2 = -a^2 + (n-1). \quad (2.24)$$

The first two equations decouple in three special cases [18]. The obvious first case is $a^2 = n-1$ ($\Delta_2 = 0$). The other two possibilities correspond both to

$$G_1 - G_2 \equiv 2a\phi = 2a\phi_0 \quad (2.25)$$

constant. Substracting (2.22) from (2.21), we find that this is possible if

$$0 = a^2(b_1^2 e^{a\phi_0} - b_2^2 e^{-a\phi_0})e^{(G_1+G_2)/2}, \quad (2.26)$$

that is, if either a is arbitrary with $b_1^2 e^{a\phi_0} = b_2^2 e^{-a\phi_0} = b_1 b_2$ (we assume without loss of generality $b_1 b_2 > 0$), or $a = 0$ ($\Delta_1 = \Delta_2$). In the first case ($a^2 = n-1$) the equations for G_1 and G_2 separate and give

$$\ddot{G}_{1,2} = b_{1,2}^2(n-1)e^{G_{1,2}}, \quad (2.27)$$

so the solution will be

$$G_{1,2} = \ln\left[\frac{\alpha_{1,2}^2}{2(n-1)b_{1,2}^2}\right] - \ln\left[\sinh^2\left(\frac{\alpha_{1,2}}{2}(\rho - \rho_{1,2})\right)\right], \quad (2.28)$$

with integration constants $\alpha_{1,2}$ (real or imaginary), and $\rho_{1,2}$. In the second case ($a \neq 0$) and the third case ($a = 0$), $G_{1,2} = G \pm a\phi_0$, with G obeying the equation

$$\ddot{G} = b^2(n-1)e^G, \quad (2.29)$$

where $b^2 = b_1 b_2$ in the second case, and $b^2 = (b_1^2 + b_2^2)/2$ in the third case. The solution can be written in the form (2.28) with $\rho_1 = \rho_2$, $\alpha_1 = \alpha_2$, and $b_{1,2}$ replaced by $b^2 = b_1 b_2$ in the second case and by $b = \sqrt{(b_1^2 + b_2^2)/2}$ in the third case. The second case is a subset of the third for $b_1 = b_2$ which can be recovered by a dilaton shift and a redefinition of $b_{1,2}$ as follows:

$$\phi \to \phi - \phi_0, \qquad b_{1,2} \to b_{1,2}e^{\pm a\phi_0}. \quad (2.30)$$

In all cases, the solution of the Eq. (2.23) is

$$H = \begin{cases} \ln\left[\frac{\beta^2}{4(k-1)^2}\right] - \ln\left[\sinh^2\left(\frac{\beta}{2}\rho\right)\right], & \sigma = 1, \\ \beta\rho, & \sigma = 0, \\ \ln\left[\frac{\beta^2}{4(k-1)^2}\right] - \ln\left[\cosh^2\left(\frac{\beta}{2}\rho\right)\right], & \sigma = -1, \end{cases} \quad (2.31)$$

with an integration constant β (real or imaginary for $\sigma = +1$, real for $\sigma = 0$ or -1); we have used the translation freedom inherent in the definition (2.12) of ρ to set the second integration constant to zero. In the limiting cases $\alpha_{1,2} = 0$ and $\beta = 0$ the solutions (2.28) and (2.31) should be replaced by

$$G_{1,2} = -\ln[(n-1)b_{1,2}^2(\rho - \rho_{1,2})^2/2], \quad (2.32)$$

$$H = \begin{cases} -\ln(\rho^2), & \sigma = 1, \\ H_0, & \sigma = 0. \end{cases} \quad (2.33)$$

The final solution is given in terms of G_1, G_2 and H as

$$B = \frac{1}{4(n-1)}\{G_1 + G_2\} + (n-2)\{d_1\rho + d_0\}, \quad (2.34)$$

$$D = \frac{1}{4(n-1)}\{G_1 + G_2\} - \{d_1\rho + d_0\}, \quad (2.35)$$

$$A = -\frac{1}{4(n-1)}\{G_1 + G_2\} + \frac{k}{2(k-1)}H - \frac{n-k}{k-1}(c_1\rho + c_0), \quad (2.36)$$

$$C = -\frac{1}{4(n-1)}\{G_1 + G_2\} + \frac{1}{2(k-1)}H - \frac{n-k}{k-1}(c_1\rho + c_0), \quad (2.37)$$

$$E = -\frac{1}{4(n-1)}\{G_1 + G_2\} + c_1\rho + c_0, \quad (2.38)$$

$$a\phi = \tfrac{1}{2}\{G_1 - G_2\}. \quad (2.39)$$

The integration constants are related by the constraint equation

$$\frac{1}{4(n-1)}(\alpha_1^2 + \alpha_2^2) - \frac{k}{4(k-1)}\beta^2 + \frac{(n-1)(n-k)}{k-1}c_1^2 + (n-1)(n-2)d_1^2 = 0. \quad (2.40)$$

Furthermore, we can always rescale the x and y coordinates so that

$$c_0 = d_0 = 0. \tag{2.41}$$

III. BLACK DYONS

From now on we will consider the case of the spherical topology of the transverse space, $\sigma = 1$. Generically, the spacetime will contain an event horizon which can be identified with the surface $\rho = -\infty$. On the horizon, choosing the affine parameter λ along the radial geodesic as

$$d\lambda = \mathrm{e}^{A+B} d\rho, \tag{3.1}$$

one must have for the metric function B

$$\mathrm{e}^{2B} \sim \lambda^m, \tag{3.2}$$

where $m = 1$ for a nondegenerate horizon, and $m = 2$ in the degenerate case. Assuming first $\alpha_{1,2}$ and β real positive, we find near the horizon

$$G_{1,2} \approx \alpha_{1,2}\rho + \text{const}, \qquad H \approx \beta\rho + \text{const}, \tag{3.3}$$

Therefore, e^{2B} vanishes on the horizon provided

$$\alpha_1 + \alpha_2 + 4(n-1)(n-2)d_1 > 0. \tag{3.4}$$

Nondegenerate case.—Differentiating (3.2), one obtains

$$\frac{d\mathrm{e}^{2B}}{d\lambda} = 2\mathrm{e}^{B-A}\dot{B} \sim 1, \tag{3.5}$$

and hence $B - A \rightarrow$ const. It follows that the left-hand side of the constraint equation (2.20) reduces on the horizon to a sum of squares, while the right-hand side (rhs) goes to zero, so equating the separate terms to zero one obtains

$$\alpha_1 = \alpha_2 = \beta = 2(n-1)c_1 = 2(n-1)d_1. \tag{3.6}$$

The black solution can be transformed to a Schwarzschild-like gauge by the map

$$\mathrm{e}^{\alpha\rho} = \frac{f_+(\xi)}{f_-(\xi)}, \tag{3.7}$$

with

$$f_\pm(\xi) = 1 - \frac{\xi_\pm}{\xi}, \tag{3.8}$$

so that the horizon $\rho \rightarrow -\infty$ maps to a finite value $\xi = \xi_+$. This map defines ξ (and its special values $\xi_\pm$) only up to a scale, which we shall fix such that

$$\xi_+ - \xi_- = \frac{\alpha}{k-1}. \tag{3.9}$$

The images of ρ_1 and ρ_2 under the map (3.7) define the new integration constants ξ_1 and ξ_2,

$$\mathrm{e}^{\alpha\rho_{1,2}} = \frac{f_+(\xi_{1,2})}{f_-(\xi_{1,2})}, \tag{3.10}$$

which (due to the positivity of the left-hand side) lie both outside the interval $[\xi_-, \xi_+]$, i.e.

$$\xi_{1,2} \subset (-\infty, \xi_-) \cup (\xi_+, +\infty). \tag{3.11}$$

It is convenient to further transform the radial coordinate to r, with

$$\xi = r^{k-1}. \tag{3.12}$$

This corresponds to fixing the following gauge function:

$$\mathcal{F} = r^k f_+ f_-. \tag{3.13}$$

The solution then takes the form

$$ds^2 = \left[\frac{\mathrm{e}^{G_0} f_-^2}{f_1 f_2}\right]^{1/(n-1)} \left\{-\frac{f_+}{f_-}dt^2 + d\mathbf{x}^2\right\} + \left[\frac{\mathrm{e}^{G_0} f_-^2}{f_1 f_2}\right]^{-[1/(n-1)]} \times \left[(f_-^2)^{1/(k-1)}\left\{\frac{dr^2}{f_+ f_-} + r^2 d\Omega_k^2\right\} + d\mathbf{y}^2\right], \tag{3.14}$$

$$e^{a\phi} = e^{a\phi_0}\frac{f_2}{f_1}, \tag{3.15}$$

$$F_{[n]} = b_1 \mathrm{vol}(\Omega_k) \wedge dy_1 \wedge \cdots \wedge dy_{n-k} - b_2 \mathrm{e}^{G_0 - a\phi_0}\frac{dr}{f_2^2 r^k} \wedge dt \wedge dx_1 \wedge \cdots \wedge dx_{n-2}, \tag{3.16}$$

with

$$f_{1,2} = 1 - \frac{\xi_{1,2}}{\xi}, \tag{3.17}$$

and

$$\mathrm{e}^{G_0} = \frac{2(k-1)^2}{(n-1)b_1 b_2}[(\xi_+ - \xi_1)(\xi_- - \xi_1) \times (\xi_+ - \xi_2)(\xi_- - \xi_2)]^{1/2}, \tag{3.18}$$

$$e^{a\phi_0} = \frac{b_2}{b_1}\left[\frac{(\xi_+ - \xi_1)(\xi_- - \xi_1)}{(\xi_+ - \xi_2)(\xi_- - \xi_2)}\right]^{1/2}. \tag{3.19}$$

The values $\xi = \xi_{1,2}$ correspond to curvature singularities. This follows from the fact that the Ricci scalar, which from the Einstein equations for our dyons is simply

$$R = \mathrm{e}^{-2A}\frac{\dot{\phi}^2}{2}, \tag{3.20}$$

behaves as

$$R \sim [f_1 f_2]^{-[(2n-1)/(n-1)]} \tag{3.21}$$

for $a^2 = n - 1$, while the Kretschmann scalar (in both cases $a = 0$ and $a^2 = n - 1$) behaves as

$$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} \sim [f_1 f_2]^{-2[(2n-1)/(n-1)]}. \tag{3.22}$$

So, for finite $\xi_{1,2}$ ($\rho_{1,2} \neq 0$), the black solution will be regular only if both ξ_1 and ξ_2 lie behind the outer horizon ξ_+, which [owing to (3.11)] implies

$$\xi_1 \leq \xi_2 < \xi_- < \xi_+ \tag{3.23}$$

(the permutation of ξ_1 and ξ_2 will lead only to the sign change of the dilaton $\phi \to -\phi$).

Degenerate horizon.—In terms of the affine parameter, the metric function e^{2B} and its derivative behave near the horizon as

$$e^{2B} \sim \lambda^2, e^{B-A}\dot{B} \sim \lambda, \tag{3.24}$$

so one obtains

$$e^{-A}\dot{B} \sim O(1). \tag{3.25}$$

Two possibilities then exist: either both terms are regular, or $\dot{B}$ and e^A both vanish on the horizon. Assuming the regularity of e^A, one finds from (2.36) that on the horizon $\dot{A} = 0$, and the same for $\dot{B}$ from the constraint equation, which contradicts the assumption, so we have the second case.

If both $\dot{B} = 0$ and $e^B = 0$, then at least one of the parameters $\alpha_{1,2}$ must vanish. Then $e^A = 0$ on the horizon is possible only if $\beta = 0$. It then follows from the constraint equation (2.40) that the remaining parameters vanish. So the degenerate solution corresponds to the conditions

$$\alpha_1 = \alpha_2 = \beta = c_1 = d_1 = \varphi_1 = 0.$$

One can check that this is equivalent to taking the limit $\xi_- \to \xi_+$ in the previous solution.

IV. ASYMPTOTIC BEHAVIOR: THE THREE CLASSES OF BLACK DYONS

We have not yet discussed the asymptotic behavior of our solutions at spatial infinity. Inspection of (3.14) shows that spatial infinity corresponds to $\xi \to +\infty$ ($\rho \to 0$), and that the solution is asymptotically flat. Note however that the general black dyonic solution (3.14), (3.15), and (3.16) was written down for nonzero values of the integration constants $\rho_{1,2}$, according to (3.10). In the special cases in which one or both of these integration constants vanish, the solution, which is no longer asymptotically flat, can be recovered from (3.14), (3.15), and (3.16) by taking the limit in which one or both of the image constants $\xi_{1,2}$ is sent to $-\infty$ [10]. In the following, we discuss the three classes of solutions: (1) ρ_1 and ρ_2 are both nonzero; (2) $\rho_1 = 0$, $\rho_2 \neq 0$ (this can occur only in the case $a^2 = n - 1$); (3) $\rho_1 = \rho_2 = 0$.

First class: asymptotically flat dyons.—We first consider the generic case with two nonvanishing parameters ρ_1 and ρ_2. The metric (3.14) is asymptotically Minkowskian provided

$$G_0 = 0. \tag{4.1}$$

Also, the map (3.7) defines ξ only up to an additive constant, which we can choose so that $\xi_2 = 0$. Furthermore, the value of the dilaton at infinity ϕ_0 can be set to zero by the dilaton shift, together with the form rescaling

$$\phi \to \phi - \phi_0, \qquad F_{[n]} \to e^{a\phi_0/2}F_{[n]}, \tag{4.2}$$

leading to the form of the asymptotically flat solution, for $a^2 = n - 1$,

$$ds^2 = \left[\frac{f_-^2}{f_1}\right]^{1/(n-1)}\left\{-\frac{f_+}{f_-}dt^2 + d\mathbf{x}^2\right\} + \left[\frac{f_-^2}{f_1}\right]^{-[1/(n-1)]} \times \left[(f_-^2)^{1/(k-1)}\left\{\frac{dr^2}{f_+f_-} + r^2 d\Omega_k^2\right\} + d\mathbf{y}^2\right], \tag{4.3}$$

$$e^{a\phi} = \frac{1}{f_1}, \tag{4.4}$$

$$F_{[n]} = \sqrt{\frac{2}{n-1}}(k-1)\Big[\sqrt{(\xi_+ - \xi_1)(\xi_- - \xi_1)}\mathrm{vol}(\Omega_k) \wedge dy_1 \wedge \cdots \wedge dy_{n-k} - \sqrt{\xi_+\xi_-}\frac{dr}{r^k} \wedge dt \wedge dx_1 \wedge \cdots \wedge dx_{n-2}\Big], \tag{4.5}$$

depending on the three independent parameters ξ_+, ξ_- and ξ_1. Note that the original parameters b_1 and b_2 have been eliminated altogether from the solution. The "magnetic" and "electric" charges associated with the solution (4.3), (4.4), and (4.5) are

$$P = L_p L_{q-k}\Omega_k\sqrt{\frac{2}{n-1}}(k-1)\sqrt{(\xi_+ - \xi_1)(\xi_- - \xi_1)}, \tag{4.6}$$

$$Q = L_p L_{q-k}\Omega_k\sqrt{\frac{2}{n-1}}(k-1)\sqrt{\xi_+\xi_-}, \tag{4.7}$$

where L_p and L_{q-k} are the normalization volumes of the spaces spanned by x and y, and Ω_k is the volume of the unit sphere. The Ricci scalar for this spacetime is

$$R = \frac{(k-1)^2\xi_1^2}{2(n-1)}\xi^{-2-[1/(n-1)]}(\xi - \xi_1)^{-2-[1/(n-1)]} \times (\xi - \xi_-)^{(nk-3n+k+1)/[(n-1)(k-1)]}(\xi - \xi_+). \tag{4.8}$$

This diverges on the inner horizon $\xi = \xi_-$ only for $k = 2$ and $n > 3$. However, consideration of the Kretschmann scalar shows that the inner horizon is regular only for $k = n = 2$ or 3. In these cases the timelike singularity is located at $\xi = \sup(\xi_1, 0)$. In all other cases $\xi = \xi_-$ is a spacelike singularity.

In the second and third cases ($\phi = \phi_0$), $\rho_1 = \rho_2$ implies that also $\xi_1 = 0$, so that $f_1 = 1$ in (4.3) and (4.4). The only difference between these two cases lies in the number of independent parameters. In the second case (a arbitrary), the solution given by (4.3), (4.4), and (4.5) with $\xi_1 = 0$

depends only on the two parameters ξ_+ and ξ_-. In the third case ($a = 0$), the dilaton shift is irrelevant, so that the form field is simply

$$F_{[n]} = b_1 \mathrm{vol}(\Omega_k) \wedge dy_1 \wedge \cdots \wedge dy_{n-k} - b_2 \frac{dr}{r^k} \wedge dt \wedge dx_1 \wedge \cdots \wedge dx_{n-2}, \tag{4.9}$$

which replaces (4.5). The three parameters are in this case b_1 and b_2 (proportional to the magnetic and electric charges), and the horizon radius ξ_+, the constant ξ_- being related to these by the condition $\mathrm{e}^{G_0} = 1$ which reads in this case

$$\xi_+ \xi_- = \frac{(n-1)(b_1^2 + b_2^2)}{4(k-1)^2}. \tag{4.10}$$

Second class: asymptotically LDB dyons. ($a^2 = n - 1$).—The solutions of this class are obtained by taking the limit $\xi_1 \to -\infty$ in (3.14), (3.15), and (3.16). The function f_1 diverges in this limit, however it enters the solution only through the combinations e^{G_0}/f_1 and $e^{a\phi_0}/f_1$ which go to the finite limits

$$\frac{\mathrm{e}^{G_0}}{f_1} \to \frac{\xi}{\xi_0}, \qquad \frac{e^{a\phi_0}}{f_1} \to \mathrm{e}^{a\phi_1} \frac{\xi}{\xi_0}, \tag{4.11}$$

where we have put

$$\xi_0 = \frac{(n-1)b_1 b_2}{2(k-1)^2(\xi_+\xi_-)^{1/2}}, \qquad \mathrm{e}^{a\phi_1} = \frac{2(k-1)^2 \xi_0^2}{(n-1)b_1^2}. \tag{4.12}$$

Choosing again $\xi_2 = 0$, and performing the shift on the dilaton, together with the form rescaling

$$\phi \to \phi - \phi_1, \qquad F_{[n]} \to \mathrm{e}^{a\phi_1/2} F_{[n]}, \tag{4.13}$$

the resulting solution is

$$ds^2 = \left[f_-^2 \frac{r^{k-1}}{\xi_0}\right]^{1/(n-1)} \left\{-\frac{f_+}{f_-} dt^2 + d\mathbf{x}^2\right\} + \left[f_-^2 \frac{r^{k-1}}{\xi_0}\right]^{-[1/(n-1)]} \times \left[(f_-^2)^{1/(k-1)} \left\{\frac{dr^2}{f_+ f_-} + r^2 d\Omega_k^2\right\} + d\mathbf{y}^2\right], \tag{4.14}$$

$$e^{a\phi} = \frac{r^{k-1}}{\xi_0} \tag{4.15}$$

$$F_{[n]} = \sqrt{\frac{2}{n-1}}(k-1)\left[\xi_0 \mathrm{vol}(\Omega_k) \wedge dy_1 \wedge \cdots \wedge dy_{n-k} - \sqrt{\xi_+\xi_-} \frac{dr}{r^k} \wedge dt \wedge dx_1 \wedge \cdots \wedge dx_{n-2}\right], \tag{4.16}$$

This depends on the three independent parameters ξ_+ and ξ_- (the locations of the outer and the inner horizons), and ξ_0 (the overall scale). Again, the original parameters b_1 and b_2 have been eliminated from the solution. The magnetic and electric charges associated with the solution (4.14), (4.15), and (4.16) are

$$P = L_p L_{q-k} \Omega_k \sqrt{\frac{2}{n-1}}(k-1)\xi_0, \tag{4.17}$$

$$Q = L_p L_{q-k} \Omega_k \sqrt{\frac{2}{n-1}}(k-1)(\xi_+\xi_-)^{1/2}. \tag{4.18}$$

Note that the magnetic charge does not depend on the horizon radii ξ_+, ξ_-, but only on the overall scale ξ_0, so that it is a property of the linear dilaton background rather than of the black brane. On the other hand, the electric charge does depend on these parameters, and goes to zero in the limit $\xi_+ = \xi_- = 0$. The LDB metric is recovered in this limit:

$$ds^2 = \left[\frac{r^{k-1}}{\xi_0}\right]^{1/(n-1)} [-dt^2 + d\mathbf{x}^2] + \left[\frac{r^{k-1}}{\xi_0}\right]^{-[1/(n-1)]} \times [dr^2 + r^2 d\Omega_k^2 + d\mathbf{y}^2]. \tag{4.19}$$

The dual solutions, corresponding to the limit $\xi_2 \to +\infty$, may be obtained from (4.14), (4.15), and (4.16) by the discrete S-duality:

$$g_{\mu\nu} \to g_{\mu\nu}, \qquad F \to \mathrm{e}^{-a\phi} * F, \qquad \phi \to -\phi. \tag{4.20}$$

The roles of the electric and magnetic charges are then exchanged, the electric charge being associated with the background, and the magnetic charge with the black brane.

In both cases, we find that $\xi = \xi_-$ is generically a spacelike singularity. However, for $k = n = 2$ or 3, $\xi = \xi_-$ is a regular inner horizon hiding a timelike singularity at $\xi = 0$.

Third class.—This is obtained by taking the limit $\xi_1 = \xi_2 \to -\infty$. In this limit the combination $\mathrm{e}^{G_0}/f_1 f_2$ goes to the finite limit

$$\frac{\mathrm{e}^{G_0}}{f_1 f_2} \to \left(\frac{\xi}{\xi_0}\right)^2, \tag{4.21}$$

where now

$$\xi_0^2 = \frac{(n-1)b_1 b_2}{2(k-1)^2}. \tag{4.22}$$

Taking the limit of (3.15) and (3.19), we obtain that the dilaton is frozen:

$$\mathrm{e}^{a\phi} = \mathrm{e}^{a\phi_0} = \frac{b_2}{b_1}. \tag{4.23}$$

Thus, this third class of black dyons arises in the two cases $\phi = \phi_0$ with either $a \neq 0$, or $a = 0$. In the case $a \neq 0$, choosing $\xi_- = 0$ and performing the dilaton shift (4.2), we obtain the solution

$$ds^2 = \left[\frac{\xi}{\xi_0}\right]^{2/(n-1)}\{-f_+ dt^2 + d\mathbf{x}^2\} + \left[\frac{\xi}{\xi_0}\right]^{-[2/(n-1)]}\left[\frac{dr^2}{f_+} + r^2 d\Omega_k^2 + d\mathbf{y}^2\right], \quad (4.24)$$

$$e^{a\phi} = 1, \quad (4.25)$$

$$F_{[n]} = \sqrt{\frac{2}{n-1}}(k-1)\xi_0\left[\mathrm{vol}(\Omega_k) \wedge dy_1 \wedge \cdots \wedge dy_{n-k} - \left(\frac{\xi}{\xi_0}\right)^2 \frac{dr}{r^k} \wedge dt \wedge dx_1 \wedge \cdots \wedge dx_{n-2}\right], \quad (4.26)$$

depending on only two parameters, the horizon location ξ_+ and the scale ξ_0. In the case $a = 0$, the product $b_1 b_2$ should be replaced by $(b_1^2 + b_2^2)/2$ in the definition of the parameter ξ_0, while the magnetic and electric form field strengths stay arbitrary, so that the solution given by (4.24) and (4.25) and

$$F_{[n]} = \frac{2(k-1)}{\sqrt{n-1}}\xi_0\left[\cos\alpha\,\mathrm{vol}(\Omega_k) \wedge dy_1 \wedge \cdots \wedge dy_{n-k} - \sin\alpha\left(\frac{\xi}{\xi_0}\right)^2 \frac{dr}{r^k} \wedge dt \wedge dx_1 \wedge \cdots \wedge dx_{n-2}\right], \quad (4.27)$$

now depends on a third parameter α. In all cases, the electric and magnetic charges are both independent of the horizon parameter ξ_+, and so are associated with the background rather than with the black brane. The background metric is obtained by putting $\xi_+ = 0$ in (4.24):

$$ds^2 = \left[\frac{\xi}{\xi_0}\right]^{2/(n-1)}\{-dt^2 + d\mathbf{x}^2\} + \left[\frac{\xi}{\xi_0}\right]^{-[2/(n-1)]} \times [dr^2 + r^2 d\Omega_k^2 + d\mathbf{y}^2]. \quad (4.28)$$

This dyonic LDB is supported only by the antisymmetric form electric and magnetic fluxes, the dilaton being frozen. It has a particularly simple form if $k = n$, reducing to $\mathrm{AdS}_n \times S^n$:

$$ds^2 = \left(\frac{r}{r_0}\right)^2(-dt^2 + d\mathbf{x}_{n-2}^2) + \left(\frac{r_0}{r}\right)^2 dr^2 + r_0^2 d\Omega_k^2. \quad (4.29)$$

Returning to the spacetime (4.24), we see that the two regular cases $n = 2$ or 3 now correspond to geodesically complete spacetimes, $\mathrm{AdS}_2 \times S^2$ (in a noncomplete chart) for $n = k = 2$, and $\mathrm{BTZ} \times S^3$ (with BTZ the Bañados-Teitelboim-Zanelli black hole [27]) for $n = k = 3$.

V. MASS, ENTROPY, TEMPERATURE AND THE FIRST LAW OF THERMODYNAMICS

In order to give a correct interpretation to the black branes obtained it is useful to develop the corresponding thermodynamics. To compute the mass in the case of non-asymptotically flat configurations, we shall use the quasilocal approach [24–26,28] as extended to the case of the Einstein-dilaton-antisymmetric form theory in d dimensions in [9]. The Arnowitt-Deser-Misner (ADM) decomposition

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \quad (5.1)$$

leads to a foliation of the spacetime by spacelike $(d-1)$-surfaces Σ_t of metric $h_{\mu\nu}$. The surfaces Σ_t are themselves foliated by $(d-2)$-surfaces Σ_t^r (t, r constant) of metric $\sigma_{\mu\nu} = h_{\mu\nu} - n_\mu n_\nu$, with n^μ the unit spacelike normal to S_t^r. A careful evaluation of the field-theoretical Hamiltonian for the theory (2.1) in a spacetime volume bounded by initial and final spacelike surfaces Σ_{t_i} and Σ_{t_f}, and a timelike surface $r =$ constant, leads to the sum of a volume contribution which vanishes on shell, plus a surface contribution, the quasilocal energy, which is given in the static case ($N^i = 0$) by

$$E = \int_{\Sigma_t^r}(2\sqrt{\sigma}N\kappa + (n-1)A_{ti_1\ldots i_{n-2}}\Pi^{ri_1\ldots i_{n-2}})d^{d-2}x. \quad (5.2)$$

In (5.2),

$$\kappa = -\sigma^{\mu\alpha}D_\alpha n_\mu \quad (5.3)$$

(with D_α the covariant derivative compatible with the metric $h_{\mu\nu}$) is the extrinsic curvature of Σ_t^r embedded in Σ_t, $A_{ti_1\ldots i_{n-2}}$ are the electric components of the $(q-1)$-potential form A ($F = dA$), and

$$\Pi^{ri_1\ldots i_{n-2}} = -\sqrt{-g}e^{a\phi}F^{tri_1\ldots i_{n-2}} \quad (5.4)$$

are the conjugate momenta, equal to the constant electric charges per brane volume.

The quasilocal mass may formally defined as the quasilocal energy evaluated in the limit $r \to \infty$. However, the quasilocal energy generically diverges in this limit. This divergence may be regularized by subtracting the contribution of a background solution (the zero-point energy), provided one can impose the same Dirichlet boundary conditions on Σ_t^r for the black solution under consideration and for the background solution. Specifically, this means that the boundary metric σ_{ij} and the nondynamical fields (Lagrange multipliers) N, N^i and $A_{ti_1\ldots i_{n-2}}$ of the black solution and of the background solution should coincide (or asymptotically coincide to a sufficient accuracy) on the boundary [25]. For the nondynamical fields, this requirement can be taken care of by a rescaling of time for N, or a gauge transformation for $A_{ti_1\ldots i_{n-2}}$. On the other hand, the requirement on the boundary metric strongly constrains the choice of the background solution, which in practice must be an extreme member of the black family of solutions. After regularization, the quasilocal mass is

$$M = \lim_{r_b\to\infty}\int_{\Sigma_t^r}[2\sqrt{\sigma}N(\kappa-\kappa_0) + (n-1)A_{ti_1\ldots i_{n-2}}(\bar{\Pi}^{ri_1\ldots i_{n-2}} - \bar{\Pi}_0^{ri_1\ldots i_{n-2}})]d^{d-2}x, \quad (5.5)$$

where the quantities with the subscript 0 are associated with the background solution, and the equation of the boundary Σ_t^r is $t =$ constant, $r = r_b$.

First class.—The electric potential $A_{ti_1\ldots i_{n-2}}$ goes to zero as r^{1-k}, so that the quasilocal mass is given by the purely metric contribution [the first term in the rhs of (5.5)]. The natural background for asymptotically flat black dyons is the Minkowski spacetime, which is obtained from (4.3) by putting $\xi_+ = \xi_- = 0$,

$$ds_0^2 = -dt^2 + d\mathbf{x}^2 + d\rho^2 + \rho^2 d\Omega_k^2 + d\mathbf{y}^2. \quad (5.6)$$

Note that we take care to distinguish between the generic black radial coordinate r and the background radial coordinate ρ. The equation of the boundary Σ_t^r is $r = r_b$ in black coordinates, or $\rho = \rho_b$ in background coordinates. Because this boundary is common, the identification of the k-spheres leads to

$$C_0(\rho_b) = C(r_b), \quad (5.7)$$

which can be solved to lead to a function $\rho_b(r_b)$. The adjustment of the (xx) or (yy) components can be simply taken care of by a radius-dependent rescaling of the x or y coordinates. Then, the computation of the extrinsic curvature leads to

$$\kappa(r_b) = -e^{-A}\partial_r[kC + pD + (n-k)E]|_{r=r_b} \quad (5.8)$$

for the black metric, and

$$\kappa_0(\rho_b) = -e^{-A_0}\partial_\rho[kC_0 + pD_0 + (n-k)E_0]|_{\rho=\rho_b} \quad (5.9)$$

for the background metric.

We obtain asymptotically, for the black metric

$$\kappa(r_b) \simeq -k\xi_b^{-[1/(k-1)]} \times \left[1 + \left(-\frac{\xi_+}{2} + \frac{-4 + k(n+9) + k^2(n-7)}{2k(k-1)(n-1)}\xi_- + \frac{3k-2}{2k(n-1)}\xi_1\right)\frac{1}{\xi_b}\right], \quad (5.10)$$

and for the Minkowski background

$$\kappa_0(\rho_b) \simeq -k\xi_b^{-[1/(k-1)]}\left[1 + \left(\frac{n-k}{(k-1)(n-1)}\xi_- + \frac{1}{2(n-1)}\xi_1\right)\frac{1}{\xi_b}\right], \quad (5.11)$$

leading to the quasilocal mass

$$M = L_pL_{q-k}\Omega_k\left[k(\xi_+ - \xi_-) + \frac{2(k-1)}{n-1}(2\xi_- - \xi_1)\right]. \quad (5.12)$$

This result coincides with the ADM mass.

Now we check that this value of the mass, together with the other physical parameters of the dyonic black branes, satisfy the generalized first law of black hole thermodynamics [7,29]

$$dM = TdS + W_h dP + V_h dQ, \quad (5.13)$$

where T and S are the Hawking temperature and the black hole entropy, W_h and V_h are the values of the magnetic and electric potentials on the horizon $\xi = \xi_+$.

The entropy and the temperature are found locally in a standard way:

$$S = 4\pi L_pL_{q-k}\Omega_k\xi_+^{1/(n-1)}(\xi_+ - \xi_-)^{[k/(k-1)]-[2/(n-1)]} \times (\xi_+ - \xi_1)^{1/(n-1)}, \quad (5.14)$$

$$T = \frac{k-1}{4\pi}\xi_+^{-[1/(n-1)]}(\xi_+ - \xi_-)^{[2/(n-1)]-[1/(k-1)]} \times (\xi_+ - \xi_1)^{-[1/(n-1)]}. \quad (5.15)$$

The electric potential and the magnetic potential (the electric potential of the dual form) can be written out from the form field (4.5) or (4.9) as follows:

$$W = \sqrt{\frac{2}{n-1}}\frac{\sqrt{(\xi_+ - \xi_1)(\xi_- - \xi_1)}}{\xi - \xi_1}, \qquad V = \sqrt{\frac{2}{n-1}}\frac{\sqrt{\xi_+\xi_-}}{\xi}, \quad (5.16)$$

for $a^2 = n-1$, and for arbitrary $a \neq 0$ with $\xi_1 = 0$, or

$$W = \frac{1}{k-1}\frac{b_1}{\xi}, \qquad V = \frac{1}{k-1}\frac{b_2}{\xi}, \quad (5.17)$$

for $a = 0$ (implying $\xi_1 = 0$). In all cases we find that the generalized first law (5.13) is satisfied under independent variation of the parameters ξ_+, ξ_- and ξ_1 (case $a^2 = n-1$); ξ_+ and ξ_- (case $a \neq 0$ with $\xi_1 = 0$); or ξ_+, b_1 and b_2 [case $a = 0$, where the relation (4.10) between the parameters should be taken into account].

Second class.—Again the electric potential of the electric black dyon (4.16) goes to zero as r^{1-k}, so that the quasilocal mass is given by the purely metric contribution. The natural background is the magnetically charged LDB $\xi_+ = \xi_- = 0$ (4.19). We obtain asymptotically, for the extrinsic curvature of the black metric

$$\kappa(r_b) \simeq -\frac{1+k(n-2)}{n-1}\left(\frac{\xi_b}{\xi_0}\right)^{1/[2(n-1)]}\xi_b^{-[1/(k-1)]}\left[1-\left(\frac{\xi_+}{2} - \frac{n+1+k(n-3)}{2(k-1)(n-1)}\xi_- + \frac{k-1}{1+k(n-2)}\xi_-\right)\frac{1}{\xi_b}\right], \tag{5.18}$$

and for that of the linear dilaton background

$$\kappa_0(\rho_b) \simeq -\frac{1+k(n-2)}{n-1}\left(\frac{\xi_b}{\xi_0}\right)^{1/[2(n-1)]}\xi_b^{-[1/(k-1)]} \times \left(1+\frac{n-k}{(k-1)(n-1)}\frac{\xi_-}{\xi_b}\right), \tag{5.19}$$

leading to the quasilocal mass

$$M = L_p L_{q-k}\Omega_k\left[\left(k-\frac{k-1}{n-1}\right)(\xi_+ - \xi_-) + 2\frac{k-1}{n-1}\xi_-\right]. \tag{5.20}$$

Bearing in mind that the magnetic charge is a property of the linear dilaton background and thus should not be varied, the first law for asymptotically LDB black dyons reads

$$dM = TdS + V_h dQ. \tag{5.21}$$

The entropy and temperature are

$$S = 4\pi L_p L_{q-k}\Omega_k(\xi_0\xi_+)^{1/(n-1)} \times (\xi_+ - \xi_-)^{[k/(k-1)]-[2/(n-1)]}, \tag{5.22}$$

$$T = \frac{k-1}{4\pi}(\xi_0\xi_+)^{-[1/(n-1)]}(\xi_+ - \xi_-)^{[2/(n-1)]-[1/(k-1)]}, \tag{5.23}$$

the electric potential is

$$V = \frac{1}{k-1}\frac{b_2}{\xi}, \tag{5.24}$$

and the electric charge is given by (4.18). These quantities satisfy the first law (5.21) under independent variation of the parameters ξ_+ and ξ_-, the scale parameter ξ_0, associated with the linear dilaton background, being held fixed.

In the case of the dual magnetic black dyon, the constant electric field $\Pi^{ri_1\ldots i_{n-2}}$ is identical to that of the electric linear dilaton background, so that the electric contribution to the quasilocal mass (5.5) is identically zero, and the quasilocal mass is again given by (5.20). The first law appropriate for this case,

$$dM = TdS + W_h dP, \tag{5.25}$$

is again satisfied [with the magnetic potential W and the magnetic charge P given by (5.24) and (4.18)] provided the scale parameter ξ_0, proportional to the electric charge of the linear dilaton background, is held fixed.

Third class.—In this case, the constant electric field is again identical to that of the background $\xi_+ = 0$, so that the quasilocal mass is given by the sole metric contribution. From the asymptotic extrinsic curvature of the black metric

$$\kappa(r_b) \simeq -\frac{2+k(n-3)}{n-1}\left(\frac{\xi_b}{\xi_0}\right)^{1/(n-1)}\xi_b^{-[1/(k-1)]}\left[1-\frac{\xi_+}{2\xi_b}\right], \tag{5.26}$$

we obtain (in this case $\rho_b = r_b$)

$$M = L_p L_{q-k}\Omega_k\left(k - 2\frac{k-1}{n-1}\right)\xi_+. \tag{5.27}$$

The mass is zero for $n = k = 2$ (the solution in this case is Bertotti-Robinson), and positive in the other cases ($n > 2$ and $k \geq 2$, with $n \geq k$).

The entropy and temperature are

$$S = 4\pi L_p L_{q-k}\Omega_k\left(\frac{\xi_+}{\xi_0}\right)^{-[2/(n-1)]}\xi_+^{k/(k-1)}, \tag{5.28}$$

$$T = \frac{k-1}{4\pi}\left(\frac{\xi_+}{\xi_0}\right)^{2/(n-1)}\xi_+^{-[1/(k-1)]}, \tag{5.29}$$

and the first law takes the simple form

$$dM = TdS, \tag{5.30}$$

since both charges belong to the background, and so should not be varied.

VI. CONCLUSIONS

In this paper we have constructed new nonasymptotically flat p-brane solutions which possess a regular event horizon and which approach the linear dilaton background at spatial infinity. The latter is a supersymmetric solution of the supergravity equations with a nonzero flux of the antisymmetric form field. More precisely, we have shown that there exist magnetically charged p-branes on an electric LDB, electrically charged branes on a magnetic LDB and uncharged branes on a LDB with both electric and magnetic fluxes. Together with the usual asymptotically flat dyonic branes, these configurations exhaust all possibilities for brane solutions free of naked singularities and involving both electric and magnetic sectors of the (unique) form field.

The physical interpretation of nonasymptotically flat dyonic solutions has some subtleties related to the nature of the charge parameters. It turns out that at least one of the two charge parameters must be attributed to the background, not to the brane itself. This is clearly seen from the first law of thermodynamics, which is derived using the generalized formalism of quasilocal charges.

Black brane solutions on a fluxed linear dilaton background describe the thermal phase of the QFT involved into the corresponding DW/QFT correspondence [30]. Previously [9] we have found configurations of this kind involving neutral branes on a purely magnetic or purely electric LDB. Now we see that there exist also charged branes on a LDB with a dual flux (i.e. electric branes on a magnetic LDB and vice versa) as well as uncharged branes on a dyonic LDB. Their role in the DW/QFT correspondence requires further study.

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[1] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); Int. J. Theor. Phys. **38**, 1113 (1999); S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998); O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, Phys. Rep. **323**, 183 (2000).
[2] N. Itzhaki, J. M. Maldacena, J. Sonnenschein, and S. Yankielowicz, Phys. Rev. D **58**, 046004 (1998).
[3] O. Aharony, Classical Quantum Gravity **17**, 929 (2000); O. Aharony, A. Giveon, and D. Kutasov, Nucl. Phys. **B691**, 3 (2004).
[4] H. J. Boonstra, K. Skenderis, and P. K. Townsend, J. High Energy Phys. 01 (1999) 003.
[5] K. C. K. Chan, J. H. Horne, and R. B. Mann, Nucl. Phys. **B447**, 441 (1995).
[6] G. Clément, D. Gal'tsov, and C. Leygnac, Phys. Rev. D **67**, 024012 (2003).
[7] G. Clément and C. Leygnac, Phys. Rev. D **70**, 084018 (2004).
[8] C. Leygnac, gr-qc/0409040.
[9] G. Clément, D. Gal'tsov, and C. Leygnac, Phys. Rev. D **71**, 084014 (2005).
[10] D. Gal'tsov, S. Klevtsov, D. Orlov, and G. Clément, hep-th/0508070 [Int. J. Mod. Phys. A (to be published)].
[11] C. M. Chen, D. V. Gal'tsov, and N. Ohta, Phys. Rev. D **72**, 044029 (2005).
[12] Miguel S. Costa, Nucl. Phys. **B490**, 202 (1997).
[13] J. M. Izquierdo, N. D. Lambert, G. Papadopoulos, and P. K. Townsend, Nucl. Phys. **B460**, 560 (1996).
[14] M. A. Grebeniuk and V. D. Ivashchuk, Phys. Lett. B **442**, 125 (1998).
[15] V. D. Ivashchuk and V. N. Melnikov, Gravitation Cosmol. **5**, 313 (1999); Gravitation Cosmol. **6**, 27 (2000); Classical Quantum Gravity **18**, R87 (2001).
[16] N. Ohta, Phys. Lett. B **403**, 218 (1997); Y.-G. Miao and N. Ohta, Phys. Lett. B **594**, 218 (2004).
[17] M. Cvetic, H. Lü, and C. N. Pope, Nucl. Phys. **B600**, 103 (2001).
[18] M. J. Duff, H. Lü, and C. N. Pope, Phys. Lett. B **382**, 73 (1996).
[19] H. Lü, C. N. Pope, and K. W. Xu, Mod. Phys. Lett. A **11**, 1785 (1996).
[20] H. Lü and C. N. Pope, hep-th/9601089.
[21] H. Lü and C. N. Pope, Nucl. Phys. **B465**, 127 (1996).
[22] C. Grojean, F. Quevedo, G. Tasinato, and I. Zavala, J. High Energy Phys. 08 (2001) 005.
[23] S. S. Yazadjiev, Classical Quantum Gravity **22**, 3875 (2005).
[24] J. D. Brown and J. W. York, Phys. Rev. D **47**, 1407 (1993).
[25] S. W. Hawking and G. T. Horowitz, Classical Quantum Gravity **13**, 1487 (1996).
[26] C. M. Chen and J. M. Nester, Classical Quantum Gravity **16**, 1279 (1999); C. M. Chen, J. M. Nester, and R. S. Tung, Phys. Rev. D **72**, 104020 (2005).
[27] M. Bañados, C. Teitelboim, and J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992); M. Bañados, M. Henneaux, C. Teitelboim, and J. Zanelli, Phys. Rev. D **48**, 1506 (1993).
[28] I. S. N. Booth, gr-qc/0008030.
[29] D. Rasheed, Nucl. Phys. **B454**, 379 (1995).
[30] K. Behrndt, E. Bergshoeff, R. Halbersma, and J. P. van der Schaar, Classical Quantum Gravity **16**, 3517 (1999; E. Bergshoeff and R. Halbersma, hep-th/0001065.