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Localized U(1) gauge fields, millicharged particles, and holography

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We consider U(1) gauge fields in a slice of AdS_5 with bulk and boundary mass parameters. The zero mode of a bulk U(1) gauge field can be localized either on the UV or IR brane. This leads to a simple model of millicharged particles in which fermions can have arbitrarily small electric charge. In the electroweak sector we also discuss phenomenological implications of a localized $U(1)_Y$ gauge boson. Using the AdS/CFT correspondence we present the 4D holographic interpretation of the 5D model. In particular the photon is shown to be a composite particle when localized near the IR brane, whereas it is elementary when localized near the UV brane. In the dual interpretation the millicharge results from an elementary fermion coupling to a composite photon via a vector current with large anomalous dimension.

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I. INTRODUCTION

The AdS/CFT correspondence [1] has provided a simple and compelling framework in which to study four dimensional (4D) gauge theories. Gauge fields propagating in the background of an AdS_5 warped geometry provide a weak-coupling description of the nontrivial dynamics that occurs in strongly coupled 4D gauge theories. In a slice of AdS_5 [2] the best known example is a massless U(1) gauge field [3,4]. The zero mode of this bulk field is not localized and has a flat profile. In the dual 4D interpretation this corresponds to a part-elementary and part-composite photon eigenstate which resembles $\gamma - \rho$ mixing in QCD [5].

It is perhaps not so well known that the zero mode U(1) gauge field can in fact be localized anywhere in the warped bulk by adding both bulk and brane-localized mass terms [6,7]. Essentially, as first pointed out for bulk scalar fields [8], the brane-localized mass terms serve to alter the boundary conditions in such a way that a zero mode solution is still allowed. Although this requires a fine-tuning between bulk and boundary mass parameters the zero mode photon can be localized anywhere in the warped bulk.

Several interesting phenomenological scenarios are then possible. Just as separating bulk fermions from the Higgs boson in the warped bulk can lead to Yukawa coupling hierarchies [8,9], a simple model of millicharged particles by separating fermions in the warped bulk from a localized U(1) gauge boson can also be constructed. This will allow fermions to have arbitrarily small electric charge and is a new way to obtain millicharged particles. Moreover, a grand unified scenario can be considered in the warped bulk by generating kinetic mixing between two U(1) gauge fields [10,11]. In addition a localized U(1) gauge boson will lead to different possibilities in the electroweak sector from those considered so far in warped Randall-Sundrum models. In particular, stringent electroweak constraints from bulk Abelian gauge fields can be avoided without delocalizing fermions or adding brane kinetic terms.

The most interesting aspect of the localized bulk U(1) gauge field is that it can be given a 4D holographic description. Much like previous analyses for bulk fermions [12] and bulk gravitons [13], the UV (IR) brane-localized gauge fields can be shown to correspond to elementary (composite) photon eigenstates in the dual 4D theory. The boundary mass provides a continuous parameter which connects these two limiting possibilities. In particular, when the photon zero mode is localized on the IR brane this corresponds to a composite photon in the dual 4D theory. This is an example of emergent behavior since the photon only exists at large distance scales. The dual holographic description then allows us to interpret millicharged particles as resulting from elementary fermions coupling to a composite photon via a vector current with large anomalous dimension.

The outline of this paper is as follows. In Sec. II we will review Abelian gauge fields in warped space. The equations of motion for both massless and massive gauge fields have already been studied. However dealing with gauge fields in warped extra dimensions can be tricky because of additional scalar fields that arise in the dimensional reduction, and the fate of these modes is often obscured. Instead we will employ a five dimensional (5D) Stückelberg mechanism which maintains manifest gauge invariance. This provides a simple way to identify the dynamical scalar fields of the theory, while still being able to decouple the photon and preserve 4D gauge invariance. The phenomenological applications of localized U(1) gauge fields are then presented in Sec. III. This includes millicharged particles, and a model of the electroweak sector. In Sec. IV the holographic interpretation of the bulk 5D model is presented, including the dual interpretation of millicharged particles. Concluding remarks are given in Sec. V.

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II. ABELIAN GAUGE FIELDS IN WARPED SPACE

A. Massless bulk gauge fields

In a slice of AdS_5, the metric is [2]

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{1}$$

where k is the AdS curvature scale. The extra coordinate y ranges from $0 < y < \pi R$. At the boundaries $y = 0$ and $y = \pi R$ there exist two three-branes, called the ultraviolet (UV) and infrared (IR) brane, respectively. We label 5D coordinates with Latin indices $(A, B, \ldots)$ and 4D coordinates with Greek indices $(\mu, \nu, \ldots)$. Greek indices are raised and lowered strictly with the 4D Minkowski metric, $\eta_{\mu\nu}$, which has signature $(-, +, +, +)$.

Before we examine massive vector fields and the possibility of localizing the zero mode, it will be instructive to review the massless case and compare our approach to results obtained previously [3,4,14]. The 5D action for a massless gauge field in warped space is given by

$$S = \int d^5x\sqrt{-g}\left(-\frac{1}{4}g^{MN}g^{RS}F_{MR}F_{NS}\right). \tag{2}$$

Rather than choosing a particular gauge to simplify the analysis, our strategy will be to write the action in terms of gauge invariant combinations of the fields. To this end, we parametrize the 5D vector A_M in the following way:

$$A_M = (\hat{A}_\mu + \partial_\mu\varphi, A_5) \tag{3}$$

where $\partial_\mu\hat{A}^\mu = 0$. We interpret $\hat{A}_\mu$ and φ as the transverse and longitudinal components of A_μ, respectively. Under the gauge transformation $A_M \to A_M + \partial_M\lambda$, with gauge parameter λ, the transverse vector $\hat{A}_\mu$ is invariant, while the scalar longitudinal mode transforms as $\varphi \to \varphi + \lambda$. Three degrees of freedom are contained in $\hat{A}_\mu$ while the scalars φ and A_5 account for another two, making up a total of 5 degrees of freedom for the 5D vector field. However, the dynamics and the gauge symmetry imply that there are only three physical degrees of freedom, and this fact will guide our analysis.

With this parametrization, the action (2) becomes

$$S = \int d^5x\left[-\frac{1}{4}\hat{F}^2_{\mu\nu} - \frac{1}{2}e^{-2ky}(\partial_5\hat{A}_\mu)^2 - \frac{1}{2}e^{-2ky}(\partial_\mu\psi)^2\right], \tag{4}$$

where we have defined the gauge invariant field $\psi = A_5 - \varphi'$, with prime $(')$ denoting differentiation with respect to y. Note that the action (4) has decoupled in terms of the fields $\hat{A}_\mu$ and ψ. Performing a Kaluza-Klein decomposition of the vector, we find the standard result of a single massless mode (2 degrees of freedom) and a tower of Proca fields (3 degrees of freedom) [3,4]. We therefore expect that the bulk dynamics will allow only a single massless scalar mode. Indeed, the equation of motion for ψ is simply $\Box\psi(x, y) = 0$, which means that there exists only a massless mode,

$$\psi(x, y) = \psi^0(x)f^0(y). \tag{5}$$

Hence, we have found our remaining degree of freedom at the massless level.

Although we have found a massless scalar mode, the wave function of ψ is undetermined from the action. Instead to find the wave function, we can start from the 5D equation of motion, which comes from varying the action (2). This leads to the first order differential equation

$$\partial_5(e^{-2ky}\psi(x, y)) = 0. \tag{6}$$

The solution is given by

$$\psi(x, y) = C\psi^0(x)e^{2ky}, \tag{7}$$

where C is a normalization constant. Substituting the solution back into the action gives

$$S = -C^2\int d^4x\frac{1}{2}(\partial_\mu\psi)^2\int dye^{2ky} + \cdots, \tag{8}$$

and therefore the profile of ψ with respect to a flat metric is given by

$$f^0(y) \propto e^{ky}. \tag{9}$$

It is common when dealing with 5D gauge fields to expand the scalar field A_5 in terms of a derivative of a function [14], $A_5^n(x)\partial_5 f^n(y)$. This has the virtue of diagonalizing the interaction terms between A_μ^n and A_5^n. Since A_5 is contained in the gauge invariant field ψ, it is natural to ask what happens if we expand the field $\psi(x, y)$ in terms of a derivative of a function. From (6), it is clear that the equation of motion is second order and we get two solutions. One solution is a constant c, meaning that the zero mode vanishes: $\partial_5 c = 0$. The other solution gives us the same bulk profile as (9), so clearly the physics is the same regardless of which way the scalar field is expanded. Whether or not the mode actually exists depends on the boundary conditions of the theory.

Another approach is to add the following gauge fixing term to the Lagrangian [14,15]:

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}(\partial_\mu A^\mu + \xi\partial_5(e^{-2ky}A_5))^2, \tag{10}$$

which removes the interaction term between A_μ and A_5. With this choice of gauge, the equation of motion for A_5 becomes

$$(e^{2ky}\Box + \xi e^{2ky}\partial_5^2 e^{-2ky})A_5(x, y) = 0. \tag{11}$$

Performing a Kaluza-Klein decomposition, we find that the wave function of the massive modes depends on the gauge parameter ξ, indicating that these are fictitious degrees of freedom. This agrees with our analysis in which we found no massive scalar modes. However, since $\Box A_5^0(x) = 0$, the equation of motion for the zero mode is independent of ξ, which means that the massless mode is indeed a physical

particle. Its profile with respect to a flat background agrees with (9).

Working with gauge invariant fields allows us to clearly identify the dynamical degrees of freedom contained in A_M. We have seen that our analysis of the massless gauge field is consistent with other approaches. This approach will be even more helpful when we examine massive bulk vector fields.

B. Massive bulk gauge fields

We now turn to the study of massive gauge fields. Essentially, our analysis will follow [6], but we will demand that 5D gauge invariance be a symmetry of our theory. As we will see, this allows us to cleanly identify the scalar degrees of freedom in the theory. One way to restore gauge invariance to the theory of the massive gauge field is to add a degree of freedom by simply promoting the gauge parameter to a dynamical field. This is the famous Stückelberg formalism (for a review, see Ref. [16]). The action is

$$S = \int d^5x\sqrt{-g}\left(-\frac{1}{4}g^{MN}g^{RS}F_{MR}F_{NS} - \frac{1}{2}(\partial_M\phi - mA_M)^2\right), \tag{12}$$

which is invariant under the gauge transformation, $A_M \rightarrow A_M + \partial_M\lambda$ provided the field ϕ simultaneously transforms as $\phi \rightarrow \phi + m\lambda$. Following Ref. [6], we will parametrize A_M as in (3). There are 3 degrees of freedom contained in the scalar fields φ, ϕ, and A_5, while $\hat{A}_\mu$ contains 3 degrees of freedom. This makes a total of 6 degrees of freedom for the Stückelberg action (12). Of course, as in the massless case, 2 degrees of freedom are removed by gauge invariance and dynamics.

Rewriting the action using our parametrization for A_M (3), we again find that the transverse vector $\hat{A}_\mu$ decouples from the scalar fields. The action for $\hat{A}_\mu$ becomes

$$S(\hat{A}_\mu) = \int d^5x\left[-\frac{1}{4}\hat{F}^2_{\mu\nu} - \frac{1}{2}e^{-2ky}(\partial_5\hat{A}_\mu)^2 - \frac{1}{2}e^{-2ky}m^2\hat{A}^2_\mu\right]. \tag{13}$$

This will be our starting point in the next section. We will see that by adding a specific boundary mass term, it is possible to localize the zero mode of $\hat{A}_\mu$.

Before that let us analyze the scalar modes. The action for the scalar fields, which follows from (12), can be written in the form

$$S(\varphi, \phi, A_5) = \int d^5x\left[-\frac{1}{2}e^{-2ky}(\partial_\mu(A_5 - \varphi'))^2 - \frac{1}{2}e^{-2ky}(\partial_\mu(\phi - m\varphi))^2 - \frac{1}{2}m^2e^{-4ky}\left(A_5 - \frac{1}{m}\phi'\right)^2\right]. \tag{14}$$

Notice that this action is gauge invariant, which indicates that we really only have 2 degrees of freedom. Varying the action (14), we get a system of coupled differential equations in terms of the fields A_5, φ, and ϕ. To isolate the true dynamical variables, let us define the following gauge invariant scalar fields:

$$\psi = A_5 - \varphi', \qquad \rho = \phi - m\varphi, \qquad \sigma = A_5 - \frac{1}{m}\phi'. \tag{15}$$

The action (14) can then be written solely in terms of these fields. However these fields are not independent, since $\psi - \rho'/m = \sigma$. If we eliminate ρ in favor of ψ and σ then the equations of motion are

$$\Box(e^{2ky}\Box + \partial_5^2 - 2k\partial_5 - m^2)\psi(x, y) = 0, \tag{16}$$

$$(e^{2ky}\Box + \partial_5^2 - 6k\partial_5 + 8k^2 - m^2)\sigma(x, y) = 0. \tag{17}$$

The equation for σ is identical to the equation for A_5 found in Ref. [6]. This is consistent since working in the gauge $\phi = 0$ corresponds to $\sigma = A_5$ (15). It is clear from (15)–(17) that the equations of motion for A_5, φ, and ϕ are indeed dependent on the choice of gauge. This underscores the advantage of working with gauge invariant fields.

Note that the bulk equations do permit two massless scalar modes, $\sigma^0(x)$ and $\psi^0(x)$. From a phenomenological standpoint, these modes are usually undesirable because they are ruled out experimentally. However, boundary conditions can be imposed so that the zero modes vanish, and we will therefore not consider the phenomenological implications further in this paper.

One additional point deserves to be mentioned. Our approach was to define gauge invariant combinations of the fields by separating the vector and scalar fields contained in A_M. In practice, this is equivalent to choosing a gauge $\partial_\mu A^\mu = 0$. Of course, the analysis can be done in another gauge. However, from a physical standpoint, choosing to work in a particular gauge obscures the dynamics. The 5D equation of motion is dependent on the gauge choice. We know physically that the wave function of the true dynamical fields should not depend on the gauge, and working with gauge invariant fields allows us to avoid this problem.

C. Localizing the photon

Let us now return to the transverse vector modes $\hat{A}_\mu$. Varying the action (13), we find the equations of motion for $\hat{A}_\mu$:

$$(e^{2ky}\Box + \partial_5^2 - 2k\partial_5 - ak^2)\hat{A}_\mu(x, y) = 0, \tag{18}$$

where we have defined the bulk mass $m^2 = ak^2$ with dimensionless parameter a. To perform the Kaluza-Klein decomposition, we expand $\hat{A}_\mu$ into eigenmodes

$$\hat{A}_\mu(x,y) = \sum_{n=0}^{\infty} \hat{A}_\mu^n(x) f^n(y), \tag{19}$$

where f^n satisfies

$$(\partial_5^2 - 2k\partial_5 - ak^2 + e^{2ky} m_n^2) f^n(y) = 0, \tag{20}$$

and obeys the orthonormal condition

$$\int_0^{\pi R} dy f^n f^m = \delta^{nm}. \tag{21}$$

The solution for the zero mode ($m_0 = 0$) is

$$f^0(y) = C_1 e^{(1+\sqrt{1+a})ky} + C_2 e^{(1-\sqrt{1+a})ky}. \tag{22}$$

For arbitrary boundary conditions, one finds that $C_1 = C_2 = 0$, and therefore there is no zero mode. However, as for a bulk scalar field [8], consider adding the following boundary mass term to the action [6,7]:

$$S_{bdy} = -\int d^5x \sqrt{-g} \alpha k g^{\mu\nu} A_\mu A_\nu (\delta(y) - \delta(y - \pi R)), \tag{23}$$

where α is a dimensionless parameter. Note that we have chosen equal and opposite boundary mass terms. This brane-localized mass term alters the boundary conditions for $\hat{A}_\mu$, which become

$$(\partial_5 \hat{A}_\mu - \alpha k \hat{A}_\mu)|_{0,\pi R} = 0. \tag{24}$$

For generic values of α, there is again no zero mode allowed. However, if the bulk and boundary mass parameters are tuned in the following way,

$$\alpha_\pm = 1 \pm \sqrt{1+a}, \tag{25}$$

then either C_1 or C_2 is nonvanishing. Under this condition (25), there is a normalizable massless mode solution. Note that the relation (25) may be enforced by bulk supersymmetry, as happens in the case of massive scalar field in a slice of AdS [8].

We can consider both the α_+ and the α_- branches simultaneously by defining $\alpha = \alpha_\pm$. We restrict our consideration to values of the bulk mass parameter $a > -1$ so that α is real. In this case, it is clear that $\alpha_+ > 1$ and $\alpha_- < 1$ so that the boundary mass parameter α can be any real value. The case of a massless gauge boson corresponds to $a = \alpha_- = 0$. There exists a flat zero mode, and this case has been studied extensively [3,4]. All other zero mode solutions on the α_- branch are a continuous deformation of the flat mode from $\alpha = 0$ to $\infty < \alpha < 1$. Notice that on this branch it is possible to localize the massless mode on either brane. The situation on the α_+ branch is slightly different. Boundary mass terms must be present for the zero mode to exist on this branch, and the mode is only localized on the IR brane. The normalized massless mode solution for arbitrary values of α is

$$f^0(y) = \sqrt{\frac{2\alpha k}{e^{2\alpha\pi kR} - 1}} e^{\alpha k y}. \tag{26}$$

The mode is localized on the UV(IR) brane for $\alpha < 0$ ($\alpha > 0$). Interestingly, the zero mode can also be localized in the flat space limit $k \to 0$. The wave function becomes $f_0 \propto e^{\pm my}$, where m is the bulk mass.

The massive modes are found by solving (20), and are given by

$$f^n(y) = e^{ky}\left[C_1 J_{\sqrt{1+a}}\left(\frac{m_n}{k} e^{ky}\right) + C_2 Y_{\sqrt{1+a}}\left(\frac{m_n}{k} e^{ky}\right)\right], \tag{27}$$

and obey the following condition:

$$\frac{J_{\sqrt{1+a}\pm 1}(\frac{m_n}{k})}{Y_{\sqrt{1+a}\pm 1}(\frac{m_n}{k})} = \frac{J_{\sqrt{1+a}\pm 1}(\frac{m_n}{k} e^{\pi kR})}{Y_{\sqrt{1+a}\pm 1}(\frac{m_n}{k} e^{\pi kR})}. \tag{28}$$

The masses of the Kaulza-Klein excitations are obtained from solving this equation. Taking the limit in the regime $ke^{-\pi kR} \ll m_n \ll k$, we determine the mass spectrum to be

$$m_n \simeq \left(n \pm \frac{1}{2}\alpha_\pm - \frac{1}{4}\right)\pi k e^{-\pi kR}, \qquad n = 1, 2, 3, \ldots, \tag{29}$$

which agrees with [17] for the α_- branch.

D. Modification of the propagator

An interesting consequence of localizing the photon is a modification of the propagator at high energies. It is clear from (29) that below the IR scale, only the massless photon exists and we have the usual massless propagator. At energies somewhat higher than this, the fermions will exchange massive modes and the propagator will be modified. The strength of the corrections depend on where the photon is localized.

To analyze these effects, we will compute the UV-UV brane Green's function. It is convenient to do the analysis using Poincaré coordinates, $z = e^{ky}/k$. The positions of the UV and IR branes in these coordinates are $1/k$ and $L = e^{\pi kR}/k$, respectively. Using the general expressions in Ref. [18], the Green's function in momentum space is

$$G(p) = -g_5^2\left(\frac{2\alpha k}{p^2} - \frac{1}{p} \times \frac{I_\alpha(pL)K_{\alpha+1}(p/k) + K_\alpha(pL)I_{\alpha+1}(p/k)}{I_\alpha(pL)K_\alpha(p/k) - K_\alpha(pL)I_\alpha(p/k)}\right), \tag{30}$$

where we have absorbed the 5D coupling g_5 in the propagator. We can expand the propagator in powers of p to analyze the effects of the massive modes. We will always assume that $p \ll k$. As we expect, when we expand (30) in the regime $p \ll 1/L$ we find that the propagator is proportional to $1/p^2$ for all values of α. The dominant exchange

process comes from the massless mode and charged particles experience the ordinary $1/r$ Coulomb potential.

Now let us see what happens at high energies. The results depend on where the zero mode is localized in the bulk. First, consider $\alpha < 0$. Taking the limit $p \gg 1/L$, we find that the propagator is given by

$$G(p) \simeq -2\alpha g_5^2 k\left(\frac{1}{p^2} + \cdots - \frac{2^{2\alpha}\Gamma(\alpha)}{\Gamma(-\alpha)} k^{2\alpha} p^{-2(\alpha+1)}\right). \tag{31}$$

At large distance scales (compared to $1/k$) we see that the dominant contribution to the propagator comes from the zero mode. The corrections only become important at distance scales $\sim 1/k$. This is because the zero mode is localized on the UV brane and appears effectively pointlike below momentum scales of order the curvature scale k. However, for $\alpha > 0$ and in the regime $p \gg 1/L$, we find

$$G(p) \simeq -\frac{g_5^2 k}{2k^2(1-\alpha)}\left[1 - \frac{p^2}{4k^2(\alpha-1)(\alpha-2)} - \left(\frac{p}{2k}\right)^{2(\alpha-1)} \frac{\Gamma(2-\alpha)}{\Gamma(\alpha)} + \cdots\right]. \tag{32}$$

The dominant contribution now comes from the massive states. When the zero mode is localized on the IR brane, it appears to be a composite particle. Hence, at energies above the IR scale, the zero mode effectively disappears and there are only contributions from the Kaluza-Klein tower.

This correspondence will be made more explicit later where the different behavior exhibited by the propagator can be given a holographic interpretation. As we will see, the behavior of the propagator at high energies again depends on whether the photon is a composite CFT state or an elementary source field. Although the physics in the dual theory differs from the bulk theory, we will see that the result for the propagator is replicated exactly.

III. PHENOMENOLOGICAL IMPLICATIONS

We would like to examine the phenomenological implications of localizing Abelian gauge fields in the bulk. For the moment, we will consider a simple model with fermions localized on each brane and the ''photon'' residing in the bulk. We will describe a realistic setup within the context of the standard model at the end of this section in which the photon is indeed localized. Many of the results derived for this simple model are robust and will also apply in a realistic model. Moreover, as we will show later, this simple model has a very interesting dual interpretation.

A. ''Millicharged'' particles

The ability to localize the photon allows for an interesting phenomenological scenario in which fermions physically separated from the photon in the fifth dimension appear to four dimensional observers as millicharged particles. The existence of particles with fractional electric charge is not forbidden in the standard model because of the trivial commutation relations of the Abelian group. Several theoretical models have therefore been proposed over the years which predict millicharged particles [10,19,20], and numerous constraints from collider experiments as well as astrophysics and cosmology exist for such particles [21–23]. We will see that we can skirt any such constraint in this model.[1]

To begin, let us consider the 5D interaction of the U(1) field with the electron on the IR brane:

$$S = -\int d^5x\sqrt{-g}[g_5\overline{\psi}(x)e_a^\mu \gamma^a A_\mu(x,y)\psi(x)]\delta(y-\pi R). \tag{33}$$

Here, g_5 is the 5D coupling constant, $e_a^\mu = e^{ky}\delta_a^\mu$ is the vierbein, and γ^a are the ordinary flat space Dirac matrices. To examine 4D physics, we insert the Kaluza-Klein expansion (19) into (33) and integrate over the extra dimension. The resulting interactions between the fermion and the Kaluza-Klein tower are

$$S = -\sum_{n=0}^{\infty} g_5 f^n(\pi R)\int d^4x\overline{\psi}(x)\gamma^\mu A_\mu^n(x)\psi(x), \tag{34}$$

where we have redefined the field $\psi(x) \to e^{-(3/2)\pi kR}\psi(x)$ to canonically normalize the fermion kinetic term. The effective 4D coupling constants can be directly read from (34):

$$g_n = g_5 f^n(\pi R), \tag{35}$$

and, in particular, the electric charge is given by

$$e = g_5 f^0(\pi R) = g_5\sqrt{\frac{2\alpha k}{1-e^{-2\alpha\pi kR}}}. \tag{36}$$

Taking the $\alpha = 0$ limit correctly reproduces the result for a 5D photon with no bulk or boundary mass terms [3,4],

$$e = \frac{g_5}{\sqrt{\pi R}}. \tag{37}$$

Next, consider a fermion ψ living on the UV brane. Following the analysis of the electron above, we find that the coupling of ψ to the photon is now given by

$$g = g_5 f^0(0) = g_5\sqrt{\frac{2\alpha k}{e^{2\alpha\pi kR}-1}}. \tag{38}$$

The only difference from the case of the electron is that the

[1]To conform with standard terminology, we will use the term ''millicharge'' to refer to fractionally charged particles ϵe with e the electron charge and $\epsilon < 1$. Furthermore it is important to realize that the effect considered here does not result from kinetic mixing and can produce charges with any value, not just $\epsilon \sim 10^{-3}$.

wave function is evaluated on the UV brane at $y = 0$. Equivalently, the electric charge of ψ can be written as $g = \epsilon e$, where ϵ is just a number. Then ϵ can be computed using (36) and (38):

$$\epsilon = \frac{f^0(0)}{f^0(\pi R)} = e^{-\alpha \pi k R}. \tag{39}$$

The most stringent limits on ϵ are derived from considering the influence of a new energy loss mechanism on stellar evolution, and apply to particles with mass $m_\epsilon < 10^4$ eV [21]. For such particles, $\epsilon > 10^{-14}$ has been ruled out. From (39), we see that this corresponds to

$$\alpha > \frac{14 \ln(10)}{\pi k R} \simeq 0.9, \tag{40}$$

and since α can be any real value [see Eq. (25)] we can clearly produce charges with $\epsilon < 10^{-14}$.

It is easy to see why the fermion on the UV brane can have a much lower charge than the electron. The photon wave function is peaked on the IR brane and is exponentially suppressed on the UV brane for $\alpha > 0$. The photon overlaps very weakly with the fermion on the UV brane resulting in the tiny coupling (39). This phenomenon is similar to what happens with gravity. The massless graviton also has an exponential profile in the bulk which explains the weakness of gravity on the IR brane. Of course, we could have also considered the reversed situation in which the electron lives on the UV brane while the fermion ψ lives on the IR brane. In this case, ''millicharges'' could be produced for $\alpha < 0$.

1. Kinetic mixing

Usually, the fact that electric charge is quantized is thought to arise from grand unification. If the standard model is embedded into a larger gauge group, electric charge quantization is a result of the nontrival commutation relations of the group. However, Holdom [10] pointed out that the existence of millicharged particles is not forbidden by grand unification if the model contains two U(1) fields. If matter couples to both fields at high energies, then kinetic mixing with strength $\chi \propto e^2/(16\pi^2)$ will be induced by quantum corrections. Fields coupling to the second ''shadow'' U(1) at high energies will appear as millicharges in the effective theory.

It is simplest to embed a massless U(1) boson into a grand unified theory and therefore we will consider a 5D version of Holdom's model. As we will show in Sec. IV C, the theory will also have a 4D dual interpretation. Consider two U(1) fields, A_1^M and A_2^M in the bulk of AdS_5 with $\alpha = 0$. A gauge invariant operator that mixes their kinetic terms can be added to the action. However, to estimate the strength of this operator, we will assume it was generated by integrating out massive fermions as in [10]. The Lagrangian is then

$$\mathcal{L} = -\frac{1}{4g_1^2}(F_1^{MN})^2 - \frac{1}{4g_2^2}(F_2^{MN})^2 + \frac{\chi}{2} k F_1^{MN} F_{2MN}, \tag{41}$$

where the mixing in units of k is given by the dimensionless parameter χ. Since we are assuming the mixing is generated perturbatively, we expect some suppression due to loop effects ($\chi \sim 10^{-3}$). Even in the presence of the mixing term, the equations of motion for A_1^μ and A_2^μ are separable. The solution for the zero mode is simply given by a constant. After compactification, the mixing of the zero modes becomes

$$\begin{aligned} S_{mix} &= \chi k \int_0^{\pi R} dy \int d^4x \frac{1}{2} F_1^{0\mu\nu} F^0_{2\mu\nu} + \cdots \\ &= \chi \pi k R \int d^4x \frac{1}{2} F_1^{0\mu\nu} F^0_{2\mu\nu} + \cdots. \end{aligned} \tag{42}$$

The strength of the zero mode kinetic mixing is then

$$\epsilon = \chi \pi k R \sim 10^{-2}. \tag{43}$$

Following the analysis in Ref. [10], the fields coupling to the shadow photon $A_2^{0\mu}$ will receive an order ϵ electric charge after diagonalizing the kinetic terms (for other possibilities on the couplings, see [24]). A charge of 10^{-2} is actually quite constrained [21]. The dedicated millicharged particle search at SLAC [25] rules out such a large electric charge for particles of mass $m_\epsilon < 100$ GeV, while demanding that the millicharge density parameter $\Omega_\epsilon = \rho_\epsilon/\rho_c < 1$ constrains the mass to be $m_\epsilon < 10$ TeV. Notice that compared to the 4D version [10], there is an enhancement of $\pi k R \sim 36$ to the millicharge in this 5D model which results from integrating out the CFT. In fact, we will see that this simple 5D generalization of millicharges generated through kinetic mixing has a 4D dual interpretation and we will later compare the result obtained in the bulk to the 4D theory.

B. Electroweak model

Until now we have been considering a simple model with a U(1) gauge field in the bulk that produces a localized zero mode. We have identified this mode as the photon. But the photon in the standard model is a mixture involving non-Abelian gauge fields. These fields would need to be similarly localized to realize the simplest model. However, it is not clear that non-Abelian gauge fields can be localized in the same manner as Abelian gauge fields. Therefore to realize an effectively localized photon in the standard model we will suppose that the $U(1)_Y$ gauge boson is a bulk field, while the $SU(2)_L$ gauge bosons and the Higgs boson are confined on the IR brane. To check that the proper mixing does indeed occur to produce a photon, $W^\pm$, and Z, consider the following 5D action:

$$\int d^5x\sqrt{-g}\Big[-\frac{1}{4}F_{MN}F^{MN}+\Big(-\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} + (D_\mu\phi)^\dagger(D^\mu\phi)-V(\phi)\Big)\delta(y-\pi R)\Big], \quad (44)$$

where F_{MN} and $G^a_{\mu\nu}$ are the field strength tensors for the $U(1)_Y$ and the $SU(2)_L$ gauge bosons, respectively. It is important to keep in mind that we are supplementing this action with the bulk and boundary mass terms in order to localize the $U(1)_Y$ field. The gauge covariant derivative is given by

$$D_\mu = \partial_\mu - igV^a_\mu(x)\frac{\sigma^a}{2} - ig_5 Y B_\mu(x,y) = \partial_\mu - igV^a_\mu(x)\frac{\sigma^a}{2} - ig_5 Y\sum_{n=0}^{\infty}B^n_\mu(x)f^n(y), \quad (45)$$

where σ^a are the Pauli matrices and V^a_μ, B_μ are the $SU(2)_L$ and $U(1)_Y$ gauge bosons with g and g_5 being their respective coupling to the Higgs. After decomposing the $U(1)_Y$ gauge boson and allowing the Higgs to acquire a vacuum expectation value:

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \quad (46)$$

the mass Lagrangian is given by

$$\mathcal{L}_m = \sum_{n=1}^{\infty}\frac{1}{2}m_n^2(B^n_\mu(x))^2 + \frac{1}{2}\Big(\frac{gv}{2}\Big)^2[(V^1_\mu(x))^2+(V^2_\mu(x))^2] + \frac{1}{2}\Big(\frac{v}{2}\Big)^2\Big(-gV^3_\mu(x)+g'B^0_\mu(x) + g'\sum_{n=1}^{\infty}\frac{f^n(\pi R)}{f^0(\pi R)}B^n_\mu(x)\Big)^2, \quad (47)$$

where we have used $Y = 1/2$ for the hypercharge of the Higgs and defined the coupling of B^0_μ to be $g' = g_5 f^0(\pi R)$. Clearly, the $W^\pm$ bosons are defined in the standard way and the nontrivial mixing occurs between the V^3 and B^n bosons. To make contact with the standard model, we change the basis from (V^3, B^0) to (Z, A) by introducing the weak mixing angle θ_w. The mass Lagrangian then reads

$$\mathcal{L}_m = \sum_{n=1}^{\infty}\frac{1}{2}m_n^2(B^n_\mu)^2 + m_W^2W^+_\mu W^{\mu-} + \frac{1}{2}\Big(m_Z Z_\mu - \frac{g'v}{2}\sum_{n=1}^{\infty}\frac{f^n(\pi R)}{f^0(\pi R)}B^n_\mu\Big)^2, \quad (48)$$

where m_W and m_Z are defined in the usual way. We see that the photon is massless, but there is still mixing between the Z and the Kaluza-Klein modes of the B. Since $m_n \gg m_Z$, we can diagonalize this Lagrangian order by order. We will not do this here, but see Ref. [26] for an example in flat space. The result is that the physical Z contains a small admixture of Kaluza-Klein modes and its mass is shifted. Thus, there is a Kaluza-Klein tower of Z bosons in this setup rather than photons. To leading order, the physical mass of the Z is

$$m_Z^{(ph)2} = m_Z^2\Big[1-\sum_{n=1}^{\infty}\Big(\frac{f^n(\pi R)}{f^0(\pi R)}\Big)^2\Big(\frac{m_Z^2-m_W^2}{m_n^2}\Big)\Big], \quad (49)$$

where $m_Z^2 - m_W^2 = (g'v/2)^2$. Only the $U(1)_Y$ gauge boson is in the bulk and therefore the mass corrections depend only on its coupling to the Higgs.

Although we can reproduce the standard model, we have not yet shown that any of the gauge fields, in particular, the photon, are localized in the bulk. What is actually happening is those components of the photon and the Z that come from the B boson exist in the bulk and have the exponential profile given by (26). The remaining components are confined to the IR brane. Therefore, mixing only occurs on the IR brane, and we cannot strictly define a wave function for the standard model gauge bosons. The most straightforward way to see that the photon and Z are effectively localized is to examine their interactions with fermions. For simplicity, consider a $SU(2)_L$ singlet fermion on the IR brane. Its interaction with the B boson is given by (33) with A_μ replaced with B_μ. Performing a Kaluza-Klein decomposition and changing basis to the standard model gauge bosons, we find the following effective interaction Lagrangian:

$$\mathcal{L}_{int} = -g'\cos\theta_w\overline{\psi}\gamma^\mu\psi A_\mu + g'\sin\theta_w\overline{\psi}\gamma^\mu\psi Z_\mu - g_5\sum_{n=1}^{\infty}f^n(\pi R)\overline{\psi}\gamma^\mu\psi B^n_\mu. \quad (50)$$

Note that the interaction has not yet been written in terms of the physical mass eigenstates of the Z boson Kaluza-Klein tower. Concentrating now on the electromagnetic force, we define the electric charge of the fermion to be

$$e = g'\cos\theta_w = g_5 f^0(\pi R)\cos\theta_w. \quad (51)$$

We can already guess that the electric charge of a fermion on the UV brane is given by $g = \epsilon e$, where ϵ is defined in (39), and our intuition is correct. Therefore, the photon and the Z boson are effectively localized in the bulk due to the fact that their couplings to fermions depend on the exponential profile of B^0_μ.

We have shown that it is indeed possible to localize a U(1) gauge boson in a realistic context. In fact, the model we have been considering is an extension of the original Randall-Sundrum model (RS1) [2], where we only delocalize the $U(1)_Y$ gauge boson. In the limit $\alpha \to \infty$, the photon and Z-boson are confined to the IR brane and we smoothly reproduce RS1. Note also that, since it is not clear that non-Abelian fields can be localized, we have chosen to confine $SU(2)_L$ gauge bosons to the IR brane. However, it seems likely that another realistic model could also be constructed in which massless (flat) $SU(2)_L$ gauge

bosons propagate in the bulk while the $U(1)_Y$ gauge boson is still localized.

1. Electroweak constraint

The fact that we can localize the $U(1)_Y$ gauge boson on the IR brane has some desirable phenomenological consequences. Placing massless gauge fields in the bulk within the context of the original RS1 model (i.e. fermions on the IR brane) was analyzed in Ref. [3]. One of the problems of this scenario was that it was necessary to push the IR scale above 100 TeV in order to preserve the necessary condition that the bulk curvature be less than the 5D Planck scale. Basically, this dilemma can be traced to the fact that the Kaluza-Klein modes couple to matter roughly 8 times stronger than does the zero mode. Stated another way, the problem is that the zero mode is flat in the bulk, while the Kaluza-Klein modes are localized near the IR brane. A similar problem will also occur if the $U(1)_Y$ gauge boson is in the bulk as considered in the previous section. One way to avoid this problem is to delocalize the fermions as well [8]. However, since we can control the degree of localization of the U(1) bulk field with the boundary mass parameter, we will also be able to avoid any undesirable constraint on the IR scale.

At low energies, four-fermion operators will be produced by integrating out the Kaluza-Klein tower. The strengths of these operators will be proportional to a parameter V defined to be

$$V = \sum_{n=1}^{\infty} \frac{g_n^2}{g_0^2} \frac{m_W^2}{m_n^2}, \tag{52}$$

where m_n are the Kaluza-Klein masses. The exchange of Kaluza-Klein modes will affect electroweak observables, and thus an upper limit can be placed on V. However, the size of V depends on the ratio of the couplings, g_n/g_0. If this ratio is large, we must push the Kaluza-Klein mass scale and hence the IR scale to high energies to comply with the upper limit on V. Therefore, let us compute this ratio for arbitrary values of α. The coupling of the zero mode is given by Eq. (36). The Kaluza-Klein tower couplings depend on $f^n(\pi R)$, which can be computed from (27) and are given by

$$f^n(\pi R) = \sqrt{2k \frac{Y_\alpha^2(\frac{m_n}{k})}{Y_\alpha^2(\frac{m_n}{k}) - Y_\alpha^2(\frac{m_n}{ke^{-\pi kR}})}} \simeq \sqrt{2k}. \tag{53}$$

The ratio of couplings is therefore

$$\frac{g^n}{g^0} = \frac{f^n(\pi R)}{f^0(\pi R)} \simeq \sqrt{\frac{1 - e^{-2\alpha\pi kR}}{\alpha}}. \tag{54}$$

Taking the limit as α goes to zero, we find that $g^n/g^0 = \sqrt{2\pi kR} \simeq 8.4$, which agrees with [3,4]. This large coupling forces us to push the IR scale to energies much greater than a TeV. Obviously if we localize the photon on the UV brane ($\alpha < 0$), the problem only becomes more severe. However, in the opposite limit of $\alpha > 0$ when the photon is localized on the IR brane the ratio becomes

$$\frac{g^n}{g^0} \simeq \sqrt{\frac{1}{\alpha}}. \tag{55}$$

We see that for $\alpha > 1$, this ratio is actually a small number, and therefore the upper limit on V becomes a weak constraint. This corresponds to the gauge boson being localized near the IR brane. Clearly in the limit that $\alpha \to \infty$ the constraint disappears and we recover the original RS1 model, with all gauge bosons confined on the IR brane.

Finally, we should point out that when computing the limit on the parameter V, Ref. [3] considered the effects of all standard model fields propagating in the extra dimension, whereas we have only considered a U(1) bulk field. Indeed, problems could arise in a model in which flat SU(2) gauge bosons are bulk fields even if the $U(1)_Y$ gauge boson is localized on the IR brane. Of course, in this case one would have to suppress dangerous four-fermion operators in other ways, such as localizing fermions in the bulk [8] and adding brane-localized kinetic terms [27].

IV. HOLOGRAPHIC INTERPRETATION

Remarkably bulk models in a slice of AdS_5 can be interpreted through a regularized AdS/CFT correspondence as being dual to a strongly coupled CFT [1,5,28–31]. In this modified conjecture, the fifth coordinate corresponds to a momentum scale in the dual 4D theory. Boundary values of bulk fields on the UV brane source corresponding CFT operators. The UV brane boundary condition implies nontrivial bulk dynamics which in the dual theory leads to a discrete CFT spectrum, induced dynamics for source fields, and mixing between the source and CFT sectors. Moreover, the presence of the IR brane in the bulk theory is holographically interpreted as a deformation of the CFT, with conformal invariance spontaneously broken in the IR.

We will therefore be interested in giving the holographic interpretation of the localized U(1) gauge field in a slice of AdS_5. First, however, it will be useful to review several aspects regarding the duality of massless gauge fields (flat-profile zero mode) that have been discussed in the literature [5,32]. The dual theory is a strongly coupled CFT gauged by an external source field. The theory contains a massless spin one field (photon) that is a mixture of the source and CFT fields. The situation is somewhat analogous to $\gamma - \rho$ mixing in QCD; however, in this case there is strong mixing between the source and CFT because the zero mode is flat in the bulk. The 4D coupling constant resulting from the overlap integral of the zero mode wave functions is interpreted as a logarithmically running coupling constant evaluated at the IR scale. Also, corrections to the source propagator induced by CFT loops are seen in the

bulk theory as contributions of the Kaluza-Klein tower to the UV-UV brane propagator.

The fact that the massless mode can be localized anywhere in the bulk has interesting consequences for the dual picture. As we will see, when the mode is localized on the UV brane, the photon eigenstate in the dual theory is primarily composed of the source field. To continue the analogy with QCD, in this case the photon eigenstate is mostly the elementary QED photon, with a tiny admixture of the QCD composite state (''ρ''). In fact this case mimics quite well the situation in QCD. However, if the massless mode is localized on the IR brane, the situation is reversed. The photon eigenstate is then mostly a massless composite state while the source field (''QED photon'') becomes massive and contributes very little to the mass eigenstate. The dual picture is in fact qualitatively similar to that derived for localized fermions [12] and localized gravitons [13].

In addition to seeing the different aspects of elementarity and compositeness, it will also be interesting to give the dual interpretation of the bulk couplings and interactions with fermions living on the UV brane. To this end, we will examine the two-point function of the dual CFT operator J and describe its behavior in different energy regimes. The analysis will be done using Poincaré coordinates, and the position of the UV and IR branes will now be at $z = L_0$ and $z = L_1$, respectively.

Our starting point is the 5D homogeneous equation of motion (written in momentum space):

$$\left[\partial_5^2 - \frac{1}{z}\partial_5 - \left(p^2 + \frac{a}{z^2}\right)\right]A_\mu(p, z) = 0. \tag{56}$$

The solution is given by

$$\begin{aligned} A_\mu(p, z) &= C_\mu(p) pz[K_{\alpha-1}(pz) + bI_{\alpha-1}(pz)] \\ &\equiv C_\mu(p) A(z), \end{aligned} \tag{57}$$

where $\alpha = 1 \pm \sqrt{1 + a}$ is the boundary mass parameter and the coefficient b is determined from the IR boundary condition,

$$b = \frac{K_\alpha(pL_1)}{I_\alpha(pL_1)}. \tag{58}$$

Next, we evaluate the bulk action for an arbitrary UV boundary condition for the gauge field, $A_\mu(x, L_0) = \hat{A}_\mu(x)$. The regularized correspondence states that in the dual theory, the boundary value acts as a source for a CFT current J:

$$\left\langle \exp\left(-\int d^4x \hat{A}_\mu J^\mu\right)\right\rangle_{\text{CFT}} = \exp[-\Gamma(\hat{A})]. \tag{59}$$

Here the left-hand side is the generating functional of CFT correlation functions. The effective action is

$$\begin{aligned} \Gamma(\hat{A}) &= -\frac{1}{2g_5^2}\int d^4x \eta^{\mu\nu}\left(\frac{1}{kz}A_\mu(x, z)\partial_5 A_\nu(x, z)\right. \\ &\quad \left. - \frac{1}{(kz)^3}\alpha k(kL_0)A_\mu(x, z)A_\nu(x, z)\right)\Big|_{z=L_0} \\ &= -\frac{1}{2g_5^2 k}\frac{1}{L_0}\int \frac{d^4p}{(2\pi)^4}\eta^{\mu\nu}\hat{A}_\mu(p)\hat{A}_\nu(-p) \\ &\quad \times \left(\frac{\partial_5 A}{A} - \frac{\alpha}{L_0}\right)\Big|_{z=L_0} \\ &= -\frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\eta^{\mu\nu}\hat{A}_\mu(p)\Sigma(p)\hat{A}_\nu(-p), \end{aligned} \tag{60}$$

where

$$\begin{aligned} \Sigma(p) &= -\frac{1}{g_5^2 k}\frac{p}{L_0} \\ &\quad \times \frac{K_\alpha(pL_0)I_\alpha(pL_1) - I_\alpha(pL_0)K_\alpha(pL_1)}{K_{\alpha-1}(pL_0)I_\alpha(pL_1) + I_{\alpha-1}(pL_0)K_\alpha(pL_1)}. \end{aligned} \tag{61}$$

Differentiating twice with respect to the source $\hat{A}_\mu$ yields the $\langle JJ\rangle$ correlator which is contained (up to the overall tensor structure) in $\Sigma(p)$.

First note that the expression for $\Sigma(p)$ is only valid for momentum scales below the UV cutoff, $p < 1/L_0$. However, we can examine the behavior of the correlator at energies above and below the IR scale by expanding in the momentum p. At high energy $pL_1 \gg 1$ we can essentially neglect the presence of the IR brane; the dual theory is conformal in this regime and the leading nonanalytic piece results from CFT dynamics. In the 5D picture, this behavior results from the exchange of Kaluza-Klein modes at high energies. Also present are terms analytic in p, which we interpret as source dynamics. At low energies, $pL_1 \ll 1$, we no longer have the Kaluza-Klein tower in the bulk and the conformal piece vanishes. As we will see, the interesting features of this behavior in these two distinct regimes depends heavily on where the gauge field is localized in the extra dimension.

A. α_- branch holography

Consider first the correlator $\Sigma(p)$ on the α_- branch, where $-\infty < \alpha < 1$. At momentum scales far above the IR, $pL_1 \gg 1$, but below the UV, $pL_0 \ll 1$, we find for noninteger α

$$\Sigma(p) \simeq \frac{1}{g_5^2 k}\frac{1}{2\alpha}\left[p^2 + \cdots + \frac{2^{2\alpha}\Gamma(\alpha)}{\Gamma(-\alpha)}L_0^{-2\alpha}p^{2-2\alpha} + \cdots\right], \tag{62}$$

where we have included only the leading terms in the expansion. Note that for integer α the nonanalytic terms will instead be logarithmic. We will not treat integer α here, since the analysis is qualitatively similar to the noninteger case. As we expect, the conformal (nonanalytic)

term is present and the effects of the IR brane are irrelevant. We can isolate the two-point function $\langle JJ\rangle$ by defining a rescaled source field $\hat{A}_\mu \to L_0^\alpha \hat{A}_\mu$ in the effective action (60) and taking the limit $L_0 \to 0$. In this limit, terms analytic in p that are divergent can be canceled by adding appropriate counterterms in the boundary action. This is the customary prescription used in the stringy correspondence to make contact with the CFT on the AdS boundary. The correlator is then given by

$$\langle JJ\rangle(p) = \frac{1}{g_5^2 k}\frac{2^{2\alpha}\Gamma(\alpha)}{2\alpha\Gamma(-\alpha)}p^{2(1-\alpha)}. \tag{63}$$

The scaling dimension of J can be found by Fourier transforming this term, and is given by

$$\Delta_J = 3 - \alpha, \tag{64}$$

which can be as low as 2 when $\alpha = 1$. This deviates from the canonical dimension of J, namely $[J] = 3$, as occurs for the case of a flat zero mode ($\alpha = 0$) and leads to an anomalous dimension $-\alpha$. The leading analytic piece is interpreted as a kinetic term for the source field that is induced via interactions with the CFT. The absence of a constant term in $\Sigma(p)$ tells us that the source field is massless in the 4D theory.

Previously we considered the interaction of the bulk gauge field with fermions on the UV brane, and we can directly include this interaction in the dual theory. Hence, the Lagrangian of our dual theory below the cutoff scale $\Lambda = 1/L_0 \sim k$ is given by

$$\mathcal{L}_{4D} = -\tfrac{1}{4}Z_0 F_{\mu\nu}^2 + \Lambda^\alpha A_\mu J^\mu + \overline{\psi}\gamma^\mu A_\mu \psi + \mathcal{L}_{\rm CFT}, \tag{65}$$

where Z_0 is a dimensionless coupling. In fact from (62) one can read off $Z_0 = -1/(2g_5^2 k\alpha)$. Because of the anomalous dimension of J (64), it is clear that the coupling of the source to the CFT current is relevant for positive α, marginal for $\alpha = 0$, and irrelevant for negative α. Thus for negative α we can neglect the source coupling to the CFT and the mass eigenstate of the photon is primarily composed of the source field. Instead for the marginal or relevant couplings the mixing between the source and CFT sector will result in a part-elementary and part-composite photon eigenstate.

Below the IR scale, we expect that conformal invariance will be broken. Physically, we have integrated out the massive CFT degrees of freedom at the IR scale. This will induce an extra contribution to the kinetic term of the photon. We can see this effect exactly by calculating $\Sigma(p)$ for energies $p \ll 1/L_1$:

$$\Sigma(p)_{\rm IR} \simeq \frac{1}{g_5^2 k}\frac{1}{2\alpha}[1 - (L_1/L_0)^{2\alpha}]p^2 + \cdots. \tag{66}$$

The disappearance of the nonanalytic piece signals the breaking of conformal invariance in the IR. Moreover, we now see a contribution to the kinetic term arising from integrating out CFT dynamics. This suggests that one can define a running wave function $Z(\mu)$ where

$$Z(1/L_1) = \frac{1}{g_5^2 k}\frac{1}{2\alpha}((L_1\Lambda)^{2\alpha} - 1). \tag{67}$$

Canonically normalizing the Lagrangian (65), we find the low energy effective coupling of the source field to matter is given by

$$g = \frac{1}{\sqrt{Z(1/L_1)}} = g_5\sqrt{\frac{2\alpha k}{e^{2\alpha\pi kR} - 1}}. \tag{68}$$

This precisely matches the bulk calculation for the effective 4D charge (38) for fermions on the UV brane when $L_0 = 1/k$. The strength of the coupling depends on whether or not the photon is mostly elementary ($\alpha < 0$) or composite ($0 < \alpha < 1$).

Using the running wave function $Z(\mu)$ [obtained from (62)] we can write down a renormalization group equation which encodes the mixing behavior as was done for fermions [12].[2] Define the dimensionless coupling $\omega(\mu) = 1/\sqrt{Z(\mu)}(\mu/\Lambda)^{-\alpha}$, then we obtain

$$\mu\frac{d\omega}{d\mu} = -\alpha\omega + c\frac{N}{16\pi^2}\omega^3, \tag{69}$$

where $1/(g_5^2 k) = N/(16\pi^2)$ and c is a constant. The second term in (69) arises from the CFT contribution to Z_0. When $0 < \alpha < 1$, the constant $c > 0$ and there is a fixed point $\omega_* \sim 4\pi\sqrt{\alpha/(cN)}$, corresponding to the fact that the L_1/L_0 term in (66) dominates the kinetic term at low energies. This corresponds to non-negligible mixing between the source and CFT sector. On the other hand, when $\alpha < 0$ we can neglect the second term in (69) and the solution corresponds to the simple scaling behavior $\omega \sim 4\pi\sqrt{-\alpha/N}(\mu/\Lambda)^{-\alpha}$, where we have matched to the low energy value (67). Clearly at low energies ($\mu \ll \Lambda \sim k$) the mixing will diminish and the contribution from the CFT sector is not important.

1. Interactions with external fermions

The mixing between the source and CFT sector has important effects on fermionic interactions. Because the source couples directly to external fermions, interactions are mediated through the source propagator. It is important to realize that in the dual 4D theory the physical photon is a combination of source and CFT states. We will therefore examine corrections to the propagator which arise from insertions of CFT correlators as shown in Fig. 1. The infinite series of Feynman diagrams can easily be summed in the following way:

[2]We thank Roberto Contino for helpful discussions on the fermion case.

FIG. 1. The Feynman diagrams in the 4D dual theory responsible for the corrections to the propagator. The source field A_μ interacts with the CFT contribution, indicated by the blob.

$$G(p) = \frac{1}{Z_0 p^2}\left[1 - \Lambda^{2\alpha}\frac{\langle JJ\rangle(p)}{Z_0 p^2} + \left(\Lambda^{2\alpha}\frac{\langle JJ\rangle(p)}{Z_0 p^2}\right)^2 - \cdots\right] = \frac{1}{Z_0 p^2 + \Lambda^{2\alpha}\langle JJ\rangle(p)} = \frac{1}{\Sigma(p)}. \tag{70}$$

This is what we would expect from examining the bulk effective action (60), and provides a nontrival check of the holographic correspondence.

How can we physically see what is happening in the 4D theory? Let us first consider $\alpha < 0$, in which case the coupling of the source field to the CFT is irrelevant. In this case, the photon is mostly comprised of the source field. Thus, the correction arising from the CFT is small, and we can expand the denominator in (70) to find

$$G(p) \simeq \frac{1}{Z_0 p^2} - \Lambda^{2\alpha}\frac{\langle JJ\rangle(p)}{Z_0^2 p^4}. \tag{71}$$

By inserting $\langle JJ\rangle$ (63) into (71) and using (67), we recover the result previously obtained in the bulk gravity calculation (31). Contributions from the CFT are only important in the UV. As we flow to the IR, the coupling between the source and the CFT becomes negligible and the interaction is mediated solely by the source field. The marginal case ($\alpha = 0$) is special and can be treated in a similar fashion as was done for gravity [13].

For relevant couplings ($\alpha > 0$), the CFT contribution dominates, and we can neglect the source contribution:

$$G(p) \simeq \frac{1}{\Lambda^{2\alpha}\langle JJ\rangle(p)}. \tag{72}$$

Strong mixing between the source and CFT fields produces the massless photon for $0 < \alpha < 1$. It therefore makes sense that the CFT contribution is indeed important to the interaction between external fermions. Again rewriting the couplings in terms of the 4D charge, it is easy to verify that (72) matches the bulk gravity calculation (32) exactly.

It is remarkable that both the couplings and the propagators of the 4D and 5D theories match precisely, and complement those found for the Newtonian potential [13]. The tree level effects in the classical gravity theory are realized as first order corrections in the dual CFT. Although the dynamics of the dual theory are a mystery, we can compute quantum effects directly using holography.

B. α_+ branch holography

We now consider the α_+ branch ($\alpha > 1$), in which the photon is always localized on the IR brane. Expanding the two-point function in the regime $1/L_1 \to 0$ and $1/L_0 \to \infty$, we find

$$\Sigma(p) \simeq -\frac{1}{g_5^2 k}\left[2(\alpha-1)L_0^{-2} + \frac{1}{2(\alpha-2)}p^2 + \cdots + \frac{\Gamma(2-\alpha)}{2^{2\alpha-3}\Gamma(\alpha-1)}L_0^{2\alpha-4}p^{2(\alpha-1)} + \cdots\right]. \tag{73}$$

Following the same renormalization procedure as in the previous section, we can extract the $\langle JJ\rangle$ correlator from (73):

$$\langle JJ\rangle = -\frac{1}{g_5^2 k}\frac{\Gamma(2-\alpha)}{2^{2\alpha-3}\Gamma(\alpha-1)}p^{2(\alpha-1)}. \tag{74}$$

The scaling dimension of J is therefore

$$\Delta_J = \alpha + 1. \tag{75}$$

In this case the anomalous dimension is $\alpha - 2$. It also appears that the source field has become massive, as indicated by the leading constant analytic piece.

To ascertain what happened to the massless particle, let us expand $\Sigma(p)$ in the low energy regime, $pL_1 \ll 1$:

$$\Sigma(p)_{IR} = -\frac{1}{g_5^2 k}\left[2(\alpha-1)L_0^{-2} + \frac{1}{2(\alpha-2)}p^2 - 8\alpha(\alpha-1)^2\frac{L_0^{2\alpha-4}}{L_1^{2\alpha}}\frac{1}{p^2} + \cdots\right]. \tag{76}$$

We notice the appearance of a pole at $p^2 = 0$. This implies that the photon is primarily a CFT bound state. Similar massless bound states have also been found for bulk scalars [30,33], fermions [12], and gravitons [13]. Notice what happens as we transition from the α_- to the α_+ branch. The source, which was massless on the α_- branch, obtains a mass at the same point that the CFT produces a composite massless vector field.

We can now write the Lagrangian of the dual theory as

$$\mathcal{L}_{4D} = -\frac{1}{4}\tilde{Z}_0 F_{\mu\nu}^2 - \frac{1}{2}m_0^2 A_\mu A^\mu + \frac{1}{\Lambda^{\alpha-2}}A_\mu J^\mu + \overline{\psi}\gamma^\mu A_\mu \psi, \tag{77}$$

where $\tilde{Z}_0$ is a dimensionless parameter and m_0 is a mass parameter of order the curvature scale. In fact from (73) we can read off that $\tilde{Z}_0 = 1/(2g_5^2 k(\alpha-2))$ and $m_0^2 = 2\Lambda^2(\alpha-1)/g_5^2 k$. In contrast to the α_- branch, the source-CFT interaction remains in the IR due to the fact that CFT contains a massless composite particle. Because the source has become heavy, it effectively decouples at low energy from matter. The photon propagates through its interactions with the source field. Therefore, the dominant contribution to the propagator is given by a single insertion of the $\langle JJ\rangle$ correlator. Noting that, at large distances, the

correlator is given by

$$\langle JJ\rangle(p) \simeq \frac{1}{g_5^2 k} 8\alpha(\alpha-1)^2 L_1^{-2\alpha} \frac{1}{p^2}, \tag{78}$$

we can calculate the propagator in a straightforward manner:

$$G(p) \simeq \frac{1}{\tilde{Z}_0 p^2 + m_0^2}\left(\frac{\langle JJ\rangle(p)}{\Lambda^{2(\alpha-2)}}\right)\frac{1}{\tilde{Z}_0 p^2 + m_0^2} \simeq \frac{\langle JJ\rangle(p)}{m_0^4 \Lambda^{2(\alpha-2)}}$$
$$= 2\alpha g_5^2 k(\Lambda L_1)^{-2\alpha}\frac{1}{p^2}. \tag{79}$$

In the second line we have neglected the p^2 part in the source propagator which is valid for the momentum scales we are considering. Taking the nonrelativistic limit, we see that the Coulomb potential emerges at low energy:

$$V(r) = \int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{x}} G(\mathbf{p})$$
$$= 2\alpha g_5^2 k(\Lambda L_1)^{-2\alpha}\int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{x}}\frac{1}{\mathbf{p}^2} = \frac{g^2}{4\pi r}, \tag{80}$$

where we have written the electric charge as defined in (38) when the UV brane is at $L_0 = 1/k$. Thus, we see that in the dual 4D theory the millicharge arises because the UV fermion must now couple to a composite photon. This coupling to the CFT vector current (with large anomalous dimension) can only occur via the massive source field.

Above the IR scale, it is clear from (73) that although the source remains massive, the pole disappears, indicating the absence of a massless particle. This is what we would expect from the bulk calculation of the propagator, in which the $1/p^2$ term vanishes at high energies. Again, the $\langle JJ\rangle$ correlator yields the dominant contribution to the propagator:

$$G(p) \simeq \frac{\langle JJ\rangle(p)}{m_0^4\Lambda^{2(\alpha-2)}}$$
$$= -g_5^2 k\frac{\Gamma(2-\alpha)}{2^{2\alpha-1}(\alpha-1)\Gamma(\alpha)}\Lambda^{-2\alpha}p^{2(\alpha-1)}. \tag{81}$$

which agrees with the result obtained in the gravity dual (32).

Therefore, on the α_+ branch, CFT dynamics produces a nontrivial effective interaction at high energies. However, large distance interactions are mediated by a massless vector particle which is primarily a CFT bound state. As we transition to the IR, the photon emerges from the CFT and the standard low energy theory is reproduced. This emergent photon behavior is similar to the emergent gravity behavior obtained in Ref. [13].

C. Kinetic mixing dual interpretation

Let us consider the dual interpretation of the bulk kinetic mixing discussed in Sec. III A 1. If we consider the fields A_1^M and A_2^M in our bulk theory, there will exist corresponding operators J_1 and J_2 in the dual CFT. As we will show, if there is kinetic mixing in the bulk, a corresponding kinetic mixing will be induced in the dual theory. Hence, this theory can be considered analogous to the mechanism in Ref. [10], but coupled to a strongly interacting sector.

The bulk theory is governed by the 5D Lagrangian (41). Again for simplicity we assume the bulk fields are massless to ensure that both U(1)'s can be embedded in a grand unified theory. Using the bulk solution to the homogeneous equations of motion for A_1^μ and A_2^μ, which is given (up to an overall constant) in (57) with $\alpha = 0$, we can calculate the effective gravity action. Here we only consider the portion contributing to the $\langle J_1 J_2\rangle$ correlator, which is given by

$$\Gamma(\hat{A}_1,\hat{A}_2) = \frac{\chi k}{2}\int\frac{d^4p}{(2\pi)^4}\eta_{\mu\nu}\hat{A}_1^\mu\left(\frac{\partial_5 A_1}{A_1}+\frac{\partial_5 A_2}{A_2}\right)\hat{A}_2^\nu$$
$$= \frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\eta_{\mu\nu}\hat{A}_1^\mu\Sigma_{12}(p)\hat{A}_2^\nu, \tag{82}$$

with $\Sigma_{12}(p)$ defined by

$$\Sigma_{12}(p) = -\chi\frac{p}{L_0}\frac{K_0(pL_0)I_0(pL_1) - I_0(pL_0)K_0(pL_1)}{K_1(pL_0)I_0(pL_1) + I_1(pL_0)K_0(pL_1)}. \tag{83}$$

Note that the calculation is similar to that performed in Ref. [5] for a single massless gauge field. The correlator is found by differentiating with respect to $\hat{A}_1^\mu$ and $\hat{A}_2^\mu$

The existence of a nonvanishing $\langle J_1 J_2\rangle$ correlator implies that a kinetic mixing for the source fields will receive corrections from CFT loops. Expanding $\Sigma_{12}(p)$ for low momentum scales, $pL_1 \ll 1$:

$$\Sigma_{12}(p) \simeq \chi\log(L_1/L_0)p^2 + \cdots, \tag{84}$$

we can read off the strength of the mixing as

$$\epsilon = \chi\log(L_1/L_0) = \chi\pi kR \sim 10^{-2}, \tag{85}$$

which agrees identically with the bulk calculation. Thus, fermions coupling to the shadow U(1) will acquire an electric charge of order $10^{-2}e$ from the CFT sector.

V. CONCLUSION

The zero modes of 5D U(1) gauge fields in a slice of AdS can be localized anywhere in the bulk. We employed a 5D Stückelberg mechanism in order to maintain gauge invariance even though bulk and boundary masses are added to the 5D action. A simple model of millicharged particles can then be constructed, which allows fermions to have arbitrarily small electric charge. In the electroweak sector only the $U(1)_Y$ gauge boson can be localized, but leads to the effective localization of electric charge. We have also shown that stringent electroweak constraints on the IR scale from bulk Abelian gauge fields can be avoided by localizing the $U(1)_Y$ gauge boson close to the IR brane.

We have also presented the detailed holographic interpretation of the localized U(1) gauge field in the warped 5D bulk. When the zero mode is localized near the UV (IR) brane the photon eigenstate in the 4D dual theory is predominantly an elementary (composite) state. The composite photon is an example of emergent behavior because above the compositeness scale (at short distances) the photon disappears. We also verified that when the CFT has a massless pole (corresponding to the composite photon) the source field receives a mass of order the curvature scale. In this way the dual theory is consistent and there is always only one massless state. Furthermore in the dual theory, millicharged particles are understood as arising from fermions which couple to vector currents with large anomalous dimensions. The electric charge is then proportional to this coupling and can be arbitrarily small. Thus, all the physics of localized Abelian gauge fields in the warped bulk can be given a purely 4D holographic description.

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