Asymptotic perfect fluid dynamics as a consequence of AdS/CFT correspondence

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We study the dynamics of strongly interacting gauge-theory matter (modeling quark-gluon plasma) in a boost-invariant setting using the AdS/CFT correspondence. Using Fefferman-Graham coordinates and with the help of holographic renormalization, we show that perfect fluid hydrodynamics emerges at large times as the unique nonsingular asymptotic solution of the nonlinear Einstein equations in the bulk. The gravity dual can be interpreted as a black hole moving off in the fifth dimension. Asymptotic solutions different from perfect fluid behavior can be ruled out by the appearance of curvature singularities in the dual bulk geometry. Subasymptotic deviations from perfect fluid behavior remain possible within the same framework.

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I. INTRODUCTION

From the first years of the running of heavy-ion collisions at RHIC, evidence has been found that various observables are in good agreement with models based on hydrodynamics [1] and with quark-gluon plasma (QGP) in a strongly coupled regime [2]. To a large extent it seems that the QGP behaves approximately as a perfect fluid as was first considered in [3].

It is a challenge in QCD to derive from first principles the properties of the dynamics of a strongly interacting plasma formed in heavy-ion collisions and, in particular, to understand why the perfect fluid hydrodynamic equations appear to be relevant.

Even if the experimental situation is still developing and rather complex, it is worth simplifying the problem in order to be able to attack it with appropriate theoretical tools. Recently the AdS/CFT correspondence [4,5] emerged as a new approach to study strongly coupled gauge theories. This has been largely worked out in the supersymmetric case and, in particular, for the conformal case of $\mathcal{N} = 4$ super Yang-Mills theory (SYM). Interestingly enough, since the QGP is a deconfined and strongly interacting phase of QCD we could expect that results for the nonconfining $\mathcal{N} = 4$ theory may be relevant. We will make this assumption in our work.

The AdS/CFT correspondence has already been advocated in theoretical studies in the context of heavy-ion collisions [6–9]. Transport coefficients at finite temperature have been calculated using the static black hole dual geometry and some generalizations [6], thermalization has been suggested to be described by a black hole formation process [7], and proposals have been put forward for the gravity dual description of various processes during heavy-ion collisions [9] e.g. cooling as black hole motion in the 5th direction.

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In this paper we focus on the spacetime evolution of the gauge-theory (4D) energy-momentum tensor, and derive its asymptotic behavior from the solutions of the nonlinear Einstein equations of the gravity dual.

Imposing the absence of curvature singularities in the gravity dual, we will show that, in the boost-invariant setting (as in [3]), perfect fluid hydrodynamics emerges from the AdS/CFT solution at large times. The corresponding asymptotic solution of the Einstein equations is given by formula (39) of our paper.

The plan of our paper is as follows. In Sec. II we review the Bjorken hydrodynamics on the gauge theory side. Then, in Sec. III, we set up a general framework of deriving a gravity dual for a given energy-momentum tensor on the boundary, based on the holographic renormalization method. In Sec. IV we derive the large proper-time behavior of the boost-invariant gravity duals by solving analytically the corresponding nonlinear Einstein equations in the bulk. In Sec. V we arrive at the physical solution by requiring the absence of curvature singularities. This constraint selects perfect fluid hydrodynamics in the 4D gauge theory. We close the paper with conclusions and outlook.

II. BJORKEN HYDRODYNAMICS

As is well known, a model of the central rapidity region of heavy-ion reactions based on hydrodynamics was pioneered in [3] and involved the assumption of boost invariance. In this paper we study the dynamics of strongly interacting gauge-theory matter assuming boost invariance. Let us review now the picture which will serve as a basis of our theoretical investigation.

We will be interested in the spacetime evolution of the energy-momentum tensor $T_{\mu\nu}$ of the gauge-theory matter. It is convenient to introduce proper-time (τ) and rapidity (y) coordinates in the longitudinal position plane:

$$x^0 = \tau \cosh y \qquad x^1 = \tau \sinh y. \tag{1}$$

In these coordinates the Minkowski metric has the form

$$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2. \tag{2}$$

Assuming for simplicity, $y \to -y$ symmetry and translational and rotational symmetry in the transverse plane, the energy-momentum tensor has only three nonzero components $T_{\tau\tau}$, T_{yy} and $T_{x_2x_2} = T_{x_3x_3} \equiv T_{xx}$, which depend only on τ. Since we are dealing with a *conformal* gauge theory, $T_{\mu\nu}$ is necessarily traceless

$$-T_{\tau\tau} + \frac{1}{\tau^2}T_{yy} + 2T_{xx} = 0. \tag{3}$$

Energy-momentum conservation $D_\nu T^{\mu\nu} = 0$ gives a further relation between the components:

$$\tau\frac{d}{d\tau}T_{\tau\tau} + T_{\tau\tau} + \frac{1}{\tau^2}T_{yy} = 0. \tag{4}$$

So using relations (3) and (4), all components of the energy-momentum tensor can be expressed in terms of a *single* function $f(\tau)$:

$$T_{\mu\nu} = \begin{pmatrix} f(\tau) & 0 & 0 & 0 \\ 0 & -\tau^3 \frac{d}{d\tau}f(\tau) - \tau^2 f(\tau) & 0 & 0 \\ 0 & 0 & f(\tau) + \frac{1}{2}\tau\frac{d}{d\tau}f(\tau) & 0 \\ 0 & 0 & 0 & f(\tau) + \frac{1}{2}\tau\frac{d}{d\tau}f(\tau) \end{pmatrix}, \tag{5}$$

where the matrix $T_{\mu\nu}$ is expressed in (τ, y, x_1, x_2) coordinates.

Furthermore the function $f(\tau)$ is constrained by the positive energy condition which states that for any *timelike* vector t^μ, the energy in the frame whose timelike axis is t^μ should be positive, i.e.

$$T_{\mu\nu}t^\mu t^\nu \geq 0. \tag{6}$$

Using $t^\mu = (\sqrt{s^2 + \tau^2 v^2 + 2w^2}, v, w, w)$ we are led to the following restrictions

$$f(\tau) \geq 0 \qquad f'(\tau) \leq 0 \qquad \tau f'(\tau) \geq -4f(\tau). \tag{7}$$

Note that all the above structure (5) is purely based on kinematics. The dynamics of the gauge theory should pick a specific $f(\tau)$. A perfect fluid or a fluid with nonzero viscosity and/or other transport coefficients will lead to different choices of $f(\tau)$.

The main aim of this paper is to address the problem of determination of the function $f(\tau)$ from the AdS/CFT correspondence. Let us first describe two distinct cases of physical interest:

A. Perfect fluid: Bjorken hydrodynamics

Let us assume that the gauge-theory matter behaves like a perfect fluid. This means that the energy-momentum tensor has the form[1]

$$T_{\mu\nu} = (E + p)u_\mu u_\nu + p\eta_{\mu\nu} \tag{8}$$

where u^μ is the (local) 4-velocity of the fluid ($u^2 = -1$), E is the energy density and p is the pressure. Boost-invariant kinematics then forces[2] $u^\mu = (1, 0, 0, 0)$ and the comparison with (5) leads to

$$f(\tau) = \frac{e}{\tau^{4/3}} \tag{9}$$

which is the result for the ideal relativistic fluid [3] satisfying $E = 3p$. Moreover the entropy per unit rapidity remains constant, while the temperature cools down as

$$T \sim \tau^{-1/3}. \tag{10}$$

B. Free streaming case

Let us now consider the *free streaming case*, where the longitudinal pressure vanishes. This property is expected to be valid in the first stages of heavy-ion collisions when the QCD coupling is small (see [10] for further comments). Using (5) this leads to

$$f(\tau) = \frac{\tilde{e}}{\tau}, \tag{11}$$

where $\tilde{e}$ is a dimensionful constant.

In the following we will more generally introduce a family of $f(\tau)$ with the large τ behavior of the form

$$f(\tau) \sim \tau^{-s}. \tag{12}$$

Note that using the energy positivity constraint (7) we are led to consider[3] $0 < s < 4$.

III. HOLOGRAPHIC RENORMALIZATION

Let us now turn to the AdS/CFT correspondence and describe the dual bulk geometry corresponding to a given configuration of the gauge-theory energy-momentum tensor $T_{\mu\nu}$.

According to the AdS/CFT correspondence, vacuum expectation values of (a class of) local operators in the gauge theory can be reconstructed from the asymptotics of

[1]Note that in Ref. [3] the Minkowski metric has $(+, -, -, -)$ signature instead of $(-, +, +, +)$ that we use.

[2]In the (τ, y, x_1, x_2) coordinates.

[3]An interesting limiting case $s = 4$ satisfies also the constraint but requires a specific treatment which is beyond the scope of our paper.

the dual supergravity fields near the boundary [11]. In the case of the energy-momentum tensor, the dual field is just the metric. The reconstruction of the vacuum expectation value (VEV) $\langle T_{\mu\nu}\rangle$ from the near-boundary asymptotics of the gravity solution has been first studied in [12,13], and in a systematic way in Ref. [14].

Following [14] we consider general asymptotically AdS metrics in the so-called Fefferman-Graham coordinates [15]:

$$ds^2 = \frac{g_{\mu\nu}dx^\mu dx^\nu + dz^2}{z^2}. \tag{13}$$

Note that this choice leaves, in general, no remaining diffeomorphism (coordinate) freedom since five conditions on the metric have already been imposed.

One then considers solutions of *vacuum* Einstein equations[4] with negative cosmological constant $\Lambda = -6$ (which corresponds to standard AdS_5 [14]) and their expansion[5] near the boundary at $z = 0$:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + z^6 g^{(6)}_{\mu\nu} + \ldots. \tag{14}$$

Here $g^{(0)}_{\mu\nu}$ is the *physical* 4D metric for the gauge theory on the boundary. In the following we set it to the flat Minkowski metric $g^{(0)}_{\mu\nu} = \eta_{\mu\nu}$. Then $g^{(2)}_{\mu\nu}$ is found to be zero, while $g^{(4)}_{\mu\nu}$ is proportional to the VEV of the energy-momentum tensor[6]:

$$\langle T_{\mu\nu}\rangle = \text{const} \cdot g^{(4)}_{\mu\nu}. \tag{15}$$

Hence given some metric in the bulk, i.e. a solution of the supergravity equations, the VEV of the gauge-theory energy-momentum tensor can be directly read off.

In [14] one also considers the inverse problem, namely, how to construct general supergravity solutions of the form (13). It turns out that one has to give as inputs *both* $g^{(0)}_{\mu\nu}$ and $g^{(4)}_{\mu\nu}$ in order to generate a solution. Einstein equations impose on $g^{(4)}_{\mu\nu} \equiv \langle T_{\mu\nu}\rangle$ just two consistency constraints:

$$\langle T^\mu_\mu\rangle = 0, \qquad D_\nu\langle T^{\mu\nu}\rangle = 0, \tag{16}$$

namely tracelessness (since we are in a conformal theory) and energy-momentum conservation[7] in the gauge theory. Then from these data, the Einstein equations allow one to recursively reconstruct in principle all higher terms $g^{(n)}_{\mu\nu}$ in (14). This general procedure goes under the name of *holographic renormalization* [14,16].

[4]Note that there is no energy-momentum tensor in the dual gravity construction.

[5]In (14) additional logarithmic terms might in principle appear (see [14]), yet we find them absent for the cases (Minkowski metric on the boundary) considered in this paper.

[6]For a generic background metric on the boundary $g^{(0)}_{\mu\nu}$, the formula is more complicated [14].

[7]The covariant derivative here is the one for the gauge-theory metric $g^{(0)}_{\mu\nu}$.

It is crucial to note that Einstein equations by themselves do not impose *any* further local constraints on $g^{(4)}_{\mu\nu}$, or equivalently on $\langle T_{\mu\nu}\rangle$. This raises a question about the predictive power of the gauge/gravity correspondence in this context. It would seem that *a priori* any conserved energy-momentum tensor gives a viable gravity background. This would be unacceptable from the gauge-theory point of view, since the specific form of $\langle T_{\mu\nu}\rangle$ should be determined by the gauge-theory dynamics.

Therefore in order to proceed further we need to look for a global condition which would allow us to determine a physically acceptable solution and hence a physical energy-momentum tensor profile selected among all formal possibilities. One natural criterion is to require the absence of a "naked" singularity in the bulk.

In this respect we are inspired by the AdS/CFT correspondence for gauge theories with $N_f \neq 0$ flavors [17–19]. There, the embedding of a D7 brane is constructed with a coordinate y_6, behaving asymptotically[8] for $\rho \to \infty$ as

$$y_6 = m + \frac{c}{\rho^2} + \ldots. \tag{17}$$

In Eq. (17) the leading term is the current quark mass m which is fixed [alike to the gauge-theory metric $g^{(0)}_{\mu\nu}$ in our case], while the first subleading term c corresponds to the quark condensate $\langle\bar{\psi}\psi\rangle$ [alike to the $g^{(4)}_{\mu\nu}$ term in (14)]. *A priori* for fixed m one can construct locally an embedding for *any* condensate c. The requirement that the embedding is *nonsingular* picks [18,19] the unique physical value of the $\langle\bar{\psi}\psi\rangle$ condensate.

We will see that similar reasoning can be applied for the case of energy-momentum tensor. One can construct, order by order, a gravity solution for any $f(\tau)$. The requirement of nonsingularity for the dual geometry will allow to pick up the physical $f(\tau)$.

Before we proceed to describe boost-invariant geometries let us mention two examples where exact solutions of the Einstein equations exist for certain *non–boost-invariant* energy-momentum tensors.

Example I: The static black hole

Let us consider a static isotropic energy-momentum tensor with $E = 3p = \text{const}$. The corresponding geometry that we obtain from solving the Einstein equations with the boundary condition (15) and a metric of the Fefferman form

$$ds^2 = \frac{-A(z)dt^2 + B(z)dx^2}{z^2} + \frac{dz^2}{z^2} \tag{18}$$

is

[8]For precise definitions of the variables and the geometrical setting see e.g. [19].

$$ds^2 = -\frac{(1-z^4/z_0^4)^2}{(1+z^4/z_0^4)z^2}dt^2 + (1+z^4/z_0^4)\frac{dx^2}{z^2} + \frac{dz^2}{z^2} \quad (19)$$

and the VEV of the energy-momentum tensor can be read off from the expansion of the metric (14)

$$\langle T_{\mu\nu}\rangle \propto g^{(4)}_{\mu\nu} = \begin{pmatrix} 3/z_0^4 & 0 & 0 & 0 \\ 0 & 1/z_0^4 & 0 & 0 \\ 0 & 0 & 1/z_0^4 & 0 \\ 0 & 0 & 0 & 1/z_0^4 \end{pmatrix}. \quad (20)$$

The geometry (19) does not look very familiar at first sight. However by performing a change of coordinates

$$\tilde{z} = \frac{z}{\sqrt{1+\frac{z^4}{z_0^4}}} \quad (21)$$

we can see that it is exactly the standard AdS static black hole

$$ds^2 = -\frac{1-\tilde{z}^4/\tilde{z}_0^4}{\tilde{z}^2}dt^2 + \frac{dx^2}{\tilde{z}^2} + \frac{1}{1-\tilde{z}^4/\tilde{z}_0^4}\frac{d\tilde{z}^2}{\tilde{z}^2} \quad (22)$$

with $\tilde{z}_0 = z_0/\sqrt{2}$. In this way, via the Fefferman-Graham coordinates we recover the result of [12,13].

For later reference let us quote the Hawking temperature (equal to the gauge-theory temperature)

$$T = \frac{1}{\pi\tilde{z}_0} = \frac{\sqrt{2}}{\pi z_0} \quad (23)$$

and the entropy

$$S = \frac{\text{Area}}{4G_N^{(5)}} = \frac{\tilde{z}_0^{-3}V_3}{4\cdot\frac{\pi}{2}N^{-2}} = \frac{\pi^2}{2}N^2V_3T^3. \quad (24)$$

Example II : the planar shock wave

The second example of an exact solution using our method is the geometry dual to a gauge-theory shock wave (on the boundary). Shock-wave solutions have been constructed in AdS spaces [20,21] whose sources were in the bulk, or particles on the boundary [22].

If we introduce light-cone coordinates $x^- = t - y$ and $x^+ = t + y$ then the background dual to the VEV of the energy-momentum tensor

$$T_{--} = \mu\delta(x^-) \quad (25)$$

is (in the Fefferman-Graham coordinates)

$$ds^2 = \frac{-dx^-dx^+ + \mu z^4\delta(x^-)dx^{-2} + dx_\perp^2}{z^2} + \frac{dz^2}{z^2}. \quad (26)$$

One can check that this is an exact solution of the Einstein equations[9].

[9]The same holds for the more general case where $\delta(x^-)$ is replaced by any function of x^-.

The metric (26) represents the gravity background dual to a plane shell of matter moving at a speed of light which is an interesting model of e.g. an ultrarelativistically boosted large nuclei. This dual background may be a good starting point to extend the study of saturation effects to strong coupling (as it mimics closely the setup at the origin of the Colour Glass Condensate/JIMWLK picture [23]).

Ultimately one is interested in the collisions of two such shock waves approaching along two light-cone directions which is the setting corresponding to heavy-ion reactions. We leave this difficult but interesting problem for subsequent work. We will concentrate here on an idealized boost-invariant description which would correspond to the description of the central rapidity region [3].

IV. BOOST-INVARIANT GEOMETRIES

Let us now come to the main issue of this paper, namely, the study of dual geometries in the boost-invariant case. We impose boost invariance, together with $y \to -y$ symmetry plus translation and rotation invariance in the transverse plane. The most general form of the bulk metric respecting these symmetries in the Fefferman-Graham coordinates reads

$$ds^2 = \frac{-e^{a(\tau,z)}d\tau^2 + \tau^2e^{b(\tau,z)}dy^2 + e^{c(\tau,z)}dx_\perp^2}{z^2} + \frac{dz^2}{z^2}. \quad (27)$$

The three coefficient functions $a(\tau, z)$, $b(\tau, z)$ and $c(\tau, z)$ must start off at small z as z^4 according to (5), (14), and (15). In this paper we will restrict ourselves to the energy density behaving like

$$f(\tau) = \frac{1}{\tau^s} \quad (28)$$

for $0 < s < 4$ and we concentrate on the resulting *leading* behavior for $\tau \to \infty$. Let us emphasize that there is a lot of physical content also in the subleading behavior and this problem certainly deserves further study.

First we solve the Einstein equations

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R - 6g_{\mu\nu} = 0 \quad (29)$$

order by order in z as in (14), starting from (28) and following the holographic renormalization procedure. We have implemented the iterative procedure using Maple [24] to obtain exact coefficients of the power series expansions like

$$a(\tau, z) = \sum_{n=0}^{N} a_n(\tau)z^{4+2n} \quad (30)$$

to some order N. This method calls for comments.

On the one hand this form is difficult to use in order to analyze possible singularities in the bulk since these occur at the edge of the radius of convergence and it is difficult to disentangle unambiguously whether the effect comes from a finite radius of convergence or is a mark of a genuine curvature singularity.

On the other hand, the knowledge of the power series solution helps us to find the large τ asymptotics of the exact solutions in an analytic form. Namely, by analyzing the structure of the power series solutions (30), we find that after introducing the scaling variable

$$v = \frac{z}{\tau^{s/4}} \tag{31}$$

the exact solutions behave like

$$a(\tau, z) = a(v) + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right), \qquad b(\tau, z) = b(v) + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right),$$
$$c(\tau, z) = c(v) + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right), \tag{32}$$

where we denoted by "#" a *positive* (here unspecified) power.

In order to find $a(v)$, $b(v)$ and $c(v)$ in an analytical form we insert the metric (27) into the Einstein equations. (29) and take the limit $\tau \to \infty$ keeping v fixed. We obtain the following set of coupled nonlinear equations:

$$v(2a'(v)c'(v) + a'(v)b'(v) + 2b'(v)c'(v)) - 6a'(v) - 6b'(v) - 12c'(v) + vc'(v)^2 = 0,$$
$$3vc'(v)^2 + vb'(v)^2 + 2vb''(v) + 4vc''(v) - 6b'(v) - 12c'(v) + 2vb'(v)c'(v) = 0,$$
$$2vsb''(v) + 2sb'(v) + 8a'(v) - vsa'(v)b'(v) - 8b'(v) + vsb'(v)^2 + 4vsc''(v) + 4sc'(v) - 2vsa'(v)c'(v) + 2vsc'(v)^2 = 0. \tag{33}$$

Taking a suitable linear combination of these equations and integrating, we find that the functions $a(v)$, $b(v)$, and $c(v)$ satisfy a linear relation

$$(4 - 3s)a(v) + (s - 4)b(v) + 2sc(v) = 0. \tag{34}$$

After nontrivial transformations, the remaining equations may be solved giving the solution

$$a(v) = A(v) - 2m(v), \qquad b(v) = A(v) + (2s - 2)m(v),$$
$$c(v) = A(v) + (2 - s)m(v) \tag{35}$$

where

$$A(v) = \tfrac{1}{2}(\log(1 + \Delta(s)v^4) + \log(1 - \Delta(s)v^4)), \tag{36}$$

$$m(v) = \frac{1}{4\Delta(s)}(\log(1 + \Delta(s)v^4) - \log(1 - \Delta(s)v^4)) \tag{37}$$

with

$$\Delta(s) = \sqrt{\frac{3s^2 - 8s + 8}{24}}. \tag{38}$$

As a cross check of this solution we have verified that performing a power series expansion of (35) indeed coincides with the scaling $\tau \to \infty$ limit of the exact power series solutions.

Let us first specialize to the two cases singled out in Sec. II, especially since the perfect fluid case will turn out to be the only one physically relevant.

A. Perfect fluid case

The perfect fluid corresponds to $s = 4/3$ in (28). Plugging in this value in the above equations leads to the following *asymptotic* geometry

$$ds^2 = \frac{1}{z^2}\left[-\frac{(1 - \frac{e_0}{3}\frac{z^4}{\tau^{4/3}})^2}{1 + \frac{e_0}{3}\frac{z^4}{\tau^{4/3}}}d\tau^2 + \left(1 + \frac{e_0}{3}\frac{z^4}{\tau^{4/3}}\right)(\tau^2 dy^2 + dx_\perp^2)\right] + \frac{dz^2}{z^2} \tag{39}$$

where we reinstated the dimensionful parameter e_0 so that $f(\tau) = e_0/\tau^{4/3}$.

Remarkably enough this geometry is of a form similar to the black hole solution (19) but with the location of the horizon *moving* in the bulk according to

$$z_0 = \left(\frac{3}{e_0}\right)^{1/4} \cdot \tau^{1/3}. \tag{40}$$

From the similarity of the geometry (39) to the black hole solution (19), we may qualitatively infer the scaling of the temperature i.e.

$$T(\tau) \sim \frac{1}{z_0} \sim \tau^{-1/3} \tag{41}$$

and similarly for the entropy per unit rapidity and transverse area

$$S(\tau) \sim \text{area} \sim \tau \cdot \frac{1}{z_0^3} \sim \text{const} \tag{42}$$

in agreement with Bjorken hydrodynamics.

A word of caution is necessary at this stage. It is not clear whether it is possible in the general *evolving* setting to identify in a precise way temperature and entropy of such a geometry (see e.g. discussions in the context of dynamical horizons in general relativity [25]). In addition, in the AdS/CFT context, a change of coordinates in the bulk involving both τ and z will modify which point of the horizon lies "above" which point on the boundary thus making the identification of a *local* temperature and entropy density problematic. Nevertheless it is quite probable that some approximate notions do exist.

Finally let us note that our geometry (39) may be a reliable tool to study gauge-theory observables which are sensitive to the bulk geometry not far away from the boundary.

In the next section we will indeed show that the solution (39) is selected by a criterion of absence of curvature singularities for large proper times.

B. Free streaming case

Inserting $s = 1$ into Eqs. (35) we find that the resulting metric is no longer similar to a moving black hole even with some different functional form of $z_0(\tau)$. Namely one gets

$$ds^2 = \frac{\left(-\left(1+\frac{v^4}{\sqrt{8}}\right)^{(1-2\sqrt{2})/2}\left(1-\frac{v^4}{\sqrt{8}}\right)^{(1+2\sqrt{2})/2}dt^2 + \left(1+\frac{v^4}{\sqrt{8}}\right)^{1/2}\left(1-\frac{v^4}{\sqrt{8}}\right)^{1/2}\tau^2 dy^2 + \left(1+\frac{v^4}{\sqrt{8}}\right)^{(1+\sqrt{2})/2}\left(1-\frac{v^4}{\sqrt{8}}\right)^{(1-\sqrt{2})/2}dx_\perp^2\right)}{z^2} + \frac{dz^2}{z^2}, \tag{43}$$

where $v = z/\sqrt[4]{\tau}$. It is qualitatively different from the perfect fluid case, in particular it displays singularities or zeroes at $v^4 = \sqrt{8}$ in all coefficients. On a more quantitative ground we will now perform an analysis of the curvature properties of the whole above class of metrics for $0 < s < 4$.

V. SINGULARITIES AND CURVATURE

Looking at the general form of (35) we see that there is a potential singularity for $v = \Delta(s)^{-1/4}$. However as is often the case in general relativity such a singularity may be a purely coordinate singularity as indeed happens in the vicinity of the static black hole horizon. In order to unambiguously locate a physical singularity we calculate a *scalar* invariant formed out of the Riemann curvature tensor. The simplest one, the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$ is actually by definition equal to -20 for *any* solution of the Einstein equations (29) as can be directly calculated. Let us then calculate the square of the Riemann tensor

$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} \tag{44}$$

as a probe of curvature singularities.

It turns out that we cannot directly reach $v = \Delta(s)^{-1/4}$ for fixed τ since we have only an asymptotic solution. Therefore we perform the calculations of R and $\mathfrak{R}^2$ by taking $\tau \to \infty$ while keeping v fixed.

Performing first the calculations for R we find that indeed

$$R = -20 + \mathcal{O}\left(\frac{1}{\tau^\#}\right). \tag{45}$$

This shows that we can trust the leading asymptotic term and thus that the asymptotic approximation is self-consistent.

The calculation of $\mathfrak{R}^2$ gives a nontrivial result:

$$\begin{aligned}\mathfrak{R}^2 = \frac{4}{(1-\Delta(s)^2v^8)^4}[&10\Delta(s)^8v^{32} - 88\Delta(s)^6v^{24} + 42v^{24}s^2\Delta(s)^4 + 112v^{24}\Delta(s)^4 - 112v^{24}\Delta(s)^4s + 36v^{20}s^3\Delta(s)^2\\ &- 72v^{20}s^2\Delta(s)^2 + 828\Delta(s)^4v^{16} + 288v^{16}\Delta(s)^2s - 288v^{16}\Delta(s)^2 - 108v^{16}s^2\Delta(s)^2 - 136v^{16}s^3 + 27v^{16}s^4\\ &- 320v^{16}s + 160v^{16} + 296v^{16}s^2 + 36v^{12}s^3 - 72v^{12}s^2 - 88\Delta(s)^2v^8 + 42v^8s^2 + 112v^8 - 112v^8s + 10]\\ &+ \mathcal{O}\left(\frac{1}{\tau^\#}\right).\end{aligned} \tag{46}$$

The striking fact is that in the range $0 < s < 4$, the above asymptotic expression for $\mathfrak{R}^2$ diverges for $v = \Delta(s)^{1/4}$ for all s apart from the perfect fluid case $s = \frac{4}{3}$ (see e.g. Fig. 1). Remarkably enough, this result requires a cancellation of the fourth order pole in front of (46). We checked this analytically by performing a Laurent expansion near that pole. The final result for the perfect fluid case is

$$\mathfrak{R}^2_{\text{perfectfluid}} = \frac{8(5w^{16} + 20w^{12} + 174w^8 + 20w^4 + 5)}{(1+w^4)^4}, \tag{47}$$

where $w = v/\Delta(\frac{4}{3})^{1/4} \equiv \sqrt[4]{3}v$. $\mathfrak{R}^2$ reaches just a finite maximum of $\mathfrak{R}^2 = 112$ at $v = 1/\sqrt[4]{3}$, which plays the

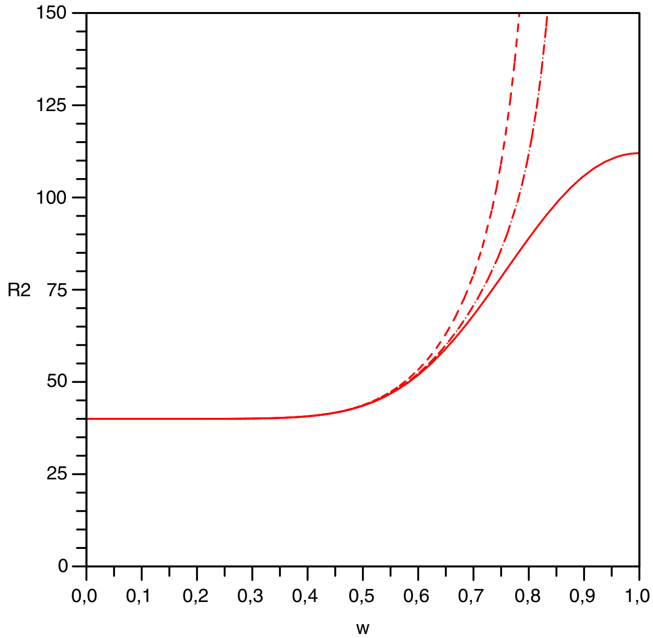

FIG. 1 (color online). The curvature scalar $\Re^2$ calculated as a function of $w = v/\Delta(s)^{1/4}$ for the perfect fluid case $s = 4/3$ (solid line), $s = 4/3 - 0.1$ (dash-dotted line) and $s = 4/3 + 0.2$ (dashed line).

role of the horizon, starting from the boundary value of $\Re^2 = 40$ (see Fig. 1.).

This result means that for asymptotic times and for $s \neq \frac{4}{3}$ one can reach arbitrarily large curvatures in the bulk. This violates our criterion of nonsingular bulk geometry. Thus we should conclude that the asymptotic behavior for large τ of the gauge-theory energy-momentum tensor should be of the perfect fluid type.

Note that our result does *not* mean that we have an *exact* perfect fluid. The above analysis was done only in the asymptotic $\tau \to \infty$ regime. We showed that gauge-theory dynamics rules out such behaviors like streaming behavior which have quite distinct asymptotic $f(\tau) \sim 1/\tau^s$ with $s \neq \frac{4}{3}$. It is quite probable, however, that there would be subleading corrections due to e.g. viscosity. In order to detect them one would have to perform a more detailed analysis. This certainly deserves further investigation.

VI. CONCLUSIONS AND OUTLOOK

Let us summarize our main results.

(i) We propose a general framework for studying the dynamics of matter (plasma) in strongly coupled gauge theory using the AdS/CFT correspondence for the $\mathcal{N} = 4$ SYM theory.

(ii) We use tools related to holographic renormalization for constructing dual geometries for given gauge-theory energy-momentum tensor profiles. We illustrate this method with the static black hole and a planar shock-wave solution.

(iii) Further imposing boost-invariant dynamics inspired by the Bjorken hydrodynamic picture, we derive the corresponding asymptotic solutions of the nonlinear Einstein equations.

(iv) Among the family of asymptotic solutions, the only one with bounded curvature scalars is the gravity dual of a perfect fluid through its energy-momentum tensor profile.

(v) This selected nonsingular solution, given by the metric (39), is similar to a black hole moving off in the 5th dimension as a function of the physical proper time.

Let us add some comments on the specific features of our approach and results. In this paper we concentrate on looking for solutions of the full nonlinear Einstein equations. It would be interesting to confront this approach with the linearization methods of Refs. [6]. In particular viscosity terms are expected to appear in the study of subasymptotic terms. Note that the possibility of black hole formation in the *dual* geometry has been argued in Ref. [7]. More specifically, the geometry of a brane moving with respect to a black hole background has been advocated in Ref. [9] for the dual description of the cooling and expansion of a quark-gluon plasma. In our case we could interpret the solution (39) as a kind of "mirror" situation in terms of a black hole moving off from the AdS boundary. Note however that the precise geometrical identification of the full solution would require further work, in particular, for the structure near the horizon and for subasymptotic proper times.

The method and solution presented in this paper raise a lot of stimulating questions for future investigation. In particular, one would like to address the key problem of connecting, on general grounds, *local* physical temperature and entropy of the gauge-theory matter to features of the dual *evolving* gravity solution.

As a natural outlook it would be interesting to study within the same framework possible deviations from perfect fluid behavior through subasymptotic gravity dual solutions. It would be also interesting to somewhat relax boost invariance and study the possible corresponding modifications involving rapidity dependence. In all cases we expect that the condition of nonsingularity remains essential to select the proper physical solution. Finally it would be stimulating to address the initial problem of two colliding gauge-theory shock waves using the framework of this paper.

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