Lorentz violation and Faddeev-Popov ghosts

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We consider how Lorentz-violating interactions in the Faddeev-Popov ghost sector will affect scalar QED. The behavior depends sensitively on whether the gauge symmetry is spontaneously broken. If the symmetry is not broken, Lorentz violations in the ghost sector are unphysical, but if there is spontaneous breaking, radiative corrections will induce Lorentz-violating and gauge-dependent terms in other sectors of the theory.

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I. INTRODUCTION

Recently there has been quite a bit of interest in the suggestion that Lorentz symmetry may not be exact in nature. Small violations of this fundamental symmetry could arise in connection with the novel physics of the Planck scale. There are a number of possible mechanisms through which Lorentz violations could arise. These include spontaneous violations in string theory [1,2] and elsewhere [3], mechanisms in loop quantum gravity [4,5] and noncommutative geometry [6,7], Lorentz violation through spacetime-varying couplings [8], and anomalous breaking of Lorentz and CPT symmetries [9].

One major focus of research has been the embedding of possible Lorentz-violating effects in effective field theories. The general local Lorentz-violating standard model extension (SME) has been developed [10–12], and the stability [13] and renormalizability [14] of this extension have been studied. The general SME contains all possible local operators that may be constructed out of existing standard model fields. However, typically one will consider only a more limited subcollection of these operators, such as the minimal SME, which contains only superficially renormalizable operators that are invariant under the standard model gauge group.

The minimal SME provides an excellent framework within which to analyze the results of experimental tests of special relativity. To date, such experimental tests have included studies of matter-antimatter asymmetries for trapped charged particles [15–18] and bound state systems [19,20], determinations of muon properties [21,22], analyses of the behavior of spin-polarized matter [23,24], frequency standard comparisons [25–27], measurements of neutral meson oscillations [28–31], polarization measurements on the light from distant galaxies [32–34], and others.

There can be a very subtle interplay between Lorentz violation and gauge invariance. For example, a Lorentz-violating Chern-Simons term $\mathcal{L}_{\rm CS} = \frac{1}{2}k_\mu \epsilon^{\mu\alpha\beta\gamma}F_{\alpha\beta}A_\gamma$ [32,35,36] in the Lagrange density is not gauge invariant. However, since $\mathcal{L}_{\rm CS}$ changes by a total derivative under a gauge transformation, the integrated action is gauge invariant, and the equations of motion involve only the field strength $F^{\mu\nu}$. The quantum corrections to $\mathcal{L}_{\rm CS}$ in spinor QED are even more complicated, and what kind of gauge invariance the final theory possesses depends sensitively on how the theory is regulated [37–43]. In particular, the radiatively induced Chern-Simons term is necessarily finite, but its value is not uniquely determined.

In this paper, we shall examine some further properties of Lorentz-violating quantum field theory. The focus will be on quantum corrections, particularly those associated with the Faddeev-Popov ghosts that arise in the quantization of gauge theories. Since the presence of these ghosts is subtly entwined with the symmetry properties of these theories, we expect that any changes to the ghosts' sector's structure could have a significant impact on gauge invariance. In this section, we shall review the structure of the gauge-fixed scalar QED Lagrangian. In Sec. II, we shall introduce Lorentz-violating modifications to this gauge theory and calculate the scalar field self-energy in their presence. The physical interpretation of our results is discussed in Sec. III, and our conclusions summarized in Sec. IV.

Our starting point will be the gauge-fixed Faddeev-Popov Lagrange density for scalar QED (in a generic R_ξ gauge),

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + |D^\mu\Phi|^2 - V(\Phi) - \frac{1}{2\xi}(\partial^\mu A_\mu - \xi e v\varphi)^2 + \bar{c}\left[-\partial^2 - \xi m_A^2\left(1+\frac{h}{v}\right)\right]c. \tag{1}$$

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the Abelian field strength, $D^\mu = \partial^\mu + ieA^\mu$ is the covariant derivative, and $V(\Phi)$ is the scalar field potential. The complex scalar field Φ itself we parameterize as

$$\Phi(x) = \frac{1}{\sqrt{2}}[v + h(x) + i\varphi(x)]. \tag{2}$$

The potential might or might not induce spontaneous sym-

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metry breaking, and v is the (possibly vanishing) vacuum expectation value of the field, which we have taken to be in the real direction. The gauge-fixing terms then depend on the fields h and φ; if $v \neq 0$, these are the Higgs and Goldstone boson fields, respectively. The parameter ξ determines the choice of gauge, and in the absence of explicit symmetry-breaking terms, it should cancel out in all physical results. Finally, $m_A^2 = e^2 v^2$ is the mass of the physical gauge field, and c and $\bar{c}$ are Grassmann-valued ghost fields.

The potential $V(\Phi)$ may be left fairly general. We shall not make use of any of its properties, except the value of v it induces. So $V(\Phi)$ need not be the bare scalar potential of the theory. If the potential possesses a symmetry-breaking minimum, that minimum could be the result of radiative corrections (as in the Coleman-Weinberg model [44]) or strong interactions in the matter sector.

The R_ξ gauge Lagrange density (1) may be obtained by the Faddeev-Popov procedure [45]. We begin with the conventional scalar QED Lagrange density,

$$\mathcal{L}_0 = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + |D^\mu \Phi|^2 - V(\Phi). \tag{3}$$

Then we introduce a gauge-fixing function

$$G = \frac{1}{\sqrt{\xi}} (\partial^\mu A_\mu - \xi e v \varphi). \tag{4}$$

Following standard procedure, we integrate over different values of G, weighted by a Gaussian. This transforms the Lagrange density to $\mathcal{L}_0 - \frac{1}{2} G^2$.

To complete the Faddeev-Popov quantization procedure however, we must also include the ghosts. The ghost Lagrangian is determined by the gauge variation of G. That gauge variation is represented by the determinant of the operator $\delta G / \delta \alpha$, where $\alpha(x)$ is the parameter of a local gauge transformation,

$$\delta h = -\alpha \varphi \tag{5}$$

$$\delta \varphi = \alpha (v + h) \tag{6}$$

$$\delta A^\mu = -\frac{1}{e} \partial^\mu \alpha. \tag{7}$$

Since

$$\frac{\delta G}{\delta \alpha} = \frac{1}{\sqrt{\xi}} \left[-\frac{1}{e} \partial^2 - \xi m_A (v + h) \right], \tag{8}$$

the ghost term in (1) reproduces the functional determinant of $\delta G / \delta \alpha$ when it is integrated out. This term therefore completes the gauge-fixed R_ξ Lagrange density.

In the conventional framework that we have outlined, the ghosts c and $\bar{c}$ are seen as auxiliary fields; they are introduced as part of the gauge-fixing procedure, the purpose of which is to reorganize the action, so that the zero modes of the gauge action do not interfere with the derivation of the propagator. So the Faddeev-Popov ghosts can be seen as further manifestations of the fundamental gauge field; in addition to the gauge-fixed A^μ, the gauge sector contains these anticommuting fields.

However, another slightly different viewpoint is also possible. Since (1) provides a correct and complete description of scalar QED, we could take this gauge-fixed action as our basic description of the physics. Then if we are interested in describing all possible Lorentz-violating modifications of scalar QED, we should include those Lorentz-violating operators that involve ghosts, even though no form of Lorentz violation involving only the ghosts has thus far been suggested as a signature for a specific mechanism of Lorentz breaking.

The Lagrange density (1), since it is gauge fixed, does not possess a conventional $U(1)$ local symmetry. So we must be a little careful when we speak of the gauge invariance of this Lagrangian. However, since (1) can be derived from a truly gauge invariant expression by the Faddeev-Popov procedure, this Lagrange density does have gauge invariance in a certain sense. Yet because the gauge symmetry is somewhat obscured, it may not necessarily be clear whether a particular term, if added to $\mathcal{L}$, will break the symmetry or not. One might hope that this difficulty could be resolved by examining the BRST symmetry of the gauge-fixed Lagrangian [46,47]. However, this turns out not to suffice. We shall encounter operators which break BRST symmetry when added to (1), yet which do not change the fact that the physical theory one would observe is an Abelian gauge theory.

Before we introduce Lorentz violation and begin calculating loop diagrams, we should point out one further point. If the gauge symmetry is not spontaneously broken, then the gauge-fixed Lagrange density reduces to

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + |D^\mu \Phi|^2 - V(\Phi) - \frac{1}{2\xi} (\partial^\mu A_\mu)^2 + (\partial^\mu \bar{c})(\partial_\mu c). \tag{9}$$

What is important about this expression is the well-known fact that the Faddeev-Popov ghosts decouple completely. They can be completely ignored, and even if they are included in the theory's Feynman diagrams, they will only appear in unconnected vacuum bubbles. So the structure of the ghost sector has no effect on the S matrix. This is in sharp contrast with the case in which the gauge symmetry is spontaneously broken, because if v is nonzero, (1) implies that there is a coupling between the ghosts and the physical Higgs field h. It is this difference that will be at the crux of our discussions.

II. LORENTZ VIOLATION FROM THE GHOST SECTOR

A. Lorentz-violating ghost Lagrangian

We shall now consider modifying (1) to include Lorentz violation in the ghost sector. However, not all the Lorentz-violating terms that we may add to $\mathcal{L}$ are physically meaningful. There are actually relatively few superficially renormalizable couplings one can write down involving only the c and $\bar{c}$ fields, because these fields are Lorentz scalars. There is only one such CPT-odd modification of the ghost sector; adding it changes the Lagrange density for c and $\bar{c}$ into

$$\mathcal{L}_a = [(\partial^\mu + ia^\mu)\bar{c}][(\partial^\mu - ia^\mu)c] - \xi m_A^2\left(1 + \frac{h}{v}\right)\bar{c}c. \tag{10}$$

However, the presence of a^μ actually has no physical consequences. A field redefinition

$$c \rightarrow e^{ia\cdot x}c, \qquad \bar{c} \rightarrow e^{-ia\cdot x}\bar{c} \tag{11}$$

eliminates a^μ from the theory. This shift is equivalent to a change in the origin of the momentum integration for all ghost loops. In more general theories, field redefinitions may be used to eliminate a number of other apparently Lorentz-violating terms [48].

A superficially renormalizable, CPT-even modification of the ghost sector is also possible. In this case, the Lorentz violation changes the ghost Lagrange density to

$$\mathcal{L}_c = \bar{c}\left[-\partial^2 - c^{\nu\mu}\partial_\nu\partial_\mu - \xi m_A^2\left(1 + \frac{h}{v}\right)\right]c. \tag{12}$$

This does not represent a Lorentz-violating choice of gauge; it is something entirely different. Using a Lorentz-violating gauge would mean choosing a function G that transforms nontrivially under particle Lorentz transformations. Doing this would induce Lorentz violation in the A^μ sector, which any violations in the ghost sector should then cancel out. Here, we have added Lorentz violation to the ghost Lagrangian without making any corresponding changes to the Lagrangian for A^μ.

The main question that this paper shall address is whether the Lorentz-violating coefficient $c^{\nu\mu}$ is physical. It is obvious from the form of the interaction that the antisymmetric part of $c^{\nu\mu}$ does not contribute. However, whether the symmetric part will manifest itself physically is not so obvious. In fact, we shall show that the question of whether the $c^{\nu\mu}$ Lorentz violation contributes to real effects cannot be answered by looking at the gauge sector alone. The behavior of the matter fields affects things in a crucial way.

Ad hoc modifications of the ghost sector like (12) also will be expected to damage the gauge invariance properties of our theory. However, if $c^{\nu\mu}$ turns out to be unphysical (like a^μ), then the gauge symmetry is effectively restored. If the Lorentz-violating interactions do not contribute to physical effects, then they may be ignored, and only the gauge-symmetric part of the theory need be retained. So in this sense, the antisymmetric part of $c^{\nu\mu}$ does not violate gauge invariance, just as it does not violate physical Lorentz invariance.

Before proceeding, we should examine a number of field and coordinate redefinitions that may often be used to simplify Lorentz-violating field theories. The trace of $c^{\nu\mu}$ can be eliminated from (12) without changing the overall form of the Lagrangian, by making a field redefinition

$$c \rightarrow \frac{c}{\sqrt{1 + \frac{1}{4}c^\alpha{}_\alpha}}, \qquad \bar{c} \rightarrow \frac{\bar{c}}{\sqrt{1 + \frac{1}{4}c^\alpha{}_\alpha}}. \tag{13}$$

However, this also changes the gauge parameter ξ, unless $\xi = 0$. If the final theory is ξ independent, $c^\alpha{}_\alpha$ is definitively unphysical. Yet we shall find that under certain circumstances, ξ does affect the theory, so the trace of $c^{\nu\mu}$ can lead to physical effects.

Also, the symmetric $c^{\nu\mu}$ can be removed from the ghost sector at leading order by redefining the coordinates,

$$x^\mu \longrightarrow x^\mu + \tfrac{1}{2}c^{\nu\mu}x_\nu. \tag{14}$$

This exports the Lorentz violation to the other sectors of the theory. The field Φ acquires a $c^{\nu\mu}$-type modification of its kinetic energy, and the gauge sector acquires a k_F term

$$\mathcal{L}_F = -\tfrac{1}{4}(k_F)_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}, \tag{15}$$

with $(k_F)_{\mu\nu\rho\sigma}$ proportional to $g_{\mu\rho}c_{\nu\sigma} + g_{\nu\sigma}c_{\mu\rho}$ [49,50]. Because the Lagrangian is gauge fixed, the $-\frac{1}{2\xi}(\partial^\mu A_\mu - \xi ev\varphi)^2$ term also receives a Lorentz-violating modification. For diagrams not involving ghosts, there is then an intricate cancellation of all the Lorentz-violating effects. However, after the transformation, the ghost sector contains no leading order Lorentz violation. What may lead to physical Lorentz-violating effects is the mismatch between the Lorentz violations for the ghosts and for the Higgs field to which the ghosts are coupled; certain cancellations that would exist if c and Φ had the same $c^{\nu\mu}$ are missing. Ghost loops may generate Lorentz-invariant contributions to the Higgs field strength renormalization; these can combine with the tree-level Lorentz violation for Φ to produce the same effects as we shall discuss below. However, we shall henceforth consider only the formulation of the theory in which all the Lorentz violation resides in the ghost Lagrangian, just because doing so is simpler.

B. Unbroken $U(1)$ phase

Since the ghosts are not supposed to appear as external particles, addressing the issues we wish to discuss must necessarily involve consideration of quantum corrections. The Feynman rules for the $c^{\nu\mu}$-modified theory depend on whether the scalar field potential induces spontaneous symmetry breaking. If it does not, then v and m_A vanish, so the Faddeev-Popov ghosts remain decoupled from the rest of the theory. The presence of the Lorentz violation does not change this. So c and $\bar{c}$ still only appear in vacuum bubble diagrams. These disconnected diagrams will be modified, but this fact does not have any physical meaning (since the theory as we are considering it is not coupled to gravity). We may therefore conclude that $c^{\nu\mu}$ does not correspond to any physical Lorentz violation, as long as the theory is in a phase with no spontaneous breaking of the $U(1)$ symmetry. Nor is there any meaningful breaking of gauge invariance under these circumstances (even though the addition of $c^{\nu\mu}$ destroys the BRST symmetry).

C. Broken $U(1)$ Phase

The case in which the gauge symmetry is broken by a nonzero v is much more complicated, because the ghosts do not decouple. Instead, they are coupled to the physical Higgs field h. The ghosts will contribute to the n-point correlation functions for h; the important diagrams have n Higgs lines attached to a ghost loop. We shall compute the simplest such diagrams, which give a one-loop contribution to the Higgs two-point function. (The one-loop diagrams with $n > 2$ external lines are all finite by power counting.)

We shall work to leading order in $c^{\nu\mu}$. It is common practice to ignore higher-order Lorentz-violating effects, because any physical Lorentz violation is known to be small. At first order in $c^{\mu\nu}$, there are two diagrams, which we shall evaluate by dimensional regularization. Without Lorentz violation, the one-loop ghost contribution to the Higgs self-energy $\Pi_h(p)$ comes from a single diagram with two ghost propagators. The leading Lorentz-violating contributions then come from adding $c^{\nu\mu}$ insertions on one or the other of the ghost lines. This splits the $\mathcal{O}(c^{\nu\mu})$ contribution into two terms, Π_h^1 and Π_h^2, and the first diagram of this type gives

$$i\Pi_h^1(p) = (-1)\left(-i\xi\frac{m_A^2}{v}\right)^2 \int \frac{d^dk}{(2\pi)^d}\frac{i}{k^2-\xi m_A^2} \times(-ic^{\nu\mu}k_\nu k_\mu)\frac{i}{k^2-\xi m_A^2}\frac{i}{(k+p)^2-\xi m_A^2}, \tag{16}$$

where p is the external momentum of the h field. The factor of -1 comes from the Grassmann nature of the ghosts. This whole expression depends strongly on the gauge parameter ξ, and at $\xi = 0$ it vanishes. This dependence just indicates that if we find a nonzero physical result, it will break gauge as well as Lorentz symmetry, just as previously discussed.

We may combine the denominators with a Feynman parameter x to get

$$i\Pi_h^1(p) = -\xi^2\frac{m_A^4}{v^2}\int_0^1 dx \int\frac{d^dk}{(2\pi)^d}c^{\nu\mu}p_\nu p_\mu \times \frac{2(1-x)}{\{(1-x)(k^2-\xi m_A^2)+x[(k+p)^2-\xi m_A^2]\}^3}. \tag{17}$$

In terms of $\ell = k + xp$ and $\Delta = -x(1-x)p^2 + \xi m_A^2$, and dropping all terms odd in ℓ, this is

$$i\Pi_h^1(p) = -2\xi^2\frac{m_A^4}{v^2}\int_0^1 dx\int\frac{d^d\ell}{(2\pi)^d}c^{\nu\mu} \times\frac{(1-x)(\ell_\nu\ell_\mu + x^2p_\nu p_\mu)}{(\ell^2-\Delta)^3}. \tag{18}$$

The $\ell_\nu\ell_\mu$ term makes a contribution proportional to $g_{\nu\mu}$. When contracted with $c^{\nu\mu}$, this gives only a Lorentz scalar. The momentum-independent part of this becomes part of the mass renormalization and does not result in any Higgs sector Lorentz violation. Moreover, despite its dependence on ξ, the p-independent term cannot result in a physical failure of gauge invariance, because the measurable value of the Higgs mass is a free parameter in the renormalized theory. We may absorb the dependence on ξ into the unphysical bare mass of the Higgs. We shall therefore not consider the divergent part of this expression any further, although we shall comment on the p^2-dependent part of the Lorentz-invariant term at the end of this section.

The $p_\nu p_\mu$ term gives the potentially Lorentz-violating part of $\Pi_h^1(p)$, which we shall denote as $\Pi_{h,\mathrm{LV}}^1(p)$. Evaluating this as $d \to 4$, we find

$$i\Pi_{h,\mathrm{LV}}^1(p) = \frac{i}{16\pi^2}\xi^2\frac{m_A^4}{v^2} \times\int_0^1 dx\frac{x^2(1-x)}{\xi m_A^2 - x(1-x)p^2}c^{\nu\mu}p_\nu p_\mu. \tag{19}$$

The Lorentz-violating contribution $\Pi_{h,\mathrm{LV}}^2(p)$ coming from the other diagram is the same, but with $x \to (1-x)$. So adding these together, we get

$$i\Pi_{h,\mathrm{LV}}(p) = \frac{i}{16\pi^2}\xi^2\frac{m_A^4}{v^2} \times\int_0^1 dx\frac{x(1-x)}{\xi m_A^2 - x(1-x)p^2 - i\eta}c^{\nu\mu}p_\nu p_\mu, \tag{20}$$

where we have inserted an infinitesimal $i\eta$ ($\eta > 0$) to give the correct behavior when Δ vanishes. If $0 < p^2 < 4\xi m_A^2$, the integral over x may be evaluated in closed form:

$$\int_0^1 dx \frac{x(1-x)}{\xi m_A^2 - x(1-x)p^2} = -\frac{1}{p^2} + \frac{4\xi m_A^2}{(p^2)^{3/2}\sqrt{4\xi m_A^2 - p^2}} \times \tan^{-1}\left(\frac{\sqrt{p^2}}{\sqrt{4\xi m_A^2 - p^2}}\right). \tag{21}$$

We can also combine the Lorentz-invariant parts of $\Pi_h^1(p)$ and $\Pi_h^2(p)$ (coming from the $\ell_\nu \ell_\mu$ terms), to obtain another expression, $\Pi_{h,\mathrm{LI}}(p)$. As previously stated, the divergent, p^2-independent part of $\Pi_{h,\mathrm{LI}}(p)$ is just absorbed into the mass renormalization. However, there is a finite, momentum-dependent part as well. The momentum dependence is given by

$$i\Pi_{h,\mathrm{LI}}(p) = -\frac{i}{32\pi^2}\xi^2 \frac{m_A^4}{v^2} c^\mu{}_\mu \times \int_0^1 dx \log[\xi m_A^2 - x(1-x)p^2 - i\eta] + C. \tag{22}$$

C is the unphysical infinite constant (which, however, contains the scale of the logarithm). Although the momentum-dependent part of $\Pi_{h,\mathrm{LI}}(p)$ is Lorentz invariant, it is finite, and most of the remarks in Sec. III B will apply to this expression as well as to $\Pi_{h,\mathrm{LV}}(p)$.

III. INTERPRETATION OF RESULTS

A. Unbroken phase

We shall now look at the physical implications of these results, beginning in the phase without spontaneous symmetry breaking. Of course, matters are very simple in this situation, because we have already established that there can be no contribution from the ghosts to any connected diagram with physical external particles.

The physical theory remains an Abelian gauge theory. The $c^{\nu\mu}$ modification of the ghost sector, although it apparently violates Lorentz and BRST symmetries, does not affect the symmetries of any physical process. Indeed, it does not affect physical processes in any way. The S matrix for the theory is unchanged by the modification of the Lagrangian.

It is even possible to see how $c^{\nu\mu}$ could be defined away in the path integral. However, this is not accomplished by a field redefinition, but rather by replacing the existing ghosts with an entirely new set of ghost fields. c and $\bar{c}$ can simply by integrated out of the theory; this will only change the normalization of the measure. Then a new set of ghost fields c' and $\bar{c}'$ may be introduced, with a rescaled functional measure and a Lagrange density $(\partial^\mu \bar{c}')(\partial_\mu c')$. This restores the Lagrangian to its Lorentz-invariant form. Of course, all these formal manipulations are rather trivial, but this just underscores the fact that the ghosts are entirely superfluous in this theory.

B. Broken phase

Things are not trivial in the Higgs phase, however. Unless $\xi = 0$, $\Pi_{h,\mathrm{LV}}(p)$ makes a physical Lorentz-violating contribution to the Higgs propagator. Let us examine a few special cases in the parameter space and see how this radiative correction will affect particle propagation. For an on shell Higgs boson, $p^2 = m_h^2$, where m_h is the Higgs mass. Then, if $|\xi m_A^2| \gg m_h^2$, we have

$$\Pi_{h,\mathrm{LV}}(p^2 = m_h^2) = \frac{1}{96\pi^2}\xi \frac{m_A^2}{v^2} c^{\nu\mu} p_\nu p_\mu. \tag{23}$$

In the opposite limit, $|\xi m_A^2| \ll m_h^2$, the contribution to the on-shell propagator is given by

$$\Pi_{h,\mathrm{LV}}(p^2 = m_h^2) = -\frac{1}{16\pi^2}\xi^2 \frac{m_A^4}{v^2 m_h^2} c^{\nu\mu} p_\nu p_\mu. \tag{24}$$

(Note that the relationship between ξm_A^2 and m_h^2 is not determined merely by the ordinarily physical masses. The relative sizes of these terms depend on the normally unphysical gauge parameter ξ.) In either limit, there is a nonzero Lorentz-violating term in the effective action for the Higgs. This term will affect the propagation states of the theory, just as would a Lorentz-violating tensor appearing in the fundamental Lagrangian. The particular terms that we have found would, for example, change the energy-momentum relation and hence velocity for physical Higgs excitations [51].

So in the Higgs phase, when the $U(1)$ gauge symmetry is spontaneously broken, the symmetric part of $c^{\nu\mu}$ has a real physical effect. The magnitude of the induced violation in the Higgs sector is controlled by ξ. In the Lorenz-Landau gauge ($\xi = 0$), $\Pi_{h,\mathrm{LV}}(p)$ actually vanishes; but otherwise, the Lorentz violation will be nonzero. This explicit ξ dependence signals that there is also a breakdown of gauge invariance. The Higgs propagator has acquired a new gauge dependence, which will not be cancelled by effects in other sectors of the theory.

However, although $\Pi_{h,\mathrm{LV}}$ and $\Pi_{h,\mathrm{LI}}$ both depend on p, the momentum of the Higgs particle, rather than $p - eA$, this should not be taken as a further indication of gauge invariance violation. We have not considered any diagrams with external photons, and in fact, when such higher-order terms are included, the proper dependence on covariant derivatives should appear.

The induced Lorentz violation in the unitarity gauge cannot be studied by taking the $\xi \to \infty$ limit of $\Pi_{h,\mathrm{LV}}(p)$. That limit would have to be taken before any loop integrations are performed [52]. This limiting process will cause any diagrams containing more ghost propagators than Higgs-ghost vertices to vanish. All diagrams involving $c^{\nu\mu}$ have this property, so there is again no Lorentz violation in this singular limit. However, this is fairly unsurprising, since for $\xi = \infty$, the propagating ghosts effectively disappear.

What we have found is a somewhat unexpected connection between the gauge sector of the theory (of which the ghosts are a part) and the Higgs potential. Some aspects of the entanglement between the gauge and Higgs sectors in spontaneously broken gauge theories are already well known. The most obvious example is the "eating" of the Goldstone boson by the gauge field; the fundamental scalar becomes the longitudinal component of the vector boson. We can see in (1) where information about the Higgs potential has been fed back into the gauge sector. The ghost Lagrangian depends on v, which is not a quantity that can be determined within the gauge sector. However, we should keep in mind that (1) is valid in either the Higgs or unbroken phase of the theory; the appearance of v in the ghost Lagrangian is not alone responsible for the difference in behavior between the two phases of the theory.

In actuality, the theory may not even be well-defined in the broken phase. The gauge-dependent corrections could destroy renormalizability, and without BRST symmetry, we cannot necessarily ensure the unitarity of the S matrix. However, we cannot know whether either of these two problems actually exists without performing more detailed calculations. Gauge invariance is not actually a necessary requirement for the renormalizability of an Abelian vector theory, and although the BRST symmetry is also broken in the $v = 0$ case, we know that the resulting theory is definitely unitary. Moreover, we can be certain that if $\xi = 0$, there will be a well-defined theory, because all the troublesome radiative corrections vanish.

C. Relationship to finite, undetermined quantum corrections

The fact that the finite radiative corrections are not gauge invariant in the Higgs phase allows us to draw an analogy with other finite, yet undetermined, radiative corrections. The parameter ξ which controls the size of the radiative corrections does not appear in any tree-level results. Although the Feynman rules depend on its value, this dependence cancels in all physical quantities. However, once loop corrections are included, Lorentz-violating effects can appear. The size of these effects depends on ξ, which is effectively a free parameter. That is, the loop corrections cannot be uniquely determined from observations of tree-level processes.

A theoretical discussion of finite, undetermined quantum corrections is given in [53]. When a symmetry or other formal property forbids the appearance of a given operator at tree level, finite radiative contributions to this operator have been found to give definite values. For example, the anomalous magnetic moment of the electron in QED and the photon mass in the Schwinger model [54] get definite values from loop corrections. Of these two quantities, the former is forbidden at tree level by the requirement of renormalizability and the latter by gauge invariance. However, the photon mass in the chiral Schwinger model [55] is undetermined, because gauge invariance cannot be preserved in that theory. Similarly, if Lorentz and CPT invariances are abandoned, no other symmetry can prevent a Chern-Simons term from being present in the bare action; so, as discussed above, the finite radiative corrections to this term are undetermined.

In the theory considered here, both of the regimes discussed in [53] actually manifest themselves, in a self-consistent fashion. If the gauge symmetry is not spontaneously broken, then the Lorentz-violating radiative corrections are uniquely determined; they vanish. On the other hand, if there is no gauge invariance to protect the theory, gauge-dependent corrections arise. This situation is not exactly the same as in the other cases discussed above, because the parameter describing the ambiguity, ξ, is the gauge parameter itself. So the observations that the corrections are undetermined and that they depend on the gauge both come directly from the fact that there is a nontrivial ξ dependence; in other words, the gauge dependence is the ambiguity.

In this framework of the preceding paragraph, the Lorenz-Landau gauge manifests itself as a fine tuning of the model, such that the Lorentz-violating effects are made to vanish. However, if it actually turns out that the theory is only renormalizable and unitary in this particular gauge, then these conditions will again restore uniqueness to the radiative corrections. A well-defined theory can result only if $\xi = 0$, so the only radiative corrections that could be seen physically would correspond to this special value.

IV. CONCLUSION

We have demonstrated that there are subtle issues surrounding any Lorentz-violating operators that involve Faddeev-Popov ghost fields. We introduced a Lorentz-violating term into the ghost sector of the action. In addition to Lorentz symmetry, we expected this addition to break the gauge symmetry of the theory, if it actually turned out to have any physical effects. However, the question of whether such physical effects exist turns out to be quite subtle.

It is well established that some Lorentz-violating operators have no physical consequences. However, in an Abelian gauge theory, one cannot determine whether or not the $c^{\nu\mu}$ operator is physically meaningful without knowing what phase the theory is in. Moreover, when the Lorentz violation does exist, it is gauge dependent. So far, our results only apply to $U(1)$ gauge theories; however, it would be very interesting to see how they generalize to the non-Abelian case, in which the ghosts are more closely coupled to the rest of the theory.

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