Two-body problem with the cosmological constant and observational constraints

Philippe Jetzer* and Mauro Sereno†

Institut für Theoretische Physik, Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich , Switzerland

(Received 5 December 2005; published 13 February 2006)

We discuss the influence of the cosmological constant on the gravitational equations of motion of bodies with arbitrary masses and eventually solve the two-body problem. Observational constraints are derived from measurements of the periastron advance in stellar systems, in particular, binary pulsars and the solar system. Up to now, Earth and Mars data give the best constraint, $\Lambda \lesssim 10^{-36}$ km^{-2}; bounds from binary pulsars are potentially competitive with limits from interplanetary measurements. If properly accounting for the gravito-magnetic effect, this upper limit on Λ could greatly improve in the near future thanks to new data from planned or already operating space missions.

DOI: 10.1103/PhysRevD.73.044015 PACS numbers: 04.25.Nx, 04.80.Cc, 95.10.Ce, 95.30.Sf

I. INTRODUCTION

Aged nearly one century, Einstein's cosmological constant Λ still keeps unchanged its cool role to solve problems. Λ, despite being just one number, was able to respond to very different needs of the scientific community, from theoretical prejudices about the universe being static (which provided the original motivation for introducing Λ in 1917) to observational hints that the universe is dominated by unclustered energy density exerting negative pressure, as required by data of exquisite quality which became available in the last couple of decades. Although it is apparently plagued by some theoretical problems about its size and the coincidence that just in the current phase of the universe the energy contribution from Λ is of the same order of that from nonrelativistic matter, the cosmological constant still provides the most economical and simplest explanation for all the cosmological observations [1]. The interpretation of the cosmological constant is a very fascinating and traditional topic. Λ might be connected to the vacuum density, as suggested by various authors (see [2] for an historical account), and could offer the greatest contribution from cosmology to fundamental physics.

The big interest in the cosmological constant has recursively raised attempts in putting observational bounds on its absolute value from completely different phenomena. Λ, supposed to be $\sim 10^{-46}$ km^{-2} from observational cosmology analyses, is obviously of relevance on cosmological scales but it could play some role also in local problems. Up to now, no convincing methods for constraining Λ in a laboratory have been proposed [3], but interesting results have been obtained considering planetary motions in the solar system [4–6]. The effects of Λ become stronger for diluted mass conglomeration but they get enhanced also through various mechanisms [7,8]. As an example, conditions for the virial equilibrium can be affected by Λ for highly flattened objects [7]. On the scale of the Local Volume, a cosmological constant could have observable consequences by producing lower velocity dispersion around the Hubble flow [9].

Up to now, local physical consequences of the existence of a cosmological constant were investigated studying the motion of test bodies in the gravitational field of a very large mass. This one-body problem can be properly considered in the framework of the spherically symmetric Schwarzschild vacuum solution with a cosmological constant, also known as Schwarzschild-de Sitter or Kottler space-time. The rotation of the central source can also be accounted for using the so-called Kerr-de Sitter space-time [4]. Here, we carry out an analysis of the gravitational N-body problem with arbitrary masses in the weak field limit with a cosmological constant. This study is motivated by the more and more central role of binary pulsars, from the discovery of the pulsar PSR B1913 + 16 in 1974 [10], in testing gravitational and relativistic effects. The gravitational two-body equations of motion for arbitrary masses were first derived in absence of spin by Einstein, Infeld and Hoffmann (EIH) [11]. The problem was later addressed in more general cases, subsequently accounting for spins and quadrupole moments [[12] and reference therein]. Here, we take the further step to consider a cosmological constant.

The paper is as follows. In Sec. II, we discuss the gravitational weak field limit in presence of a cosmological constant and introduce the relevant approximations. Section III presents the generalization of the EIH equations of motion, whereas Sec. IV is devoted to the study of the two-body problem. In Sec. V, we review how measurements of precession of pericentre in stellar system can constrain Λ. In particular, we consider binary pulsars and the solar system. Sec. VI contains some final considerations.

*Electronic address: jetzer@physik.unizh.ch
†Electronic address: sereno@physik.unizh.ch

II. FIELD EQUATIONS WITH A COSMOLOGICAL CONSTANT IN POST-NEWTONIAN APPROXIMATION

Einstein's equations with the cosmological constant are

$$R_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} S_{\mu\nu}, \qquad (1)$$

where G is the gravitational constant, c the vacuum speed of light and

$$S_{\mu\nu} \equiv T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\lambda}_{\lambda}, \qquad (2)$$

with $T_{\mu\nu}$ being the energy-momentum tensor. The weak field expansion can start by introducing a nearly Lorentzian system for weak, quasistationary fields, in which

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1. \qquad (3)$$

Actually, the Minkowski metric $\eta_{\mu\nu}$ is not a vacuum solution of the field equations with a cosmological constant, but for $|\Lambda| \ll 1$ an approximate solution in a finite region can still be found by an expansion around $\eta_{\mu\nu}$. In the post-Newtonian (pN) approximation, metric components can be expanded in powers of

$$\varepsilon \sim \left(\frac{GM}{c^2 R}\right)^{1/2} \sim \frac{v}{c} \sim \frac{p}{\rho}, \qquad (4)$$

where M, R, v, p and ρ represent typical values for the mass, length, velocity, pressure and energy density of the system, respectively. In what follows, ${}^{(n)}g_{\mu\nu}$ and ${}^{(n)}T_{\mu\nu}$ will denote terms of order ε^n and $\varepsilon^n(M/R)$, respectively. To perform a proper treatment in presence of a Λ-term we have to consider the suitable approximation order for Λ. We assume that the size of the contributions due to the cosmological constant is at most comparable to the post-Newtonian terms, i.e, $\mathcal{O}(\Lambda g_{00}) \gtrsim \mathcal{O}(G^{(2)}T^{00}/c^2)$. This condition can be rewritten as

$$\Lambda \lesssim \frac{R_{\rm g}^2}{R^4}, \qquad (5)$$

where $R_{\rm g} \equiv GM/c^2$ is the gravitational radius. Equation (5) is easily satisfied by gravitational bound systems with $M \sim M_\odot$ and $R \sim 1 - 10^2$AU if $\Lambda \lesssim 10^{-33}$ km, a value well above the estimated one from cosmological constraints and also greater than the limits we will derive in Sec. V considering stellar systems. Hereafter, we will put $c = 1$. With such an approximation order, we can use classical results within the standard pN gauge. Following [13], the approximate field equations read

$$\Delta^{(2)} g_{00} = -8\pi G^{(0)}T^{00}, \qquad (6)$$

$$\Delta^{(4)} g_{00} = {}^{(2)}g_{ij}{}^{(2)}g_{00,ij} + {}^{(2)}g_{ij,j}{}^{(2)}g_{00,i} - \frac{1}{2}{}^{(2)}g_{00,i}{}^{(2)}g_{00,i} - \frac{1}{2}{}^{(2)}g_{00,i}{}^{(2)}g_{jj,i} - 8\pi G({}^{(2)}T^{00} - 2^{(2)}g^{(0)}_{00}T^{00} + {}^{(2)}T^{ii}) + 2\Lambda, \qquad (7)$$

$$\Delta^{(3)} g_{oi} = -\frac{1}{2}{}^{(2)}g_{jj,0i} + {}^{(2)}g_{ij,0j} + 16\pi G^{(1)}T^{i0}, \qquad (8)$$

$$\Delta^{(2)} g_{ij} = -8\pi G \delta_{ij}{}^{(0)}T^{00}. \qquad (9)$$

The components of the metric can be expressed in terms of potentials. Let $\phi_{\rm N}$ be the Newtonian potential,

$$\phi_{\rm N} = -G \int \frac{{}^{(0)}T^{00}(t, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x^i. \qquad (10)$$

According to our approximation order, the cosmological constant appears only in the equation for ${}^{(4)}g_{00}$. This can be rearranged to give

$$\Delta^{(4)}(g_{00} + 2\phi_{\rm N}^2) = -8\pi G({}^{(2)}T^{00} + {}^{(2)}T^{ii}) + 2\Lambda \qquad (11)$$

Together with the classical pN potential ψ,

$$\psi = -G \int \frac{d^3x'}{|\mathbf{x} - \mathbf{x}'|}({}^{(2)}T^{00} + {}^{(2)}T^{ii}), \qquad (12)$$

we introduce ϕ_Λ, solution of

$$\Delta\phi_\Lambda = -\Lambda. \qquad (13)$$

In presence of a cosmological constant, there is an upper limit on the maximum distance within which the Newtonian limit holds and boundary conditions must then be chosen at a finite range [14]. When these boundary conditions are chosen on a sphere whose origin coincides with the origin of the coordinate system, ϕ_Λ can be expressed as

$$\phi_\Lambda = -\frac{1}{6}\Lambda|\mathbf{x}|^2, \qquad (14)$$

where we have neglected correction terms which appear because of boundary conditions. Because of a positive cosmological constant, the origin of the coordinate system has a distinguished dynamical role with a radial force directed away from it [15]. Since the choice of the origin is arbitrary, any point in the space will experience repulsion from any other point. Finally, introducing the standard pN potentials,

$$\xi_i = -4G \int \frac{d^3x'}{|\mathbf{x} - \mathbf{x}'|} T^{i0}(t, \mathbf{x}'), (1) \qquad (15)$$

$$\chi = -\frac{G}{2} \int |\mathbf{x} - \mathbf{x}'|^{(0)}T^{00} d^3x'(t, \mathbf{x}'), \qquad (16)$$

the metric components read

$$^{(2)}g_{00} = -2\phi_{\rm N}, \tag{17}$$

$$^{(4)}g_{00} = -2(\phi_{\rm N}^2 + \psi + \phi_\Lambda), \tag{18}$$

$$^{(2)}g_{ij} = -2\delta_{ij}\phi_{\rm N}, \tag{19}$$

$$^{(3)}g_{0i} = \xi_i + \chi_{,i0}. \tag{20}$$

For a pointlike mass at the center of the coordinate system, the above expressions reduce to the weak field limit at large radii of the Kottler space-time.

A. Equations of motion for a test particle

The motion of a particle in an external gravitational field can be described by the Lagrangian

$$\mathcal{L} = 1 - \sqrt{-g_{\mu\nu}\left(\frac{dx^\mu}{dt}\right)\left(\frac{dx^\nu}{dt}\right)}. \tag{21}$$

Using the metric components in Eqs. (17)–(20), we get

$$\mathcal{L} \simeq \frac{1}{2}v^2 + \frac{1}{8}v^4 - \phi_{\rm N} - \frac{1}{2}\phi_{\rm N}^2 - \psi - \phi_\Lambda - \frac{3}{2}\phi_{\rm N}v^2 + v^i\left(\xi_i + \frac{\partial\chi}{\partial t\partial x^i}\right). \tag{22}$$

The corresponding Euler-Lagrange equations of motion in a 3-dimensional notation read,

$$\frac{d\mathbf{v}}{dt} \simeq -\nabla(\phi_{\rm N} + 2\phi_{\rm N}^2 + \psi) - \frac{\partial\boldsymbol{\xi}}{\partial t} - \frac{\partial^2}{\partial t^2}\nabla\chi + \mathbf{v}\times(\nabla\times\boldsymbol{\xi}) + 3\mathbf{v}\frac{\partial\phi_{\rm N}}{\partial t} + 4\mathbf{v}(\mathbf{v}\cdot\nabla)\phi_{\rm N} - \mathbf{v}^2\nabla\phi_{\rm N} + \frac{\Lambda}{3}\mathbf{x}. \tag{23}$$

The above expression reduces to Eq. (20) in [4] when neglecting pN corrections.

III. THE EINSTEIN-INFELD-HOFFMANN EQUATIONS

Since the contribution from the cosmological constant is of higher-order, it does not couple with other corrections. The Lagrangian of an N-body system of pointlike particles can be written as

$$\mathcal{L} \simeq \mathcal{L}_{(\Lambda=0)} + \delta\mathcal{L}_\Lambda, \tag{24}$$

where $\mathcal{L}_{(\Lambda=0)}$ is the total Lagrangian in absence of Λ. The Lagrangian $\mathcal{L}_a$ of particle a in the field of other particles is

$$\mathcal{L}_a \simeq \mathcal{L}_{a(\Lambda=0)} + \frac{\Lambda}{6}\mathbf{x}_a^2, \tag{25}$$

where $\mathcal{L}_{a(\Lambda=0)}$ is given in Eq. (5.94) in [13]. The total Lagrangian reads

$$\mathcal{L} \simeq \mathcal{L}_{(\Lambda=0)} + \sum_a \frac{\Lambda}{6} m_a \mathbf{x}_a^2, \tag{26}$$

with $\mathcal{L}_{(\Lambda=0)}$ given in [[13], Eq. (5.95)]. The corresponding Euler-Lagrange equations are the Einstein-Infeld-Hoffmann equations corrected for a Λ term,

$$\dot{\mathbf{v}}_a = -G\sum_{b\neq a}\left(\frac{\mathbf{x}_{ab}}{r_{ab}}\right) + \delta\mathbf{F}_{{\rm pN}(\Lambda=0)} + \frac{\Lambda}{3}\mathbf{x}_a \tag{27}$$

where $\mathbf{F}_{{\rm pN}(\Lambda=0)}$ is the post-Newtonian perturbing function [[13], Eq. (5.96)].

IV. THE TWO-BODY PROBLEM

The total Lagrangian for two particles can be written as

$$\mathcal{L} \simeq \frac{1}{2}m_{\rm a}v_{\rm a}^2 + G\frac{m_{\rm a}m_{\rm b}}{x} + \frac{1}{2}m_{\rm b}v_{\rm b}^2 + \delta\mathcal{L}_{{\rm pN}(\Lambda=0)} + \delta\mathcal{L}_\Lambda \tag{28}$$

where $\mathbf{x} \equiv \mathbf{x}_{\rm a} - \mathbf{x}_{\rm b}$ is the separation vector and $\delta\mathcal{L}_{{\rm pN}(\Lambda=0)}$ and $\delta\mathcal{L}_\Lambda$ are the pN and Λ-contributions, respectively. It is [13]

$$\delta\mathcal{L}_{{\rm pN}(\Lambda=0)} = \frac{1}{8}(m_{\rm a}v_{\rm a}^4 + m_{\rm b}v_{\rm b}^4) + G\frac{m_{\rm a}m_{\rm b}}{2r}[3(v_{\rm a}^2 + v_{\rm b}^2) - 7\mathbf{v}_{\rm a}\dot{\mathbf{v}}_{\rm b} - (\mathbf{v}_{\rm a}\dot{\mathbf{n}})(\mathbf{v}_{\rm b}\dot{\mathbf{n}})] - \frac{G^2}{2}\frac{m_{\rm a}m_{\rm b}(m_{\rm a}+m_{\rm b})}{x^2} \tag{29}$$

with $\mathbf{n} \equiv \mathbf{x}/x$ and

$$\delta\mathcal{L}_\Lambda = \frac{\Lambda}{6}(m_{\rm a}x_{\rm a}^2 + m_{\rm b}x_{\rm b}^2). \tag{30}$$

Because of cosmological constant, the energy of the system is modified by a contribution $-\delta\mathcal{L}_\Lambda$. The pN and Λ corrections are additive and can be treated separately. We are interested in examining the effect of a non vanishing Λ term. Let us consider the center of mass and relative motions. Introducing $\mathbf{X} \equiv (m_{\rm a}\mathbf{x}_{\rm a} + m_{\rm b}\mathbf{x}_{\rm b})/M$, with $M \equiv m_{\rm a} + m_{\rm b}$, the Lagrangian can be rewritten as

$$\mathcal{L} \simeq \frac{1}{2}MV^2 + \frac{\Lambda}{6}MX^2 + \frac{1}{2}\mu v^2 + \frac{\Lambda}{6}\mu x^2 + G\frac{M\mu}{x}, \tag{31}$$

with $\mu \equiv m_{\rm a}m_{\rm b}/M$. Because of cosmological constant, the center of mass of the system is subject to an effective repulsive force given by $\Lambda\mathbf{X}/3$ per unit mass.

The equations for the relative motion are those of a test particle in a Schwarzschild-de Sitter space-time with a source mass equal to the total mass of the two-body system. Since the perturbation due to Λ is radial, the orbital angular momentum is conserved and the orbit is planar. The main effect of Λ on the orbital motion is a precession of the pericentre [[14,16] and references therein]. Following the analysis of the Rung-Lenz vector in [4] and restoring the c factors, we get for the contribution to the precession angular velocity due to Λ,

$$\dot{\omega}_{\Lambda} = \frac{\Lambda c^2 P_b}{4\pi}\sqrt{1-e^2}, \tag{32}$$

where e is the eccentricity and P_b the Keplerian period of the unperturbed orbit. This contribution should be considered together with the post-Newtonian periastron advance, $\dot{\omega}_{pN} = 3(2\pi/P_b)^{5/3}(GM/c^3)^{2/3}(1-e^2)^{-1}$. The ratio between these two contributions can be written as,

$$\frac{\dot{\omega}_{\Lambda}}{\dot{\omega}_{pN}} = \frac{\bar{R}}{R_g}\frac{\rho_{\Lambda}}{\rho} = \frac{1}{6}\frac{\bar{R}^4}{R_g^2}\Lambda, \tag{33}$$

where $\bar{R} = a(1-e^2)^{3/8}$ is a typical orbital radius with a the semimajor radius of the unperturbed orbit, $\rho \equiv M/(4\pi\bar{R}^3/3)$ is a typical density of the system and $\rho_{\Lambda} \equiv c^2\Lambda/8\pi G$ is the energy density associated to the cosmological constant. The effect of Λ can be significant for very wide systems with a very small mass.

V. OBSERVATIONAL CONSTRAINTS

In this section, we derive observational limits on Λ from orbital precession shifts in stellar systems and in the solar system.

A. Interplanetary measures

Precessions of the perihelia of the solar system planets have provided the most sensitive local tests for a cosmological constant so far [4–6]. Estimates of the anomalous perihelion advance were recently determined for Mercury, Earth and Mars [17,18]. Such ephemerides were constructed integrating the equation of motion for all planets, the Sun, the Moon and largest asteroids and including rotations of the Earth and of the Moon, perturbations from the solar quadrupole mass moment and asteroid ring in the ecliptic plane. Extra-corrections to the known general relativistic predictions can be interpreted in terms of a cosmological constant effect. We considered the 1-σ upper bounds. Results are listed in Table I. Best constraints come from Earth and Mars observations, with $\Lambda \lesssim 10^{-36}$ km^{-2}. Major sources of systematic errors come from uncertainties about solar oblateness and from the gravito-magnetic contribution to secular advance of perihelion but their effect could be in principle accounted for [19]. In particular, the general relativistic Lense-Thirring secular precession of perihelia is compatible with the determined extra-precessions [19]. The accuracy in determining the planetary orbital motions will further improve with data from space-missions like BepiColombo, Messenger and Venus express. By considering a post-Newtonian dynamics inclusive of gravito-magnetic terms, the resulting residual extra-precessions should be reduced by several orders of magnitude, greatly improving the upper bound on Λ.

The orbital motion of laser-ranged satellites around the Earth has been also considered to confirm general relativistic predictions. Observations of the rates of change of the nodal longitude of the LAGEOS satellites allowed to probe the Lense-Thirring effect with an accuracy of ~10%, i.e. about half a milliarcsecond per year [20]. Other proposed missions, such as the LARES/WEBER-SAT satellite [21], should further increase this experimental precision. In general, since effects of Λ become significant only for very dilute systems, even very accurate measurements of orbital elements of Earth's satellites can not help in constraining the cosmological constant. For a satellite with a typical orbital semimajor axis of about 12 000 km, in order to get a bound on Λ as accurate as those inferred from Earth and Mars perihelion shifts (i.e. $\Lambda \lesssim 10^{-36}$ km^{-2}), changes in orbital elements should be measured with a today unattainable precision of a few tens of picoseconds of arc per year, about 6 order of magnitude better than today accuracy.

B. Binary pulsars

Binary pulsars have been providing unique possibilities of probing gravitational theories. Relativistic corrections to the binary equations of motion can be parameterized in terms of post-Keplerian parameters [22]. As seen before, the advance of periastron of the orbit, $\dot{\omega}$, depends on the total mass of the system and on the cosmological constant. In principle, because Keplerian orbital parameters such as the eccentricity e and the orbital period P_b can be separately measured, the measurement of $\dot{\omega}$ together with any two other post-Keplerian parameters would provide three constraints on the two unknown masses and on the cosmological constant. As a matter of fact for real systems, the effect of Λ is much smaller than $\dot{\omega}_{pN}$, so that only upper bounds on the cosmological constant can be obtained by

TABLE I. Limits on the cosmological constant due to extra-precession of the inner planets of the solar system.

Name	$\delta\dot{\omega}^a$ (arcsec/year)	$\dot{\omega}_{\Lambda}$ (deg/year)	Λ_{lim}(km^{-2})
Mercury	$-0.36(50) \times 10^{-4}$	$9.61 \times 10^{25}\Lambda/(1\ \text{km}^{-2})$	4×10^{-35}
Venus	$0.53(30) \times 10^{-2}$	$2.51 \times 10^{26}\Lambda/(1\ \text{km}^{-2})$	9×10^{-33}
Earth	$-0.2(4) \times 10^{-5}$	$4.08 \times 10^{26}\Lambda/(1\ \text{km}^{-2})$	1×10^{-36}
Mars	$0.1(5) \times 10^{-5}$	$7.64 \times 10^{26}\Lambda/(1\ \text{km}^{-2})$	2×10^{-36}

[a]From [17]

TABLE II. Binary pulsars with known post-Keplerian parameter $\dot{\omega}$ and corresponding limits on the cosmological constant. The identification of the companion is often uncertain. We refer to the original papers for a complete discussion.

PSR Name	P_b (days)	e	$\dot{\omega}$ (deg/year)	$\dot{\omega}_\Lambda$ (deg/year)	$\Lambda_{\rm lim}({\rm km}^{-2})$	Ref.
			Double neutron star binaries			
J1518 + 4904	8.634000485	0.2494849	0.0111(2)	$9.335 \times 10^{24}\Lambda/(1\ {\rm km}^{-2})$	2×10^{-29}	[23]
B1534 + 12	0.2736767	0.420737299153	1.755805(3)	$2.772 \times 10^{23}\Lambda/(1\ {\rm km}^{-2})$	1×10^{-29}	[24]
B1913 + 16	0.323	0.617	4.226595(5)	$2.838 \times 10^{23}\Lambda/(1\ {\rm km}^{-2})$	2×10^{-29}	[24]
J1756 − 2251	0.319633898	0.180567	2.585(2)	$3.510 \times 10^{23}\Lambda/(1\ {\rm km}^{-2})$	6×10^{-27}	[25]
J1811 − 1736	18.779168	0.82802	0.009(2)	$1.176 \times 10^{25}\Lambda/(1\ {\rm km}^{-2})$	2×10^{-28}	[26]
J1829 + 2456	1.176028	0.13914	0.28(1)	$1.300 \times 10^{24}\Lambda/(1\ {\rm km}^{-2})$	8×10^{-27}	[27]
B2127 + 11C	0.68141	0.335282052	4.457(12)	$7.168 \times 10^{23}\Lambda/(1\ {\rm km}^{-2})$	2×10^{-26}	[24]
B2303 + 46	12.34	0.65837	0.01019(13)	$1.037 \times 10^{25}\Lambda/(1\ {\rm km}^{-2})$	1×10^{-29}	[28]
			Neutron star/white dwarf binaries			
J0621 + 1002	8.3186813	0.00245744	0.0116(8)	$9.288 \times 10^{24}\Lambda/(1\ {\rm km}^{-2})$	9×10^{-29}	[29]
J1141 − 6545	0.171876	0.1976509587	5.3084(9)	$1.881 \times 10^{23}\Lambda/(1\ {\rm km}^{-2})$	5×10^{-27}	[24]
J1713 + 0747	67.82512987	0.0000749406	0.0006(4)[a]	$7.573 \times 10^{25}\Lambda/(1\ {\rm km}^{-2})$	8×10^{-30}	[30]
B1802 − 07	2.617	0.212	0.0578(16)	$2.856 \times 10^{24}\Lambda/(1\ {\rm km}^{-2})$	6×10^{-28}	[28]
J1906 + 0746	0.085303(2)	0.165993045(8)	7.57(3)	$9.392 \times 10^{22}\Lambda/(1\ {\rm km}^{-2})$	3×10^{-25}	[31]
			Double pulsars			
J0737 − 3039	0.087779	0.102251563	16.90(1)	$9.750 \times 10^{22}\Lambda/(1\ {\rm km}^{-2})$	1×10^{-25}	[24]
			Unknown companion			
B1820 − 11	357.7622(3)	0.79462(1)	0.01[a]	$2.425 \times 10^{26}\Lambda/(1\ {\rm km}^{-2})$	4×10^{-29}	[32]

[a]Upper limit

considering the uncertainty on the observed periastron shift. We considered binary systems with measured periastron shift, see Table II. The effect of Λ is maximum for B1820-11 and PSR J1713 + 0747. Despite of the low accuracy in the measurement of $\dot{\omega}$, PSR J1713 + 0747 provides the best constraint on the cosmological constant, $\Lambda \lesssim 8 \times 10^{-30}\ {\rm km}^{-2}$. Uncertainties as low as $\delta\dot{\omega} \gtrsim 10^{-6}$ have been achieved for very well observed systems, such as B1913 + 16 and B1534 + 12. Such an accuracy for B1820 − 11 would allow to push the bound on Λ down to $10^{-33}\ {\rm km}^{-2}$.

Better constraints could be obtained by determining post-Keplerian parameters in very wide binary pulsars. We examined systems with known period and eccentricity as reported in [33]. The binary pulsar having the most favorable orbital properties for better constraining Λ is the low eccentricity B0820 + 02, located in the Galactic disk, with $\dot{\omega}_\Lambda \sim 1.4 \times 10^{27}\Lambda/(1\ {\rm km}^{-2})$ deg/days. For binary pulsars J0407 + 1607, B1259 − 63, J1638 − 4715 and J2016 + 1948, the advance of periastron due to the cosmological constant is between 7 and $9 \times 10^{26}\Lambda/(1\ {\rm km}^{-2})$ deg/days. All of these shifts are of similar value or better than the Mars one. A determination of $\dot{\omega}$ for B0820 + 02 with the accuracy obtained for B1913 + 16, i.e. $\delta\dot{\omega} \gtrsim 10^{-6}$ deg/days would allow to push the upper bound down to $10^{-34} - 10^{-33}\ {\rm km}^{-2}$.

VI. CONCLUSIONS

We considered the N-body equations of motion in presence of a cosmological constant. The impact of Λ on the two-body system was explicitly derived. Because of the antigravity effect of the cosmological constant, the barycentre of the system drifts away. The relative motion is like that of a test particle in a Schwarzschild-de Sitter spacetime with a source mass equal to the total mass of the two-body system. The main effect of Λ is the precession of the pericentre on the orbital motion.

We determined observational limits on the cosmological constant from measured periastron shifts. With respect to previous similar analyses performed in the past on solar system planets, our estimate was based on a recent determination of the planetary ephemerides properly accounting for the quadrupole moment of the Sun and for major asteroids. The best constraint comes from Mars and Earth, $\Lambda \lesssim 1 - 2 \times 10^{-36}\ {\rm km}^{-2}$.

Because of the experimental accuracy, observational limits on Λ from binary pulsars are still not competitive with results from interplanetary measurements in the solar system. Accurate pericentre advance measurements in wide systems with orbital periods $\gtrsim 600$ days could give an upper bound of $\Lambda \lesssim 10^{-34} - 10^{-33}\ {\rm km}^{-2}$, if determined with the accuracy performed for B1913 + 16, i.e. $\delta\dot{\omega} \gtrsim 10^{-6}$ deg/years. For some binary pulsars, observations with an accuracy comparable to that achieved in the solar system could allow to get an upper limit on Λ as precise as one obtains from Mars data.

The bound on Λ from Earth or Mars perihelion shift is nearly $\sim 10^{10}$ times weaker than the determination from observational cosmology, $\Lambda \sim 10^{-46}\ {\rm km}^{-2}$, but it still gets some relevance. The cosmological constant might be the non perturbative trace of some quantum gravity aspect in the low energy limit [1]. Λ is usually related to the vacuum energy density, whose properties depends on the scale at which it is probed [1]. So that, in our opinion, it is still interesting to probe Λ on a scale of order of astronomical unit. Measurements of periastron shift should be much better in the next years. New data from space-missions should get a very high accuracy and might probe spin effects on the orbital motion [19,34]. A proper consideration of the gravito-magnetic effect in these analyses plays a central role to improve the limit on Λ by several orders of magnitude.

ACKNOWLEDGMENTS

The authors thank N. Straumann for stimulating discussions. M. S. is supported by the Swiss National Science Foundation and by the Tomalla Foundation.

Note added.— After submission of this work, L. Iorio [35] presented an analysis of solar system data similar to our results in section VA.

[1] T. Padmanabhan, astro-ph/0510492.
[2] P. J. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).
[3] P. Jetzer and N. Straumann, Phys. Lett. B **606**, 77 (2005).
[4] A. W. Kerr, J. C. Hauck, and B. Mashhoon, Classical Quantum Gravity **20**, 2727 (2003).
[5] J. N. Islam, Phys. Lett. A **97**, 239 (1983).
[6] E. L. Wright, astro-ph/9805292.
[7] M. Nowakowski, J.-C. Sanabria, and A. Garcia, Phys. Rev. D **66**, 023003 (2002).
[8] A. Balaguera-Antolínez and M. Nowakowski, Astron. Astrophys. **441**, 23 (2005).
[9] P. Teerikorpi, A. D. Chernin, and Y. V. Baryshev, Astron. Astrophys. **440**, 791 (2005).
[10] R. A. Hulse and J. H. Taylor, Astrophys. J. Lett. **195**, L51 (1975).
[11] A. Einstein, L. Infeld, and B. Hoffman, Ann. Math. **39**, 65 (1938).
[12] B. M. Barker and R. F. O'Connell, Phys. Rev. D **12**, 329 (1975).
[13] N. Straumann, *General Relativity With Applications to Astrophysics* (Springer, Berlin, 2004).
[14] M. Nowakowski, Int. J. Mod. Phys. D **10**, 649 (2001).
[15] R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity* (McGraw-Hill, New York, 1965).
[16] G. V. Kraniotis and S. B. Whitehouse, Class. Quant. Grav. **20**, 4817 (2003).
[17] E. V. Pitjeva, Astron. Lett. **31**, 340 (2005).
[18] E. V. Pitjeva, Solar System Research **39**, 176 (2005).
[19] L. Iorio, gr-qc/0507041.
[20] I. Ciufolini and E. C. Pavlis, Nature (London) **431**, 958 (2004).
[21] I. Ciufolini, gr-qc/0412001.
[22] C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1993) 2nd ed.
[23] D. J. Nice, R. W. Sayer, and J. H. Taylor, Astrophys. J. Lett. **466**, L87 (1996).
[24] C. M. Will, gr-qc/0510072.
[25] A. J. Faulkner, M. Kramer, A. G. Lyne, R. N. Manchester, M. A. McLaughlin, I. H. Stairs, G. Hobbs, A. Possenti, D. R. Lorimer, N. D'Amico *et al.*, Astrophys. J. Lett. **618**, L119 (2005).
[26] A. G. Lyne, F. Camilo, R. N. Manchester, J. F. Bell, V. M. Kaspi, N. D'Amico, N. P. F. McKay, F. Crawford, D. J. Morris, D. C. Sheppard, *et al.*, Mon. Not. R. Astron. Soc. **312**, 698 (2000).
[27] D. J. Champion, D. R. Lorimer, M. A. McLaughlin, J. M. Cordes, Z. Arzoumanian, J. M. Weisberg, and J. H. Taylor, Mon. Not. R. Astron. Soc. **350**, L61 (2004).
[28] S. E. Thorsett and D. Chakrabarty, Astrophys. J. **512**, 288 (1999).
[29] E. M. Splaver, D. J. Nice, Z. Arzoumanian, F. Camilo, A. G. Lyne, and I. H. Stairs, Astrophys. J. **581**, 509 (2002).
[30] E. M. Splaver, D. J. Nice, I. H. Stairs, A. N. Lommen, and D. C. Backer, Astrophys. J. **620**, 405 (2005).
[31] D. R. Lorimer, I. H. Stairs, P. C. C. Freire, J. M. Cordes, F. Camilo, A. J. Faulkner, A. G. Lyne, D. J. Nice, S. M. Ransom, Z. Arzoumanian *et al.*, astro-ph/0511523.
[32] A. G. Lyne and J. McKenna, Nature (London) **340**, 367 (1989).
[33] D. R. Lorimer, Living Rev. Relativity **8**, 7 (2005).
[34] R. F. O'Connell, Phys. Rev. Lett. **93**, 081103 (2004).
[35] L. Iorio (to be published).