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Quasilocal formalism and black-ring thermodynamics

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The thermodynamical properties of a dipole black ring are derived using the quasilocal formalism. We find that the dipole charge appears in the first law in the same manner as a global charge. Using the Gibbs-Duhem relation, we also provide a nontrivial check of the entropy/area relationship for the dipole ring. A preliminary study of the thermodynamic stability indicates that the neutral ring is unstable to angular fluctuations.

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I. INTRODUCTION

Not many objects in physics are as fascinating and intriguing as black holes. The relationship between thermodynamic entropy and the area of an event horizon is one of the most robust and surprising results in gravitational physics. Even more surprisingly is the fact that 4-dimensional black holes are highly constrained objects. That is, an *isolated* electrovac black hole can be characterized, *uniquely* and *completely*, by just three macroscopic parameters [1]: its mass, angular momentum, and charge.[1] There are no black objects with an electric dipole in four dimensions. The black holes have ‘‘smooth‘‘ horizons (there are no ripples or higher multipoles) and are clasically stable. Moreover, for asymptotically flat solutions, the event horizons of nonspherical topology are forbidden.

Gravity in higher dimensions—an important active area in both string theory and particle physics—has a much richer spectrum of black objects than in four dimensions. For example, the vacuum black ring found by Emparan and Reall in Ref. [2] has a nonspherical event horizon of topology $S^2 \times S^1$. It was also explicitly proved in Ref. [2] a ‘‘discrete‘‘ nonuniqueness: There is a range of values for the mass and angular momentum for which there exist *three* solutions, a black hole and two black rings. A more dramatic ‘‘continuous‘‘ violation of ‘‘uniqueness‘‘ was presented in Ref. [3]. The solution describes a stationary black ring *electrically* coupled to a 2-form potential and a dilaton. The ring creates a field analogous to a dipole, with no net charge measured at infinity. In this way, a family of black rings differing only by their dipole charge is obtained.[2] Then it is clear that, unlike four dimensions, in higher dimensions not all black hole equilibrium configurations are completely characterized by a few *asymptotic* conserved charges. It is not yet known if these solutions are stable.

The black-ring solutions satisfy the first law of black hole mechanics, thus suggesting that their entropy is also one quarter of the event horizon area. For dipole black rings, the novelty is that the dipole charge enters the first law in the same manner as an ordinary global charge [3–5]. A derivation of the first law of black-ring solutions based on the Hamiltonian formalism was presented in Ref. [4].

In this paper, we will take a slightly different route in deriving the first law for the dipole ring. Our proposal is to compute the thermodynamical quantities by employing the quasilocal formalism of Brown and York [6] supplemented by boundary counterterms. In this way, the difficulties associated with the choice of a reference background for a rotating spacetime in the presence of matter fields are avoided.

II. THE GENERAL FRAMEWORK

It is well known that the gravitational action contains divergences even at tree-level — they arise from integrating over the infinite volume of spacetime. For 5-dimensional asymptotically flat solutions with a boundary topology $S^3 \times R$, the action can be regularized by the following counterterm [7]:

$$I_{ct} = -\frac{1}{8\pi G}\int_{\partial M} d^4x\sqrt{-h}\sqrt{\frac{3}{2}\mathcal{R}}, \tag{1}$$

where $\mathcal{R}$ is the Ricci scalar of the induced metric on the boundary h_{ij}. Varying the total action (which contains the Gibbons-Hawking boundary term) with respect to the boundary metric h_{ij}, we compute the divergence-free boundary stress-tensor

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[1]The classical uniqueness results do not apply to black holes with *degenerate* horizons.

[2]In fact this is true at the classical level. Once the charges are quantized in terms of the brane numbers, a discrete family of dipole rings is obtained.

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$$T_{ij} = \frac{2}{\sqrt{-h}}\frac{\delta I}{\delta h^{ij}} = \frac{1}{8\pi G}(K_{ij} - h_{ij}K - \Psi(\mathcal{R}_{ij} - \mathcal{R}h_{ij}) - h_{ij}\Box\Psi + \Psi_{;ij}), \tag{2}$$

where K_{ij} is the extrinsic curvature of the boundary and $\Psi = \sqrt{\frac{3}{2\mathcal{R}}}$. Provided the boundary geometry has an isometry generated by a Killing vector ξ^i, a conserved charge

$$\mathfrak{Q}_\xi = \oint_\Sigma d^3 S^i \xi^j T_{ij} \tag{3}$$

can be associated with a closed surface Σ [8]. Physically, this means that a collection of observers on the hypersurface whose metric is h_{ij} all observe the same value of $\mathfrak{Q}_\xi$ provided this surface has an isometry generated by ξ. For example, if $\xi = \partial/\partial t$ then $\mathfrak{Q}$ is the conserved mass/energy M.

Upon continuation to imaginary time, the gravitational thermodynamics is then formulated via the Euclidean path integral. The thermodynamic system has a constant temperature $T = 1/\beta$ which is determined by requiring the Euclidean section be free of conical singularities. In a very basic sense, gravitational entropy can be regarded as arising from the Gibbs-Duhem relation applied to the path-integral formulation of quantum gravity [8]. The total action I is evaluated from the classical solution to the field equations, which yields an expression for the entropy

$$S = \beta(\mathfrak{M} - \mu_i \mathfrak{C}_i) - I, \tag{4}$$

upon application of the Gibbs-Duhem relation to the partition function [8] (with chemical potentials $\mathfrak{C}_i$ and conserved charges μ_i). The first law of thermodynamics is then

$$dS = \beta(d\mathfrak{M} - \mu_i d\mathfrak{C}_i). \tag{5}$$

However, we will find that a key point in our intuition about Euclidean sections does not apply to black rings—there is no real *nonsingular* Euclidean section in this case. Nevertheless, as argued in Ref. [9], these configurations still can be described by a complex geometry and a real action (for other examples, see Refs. [10,11]).

III. ASYMPTOTIC CONSERVED CHARGES FOR THE DIPOLE BLACK RING

For a detailed study of the dipole ring we refer the reader to Ref. [3], whose notation we follow. The line element of this solution is written as

$$\begin{aligned} ds^2 = &-\frac{F(y)}{F(x)}\left(\frac{H(x)}{H(y)}\right)^{N/3}\left(dt + C(\nu,\lambda)R\frac{1+y}{F(y)}d\psi\right)^2 \\ &+ \frac{R^2}{(x-y)^2}F(x)(H(x)H(y)^2)^{N/3}\Big[-\frac{G(y)}{F(y)H(y)^N}d\psi^2 \\ &- \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)H(x)^N}d\varphi^2\Big], \end{aligned} \tag{6}$$

where

$$F(\xi) = 1 + \lambda\xi, \qquad G(\xi) = (1-\xi^2)(1+\nu\xi), \qquad H(\xi) = 1 - \mu\xi, \tag{7}$$

and $C(\nu,\lambda) = \sqrt{\lambda(\lambda-\nu)(1+\lambda)/(1-\lambda)}$. The constant N which enters the above relations is related to the dilaton coupling constant α through $N = (\alpha^2/4 + 1/3)^{-1}$. The values $N = 1, 2, 3$ are of particular relevance to string and M-theory.

The coordinates x and y vary within the ranges $-1 \le x \le 1$, $-\infty < y \le -1$, while R, λ, μ and ν are real parameters with $0 < \nu \le \lambda < 1$. To avoid conical singularities at $x = -1$ and $y = -1$ one sets

$$\Delta\psi = \Delta\varphi = 2\pi\frac{(1+\mu)^{N/2}\sqrt{1-\lambda}}{1-\nu}, \tag{8}$$

while the singularity at $x = +1$ is avoided by requiring

$$\frac{1-\lambda}{1+\lambda}\left(\frac{1+\mu}{1-\mu}\right)^N = \left(\frac{1-\nu}{1+\nu}\right)^2. \tag{9}$$

With these choices, the solution has a regular horizon at $y = -1/\nu$, of topology $S^1 \times S^2$ and area $\mathcal{A}_H = 8\pi^2 R^3(1+\mu)^N \nu^{(3-N)/2}(\mu+\nu)^{N/2}\sqrt{\lambda(1-\lambda^2)}/(1-\nu)^2(1+\nu)$, an ergosurface of the same topology at $y = -1/\lambda$, and an inner spacelike singularity at $y = -\infty$. Asymptotic spatial infinity is reached as $x \to y \to -1$.

The dilaton $\tilde{\phi}$ and the twoform potential are given by

$$e^{\tilde{\phi}} = \left(\frac{H(x)}{H(y)}\right)^{N\alpha/2},$$

$$B_{t\psi} = C(\nu, -\mu)\sqrt{N}R(1+y)/H(y) + k,$$

where k is a constant. The main observation in Ref. [4] is that the constant k is not arbitrary. Usually, the gauge potential is globally defined and nonsingular everywhere outside (and on) the horizon. However, Copsey and Horowitz have shown that this is incompatibile with the assumptions that the dipole charge is nonzero and that B is invariant under the spacetime symmetries. The constant k must be chosen so that $B_{t\psi}(y = -1) = 0$ and implies that $B_{\mu\nu}$ necessarily diverge at the horizon. For our analysis it is important to note that this is just a purely gauge effect — the physical field 3-form $H = dB$ remains finite at the horizon.

To evaluate asymptotic expressions at spacelike infinity, it is convenient to introduce coordinates in which the

asymptotic flatness of the solution becomes manifest. Our choice for this transformation is

$$x = 1 - \frac{2r^2}{r^2 + R^2\cos^2\theta}, \qquad y = 1 - \frac{2(r^2 + R^2)}{r^2 + R^2\cos^2\theta}, \tag{10}$$

r corresponding to a normal coordinate on the boundary, $0 \le r < \infty$, $0 \le \theta \le \pi/2$. In these coordinates, the black ring approaches asymptotically the Minkowski background $ds^2 = d\bar{r}^2 + \bar{r}^2(d\theta^2 + \sin^2\theta d\bar{\psi}^2 + \cos^2\theta d\bar{\varphi}^2) - dt^2$, where $\bar{\varphi}$ and $\bar{\psi}$ are angular coordinates rescaled according to Eq. (8) and $\bar{r} = ((1-\lambda)/(1-\nu))^{1/2} r$.

The mass M and angular momentum J of the black-ring solution can be computed by employing the quasilocal formalism. The relevant components of the boundary stress tensor are

$$\begin{aligned} 8\pi G T^t_\psi &= \frac{4R^3 C(\nu,\lambda)}{(1-\lambda)}\sqrt{\frac{(1+\mu)^{-N}(1-\nu)}{1-\lambda}}\frac{\sin^2\theta}{r^3} \\ &\quad + O(1/r^4), \\ 8\pi G T^t_t &= \frac{R^2(1+\mu)^{-1-N/2}}{(1-\lambda)}(N\mu + \lambda(3-(N-3)\mu)) \\ &\quad \times \sqrt{\frac{1-\nu}{1-\lambda}}\frac{1}{r^3} + O(1/r^4). \end{aligned}$$

Using Eq. (3) we obtain the following expressions for mass and angular momentum of the dipole ring solution

$$M = \frac{3\pi R^2}{4G}\frac{(1+\mu)^N}{1-\nu}\left(\lambda + \frac{N}{3}\frac{\mu(1-\lambda)}{1+\mu}\right), \tag{11}$$

$$J = \frac{\pi R^3}{2G}\frac{(1+\mu)^{3N/2}\sqrt{\lambda(\lambda-\nu)(1+\lambda)}}{(1-\nu)^2}, \tag{12}$$

which match the ADM values computed in Ref. [3].

IV. THE DIPOLE RING ACTION

The partition function for the gravitational field is defined by a sum over all smooth Euclidean geometries which are periodic with period β in imaginary time. This integral is computed by using the saddle-point approximation. The energy and entropy are evaluated by standard thermodynamic relations. Naively, one may expect to find a real Euclidean section for a black-ring solution by using the analytical continuation $t \to i\tau$ supplemented with $C \to i\bar{C}$. However, it can be verified that the conical singularities at $x = \pm 1$, $y = -1$ of the Euclidean line element cannot be removed for any choice of (λ, ν, μ) which assures a real $\bar{C}$. Therefore, we are forced to work with a complex geometry.

We adopt here the "quasi-Euclidean" method of Ref. [9] in which the Wick transformations affect the intensive variables, such as the lapse and shift ($N \to -iN$ and $N^k \to -iN^k$), but for which the extensive variables (such as energy) remain invariant. It is important to be mentioned that the Cauchy data and the equations of motion remain invariant under this complexification.[3] Starting with the ring metric in the canonical (ADM) form we obtain the following "quasi-Euclidean" section:

$$ds^2 = N^2 d\tau^2 + \gamma_{ij}(dy^i - iN^i d\tau)(dy^j - iN^j d\tau).$$

No singularities are found on this "quasi-Euclidean" section—the conical singularities at $y = -1, x = \pm 1$ are avoided by taking the same periodicity for ψ and φ together with Eq. (9). One has also to identify τ with a period β to make the metric regular on the horizon. A detailed analysis shows that the periodicity β and the shift vector at the horizon N^ψ_H reproduce the inverse of the Hawking temperature and the angular velocity of the horizon, respectively, as computed in Ref. [3]

$$\begin{aligned} \beta &= \frac{4\pi R(\mu+\nu)^{N/2}}{\nu^{(N-1)/2}(1+\nu)}\sqrt{\frac{\lambda(1+\lambda)}{1-\lambda}}, \\ \Omega_H &= \frac{1}{R}\frac{1}{(1+\mu)^{N/2}}\sqrt{\frac{\lambda-\nu}{\lambda(1+\lambda)}}. \end{aligned} \tag{13}$$

It is convenient to use the r, θ coordinates, as defined by Eq. (10), in order to compute the boundary terms (Gibbons-Hawking plus the counterterm) contribution to the total action. In the large r limit we find the following finite expression:

$$\begin{aligned} I_{\partial B} &= \frac{\pi^2 R^3}{3G(1-\nu^2)}\left(3\lambda\sqrt{\frac{\lambda\nu(1+\lambda)}{(1-\lambda)}} + \sqrt{\frac{\lambda(1+\lambda)}{(1-\lambda)}}\right. \\ &\quad \times (1+\mu)^{N-1}(-N\mu + \lambda(-3+N)\mu)\nu^{1/2-N/2} \\ &\quad \left. \times (\mu+\nu)^{N/2}\right). \end{aligned} \tag{14}$$

The tree-level bulk action is computed by using the trace of the Einstein equations

$$\begin{aligned} I_B &= \frac{1}{16\pi G}\int_M d^5x\sqrt{-g}\left(R - \frac{1}{2}(\partial\tilde{\phi})^2 - \frac{1}{12}e^{-\alpha\tilde{\phi}}H^2\right) \\ &= -\frac{1}{16\pi G}\int d^5x\sqrt{-g}e^{-\alpha\tilde{\phi}}\frac{1}{9}H^2. \end{aligned} \tag{15}$$

This volume integral evaluated on the Euclidean section takes a simple form expressed in terms of the dipole charge q and the potential Φ defined as

[3]Note that since the energy and angular momentum are described by three-surface integrals over the Cauchy data, they remain invariant and real under this complexification.

$$q = \frac{1}{4\pi}\int_{S^2} e^{-\alpha\tilde{\phi}} * H = R\sqrt{N}\frac{(1+\mu)^{(N-1)/2}\sqrt{\mu(\mu+\nu)(1-\lambda)}}{(1-\nu)\sqrt{1-\mu}}, \quad (16)$$

$$\Phi = \frac{\pi}{2G}(-B_{t\tilde{\psi}}(y=-1/\nu)) = \frac{\pi R}{2G}\sqrt{N}\frac{(1+\mu)^{(N-1)/2}\sqrt{\mu(1-\mu)(1-\lambda)}}{\sqrt{\mu+\nu}}, \quad (17)$$

where S^2 is a surface of constant t, y and ψ in the metric (6). Then, the bulk contribution is $I_B = \frac{2}{3}\beta q\Phi$. It is important to precisely point out the nature of the dipole charge. A string naturally couples to a 2-form gauge potential. The special case $N = 1$ is the NS sector of low-energy string theory. Then, a fundamental string that carries *electric* Kalb-Ramond charge is a solution of our theory. The string charge is localized on the string and the charge density can be visualized as a current on the string. Since the string winds around a *contractible* circle, no monopole term will appear in the multipole expansion for the field. Therefore, the *local* charge[4] (16) has a natural interpretation as a source of the dipole field.

The total action I is given by

$$I = I_B + I_{\partial B} = \frac{\pi^2 R^3}{4G(1-\nu)}(1+\mu)^{N-1}(\lambda(1+\mu) - N\mu(1-\lambda)), \quad (18)$$

which is a strictly positive quantity. For a grand-canonical ensemble (*i.e.* for fixed temperature, angular velocity, and gauge potential), using the definition of the Gibbs potential $G(T, \Omega_H, \Phi) = I/\beta$, Eq. (12) for the angular velocity, and Eq. (17) for the potential Φ, we obtain

$$G(T, \Omega_H, \Phi) = M - \Omega_H J - TS - \Phi q, \quad (19)$$

which means that $G(T, \Omega_H, \Phi)$ is indeed the Legendre transformation of the energy $M(S, J, q)$ with respect to S, J, and q. The entropy $S = -(\partial G/\partial T)_{\Omega_H\Phi}$ is one quarter of the event horizon area $\mathcal{A}_{\mathcal{H}}$. A straightforward calculations shows that the extensive thermodynamical quantities

$$J = -\left(\frac{\partial G}{\partial \Omega_H}\right)_{T\Phi}, \qquad q = -\left(\frac{\partial G}{\partial \Phi}\right)_{T\Omega_H}, \quad (20)$$

turn out to coincide with the expressions (12) and (16), the first law of thermodynamics (5) also being satisfied.

[4]It is well defined due to the field equation $d(e^{-\alpha\tilde{\phi}} * H) = 0$ (see, e.g., Ref. [12] for a nice discussion on different notions of charge).

V. DISCUSSION

Black rings provide a novel theoretical laboratory for studying the physics associated with event horizons. Using a countertermlike method for flat spacetimes, we have explicitly shown that the first law of black dipole ring mechanics expresses the conservation of energy by relating the change in the dipole ring mass M to the change in its area $\mathcal{A}_{\mathcal{H}}$, angular momentum J, and the dipole q. An extended form of the zeroth law implies that not only the surface gravity, but also the other intensive quantities (in our case, the angular velocity and the conjugate potential of the dipole charge) should be constant over the event horizon. Indeed, we found that the potential Φ in Eq. (17), which is constant over the event horizon, appears as the conjugate potential of the dipole charge in the first law.

The dipole ring does not have a real nonsingular Euclidean section. To remedy the situation, following Ref. [9], we constructed a complex metric that transformed the lapse function and the shift vector to imaginary quantities, but which kept the gravitational Cauchy data invariant (and hence the extensive quantities). The horizon is described by the "bolt" in this complexified geometry but has invariant features (e.g., area, gravity surface, etc.) as in the Lorentzian sector. Then, the key features of the physical Lorentzian dipole ring have been preserved.

The analysis of the thermodynamic stability of the black-ring solutions turns out to be very complicated, simple results being possible in the vacuum case only. For $\mu = 0$, the Euclidean regularity at the horizon, which is equivalent to the condition that the black ring is in thermodynamical equilibrium, gives the equation of state

$$T = \frac{1}{16\pi}\sqrt{\frac{\pi^2}{J^2\Omega_H^2} - 16\Omega_H^2}, \quad (21)$$

and the Gibbs potential can be written as

$$G[T, \Omega_H] = \frac{1}{16\pi G T^2}\left(-1 + \sqrt{\frac{16\pi^2 T^2}{\Omega_H^2} - 1}\right). \quad (22)$$

The analysis of Refs. [2,3] reveals the existence of two branches of solutions in terms of the dimensionless reduced spin j and reduced area of the horizon a_H, with $j^2 \equiv \frac{27\pi}{32G}\frac{J^2}{M^3} = (1+\nu)^3/8\nu$, $a_H \equiv \frac{3\sqrt{3}}{16\sqrt{\pi}}\frac{\mathcal{A}_H}{(GM)^{3/2}} = 2\sqrt{\nu(1-\nu)}$, which join for $(j^2, a_H) = (27/32, 1)$ (corresponding to $\nu = 1/2$), and which are dubbed "large" and "small" according to their area. For a grand-canonical ensemble, the control parameter is $4\pi T/\Omega_H$, the value for which the two branches join being $\Omega_H = 4\pi T/\sqrt{3}$.

To discuss the thermodynamic stability in a grand-canonical ensemble, we consider first the specific heat at constant angular velocity at the horizon

$$C_\Omega = T\left(\frac{\partial S}{\partial T}\right)_{\Omega_H}. \quad (23)$$

It turns out that only the "large" black-ring solutions with $\Omega_H < 4\pi\sqrt{2/3^{\frac{1}{2}}}T$ (corresponding to $\nu < 2/\sqrt{4+\sqrt{3}}$) are stable against thermal fluctuations, $C_\Omega > 0$. When considering instead a canonical ensemble with $F[T, J] = M - TS$, one finds that the specific heat at constant angular momentum is always positive

$$C_J = T\left(\frac{\partial S}{\partial T}\right)_J > 0, \tag{24}$$

which implies the ensemble is thermally stable. Another "response function" of interest is the "isothermal permittivity" $\epsilon_T \equiv (\partial J/\partial \Omega_H)_T$. Since it is always negative, the neutral black string solution is unstable to angular fluctuations, both in a macrocanonical and canonical ensemble.

At this end, we would like to comment on our counterterm prescription for the asymptotically flat spacetimes. Unlike asymptotically anti–de Sitter (AdS) spaces, the *locality* of the counterterm is not *a priori* mandatory for the asymptotically flat spaces [7]—though, for our purpose it was sufficient to consider a local counterterm. We have only investigated stationary spacetimes and so, for each value of the cut-off, the slice with the induced metric h_{ij} is stationary. Since $\xi = \partial/\partial t$ is a Kiling vector of the cut-off boundary, the energy is conserved. However, it would be interesting to find a more general result for any asymptotically flat spacetime. It is also worth exploring the connection between the holographic charges and the various alternative definitions of conserved charges in asymptotically flat spacetimes (for AdS, see Refs. [13–15]). These issues are currently being investigated [16].

A detailed analysis of the thermodynamics of black-ring solutions will be presented in a companion paper [17].

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