

Shower size parameter as an estimator of extensive air shower energy in fluorescence telescopesVitor de Souza,^{*} Gustavo Medina-Tanco,[†] and Jeferson A. Ortiz[‡]¹*Instituto de Astronomia, Geofísica e Ciências Atmosféricas, Universidade de São Paulo, Rua do Matão 1226, Cidade Universitária 05508-900, São Paulo, Brazil*Federico Sanchez[§]²*Dipartimento di Fisica, Università degli Studi di Milano e I.N.F.N., Via Celoria, 16, 20133 MI, Italy*

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The fluorescence technique has been successfully used to detect ultrahigh energy cosmic rays by indirect measurements. The underlying idea is that the number of charged particles in the atmospheric shower, i.e. its longitudinal profile, can be extracted from the amount of emitted nitrogen fluorescence light. However the influence of shower fluctuations and the very possible presence of different nuclear species in the primary cosmic ray spectrum makes the estimate of the shower energy from the fluorescence data analysis a difficult task. We investigate the potential of shower size at maximum depth as estimator of shower energy. The detection of the fluorescence light is simulated in detail and the reconstruction biases are carefully analyzed. We extend our calculations to both HiRes and EUSO experiments. This approach has shown some advantages to the reconstruction of the energy when compared to the standard analysis procedure.

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I. INTRODUCTION

Cosmic rays (CR) are the highest energy particles present in nature with energies exceeding 10^{20} eV. Their origin, nature and possible acceleration mechanisms are still a mystery, despite the efforts of many cosmic ray experiments in the last decades. This challenge is, in part, due to the very low flux of high energy and the consequent few observed events to be analyzed.

Measuring extensive air showers (EAS) is currently the only way to study the cosmic ray spectrum and chemical composition at energies around and above 10^{15} eV. At energies $E \geq 10^{17}$ eV the shower development can be directly observed by measuring the fluorescence light from atmospheric nitrogen that is excited by the ionization of the secondary charged shower particles (essentially electrons and positrons). Experiments applying this technique can determine the depth of maximum air shower development (X_{\max}) and the corresponding number of charged particles (N_{\max}). Presently, the HiRes [1], Pierre Auger Observatory [2], EUSO [3], OWL [4], and Telescope Array [5] experiments are using or planning to use fluorescence detectors to investigate the ultra high energy cosmic rays.

The total amount of emitted fluorescence light in a shower is a very good approximation to the total number of charged particles $N(X)$, where X is the atmospheric depth. In this sense the number of particles at shower maximum can serve as an estimator of the shower energy. The total energy that goes into electromagnetic charged

particles is obtained by integration of the shower longitudinal profile

$$E_{\text{em}} = \alpha \int_0^{\infty} N(X) dX, \quad (1)$$

where α is the average ionization loss rate [6], and the integral on the right-hand side represents the total track length of all charged particles in the shower projected onto the shower axis.

Recently an alternative proposal to reconstruct the shower energy has been done by B. Dawson in Ref. [7]. In this method the electromagnetic energy is calculated by using the fluorescence light intensity and the fluorescence efficiency, without the need of reconstructing the number of particles as a function of the atmospheric depth. This approach is taken as a very precise measurement of the primary shower energy because it is supposed to be weakly dependent on the simulation models and on the primary particle type. However, when the shower development details are taken into account the methods given by Eq. (1) and Ref. [7] can lead to high systematic uncertainties. Of no less concern is the important fact that the fluorescence efficiency as a function of air pressure, density and humidity is only known to a certain degree of accuracy.

Although the electrons and positrons constitute the majority of the charged particles in a shower and contribute most to the production of fluorescence light, another important fraction of the shower energy (around 10%) is carried by particles which cannot be measured by the fluorescence technique, i.e., particles that are invisible to fluorescence telescopes. This so called “missing energy” has been calculated by Monte Carlo air shower simulation and contributes to the uncertainties involved in the method, being sensitive to primary composition.

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Theoretical works have shown the existence of a clear relation between the primary energy and the maximum number of particles in the shower. Recently, Alvarez-Muñiz *et al.* [8] have studied the N_{\max} shower quantity as an estimator for the primary shower energy, confirming the efficiency of this technique. Such an approach was analyzed for different primary particles and energies using a fast one-dimensional simulation program. However, telescopes particularities and reconstruction procedures must be considered due to the introduction of biases and fluctuations in the calculation of N_{\max} .

The scope of this work is to explore the possibility of estimating the primary shower energy based on N_{\max} , taking as case studies the HiRes [1] and EUSO [3] fluorescence experiments, i.e., ground and space based experiments, respectively. The telescopes' particularities and the reconstruction procedures are included in our analysis producing a very realistic context for the application of the technique.

II. N_{\max} FLUCTUATIONS

Fluctuations are intrinsic to any extensive air shower and are a cause of uncertainty in the energy reconstruction. In addition to that, the energy of the shower is calculated without the knowledge of the mass of the primary particle which initiated the shower. The nature of the primary particle affects the longitudinal development of showers and can exert an influence on the reconstruction of the air shower energy, especially when the primary particles can be photons.

On the other hand, reconstruction procedures must rely on simulation programs in order to relate the measurable parameters to energy. In addition to the choice of the particular simulation program to be used, there is a general agreement that most shower reconstruction uncertainties originated from the simulation programs are due to differences in the hadronic interaction models.

The usual procedure used by the HiRes and Auger Collaborations to reconstruct the shower energy with fluorescence telescopes correlates the integral of the energy deposited in the atmosphere to the total shower energy. As mentioned before, a certain amount of the shower energy is carried away by particles which are invisible to the fluorescence technique, i.e., muons, neutrinos and high energy hadrons which are not converted to fluorescence photons. Such "missing energy" has been estimated by Monte Carlo simulations and shown to be dependent on the hadronic interaction model, primary composition, shower energy and arrival direction.

Some works have studied the uncertainty in the reconstruction of the shower energy due to the "missing energy" correction. In reference [6], Song *et al.* have estimated that the lack of knowledge of the primary particle type translates into an energy uncertainty of about $\sim 5\%$ for showers initiated by proton and iron nuclei and around $\sim 20\%$ if

primary photons are considered. Meanwhile Barbosa *et al.* [9] and Alvarez-Muniz *et al.* [8] have calculated similar values for hadronic primaries ($\sim 5\%$) which decrease with energy.

According to [8], the hadronic interaction model is the main source of systematic fluctuation in the estimation of the missing energy at the highest energies. The most used hadronic interaction models in cosmic ray studies are QGSJET [10] and SIBYLL [11]. Both models are tuned to reproduce the available collider data and extrapolate the relevant parameters, such as cross section, to energies above the maximum measured energy. QGSJET is a program developed to describe hadronic interactions using the quasi-eikonal Pomeron parametrization for the elastic hadron-nucleon scattering amplitude where the hadronization process is treated in the quark gluon string model. On the other hand, SIBYLL is based on the QCD mini-jet approach. For a detailed comparison between the hadronic interaction models at energies above 10^{18} eV see Ref. [12].

Because of the differences in the multiplicities of secondary particles simulated by QGSJET01 and SIBYLL2.1 hadronic interaction models, the unseen energy calculated with QGSJET is about 50% higher than the value predicted by SIBYLL at 10^{20} eV, which can be translated into an uncertainty of $\sim 5\%$ in the shower energy reconstruction.

The dependence of the missing energy on the energy of the primary is a common feature among the various studies made so far. In addition, Barbosa and coworkers [9] have studied the dependence of the missing energy on the arrival direction of the shower and limited it to be at most 0.7%.

Furthermore, it has been suggested in reference [8] that the discrepancies among such studies are on the order of 1–3% and could be explained by different hadronic interaction models for lower energy particles. It might be relevant to mention that both groups used different simulation programs: Song *et al.* and Barbosa *et al.* used different versions of the well tested CORSIKA [13], while Alvarez-Muniz *et al.* used a hybrid fast one-dimensional simulation program [14].

Considering all the uncertainties in the determination of the missing shower energy listed above, one can estimate that the total systematic fluctuation related to the primary energy reconstruction is about 5% to 10%.

In order to investigate the fluctuations in the maximum number of particles due to the natural fluctuations of the shower, different primary particles and artificial fluctuations introduced by different hadronic interaction models, we have used the recently released CONEX [15] shower simulation program. This program is based on a hybrid simulation scheme combining fast numeric solutions of cascade equations and Monte Carlo calculations. The main advantages of the hybrid programs is the execution time [16].

We have simulated 5000 showers initiated by proton, iron and gamma primaries at each considered energy and

for the QGSJET01 and SIBYLL2.1 interaction hadronic models. Showers have been simulated tracing particles down to the minimum energy available, 1 GeV for hadrons and 1 MeV for electromagnetic particles.

Figure 1 shows the N_{\max} distribution for 5000 showers initiated by protons, iron nuclei and gamma at $10^{19.5}$ eV simulated with the QGSJET hadronic model. The distribution illustrates the previous discussion regarding the fluctuations due to the primary particle type. If only proton and iron nuclei are considered, the N_{\max} distribution shows a very narrow distribution with a median value of 1.90×10^{10} and a dispersion of 3.6% at the 68% of confidence level. If gamma showers are considered, the median of the combined distribution (gamma+proton+iron nuclei) increases slightly to 1.92×10^{10} while the dispersion reaches 7% at 68% of confidence level.

The same calculation has been done for proton and iron nuclei at $10^{20.5}$ eV and the fluctuations are present at the same level of 3.6%. Gamma air showers have not been studied at the energy of $10^{20.5}$ eV due to the fact that the CONEX program does not include the preshower algorithm effect [17,18] that takes into account photon interactions with the geomagnetic field affecting the longitudinal development of showers with energy above $10^{19.5}$ eV.

In order to investigate the uncertainty due to the hadronic interaction model we have simulated 5000 proton showers with the CONEX program using SIBYLL and QGSJET hadronic interaction models. Figure 2 illustrates the N_{\max} distribution for proton showers simulated with QGSJET and SIBYLL at $10^{19.5}$ eV and the corresponding fluctuation is 4.6%. Figure 2 also shows the same distribution for iron nuclei at $10^{20.5}$ eV and the corresponding fluctuation is 3.3%.

Comparing the numbers given above and Figs. 1 and 2, we are able to conclude that the systematic uncertainties in

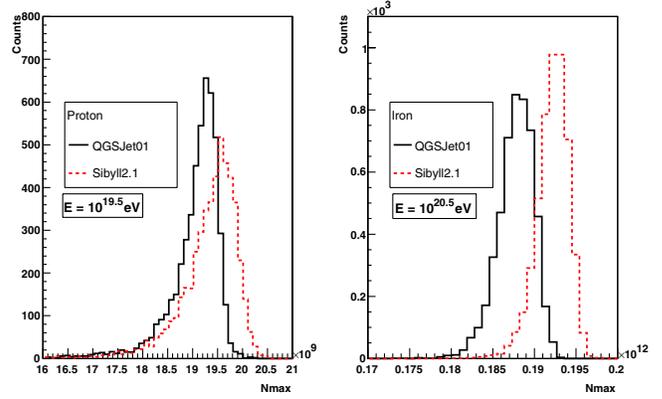


FIG. 2 (color online). Distribution of N_{\max} for 5000 showers simulated by QGSJET and SIBYLL. Simulation has been done with the CONEX program. The left-hand side shows the primary protons at $10^{19.5}$ eV while the right-hand side illustrates the primary iron nuclei at the energy of $10^{20.5}$ eV.

N_{\max} due to the hadronic interaction models are slightly larger than the uncertainties due to the mass composition if proton and iron nuclei are considered, 4.6% and 3.6%, respectively, at $10^{19.5}$ eV. If primary gamma are considered, mass composition would be more relevant than hadronic interaction models to the N_{\max} fluctuations.

In order to exemplify the increase in the uncertainty by allowing the introduction of photons in the cosmic ray flux, we also consider the peculiar case of equal abundances of protons, iron and gammas. Figure 3 shows 5000 air showers induced by gamma, proton, and iron nuclei simulated with QGSJET and SIBYLL hadronic interaction models, at the energy of $10^{19.5}$ eV. The total N_{\max} uncertainty is $\sim 7\%$ which is comparable to the systematic uncertainties introduced by the missing energy correction calculation.

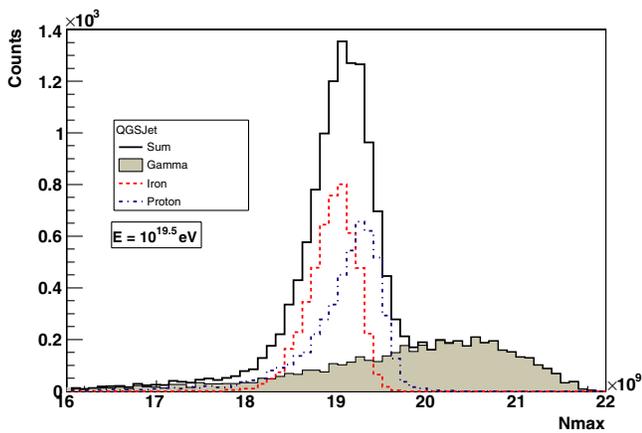


FIG. 1 (color online). Distribution of N_{\max} for 5000 showers initiated by proton, iron nuclei and gammas at $10^{19.5}$ eV. Simulation has been done with the CONEX program. The thick solid line shows the sum of the N_{\max} distributions for proton, iron nuclei and gamma primaries.

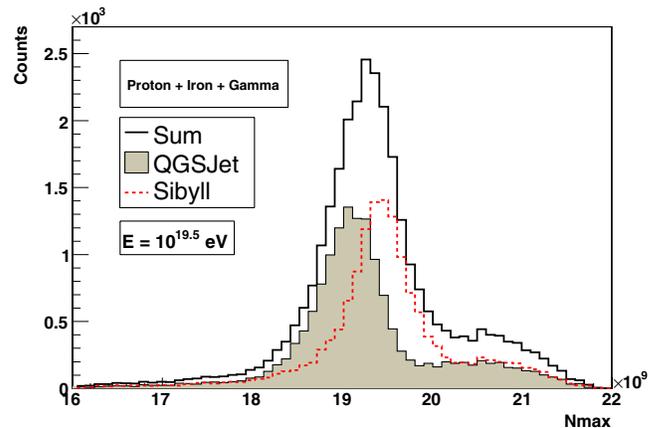


FIG. 3 (color online). Distribution of N_{\max} for 5000 showers simulated by QGSJET and SIBYLL and initiated by proton, gamma and iron nuclei at $10^{19.5}$ eV. Simulation has been done with the CONEX program.

III. SHOWER AND TELESCOPE SIMULATION

The results presented in the current section have been obtained by using the CORSIKA 6.2 simulation program. In addition, gamma showers have been simulated by considering the preshower and LPM effects. These are competing effects tending to speed up and delay the development of the shower, respectively. In principle, both effects do not change the maximum number of particles in the shower, which is the main variable to be investigated here. However, the reconstruction of the N_{\max} parameter depends on the detection of whole development of the shower and therefore the preshower and LPM effects were incorporated in the simulations of the gamma showers studied here.

The thinning factor [19] of the simulations is 10^{-6} and the longitudinal air shower profiles were sampled in steps of 5 g/cm^2 . The energy thresholds were set to 0.1 MeV, for electrons and photons, 0.3 GeV for hadrons and 0.7 GeV for muons. Protons and gamma ray showers were simulated in the energy interval from 10^{19} to $10^{20.5}$ eV in steps of 0.1 dex. For each energy and primary particle we simulated 100 events which were recycled 50 times by randomly drawing a new arrival direction and core position. For gamma showers simulated with the preshower effect only the core position was randomized since the relative direction of the primary particle with the geomagnetic field affects the development of the shower.

The simulations of the HiRes telescopes were performed following the same general procedure adopted by the HiRes Collaboration as published in [20,21]. Comparisons of our simulation with the HiRes Collaboration simulation and the HiRes data can be seen in [22].

A similar simulation program was written for the EUSO telescope in which we incorporated the geometric details of that telescope. The EUSO experiment has been proposed to operate at 430 km of altitude looking downwards to measure the fluorescence light produced by the passage of a cosmic ray shower. It is configured with a 60° field of view covered by pixels of 0.1° resulting in $200,000 \text{ km}^2$ detection area.

For the HiRes studies we have simulated shower cores within a radius of 50 km from the telescope. The zenith angle has been randomly chosen from 0 to 60 degrees. The HiRes telescopes were considered to operate in mono mode. Following the HiRes Collaboration nomenclature HiRes-I is the first fluorescence telescope they built with aperture of 15 degrees in zenith angle and 360 degrees in azimuth. HiRes-II is the second fluorescence telescope operated by the HiRes Collaboration with aperture of 30 degrees in zenith angle and 360 degrees in azimuth. Details of the HiRes telescopes can be found in Ref. [21].

For the EUSO studies we allowed showers with cores within a radius of 250 km from the axis of the telescope and the zenith angle was chosen from 0 to 90 degrees.

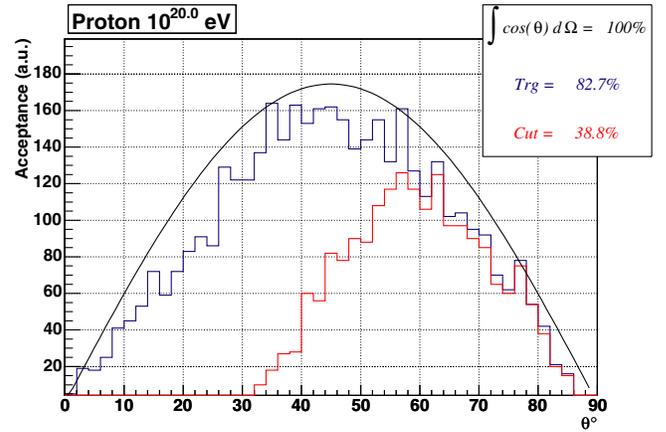


FIG. 4 (color online). EUSO acceptance for proton showers at 10^{20} eV as a function of the zenith angle according to our simulation program. The heavy smooth line shows the isotropic distribution of events, the full line histogram corresponds to the events which trigger the telescope, and the light line histogram the events which survived to the quality cuts.

Besides the configuration details, included in our simulation program, we also paid careful attention to the simulation of the atmosphere since the field of view of the EUSO telescope spans the entire atmosphere under its orbit. In order to verify our code we have the EUSO acceptance as a function of the arrival direction of the primary particle. Figure 4 illustrates the acceptance for the EUSO telescope, which is in good agreement with the predicted values given in [3].

The importance of a detailed telescope simulation in the studies of N_{\max} as an energy estimator can be seen in Fig. 5 in which the simulated N_{\max} distribution as predicted by the CORSIKA code is compared to the reconstructed N_{\max} , obtained after the detection by the EUSO and HiRes-I telescopes. It is evident from the figure that the distortion of the N_{\max} distribution after the detection and reconstruction of the shower must be taken into account properly.

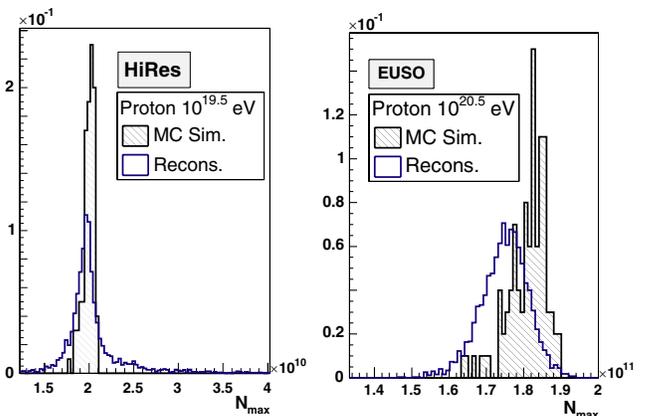


FIG. 5 (color online). Comparison between the reconstructed and simulated N_{\max} for the HiRes-I and EUSO telescopes.

Fluorescence telescopes have detection efficiency biases regarding arrival direction, primary energy and core position which effect on the N_{\max} reconstruction are very difficult to control. Moreover, the reconstruction procedure also introduces fluctuations which depend on several parameters. These effects result in a distortion of the N_{\max} distribution that depends on the configuration of the telescope and the chosen reconstruction method. Therefore the simulation of the telescope and the application of the reconstruction procedure is fundamental in order to obtain a meaningful relation between N_{\max} and energy and to properly estimate the reconstruction errors attained by this method.

IV. PRIMARY ENERGY RECONSTRUCTION

For the energy reconstruction of a shower there is one procedure which is unavoidably used in all proposed methods, especially those in investigation here. Each pixel hit in the telescopes corresponds to a given atmospheric depth and the respective signal measured by it can be correlated to the number of particles or, more directly, to the energy deposited by the shower at that depth. From a limited number of points along the longitudinal profile, a function must be fitted in order to calculate, by extrapolation, the energy deposited by the shower beyond the limits of the field of view of the telescope. The most widely used function is the Gaisser-Hillas, which efficiency have been extensively tested and confirmed [20].

In this section, we are going to investigate the influence of the fitting procedure in the uncertainty of the energy for two reconstruction methods. The analysis will be done for a ground based experiment and may differ for a space based telescope.

The standard method used by the HiRes and Auger Collaborations is based on the integration of the fitted Gaisser-Hillas function to obtain the shower energy and, from now on, it will be referred as the integral method. In addition, the method based on the N_{\max} parameter will be referred as the N_{\max} method. Our intention here is to evaluate the influence of the knowledge of a limited portion of the shower longitudinal development in the determination of the parameters related to energy, which are the integral of the longitudinal profile for the integral method and the N_{\max} parameter for the N_{\max} method.

We try to determine which parameter is less affected by the fact that the telescopes, in most of the cases, measure only a small part of the shower, and whether the extrapolation of the observed longitudinal profile depends on geometric parameters. In order to do so, we simulated 5000 CORSIKA proton showers with energy $10^{19.5}$ eV through the HiRes-II telescopes. For each shower we compare the integral of the longitudinal number of particles as calculated by CORSIKA using the whole profile of the shower with the integral of the fitted longitudinal profile based on the points actually measured by the telescope.

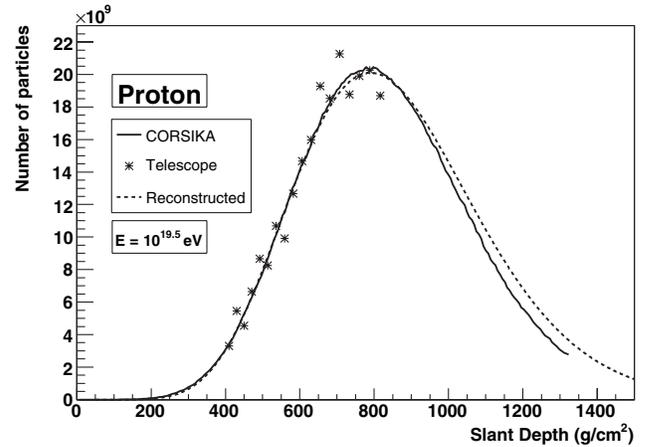


FIG. 6. Longitudinal development of a proton shower with energy $10^{19.5}$ eV as given by CORSIKA (full line), as detected by the HiRes-II telescope according to our simulation where noise has been added (points), and as reconstructed by fitting a Gaisser-Hillas function (dotted line).

The same comparison is made between the N_{\max} given by CORSIKA based on the whole development of the shower and the reconstructed N_{\max} after the restriction of the shower in the field of view of the telescope. Figures 6 and 7 illustrate this procedure. These figures show the longitudinal profile simulated by CORSIKA in solid black lines. The star symbol shows the points measured by the telescope after the inclusion of noise while the dashed line shows the Gaisser-Hillas fit to the points.

The Gaisser-Hillas fit was implemented according to the procedures adopted by the HiRes Collaboration in [20,21].

Figure 8 shows the distribution of errors in the determination of the integral of the longitudinal profile and in the

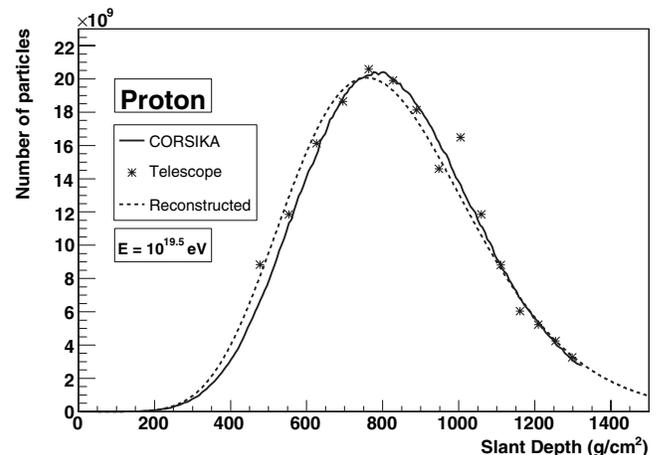


FIG. 7. Longitudinal development of a proton shower with energy $10^{19.5}$ eV as given by CORSIKA (full line), as detected by the HiRes-II telescope according to our simulation where noise has been added (points), and as reconstructed by fitting a Gaisser-Hillas function (dotted line).

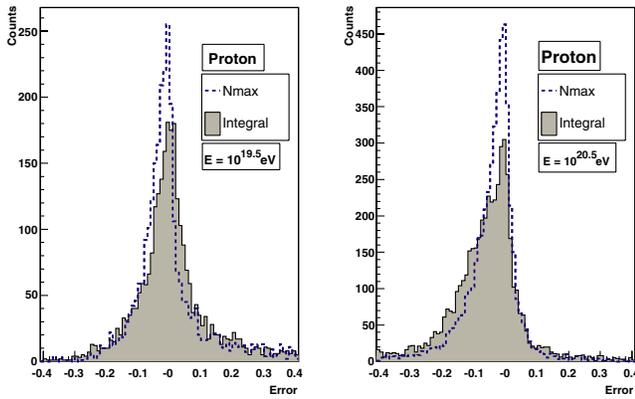


FIG. 8 (color online). Error in the integral and N_{\max} reconstructions due to the fit of a Gaisser-Hillas function using a limited number of points.

calculation of the N_{\max} parameter for proton showers at $10^{19.5}$ and $10^{20.5}$ eV. Table I shows the mean and RMS values for all the distributions. The error is defined as the reconstructed value minus the simulated value divided by the simulated value.

The same behavior is noticed in the N_{\max} and integral errors. At $10^{19.5}$ eV, the N_{\max} parameter and the integral of the longitudinal profile are underestimated and the opposite happens at $10^{20.5}$ eV where both values are overestimated. This might happen because showers with higher energy develop deeper in the atmosphere hitting ground level at an earlier stage. However, it is noticeable that the N_{\max} parameter is less affected by reconstruction showing a mean closer to zero and a slightly smaller RMS value for both energies.

The RMS values shown in Table I are the intrinsic fluctuations of the determination of the N_{\max} parameter and of the integral value of the longitudinal profile. Nevertheless, we have not applied any quality cuts to the showers analyzed so far, other than asking that X_{\max} falls inside the field of view of the telescope.

It has been claimed [8] that N_{\max} , as an energy estimator, should be superior to the integral method for showers with short track length detected by fluorescence telescopes. Other authors have also mentioned the dependence of the method on the shower zenith angle and on the distance from the telescope to the shower core. However, both the zenith angle and the distance to the core are the variables

TABLE I. Mean and RMS values for the error distributions given in Fig. 8.

Energy (eV)	N_{\max}		Integral	
	Mean	RMS	Mean	RMS
$10^{19.5}$	0.02	0.16	0.03	0.17
$10^{20.5}$	-0.04	0.10	-0.06	0.14

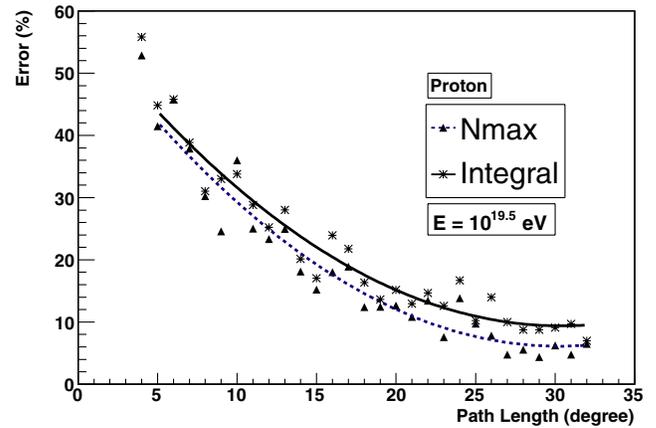


FIG. 9 (color online). Error in the integral and N_{\max} reconstruction due to the fit of a Gaisser-Hillas function using a limited number of points as a function of the path length for proton shower with energy $10^{19.5}$ eV. Lines shows polynomial fit to the points in the range from 5 to 32 degrees.

which determine the track length of the shower in the field of view of the telescope.

As illustrated in Figs. 6 and 7 the path length is the parameter which directly influences the fit of the Gaisser-Hillas function and, therefore, the energy reconstruction. The distance between telescope and shower also influences the energy reconstruction via the lateral size of the shower in the photomultiplier camera, as discussed in [23,24], but this effect should be small compared to the reduction of the path seen by the telescope.

Figure 9 illustrates the reconstruction error of the integral and of the N_{\max} parameter as a function of path length for proton showers with energy $10^{19.5}$ eV. The line is a polynomial fit to the points. In this case, a slightly better

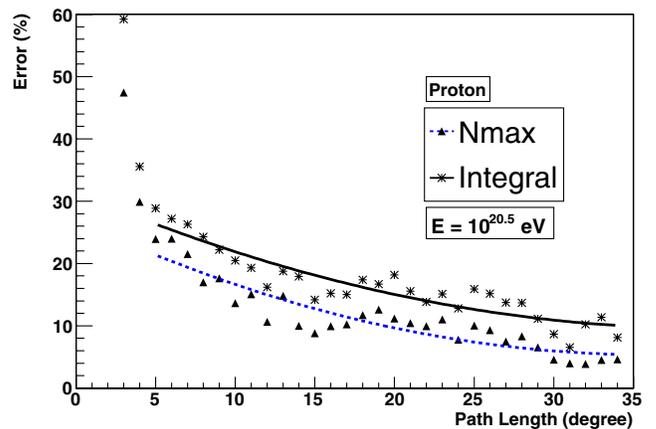


FIG. 10 (color online). Error in the integral and N_{\max} reconstruction due to the fit of a Gaisser-Hillas function using a limited number of points as a function of the path length for proton shower with energy $10^{19.5}$ eV. Lines shows polynomial fit to the points in the range from 5 to 35 degrees.

resolution for the N_{\max} parameter is seen for the entire range of path length.

Figure 10 shows the same plot for proton showers at $10^{20.5}$ eV where a small difference between the reconstruction of the N_{\max} and the integral can be seen for the range of path length from 5 to 35 degrees.

These results show that the reconstruction of the N_{\max} parameter is equivalent to the reconstruction of the integral independently of the size of the shower detected by the telescope. Our calculations suggest that the reconstruction of the N_{\max} parameter might become slightly more accurate than the integral reconstruction for the highest energies above $\sim 10^{20}$ eV.

V. RELATION BETWEEN PRIMARY ENERGY AND N_{\max}

Finally, we used the simulation programs already described to determine the relationship between the reconstructed N_{\max} and the primary energy of the shower. We used the same set of CORSIKA showers described above and simulated them through the HiRes-II and EUSO telescopes. The present are intended to assess the potential of the N_{\max} parameter as an energy estimator by means of a full simulation and reconstruction chain including all types of systematic and statistical errors.

The HiRes-II and EUSO telescopes were chosen to illustrate the method for ground and space based telescopes. The HiRes-II telescope is, in principle, very similar to the Auger telescopes regarding the variables explored here. Both experiments have 30 degrees of field of view in zenith and the pixel size for the Auger telescope is 1.5° while for the HiRes-II telescope is 1° . These small differences should not affect our conclusions.

VI. HIRES TELESCOPE

The reconstruction of the shower was also carried out according to the procedures given by the HiRes Collaboration in [20,21] of their own procedures. The X_0 and λ parameters of the Gaisser-Hillas were fixed to -60 and 70 g/cm² respectively, because their results were very insensitive to the values of X_0 and λ parameters.

The following quality cuts were applied to the showers after reconstruction:

- (a) Angular speed < 11 .
- (b) Selected tubes ≤ 7 .
- (c) $0.85 < \text{Tubes/degree} < 3.0$.
- (d) Photoelectrons/degree > 25 .
- (e) Track length $> 7^\circ$, or $> 10^\circ$ for events extending above 17° elevation.
- (f) Zenith angle $< 60^\circ$.
- (g) $150 < X_{\max} < 1200$ g/cm², and is visible in detector.
- (h) Geometry $\chi^2/\text{n.d.f.} < 10$.
- (i) Profile fit $\chi^2/\text{n.d.f.} < 10$.

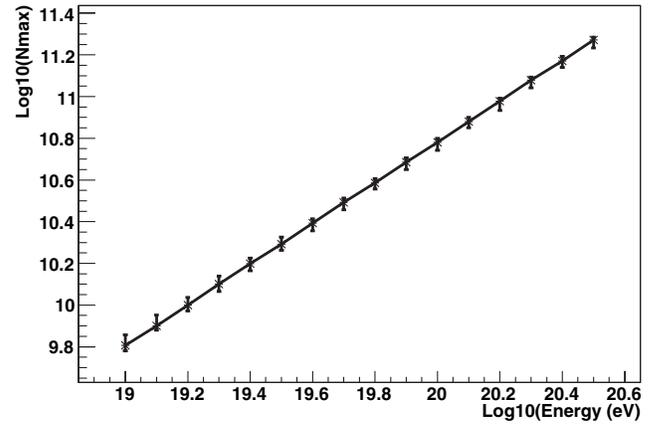


FIG. 11. Relation between the reconstructed N_{\max} and the primary energy for showers initiated by protons. Results shown for the HiRes-II telescope.

according to the HiRes Collaboration [21] this event selection guarantee a good energy and arrival direction reconstructions.

At each energy 5 000 showers were simulated and N_{\max} reconstructed. Figure 11 illustrates the relation between the median N_{\max} and the primary energy. The error bars represent the 68% confidence level of the reconstructed N_{\max} distribution. The relationship found is almost linear and can be well fitted by the equation $\log_{10}(N_{\max}) = -8.75 + 0.97 \times \log_{10}(E)$.

In order to test the reconstruction of the energy based on the N_{\max} parameter we simulated a second set of 5000 independent showers. The energy was reconstructed using the previous equation and by integrating the Gaisser-Hillas function. Figure 12 presents the error in the energy reconstruction as a function of the primary energy. As suggested in the calculations above, the N_{\max} estimator is as good as the integral procedure for energies between 10^{19} and

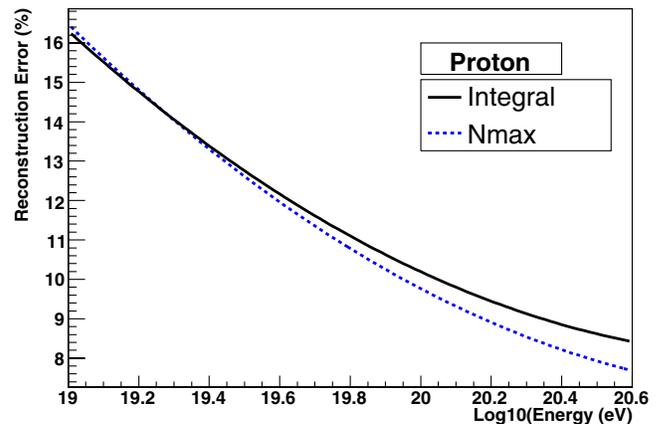


FIG. 12 (color online). Error in the energy reconstruction by the integral and N_{\max} methods as a function of the primary energy. Results shown for the HiRes-II telescope.

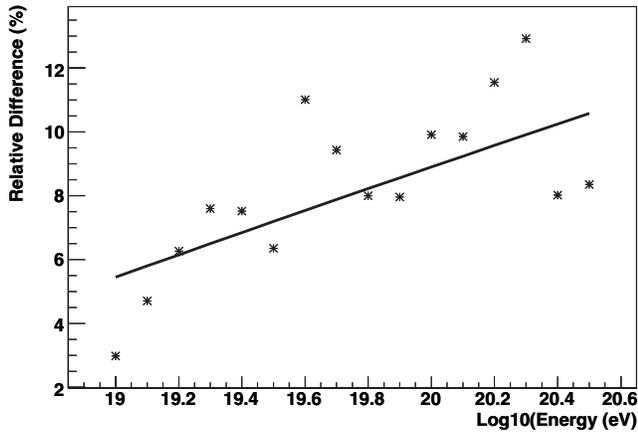


FIG. 13. Relative difference between the median N_{\max} for proton and gamma showers. Points shows the difference calculated with the median of the reconstructed N_{\max} distribution and the solid line shows the difference calculated with the fitted relation between energy and N_{\max} . Note that the line is not a linear fit to the points.

$10^{19.5}$ eV. For energies above $10^{19.5}$ eV the N_{\max} estimator becomes slightly superior to the standard integral method.

We also studied the relationship between N_{\max} and the primary energy for primary gamma showers. The relation is also linear and very similar to the one for protons shown in Fig. 11. Figure 13 shows the relative difference between the median N_{\max} for proton and gamma showers. The difference is defined as $(N_{\max}^{Pr} - N_{\max}^{Ga})/N_{\max}^{Pr}$. The points in Fig. 13 show the difference calculated with the median N_{\max} given by the reconstructed N_{\max} distribution at each energy for proton and gamma primaries. The solid line is the difference calculated using the fitted relations between energy and N_{\max} .

For the energy range $10^{19.0}$ to $10^{20.5}$ eV, the difference is smaller than 12%. This difference would influence the energy reconstruction in a mixed composition of proton and gamma primaries. The same effect would be seen in both reconstruction methods: N_{\max} and integral according to the discussion in Sec. II.

If a mixed composition of 50% proton and 50% gammas is considered the energy resolution would degrade but still would show a behavior similar to the one show in Fig. 12. At 10^{19} eV the resolution would degrade to 20% for both methods. At $10^{20.5}$ the resolution for the N_{\max} method would be 16% and for the integral procedure it would be around 18%.

VII. EUSO

The same analyses were extended to the EUSO telescope. The details of this experiment were included in the simulations as explained in Sec. III. However, not only is the telescope different, but its peculiar field of view also requires a considerable change in the shower reconstruc-

tion program. This can be verified in Fig. 4 in which we show the EUSO acceptance.

Since there are no quality cuts yet optimized for the EUSO telescope, we imposed very loose ones requiring total path length greater than 0.6° and greater than 200 g/cm^2 and X_{\max} in the field of view of the telescope. We would like to stress that different quality cuts will modify the relation between N_{\max} and energy presented here.

Figure 14 shows the relation between the reconstructed N_{\max} and the simulated energy, which can be well approximated by the equation: $\log_{10}(N_{\max}) = -8.95 + 0.90 \times \log_{10}(E)$.

In order to test the reconstruction of the energy based on N_{\max} , we simulated a second set of 5000 independent showers. The energy was reconstructed using both the equation above and the integration of the Gaisser-Hillas fitted profile. Figure 15 shows the error in the energy reconstruction as a function of the primary energy. For energies below $10^{20.2}$ eV, the N_{\max} estimator was superior to the integral method. For energies above $10^{20.2}$ eV, both methods are equivalent.

These results are different from those for the HiRes-II telescopes, which confirms that the relation between N_{\max} and primary energy depends on the telescopes and details of the analysis, mainly the quality cuts.

The exact influence of each quality cut on the energy resolution is not completely understood at present and is beyond the scope of this paper. However, the results shown here suggest that the quality cuts imposed by the HiRes Collaboration are well optimized in order to guarantee a good energy resolution for the integral method for the energy ranging from 10^{19} to $10^{20.5}$ eV. On the other hand, the loose cuts applied by us to the EUSO analysis do not achieve an optimal selection of event for energies below $10^{20.2}$ eV resulting in an increase of the reconstruction error for both techniques. It is also apparent that N_{\max} ,

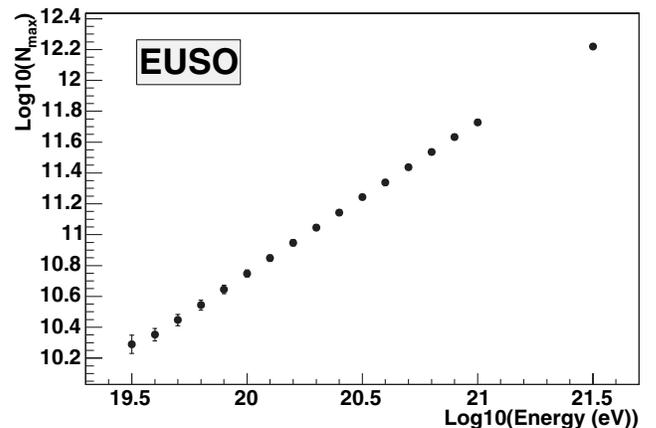


FIG. 14. Relation between the reconstructed N_{\max} and the simulated primary energy for showers initiated by protons. Results are for the EUSO telescope.

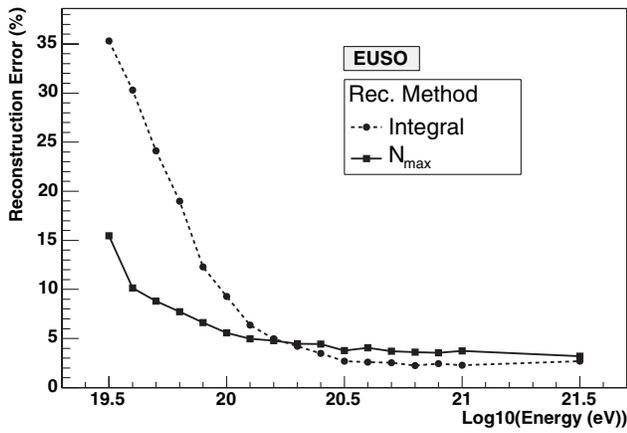


FIG. 15. Error in the energy reconstruction by the integral and N_{\max} methods as a function of the primary energy. Results are for the EUSO telescope.

for EUSO, is less sensitive to well calibrated cuts; this is because, despite the fact that the errors in this method increases at lower energies, they still remain within reasonable bounds, i.e., smaller than 15% down to 10^{19} eV.

VIII. CONCLUSION

In the present work, we tested the number of particles at the maximum air shower development, N_{\max} , as an energy estimator. The uncertainties associated with the simulation models were calculated and compared to the uncertainties in the missing energy calculation. A similar intrinsic fluctuation of around 5–10% at $10^{19.5}$ eV was verified for both approaches. These uncertainties decrease with increasing energy.

The systematic uncertainties in the determination of N_{\max} are dominated by mass composition rather than hadronic interaction models if gamma showers are taken into account. If gamma showers are excluded from the analysis, mass composition and hadronic interaction models would contribute equally to the total uncertainties.

Full telescope simulations were considered in our analysis. The first source of uncertainty in the reconstruction

procedure is the fit of the points measured by the telescope using a Gaisser-Hillas function. We investigated the statistical uncertainties involved in the evaluation of N_{\max} and of the integral of the longitudinal distribution function as a function of the path length seen by the telescope. N_{\max} appears to be slightly better reconstructed independent of the path length observed by the telescope. At $10^{19.5}$ eV the difference in reconstructing N_{\max} and the integral is very small, however with increasing energy, at $10^{20.5}$ eV for example, the difference is of the order of a few percent, depending on energy (see Fig. 10).

The relation between the N_{\max} and primary energy for proton showers was calculated for the HiRes-II telescope including the quality cuts used by the HiRes Collaboration. N_{\max} appears to be equivalent to the standard integral procedure as an energy estimator, with a small superiority of the N_{\max} approach for the highest energies (see Fig. 12). We also calculated the same relation for gamma showers and the relative difference for the proton shower is smaller than 12%. If a mixed composition of 50% proton and 50% gammas is considered the energy resolution degrades to 20% at 10^{19} eV for both methods. At $10^{20.5}$ the resolution for the N_{\max} method is 16% while for the integral procedure is $\sim 18\%$.

The same analyses were performed for the EUSO telescope and we demonstrated that the N_{\max} method also works well for space based telescopes. The specific relationships, however, depend on the telescopes characteristics and reconstruction methods.

Finally, N_{\max} was established as a good energy estimator for fluorescence telescope with some advantages over the standard integral procedure.

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