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Interpolating among the Landau, Coulomb, and maximal Abelian gauges

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A generalized gauge fixing which interpolates among the Landau, Coulomb, and maximal Abelian gauges is constructed.

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I. INTRODUCTION

The aim of this paper is that of pointing out the possibility of introducing a generalized gauge fixing which interpolates among the Landau, Coulomb, and maximal Abelian gauges. These gauges have been employed intensively through theoretical analysis and lattice numerical simulations in order to investigate several aspects of Yang-Mills theories in the infrared region. A partial list of them is given by[1]:

(i) study of the Gribov copies and their influence on the gluon and ghost propagators,

(ii) analysis of Yang-Mills theories through the Schwinger-Dyson equations,

(iii) study of renormalization group invariant effective couplings and their behavior in the low energy region,

(iv) dual superconductivity mechanism for color confinement,

(v) dimension two gauge condensates and their relevance for the infrared dynamics of gauge theories.

To some extent, this interpolating gauge can be seen as a generalization of previous results in which suitable gauge fixings interpolating between the Landau and the Coulomb gauges [3] as well as between the Landau and the maximal Abelian gauges [4] have been obtained and proven to be renormalizable to all orders of perturbation theory. As we shall also introduce a generalized interpolating dimension two mass operator, it is worthwhile here spending a few words on the issue of the gauge invariance of the dimension two condensates, a topic which is still under debate. Although the dimension two operator $\int d^4xA^2$ is not left invariant by the gauge transformations, it enjoys the property of being Becchi-Rouet-Stora-Tyutin invariant on shell in the Landau, Curci-Ferrari, and maximal Abelian gauges.[2] This property has made it possible to prove that the operator A^2 is multiplicatively renormalizable to all orders in all these gauges (see Refs. [5,6]), a feature which has been extended to the case of the more general class of the linear covariant gauges [7]. This has enabled us to construct a renormalizable effective potential for the composite operator A^2 and to investigate its condensation in all the aforementioned gauges; see Refs. [4,8–12] which provide evidence of a nonvanishing condensate, i.e. $\langle A^2\rangle \neq 0$, resulting in an effective gluon mass $m_{\mathrm{eff}} \propto \langle A^2\rangle$. Moreover, the output of our calculations shows that the condensate itself is not gauge invariant, i.e. the effective gluon mass m_{eff} depends on the gauge parameter; see, for example, Ref. [12] where the case of the linear covariant gauge has been considered. Notice that, due to color confinement, gluons cannot be observed as free particles, so that the effective gluon mass cannot be associated to a quantity which can be directly observable. Nevertheless, a well-defined gauge invariant quantity can be introduced and computed order by order, and this in the presence of a nonvanishing condensate $\langle A^2\rangle \neq 0$. This quantity is the vacuum energy of the theory, E_{vac}, its physical meaning being, of course, apparent. The construction and the computation of the vacuum energy E_{vac} in the presence of a nonvanishing condensate $\langle A^2\rangle$, as well as its independence from the gauge parameters, can be found in [4,8–12]. In this context, the possibility of having at our disposal a generalized gauge which interpolates among the Landau, Coulomb, and maximal Abelian gauges might be very useful. It could allow us to give a proof of the fact that the vacuum energy of the theory has to be the same for all three gauges. The present work is organized as follows. In Sec. II we briefly remind some basic properties of the maximal Abelian gauge. Section III is devoted to the introduction of the aforementioned interpolating gauge. A suitable dimension two operator which turns out to be BRST invariant on shell and which interpolates between the dimension two gluon mass operators already studied in the Landau, Coulomb, and maximal Abelian gauges is introduced in Sec. IV. A few remarks on possible further analytic studies and lattice numerical investigations will be outlined in the conclusion.

II. THE MAXIMAL ABELIAN GAUGE AND ITS NONRENORMALIZATION THEOREM

In order to introduce the maximal Abelian gauge, let us briefly fix the notation. According to [4], we decompose

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[1] See the reviews [1,2] and references therein.

[2] In the case of the Curci-Ferrari and maximal Abelian gauge, a slightly more general operator has to be considered, namely, $(\frac{A^2}{2} + \alpha\bar{c}c)$.

the gauge field A_μ^A, $A = 1....N^2 - 1$ into off-diagonal and diagonal components, namely,

$$A_\mu^A T^A = A_\mu^a T^a + A_\mu^i T^i, \tag{1}$$

where T^A are the generators of the gauge group $SU(N)$, $[T^A, T^B] = if^{ABC}T^C$. The indices $i, j, \ldots$ label the $N-1$ diagonal generators of the Cartan subalgebra. The remaining $N(N-1)$ off-diagonal generators will be labeled by the indices $a, b, \ldots$. For the nilpotent BRST transformations of the fields, we have

$$\begin{aligned}
sA_\mu^a &= -(D_\mu^{ab}c^b + gf^{abc}A_\mu^b c^c + gf^{abi}A_\mu^b c^i),\\
sA_\mu^i &= -(\partial_\mu c^i + gf^{abi}A_\mu^a c^b),\\
sc^a &= gf^{abi}c^b c^i + \frac{g}{2}f^{abc}c^b c^c, \qquad sc^i = \frac{g}{2}f^{abi}c^a c^b,\\
s\bar{c}^a &= b^a, \qquad sb^a = 0, \qquad s\bar{c}^i = b^i, \qquad sb^i = 0.
\end{aligned} \tag{2}$$

where $(c^i, \bar{c}^i)$, $(c^a, \bar{c}^a)$ stand for the diagonal and off-diagonal Faddeev-Popov ghosts, while (b^i, b^a) denote the diagonal and off-diagonal Lagrange multipliers. The covariant derivative D_μ^{ab} in Eq. (2) is defined as

$$D_\mu^{ab} = \delta^{ab}\partial_\mu - gf^{abi}A_\mu^i. \tag{3}$$

Also, for the field strength one gets $F_{\mu\nu}^A = (F_{\mu\nu}^i, F_{\mu\nu}^a)$, i.e.

$$\begin{aligned}
F_{\mu\nu}^a &= D_\mu^{ab}A_\nu^b - D_\nu^{ab}A_\mu^b + gf^{abc}A_\mu^b A_\nu^c,\\
F_{\mu\nu}^i &= \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + gf^{abi}A_\mu^a A_\nu^b.
\end{aligned} \tag{4}$$

Thus, for the Yang-Mills action, $S_{\rm YM}$, quantized in the maximal Abelian gauge, $S_{\rm MAG}$, we have

$$S_{\rm YM} + S_{\rm MAG}, \tag{5}$$

with

$$S_{\rm YM} = \frac{1}{4}\int d^4x F_{\mu\nu}^A F_{\mu\nu}^A = \frac{1}{4}\int d^4x (F_{\mu\nu}^a F_{\mu\nu}^a + F_{\mu\nu}^i F_{\mu\nu}^i), \tag{6}$$

and

$$\begin{aligned}
S_{\rm MAG} = s\int d^4x \Big(&\bar{c}^a\partial_\mu A_\mu^a - g\bar{c}^a f^{abi}A_\mu^i A_\mu^b + \frac{\alpha}{2}\bar{c}^a b^a\\
&- \frac{\alpha}{2}gf^{abi}\bar{c}^a\bar{c}^b c^i - \frac{\alpha}{4}gf^{abc}c^a\bar{c}^b\bar{c}^c + \bar{c}^i\partial_\mu A_\mu^i\Big),
\end{aligned} \tag{7}$$

which yields

$$\begin{aligned}
S_{\rm MAG} = \int d^4x \Big(& b^a\Big(D_\mu^{ab}A_\mu^b + \frac{\alpha}{2}b^a\Big) + \bar{c}^a D_\mu^{ab}D_\mu^{bc}c^c + g\bar{c}^a f^{abi}(D_\mu^{bc}A_\mu^c)c^i + g\bar{c}^a D_\mu^{ab}(f^{bcd}A_\mu^c c^d) - \alpha g f^{abi}b^a\bar{c}^b c^i\\
&- g^2 f^{abi}f^{cdi}\bar{c}^a c^d A_\mu^b A_\mu^c - \frac{\alpha}{2}gf^{abc}b^a\bar{c}^b c^c - \frac{\alpha}{4}g^2 f^{abi}f^{cdi}\bar{c}^a\bar{c}^b c^c c^d - \frac{\alpha}{4}g^2 f^{abc}f^{adi}\bar{c}^b\bar{c}^c c^d c^i\\
&- \frac{\alpha}{8}g^2 f^{abc}f^{ade}\bar{c}^b\bar{c}^c c^d c^e + b^i\partial_\mu A_\mu^i + \bar{c}^i\partial_\mu(\partial_\mu c^i + gf^{iab}A_\mu^a c^b)\Big).
\end{aligned} \tag{8}$$

The gauge parameter α in expression (7) has to be introduced for renormalization purposes [4]. The action (5) displays mutiplicative renormalizability. In particular, we underline that as a consequence of the Ward identities which can be established in the maximal Abelian gauge, the following nonrenormalization theorem holds; see for instance Eq. (40) of [4], i.e.

$$Z_g Z_{A^i}^{1/2} = 1. \tag{9}$$

This relationship states that the renormalization of the gauge coupling constant g is related to the renormalization factor of the diagonal components, A_μ^i, of the gauge field. Until now, Eq. (9) has been established to all orders of perturbation theory, being explicitly checked at three loops in [13].

III. THE GENERALIZED INTERPOLATING GAUGE

Following [3], let us make use of the notation

$$\begin{aligned}
\tilde{\partial}_\mu &= (\nabla, a\partial_4), \qquad \tilde{A}_\mu^i = (\vec{A}^i, aA_4^i),\\
\tilde{A}_\mu^a &= (\vec{A}^a, aA_4^a), \qquad \tilde{D}_\mu^{ab} = \delta^{ab}\tilde{\partial}_\mu - gf^{abi}\tilde{A}_\mu^i,
\end{aligned} \tag{10}$$

where a is the gauge parameter which interpolates between the Coulomb and Landau gauges. The gauge fixing, $S_{\rm CLM}$, which interpolates among the Landau, Coulomb, and maximal Abelian gauges contains three gauge parameters, (a, k, α), being given by

$$S_{\rm CLM} = s\int d^4x\Big(\bar{c}^a\tilde{\partial}_\mu A^a_\mu - g\bar{c}^a f^{abi}\tilde{A}^i_\mu A^b_\mu + \frac{\alpha}{2}\bar{c}^a b^a - \frac{\alpha}{2}gf^{abi}\bar{c}^a\bar{c}^b c^i - \frac{\alpha}{4}gf^{abc}c^a\bar{c}^b\bar{c}^c + \bar{c}^i\tilde{\partial}_\mu A^i_\mu - kgf^{iab}\tilde{A}^i_\mu A^a_\mu\bar{c}^b\Big). \tag{11}$$

Acting with the BRST operator s on the elementary fields, one obtains

$$\begin{aligned}S_{\rm CLM} = \int d^4x\Big(& b^a\Big(\tilde{D}^{ab}_\mu A^b_\mu + \frac{\alpha}{2}b^a\Big) + \bar{c}^a\tilde{D}^{ab}_\mu D^{bc}_\mu c^c + g\bar{c}^a f^{abi}(\tilde{D}^{bc}_\mu A^c_\mu)c^i + g\bar{c}^a\tilde{D}^{ab}_\mu(f^{bcd}A^c_\mu c^d) - \alpha gf^{abi}b^a\bar{c}^b c^i\\ & - g^2f^{abi}f^{cdi}\bar{c}^a c^d\tilde{A}^b_\mu A^c_\mu - \frac{\alpha}{2}gf^{abc}b^a\bar{c}^b c^c - \frac{\alpha}{4}g^2f^{abi}f^{cdi}\bar{c}^a\bar{c}^b c^c c^d - \frac{\alpha}{4}g^2f^{abc}f^{adi}\bar{c}^b\bar{c}^c c^d c^i\\ & - \frac{\alpha}{8}g^2f^{abc}f^{ade}\bar{c}^b\bar{c}^c c^d c^e + b^i\tilde{\partial}_\mu A^i_\mu + \bar{c}^i\tilde{\partial}_\mu(\partial_\mu c^i + gf^{iab}A^a_\mu c^b) + kgf^{abi}\tilde{A}^a_\mu(\partial_\mu c^i)\bar{c}^b + kg^2f^{abi}f^{cdi}\bar{c}^a c^d\tilde{A}^b_\mu A^c_\mu\\ & - kgf^{abi}\tilde{A}^i_\mu A^a_\mu(b^b - gf^{bcj}\bar{c}^c c^j) + kgf^{abi}\tilde{A}^i_\mu(D^{ac}_\mu c^c)\bar{c}^b + kg^2f^{abi}f^{acd}\tilde{A}^i_\mu A^c_\mu c^d\bar{c}^b\Big).\end{aligned} \tag{12}$$

Let us now show how the various gauges can be recovered from expression (11) when appropriate limits for the parameters (a, k, α) are taken. Let us begin with the Landau gauge.

A. The Landau gauge

The Landau gauge is recovered by taking

$$a = 1, \qquad k = 1, \qquad \alpha = 0. \tag{13}$$

In fact, from expression (11), one obtains

$$S_L = s\int d^4x(\bar{c}^a\partial_\mu A^a_\mu + \bar{c}^i\partial_\mu A^i_\mu) = s\int d^4x(\bar{c}^A\partial_\mu A^A_\mu), \tag{14}$$

which is the Landau gauge.

B. The Coulomb gauge

The Coulomb gauge is obtained from (11) by setting

$$a = 0, \qquad k = 1, \qquad \alpha = 0. \tag{15}$$

Expression (11) is easily seen to reduce to the Coulomb gauge, namely,

$$\begin{aligned}S_C &= s\int d^4x(\bar{c}^a(\nabla\cdot\vec{A}^a) + \bar{c}^i(\nabla\cdot\vec{A}^i))\\ &= s\int d^4x\bar{c}^A(\nabla\cdot\vec{A}^A).\end{aligned} \tag{16}$$

C. The maximal Abelian gauge

Finally, the maximal Abelian gauge corresponds to

$$a = 1, \qquad k = 0, \tag{17}$$

yielding

$$\begin{aligned}S_{\rm MAG} = s\int d^4x\Big(&\bar{c}^a\partial_\mu A^a_\mu - g\bar{c}^a f^{abi}A^i_\mu A^b_\mu + \frac{\alpha}{2}\bar{c}^a b^a\\ & - \frac{\alpha}{2}gf^{abi}\bar{c}^a\bar{c}^b c^i - \frac{\alpha}{4}gf^{abc}c^a\bar{c}^b\bar{c}^c + \bar{c}^i\partial_\mu A^i_\mu\Big),\end{aligned} \tag{18}$$

which is recognized to be the maximal Abelian gauge, Eq. (7).

IV. AN INTERPOLATING MASS DIMENSION TWO OPERATOR

It is worth remarking that the interpolating gauge fixing (11) allows us to introduce a generalized mass dimension two operator $O_{\rm CLM}$

$$O_{\rm CLM} = \frac{1}{2}\tilde{A}^a_\mu A^a_\mu + \frac{k}{2}\tilde{A}^i_\mu A^i_\mu + \alpha\bar{c}^a c^a, \tag{19}$$

which enjoys the property of being BRST invariant on shell. More precisely, one has

$$s\int d^4x O_{\rm CLM} = \int d^4x\Big(kc^i\frac{\delta(S_{\rm YM}+S_{\rm CLM})}{\delta b^i} + c^a\frac{\delta(S_{\rm YM}+S_{\rm CLM})}{\delta b^a}\Big). \tag{20}$$

Interestingly, the operator $O_{\rm CLM}$ interpolates among all dimension two mass operators already introduced in the Coulomb, Landau, and maximal Abelian gauges, namely,

$$O_{\rm CLM} \to O_{\rm Coulomb} = \tfrac{1}{2}A^A\cdot A^A, \quad \text{for } a = 0, \quad k = 1, \quad \alpha = 0, \tag{21}$$

$$O_{\rm CLM} \to O_{\rm Landau} = \tfrac{1}{2}A^A_\mu A^A_\mu, \quad \text{for } a = 1, \quad k = 1, \quad \alpha = 0, \tag{22}$$

$$O_{\rm CLM} \to O_{\rm MAG} = \tfrac{1}{2}A^a_\mu A^a_\mu + \alpha\bar{c}^a c^a, \quad \text{for } a = 1, \quad k = 0. \tag{23}$$

Analytic evidence for the condensation of all these mass dimension two operators have been given in Refs. [4,8–10,14].

V. CONCLUSION

In this work a generalized gauge, Eq. (11), which interpolates among the Landau, Coulomb, and maximal Abelian gauges has been introduced. It could lead to several interesting features worth being analyzed.

(i) A first aspect to be faced is the renormalizability of the interpolating gauge (11). This point is being investigated along the lines of [3], where an all orders BRST algebraic proof of the multiplicative renormalizability of the gauge fixing interpolating between the Landau and Coulomb gauges has been achieved. Also, it would be interesting to see if the algebraic setup of [3] could be generalized so as to include the dimension two mass operator $O_{\rm CLM}$, Eq. (19), as well as the nonlocal gauge invariant operator $\mathrm{Tr}\int d^4x F_{\mu\nu}\frac{1}{D^2}F_{\mu\nu}$ recently discussed in [15] within the class of the linear covariant gauges, which includes the Landau gauge as a particular case.
As already mentioned in the introduction, although the dimension two operator A^2 has been proven to be multiplicatively renormalizable to all orders in the Landau, Curci-Ferrari, maximal Abelian, and linear covariant gauges (Refs. [5–7]), a clear understanding of the aspects related to the gauge invariance of the dimension two condensate $\langle A^2\rangle$ is still under analysis. As discussed in [16], a possible gauge invariant extension of the operator A^2 could be provided by the gauge invariant operator $A^2_{\rm min}$, obtained by minimizing A^2 along the gauge orbit of A_μ. However, the operator $A^2_{\rm min}$ appears to be highly nonlocal, a feature which jeopardizes the standard perturbative renormalization procedure for an arbitrary choice of the gauge fixing [17]; see also [15]. Nevertheless, as shown in [4,11,12], the vacuum energy $E_{\rm vac}$ evaluated in the presence of the dimension two condensate $\langle A^2\rangle$ turns out to be independent from the gauge parameters. Therefore, the possibility of having at our disposal a generalized gauge which interpolates among the Landau, Coulomb, and maximal Abelian gauges might allow us to achieve the result that the vacuum energy evaluated in the presence of the generalized dimension two operator, Eq. (19), has to be same for all three gauges.

(ii) A second aspect which could be exploited is to investigate whether the nonrenormalization theorem of the maximal Abelian gauge, as expressed by Eq. (9), would remain valid beyond perturbation theory. The natural framework to discuss this issue is through lattice numerical simulations as done, for example, in the case of the nonrenormalization theorem of the ghost-gluon vertex in the Landau gauge [18]. It is worth underlining that the relationship (9) could open the interesting possibility of studying, through lattice simulations, the infrared behavior of the running coupling constant in the maximal Abelian gauge. More precisely, Eq. (9) suggests that the infrared behavior of the running coupling constant in the maximal Abelian gauge could be accessed by looking at the behavior of the diagonal component of the gluon propagator. Interestingly, a recent study of the gluon propagator in momentum space has been performed in [19], reporting an infrared suppression of the diagonal component. Moreover, according to [19], the diagonal gluon propagator could attain a finite nonvanishing value at $k \approx 0$, a feature which could signal the possible existence of an infrared fixed point for the running coupling constant in the maximal Abelian gauge.

(iii) Finally, we point out that the authors in [20] have shown that the use of the interpolating Landau-Coulomb gauge allows one to introduce two renormalization group invariant running couplings. In particular, one of these two couplings turns out to be independent from the interpolating gauge parameter, displaying an infrared fixed point whose value coincides with that already known in the Landau gauge. The existence of such an infrared fixed point extends thus to the Coulomb gauge too [20]. It would be worthwhile investigating if similar effective renormalization group invariant couplings could be introduced also in the present interpolating gauge. This might provide further indication on the possible existence of an infrared fixed point for the running coupling constant in the maximal Abelian gauge.

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