PHYSICAL REVIEW D **73,** 041301(R) (2006)

Effect of reconnection probability on cosmic (super)string network density

A. Avgoustidis* and E. P. S. Shellard†

Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom

(Received 22 December 2005; published 3 February 2006)

We perform numerical simulations of cosmic string evolution with intercommuting probability P in the range $5 \times 10^{-3} \leq P \leq 1$, both in the matter and radiation eras. We find that the dependence of the scaling density on P is significantly different than the suggested $\rho \propto P^{-1}$ form. In particular, for probabilities greater than $P \simeq 0.1$, $\rho(1/P)$ is approximately flat, but for P less than this value it is well-fitted by a power-law with exponent $0.6^{+0.15}_{-0.12}$. This shows that the enhancement of string densities due to a small intercommuting probability is much less prominent than initially anticipated. We interpret the flat part of $\rho(1/P)$ in terms of multiple opportunities for string reconnections during one crossing time, due to small-scale wiggles. We also propose a two-scale model, which satisfactorily fits our results over the whole range of P covered by the simulations.

DOI: 10.1103/PhysRevD.73.041301 PACS numbers: 98.80.Cq, 11.25.Wx

I. INTRODUCTION

Cosmic strings have recently been reincarnated in the form of "cosmic superstrings", fundamental strings and one-dimensional D-branes produced at the end of brane inflation [1–4] (for reviews see [5–8]). These objects have properties which can be significantly different than usual field theory strings, and this opens up the possibility that cosmic string observations could yield information about string theory, while providing new ways to constrain various brane inflation models [9,10]. Cosmic superstrings reconnect with probabilities which can be significantly less than unity [11,12], unlike ordinary cosmic strings [13]. This reduced reconnection probability results in an enhancement of the string number density today [4,11,14]. The evolution of cosmic superstring networks has recently been studied in Refs. [14–18]. The key issue is whether these networks reach a *scaling regime*, that is one in which their characteristic lengthscale stays constant relative to the horizon. In all studies of models of cosmic supersting networks so far, evidence for scaling behavior has been found, though the issue is not yet completely resolved.

Assuming that an attractor scaling solution exists, an interesting question to ask is how the density of the scaling network depends on the intercommuting probability P. Using a simple one-scale model [19], Jones, Stoica and Tye have argued in Ref. [11] that the expected behavior is $\rho \propto P^{-2}$. This is a dramatic effect, as a probability of 10^{-2}, for example, would lead to an enhancement of the string density by a factor of 10^4 compared to ordinary strings: we could live in a universe packed with F/D-string relics from a brane inflation era! However, string intercommuting is a small-scale process and can be expected to depend crucially on small-scale wiggles on strings, which are not captured by the simple one-scale model. One should expect that, since wiggly strings may have more than one opportunity to reconnect in each encounter, the effect of P on string density could be somewhat counterbalanced by the strings' small-scale-structure. Thus a power-law weaker than $\rho \propto P^{-2}$ may be anticipated.

Indeed, Sakellariadou recently performed flat space simulations of cosmic strings with small reconnection probabilities and found a power law $\rho \propto P^{-1}$ in the range $10^{-3} \leq P \leq 0.3$ [15]. The purpose of this letter is to investigate the dependence of the string energy density ρ on the intercommuting probability P for strings evolving in more realistic cosmological backgrounds. We perform numerical simulations in the matter and radiation era, and find a power law significantly weaker than that of Ref. [15]. We explain our results in terms of string small-scale structure and the importance of a second scale in the problem. We thus propose a two-scale analytic model which provides a good fit to our numerical results.

II. SIMULATIONS

We have performed numerical simulations of string networks, with reduced intercommuting probabilities and

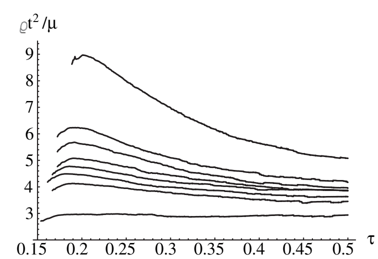

FIG. 1. Dimensionless string density plotted against conformal time τ for a network with intercommuting probability $P = 0.75$. Each curve corresponds to a different initial horizon to correlation length ratio. The asymptotic curves bracket the scaling solution, which can be estimated to be $\rho t^2/\mu = 3.6^{+0.2}_{-0.1}$.

*Electronic address: A.Avgoustidis@damtp.cam.ac.uk
†Electronic address: E.P.S.Shellard@damtp.cam.ac.uk

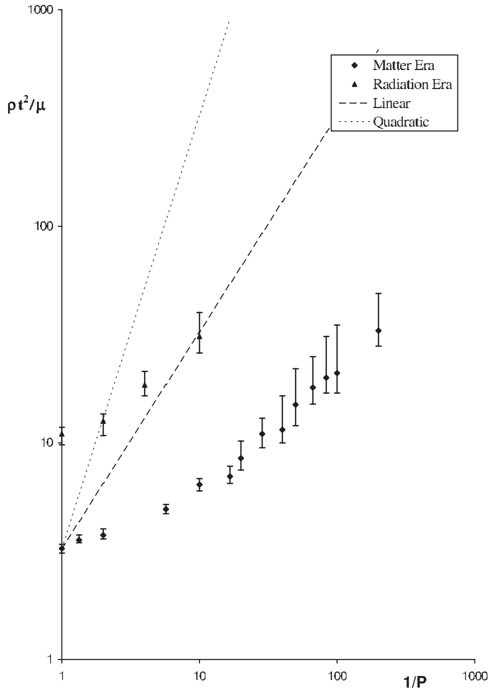

FIG. 2. Dimensionless string scaling density plotted against the inverse intercommuting probability $1/P$, for matter and radiation era runs (1σ errors displayed) . The constant slope part of the matter era data can be fitted by a power law with exponent $0.6^{+0.15}_{-0.12}$. The overall dependence of ρ on P is much weaker than the previously suggested $\rho \propto 1/P^2$ and $\rho \propto 1/P^2$ forms.

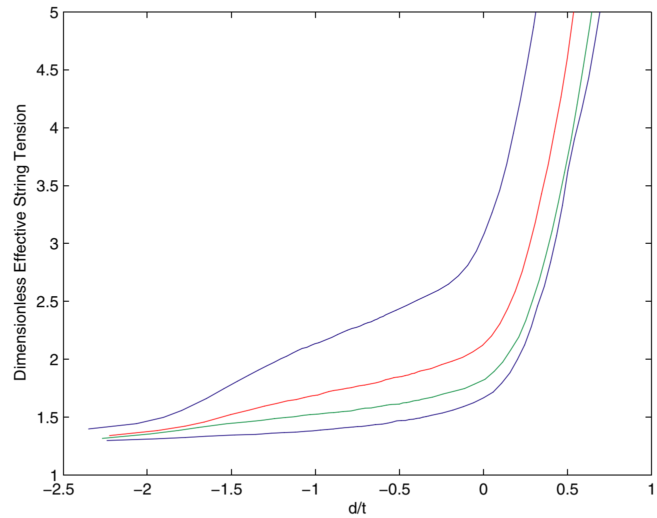

FIG. 3 (color online). Dimensionless effective string tension $\tilde{\mu}$ plotted against physical distance (measured in units of physical time t) for matter era networks with $P = 1, 0.1, 0.05$ and 0.01. As P is reduced, the effective tension increases signalling the accumulation of small-scale structure.

evolving in FRW spacetime, using a modified version of the Allen-Shellard code [22]. Each network was given a different intercommuting probability in the range $5 \times 10^{-3} \leq P \leq 1$ and was evolved both in the matter and radiation eras for a dynamical range of order 3. For each network characterized by a given intercommuting probability P, approximately ten runs were performed, each with different initial string density. By plotting the time evolution of the string density for different initial conditions, one can bracket the scaling solution, as shown in Fig. 1, getting successively more accurate convergence with subsequent runs. We have thus obtained, within errors, the scaling density ρ of these networks.

In Fig. 2 we plot the dimensionless parameter $\rho t^2/\mu$ (where μ is the string tension) versus the inverse intercommuting probability $1/P$ for matter and radiation era runs. In the former case P ranges from 5×10^{-3} to 1, but in the latter our limited dynamical range has at present only allowed us to bracket the scaling densities for P in the range $0.1 \leq P \leq 1$. We see that for probabilities greater than $P \simeq 0.1$, the function $\rho(1/P)$ is approximately flat, but for smaller P it develops a constant slope, on a log-log scale, at least in the matter era. A weighted fit gives a slope of $0.6^{+0.15}_{-0.12}$ for the matter runs, and the radiation era data are consistent with this picture, though more data points are needed to confirm the value of the slope.[1] Comparing to the $\rho \propto P^{-2}$ and $\rho \propto P^{-1}$ forms of Refs. [11,15] respectively (also plotted), we see that the enhancement of string densities due to a reduced intercommuting probability is much less prominent than what was initially anticipated. For example, a probability of 5×10^{-3} leads to an enhancement in ρ by only a factor of 10, to be contrasted with the predictions of 200 (resp. 10^4) obtained from $\rho \propto P^{-1}$ (resp. $\rho \propto P^{-2}$).

We have also investigated the effect of reducing the intercommuting probability P on the amount of string small-scale-structure [23]. This can be quantified by the dimensionless effective tension defined as $\tilde{\mu} = \mu_{\rm eff}/\mu$, where $\mu_{\rm eff}$ is the effective energy per unit length at the scale of the correlation length ξ (this is the average length beyond which string directions are not correlated). For wiggly strings $\tilde{\mu}$ is greater than unity, reflecting the fact that string structure at subcorrelation length scales "renormalizes" the tension at the scale of the correlation length to $\mu_{\rm eff} > \mu$ (see [23]). In Fig. 3 we plot the dimensionless effective tension $\tilde{\mu}$ against physical distance (in units of physical time t) for a range of intercommuting probabilities. As the intercommuting probability decreases, we see a significant increase in the effective tension, especially at the scale of the correlation length. This is strong evidence for the accumulation of small-scale structure as P is reduced.

III. DISCUSSION

We can interpret the flat part of Fig. 2 in terms of string small-scale-structure. On large scales, correlation length

[1] The function $y = (\frac{2x^3+20x-8}{1+x^2})^{0.6}$, where $x = 1/P$, gives a good fit to the matter-era data, in the whole range of intercommuting probabilities studied.

segments move together slowly and coherently ($v_c \approx 0.15$ for $P = 1$ [23]), but their small-scale structure is relativistic and rapidly oscillates ($v_{rms} \approx 1/\sqrt{2}$). One therefore expects the strings to cross several times as they pass, thus the overall process should be described by an *effective* probability of reconnection $P_{eff} > P$. Thus reducing P by a factor of a few should not have a big impact on the string density [14], as seen in Fig. 2. Furthermore we can explain the position where the slope starts to change, by using the following simple model for string interactions.

Consider two colliding straight strings, described on small-scales by a monochromatic mode of amplitude A and period T. If v_c is their relative coherent speed, the duration of the collision is approximately $\Delta t \approx 2A/v_c$. Now on small scales the strings are oscillating with period T, so the number of reconnection opportunities during their crossing time is $N \approx \Delta t/T = 2A/v_c T$. But since on these scales string modes are propagating with velocity $c \sim 1$, T is equal to the wavelength of the mode, and so A/T is a measure of string wiggliness. In particular, for large amplitudes we may write $1 + A/T \sim \tilde{\mu}$ from which we can infer that

$$N \sim \frac{2(\tilde{\mu} - 1)}{v_c}. \tag{1}$$

Interestingly, this simple estimate gives about the right value for the position where the curve changes slope in Fig. 2. Indeed, from Fig. 3 we see that for $P > 0.1$, $\tilde{\mu}$ is in the range $1.6 \leq \tilde{\mu} \leq 1.7$, while we know that string velocities at the scale of the correlation length are approximately $v_c \approx 0.15$. This gives $N \sim 10$ intercommuting opportunities and one therefore expects that, to see a significant effect on the scaling string density, P should be reduced by a factor of order 10. Furthermore, we can estimate the effective probability by considering the N reconnection opportunities, assuming each has an independent probability P, that is

$$P_{eff} = 1 - (1 - P)^N. \tag{2}$$

For $N \simeq 10$, a probability as low as 0.2 yields $P_{eff} \simeq 1$, and significant deviations from unity only occur for $P \simeq 0.1$ for which $P_{eff} \simeq 0.65$, as observed in Fig. 2. In the limit $P \ll 1$, Eq. (2) gives $P_{eff} \simeq NP \approx [2(\tilde{\mu} - 1)/v_c]P$ and so P can only be enhanced by a factor of order 10 (see Fig. 3), which results in an effective reconnection probability that is still much less than unity.

We now turn to explaining the constant slope part of Fig. 2 for small probabilities. First recall that a simple one-scale model with $P < 1$ predicts $\rho \propto P^{-2}$ in direct contradiction with our numerical results. As discussed above, the introduction of an effective intercommuting probability can dramatically weaken this but only for relatively large P (say $P > 0.1$). However, to accommodate the constant slope part for $P \ll 1$ one would need $P_{eff} \propto P^{\kappa}$, with κ a constant less than $1/2$. At present there seems no motivation for such a P_{eff}; instead, Eq. (2) suggests that P_{eff} is linear in P for $P \ll 1$. Thus, it appears that no one-scale model can fit our numerical results for $P \ll 1$: as the string intercommuting probability decreases, the one-scale approximation becomes increasingly poor. This is in agreement with the results of Ref. [24] in which long-string intercommutings were switched off in numerical simulations of evolving strings, while small-loop production was allowed. It was found that this prevented the string density from scaling, that is, the interstring distance L was no longer proportional to t (the solution being $\rho \propto L^{-2} \propto t^{-7/8}$). Nevertheless, the actual correlation length ξ along the string did scale at approximately the size of the horizon ($\xi \propto t$).

This is very similar to the situation we observe in our simulations. Reducing the intercommuting probability leads to an increase of the string density and therefore a decrease of the characteristic length associated with it (the interstring distance L). The correlation length ξ, however, is only weakly affected by this, and stays at a scale of order the horizon (Fig. 3). This can be understood by considering the two distinct mechanisms for producing loops: (i) self-intersections of the same string and (ii) collisions between long strings. The former tends to chop off small loops, straightening the strings out (thus affecting mainly the correlation length ξ) and controlling the amount of small-scale structure $\tilde{\mu}$. This alone is a significant energy loss mechanism, but not sufficient to ensure scaling [24]. On the other hand, long-string reconnections have much more dramatic effects introducing large-scale "bends" in the strings which catalyse the collapse of large regions of string and the formation of many more loops. In contrast, these energy losses are sufficient to govern the interstring distance L and cause scaling.

The first mechanism, is not greatly affected by reducing the intercommuting probability: once the string is sufficiently wiggly, left and right moving modes which fail to interact due to a small P will keep propagating and meet more incoming modes, with which they will eventually interact and form loops. It might take longer, but such interactions are inevitable even for $P \ll 1$. The relevant question is whether enough of these interactions can take place in each Hubble time in order to straighten the strings out at the horizon scale, but this seems reasonable given the much shorter timescale on which small-loop production operates. It appears to be confirmed by Fig. 3, where there is only a very weak build-up of small-scale structure (a factor of 2–3 in $\tilde{\mu}$) while P changes by over 2 orders of magnitude. On the other hand, long-string intercommuting, the second mechanism, depends more crucially on P: two colliding strings have a given interval of time in which to interact, so intercommuting is no longer inevitable for relatively small $P < 0.1$. A reduced P necessarily means less string interactions and less energy dumped from the long-string network in the form of loops. This leads to an increase in the long-string density and thus a significant decrease of the characteristic length relative to the correlation/horizon scale.

To obtain an analytic model for such networks, it is therefore necessary to introduce two scales: a characteristic length L (roughly the interstring distance) quantifying the energy density in strings, and a correlation length ξ, defined to be the distance beyond which string directions are not correlated. Indeed, similar models have appeared in the literature (e.g. the three-scale model of Ref. [25]), but for normal $P = 1$ cosmic strings these two scales are comparable so the one-scale (velocity-dependent) approximation [20,21] can be surprisingly accurate [26]. Here, we develop a two-scale velocity-dependent model which fits our numerical results, again surprisingly well given its simplicity.

Consider a string network, characterized by tension μ, correlation length ξ and energy density ρ. We define the characteristic lengthscale L of the network by $\rho \equiv \mu/L^2$ and note that the number of strings per correlation volume $V = \xi^3$ is $N_\xi = \rho V/\mu\xi = \xi^2/L^2$. If v is the typical string velocity, each string intersects $N_\xi - 1$ other strings in time $\Delta t = \xi/v$, so we have $N_\xi^2 - N_\xi$ intercommutings per correlation volume per time Δt. Assuming that each intercommuting produces a loop of length $\tilde{c}\xi$ [27] the energy loss due to the formation of loops can be written as

$$\left(\frac{\delta\rho}{\delta t}\right)_{2-\mathrm{scale}} = \frac{(N_\xi^2 - N_\xi)v}{\xi^4}\mu\tilde{c}\xi = \tilde{c}\rho\left(\frac{\xi}{L^2} - \frac{1}{\xi}\right). \quad (3)$$

This is to be contrasted with the corresponding result for the one-scale model

$$\left(\frac{\delta\rho}{\delta t}\right)_{1-\mathrm{scale}} = \tilde{c}\frac{\rho}{L}. \quad (4)$$

As an interesting aside, we note that the last term in Eq. (3) $\propto 1/\xi$ has the same form (though opposite sign) as the term which would need to be introduced to account for direct small-loop production (or string radiation); it cannot itself cause L to scale as t.

Our velocity-dependent two-scale model can therefore be constructed by using the usual VOS model equations [21], derived from the Nambu-Goto action, but using the phenomenological loop production term (3) instead of (4). The result is a system of two coupled ODEs, governing the time evolution of the characteristic length L and the average velocity v of string segments:

$$2\frac{dL}{dt} = 2\frac{\dot{a}}{a}L(1 + v^2) + \tilde{c}v\left(\frac{\xi}{L} - \frac{L}{\xi}\right) \quad (5)$$

$$\frac{dv}{dt} = (1 - v^2)\left(\frac{k}{\xi} - 2\frac{\dot{a}}{a}v\right) \quad (6)$$

In Eq. (6), k is the so-called momentum parameter, which is a measure of the angle between the curvature vector and the velocity of string segments and thus is related to the smoothness of strings [20]. Note that we have made no attempt to derive an evolution equation for the correlation length ξ as the simulations show that this depends weakly on P and moreover, unlike L, it remains comparable to the horizon (Fig. 3). We will neglect this weak dependence of ξ on P and take $\xi = t$ in Eqs. (5) and (6). To apply the model to our simulated networks, we have also introduced an effective intercommuting probability given by (2), that is, we have replaced $\tilde{c}$ in Eq. (5) by $P_{\mathrm{eff}}\tilde{c}$. As a first approximation, we have neglected the dependence of N on P and we have simply taken $N = 10$ (a rough average value over the range of P). For the loop production parameter, we have used the value $\tilde{c} = 0.23$ of Refs. [21,26] which fits both radiation and matter era runs in the $P = 1$ case.

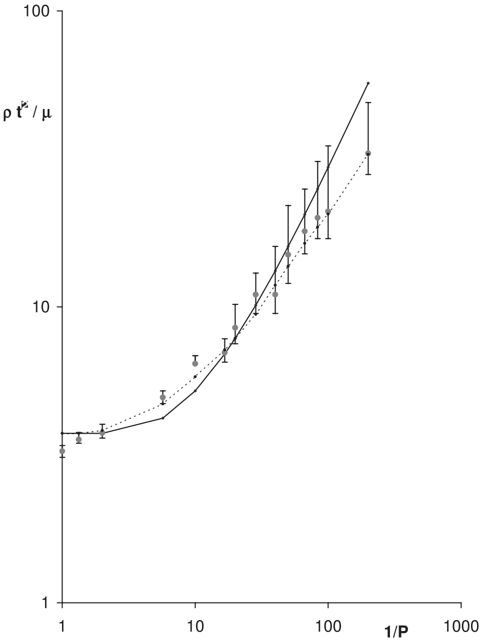

FIG. 4. Scaling string density obtained from simulations (data points with errors) and from the analytic two-scale model (5) and (6) (solid line). The fit is improved further (dashed line) by phenomenologically incorporating the dependence of the effective string tension $\tilde{\mu}$ on the reconnection probability P.

In Fig. 4 we plot the matter era scaling energy densities, obtained both from the simulations and our two-scale model (solid line). We see that the model provides a surprisingly good fit to the numerical data and it reproduces the observed change of slope around $P \approx 0.1$. We stress that we have made no special parameter choices to obtain this fit, and by modifying parameters further we could do much better. Nevertheless, it is clear from the dashed line in Fig. 4 how the model could be improved by taking into account the dependence of the collision number N on $\tilde{\mu}(P)$. In this case, we have estimated the effective string tension $\tilde{\mu}$ as a function of P from Fig. 3, which yields N from Eq. (1) and then $P_{\mathrm{eff}}(P)$ from Eq. (2) which is finally input "by hand" in our two-scale model. We note the need also to take into account the dependence of ξ on the reconnection probability, and a direct small-loop production term. Here we highlight the fact that a simple version of our two-scale model with the addition of the effective probability of Eq. (2) seems to provide a satisfactory fit to the numerical data.

IV. CONCLUSION

We have performed numerical simulations of cosmic strings with intercommuting probabilities in the range $5 \times 10^{-3} \leq P \leq 1$ evolving in a matter- or radiation-dominated FRW universe. We have found that the dependence of the string density ρ on the intercommuting probability P is much weaker than previously suggested in the literature. In particular, the function $\rho(1/P)$ has an initially flat dependence, up to probabilities of about 0.1, and then develops a constant slope (on a log-log scale) of $0.6^{+0.15}_{-0.12}$. This yields very different predictions from the generally expected $\rho \propto P^{-1}$ form; a probability of $P = 5 \times 10^{-3}$, for example, gives an energy density enhancement of a factor of 10, compared to a factor of 200 with $\rho \propto P^{-1}$. Clearly our results are important for determining the quantitative observational predictions of models with cosmic (super-)strings and various limits in the literature will have to be re-examined in light of these results.

We have also endeavoured to provide some physical explanations for why the string density depends nontrivially on the reconnection probability. We can explain the flat dependence of $\rho(1/P)$ in terms of small-scale wiggles on strings, which lead to multiple opportunities for reconnection in each crossing time, thus introducing an effective reconnection probability $P_{\rm eff} = f(P)$. By approximating the small-scale structure on long strings by a monochromatic mode, we suggested a physically motivated form for $P_{\rm eff}$, which adequately explains the large P behavior of $\rho(1/P)$.

The simulations demonstrated that, although the string density increases for small P leading to a reduction of the interstring distance L, the correlation length ξ depends only weakly on P and scales at a size comparable to the horizon. We have explained how one can understand this in terms of the two distinct mechanisms for loop production: the self-intersection of wiggly strings which produces small loops and tends to straighten the strings, determining the correlation length ξ, and long-string collisions, which cause much greater loop energy losses, determining the characteristic length L. We suggested a simple two-scale model to describe the behavior of $\rho(1/P)$ over the whole range of probabilities probed by the simulations and this provided an adequate fit to the numerical data. We leave a more detailed numerical investigation and further improvements of the analytic model for a future publication.

ACKNOWLEDGMENTS

The authors would like to thank Carlos Martins for many discussions. A. A. is supported by EPSRC, the Cambridge European Trust and the Cambridge Newton Trust. This work is also supported by PPARC grant PP/C501676/1. The code used was developed by EPSS and Bruce Allen. The simulations were performed on COSMOS supercomputer, the Altix3700 owned by the UK Computational Cosmology Consortium, and supported by SGI, Intel, HEFCE and PPARC.

[1] G. Dvali and S. H. H. Tye, Phys. Lett. B **450**, 72 (1999).
[2] C. P. Burgess, J. M. Cline, H. Stoica, and F. Quevedo J. High Energy Phys. 09 (2004) 033.
[3] S. Sarangi and S. H. H. Tye, Phys. Lett. B **536**, 185 (2002).
[4] G. Dvali and A. Vilenkin, J. Cosmol. Astropart. Phys. 03 (2004) 010.
[5] J. Polchinski, hep-th/0412244.
[6] T. W. B. Kibble, astro-ph/0410073.
[7] A. C. Davis and T. W. B. Kibble, Contemp. Phys. **46**, 313 (2005).
[8] M. Majumdar, hep-th/0512062.
[9] E. Babichev and M. Kachelriess, Phys. Lett. B **614**, 1 (2005).
[10] A. Avgoustidis and E. P. S. Shellard, J. High Energy Phys. 08 (2005) 092.
[11] N. T. Jones, H. Stoica, and S. H. H. Tye, Phys. Lett. B **563**, 6 (2003).
[12] M. G. Jackson, N. T. Jones, and J. Polchinski, J. High Energy Phys. (2005) 013.
[13] E. P. S. Shellard, Nucl. Phys. **B283**, 624 (1987).
[14] A. Avgoustidis and E. P. S. Shellard, Phys. Rev. D **71**, 123513 (2005).
[15] M. Sakellariadou, J. Cosmol. Astropart. Phys. 04 (2005) 003.
[16] C. J. A. P. Martins, Phys. Rev. D **70**, 107302 (2004).
[17] S. H. H. Tye, I. Wasserman, and M. Wyman, Phys. Rev. D **71**, 103508 (2005); **71**, 129906(E) (2005).
[18] E. J. Copeland and P. M. Saffin J. High Energy Phys. 11 (2005) 023.
[19] T. W. B. Kibble, Nucl. Phys. **B252**, 227 (1985).
[20] C. J. A. P. Martins and E. P. S. Shellard, Phys. Rev. D **53**, R575 (1996).
[21] C. J. A. P. Martins and E. P. S. Shellard, Phys. Rev. D **65**, 043514 (2002).
[22] B. Allen and E. P. S. Shellard, Phys. Rev. Lett. **64**, 119 (1990).
[23] C. J. A. P. Martins and E. P. S. Shellard, astro-ph/0511792.
[24] E. P. S. Shellard and B. Allen, in *Formation and Evolution of Cosmic Strings*, edited by G. W. Gibbons, S. W. Hawking, and T. Vachaspati, (Cambridge University Press, Cambridge, England, 1990).
[25] D. Austin, E. J. Copeland, and T. W. B. Kibble, Phys. Rev. D **48**, 5594 (1993).
[26] C. J. A. P. Martins, J. N. Moore, E. P. S. Shellard, and B. Allen, Phys. Rev. Lett. **92**, 251601 (2004).
[27] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, England, 1994).