Discrete symmetries on the light front and a general relation connecting the nucleon electric dipole and anomalous magnetic moments

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(Received 10 January 2006; published 27 February 2006)

We consider the electric dipole form factor, $F_3(q^2)$, as well as the Dirac and Pauli form factors, $F_1(q^2)$ and $F_2(q^2)$, of the nucleon in the light-front formalism. We derive an exact formula for $F_3(q^2)$ to complement those known for $F_1(q^2)$ and $F_2(q^2)$. We derive the light-front representation of the discrete symmetry transformations and show that time-reversal- and parity-odd effects are captured by phases in the light-front wave functions. We thus determine that the contributions to $F_2(q^2)$ and $F_3(q^2)$, Fock state by Fock state, are related, independent of the fundamental mechanism through which CP violation is generated. Our relation is not specific to the nucleon, but, rather, is true of spin-$1/2$ systems in general, be they lepton or baryon. The empirical values of the anomalous magnetic moments, in concert with empirical bounds on the associated electric dipole moments, can better constrain theories of CP violation. In particular, we find that the neutron and proton electric dipole moments echo the isospin structure of the anomalous magnetic moments, $\kappa^n \sim -\kappa^p$.

DOI: 10.1103/PhysRevD.73.036007 PACS numbers: 11.30.Er, 11.10.St, 11.15.Tk, 13.40.Em

I. INTRODUCTION

The electric dipole moments of particles such as the neutron, electron, muon, or neutrino, provide important windows into the fundamental origin of CP violation at the Lagrangian level. The underlying source, or sources, of CP violation in nature could arise in any of a number of ways. Such sources include not only the phase structure of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1], which describes quark mixing and provides CP violation in the standard model, but also the phase structure of the lepton-mixing matrix [2], as well as flavor-diagonal, CP-violating interactions, as could occur in theories with extended Higgs sectors, such as in supersymmetry [3]. The fundamental theory then leads to effective, higher-dimension, CP-violating operators, such as the antisymmetric product of three gluonic [4] or $SU(2)_L$ field strengths [5] or the electric-dipole interaction $\bar{\psi}\gamma_5\sigma_{\mu\nu}F^{\mu\nu}\psi$. Thus far experiment has provided upper bounds on the magnitude of the electron [6] and neutron [7] electric dipole moments; current limits imply that models with weak-scale supersymmetry and $\mathcal{O}(1)$ CP-violating parameters can produce electric dipole moments significantly in excess of experimental bounds [8]. New, improved experiments, as in Refs. [9–11], have the capacity to sharpen such constraints severely; it is our purpose to consider the ramifications of such improvements for theories of CP violation.

An essential question is how to relate the electric dipole moments of leptons and baryons to the CP-violating parameters of the underlying theory. The light-front Fock expansion [12,13] provides an exact Lorentz-invariant representation of the matrix elements of the electromagnetic current in terms of the overlap of light-front wave functions [14–16]—note Ref. [17] for a comprehensive review. The current takes an elementary form in the light-front formalism because, in the interaction picture, the full Heisenberg current can be replaced by the free quark current $J^\mu(0)$, evaluated at the light-cone time $x^+ = 0$. As first shown by Drell and Yan [14,15], such matrix elements are most readily evaluated from the matrix elements of the current $J^+(0)$ in the $q^+ = 0$ frame. In contrast to the covariant Bethe-Salpeter formalism, familiar from the analysis of hydrogenic bound states in quantum electrodynamics (QED) [18], in the light-front formalism one does not need to sum over the contributions to the current from an infinite number of irreducible kernels. Indeed, the evaluation of the current matrix elements is intractable in the standard, i.e., instant-form, Hamiltonian formalism, since the wave functions are frame-dependent, and as one must also take into account all interactions of the current with vacuum fluctuations [17].

The light-front formalism is thus ideally suited for computing electromagnetic properties of both elementary and composite states. The electric dipole form factor is rendered nonzero by time-reversal-odd and parity-odd effects in the light-front Fock-state wave functions themselves. This could occur, for example, at a fundamental level through higher Fock states which explicitly contain three generations of quarks. Alternatively, one can integrate out the effects of the heavy particles to obtain an effective chiral theory in which the light-front wave functions are expressed in terms of effective meson and baryon degrees of freedom.

In this paper we evaluate the electric dipole form factor, $F_3(q^2)$, in the light-front formalism and compare it with the well-known expressions for the Dirac and Pauli form factors, $F_1(q^2)$ and $F_2(q^2)$ [15]. In order to explore the struc-

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ture of the resulting expression for $F_3(q^2)$ we explicitly construct and classify the action of the discrete operators corresponding to the time-reversal, parity, and charge-conjugation transformations acting on wave functions realized from quantization on the light front. We then construct the general form of a light-front wave function in the presence of fundamental CP violation. This, in turn, leads to a model-independent relation which connects the time-reversal-odd and parity-odd $F_3(q^2)$ form factor to the Pauli form factor $F_2(q^2)$, Fock state by Fock state—for any spin-1/2 system. Thus we are able to relate the contribution of a particular Fock state to the electric dipole moment to a corresponding contribution to the anomalous magnetic moment. At $q^2 = 0$, the universal relation for a spin-1/2 baryon is

$$d_i = 2\kappa_i \tan\beta_i, \tag{1}$$

where repeated indices are not summed and i denotes the contribution of Fock state i. Note that the electric dipole moment d is $d \equiv \Sigma_i d_i = (e/M)F_3(0)$ and that the anomalous magnetic moment κ is $\kappa \equiv \Sigma_i \kappa_i = (e/2M)F_2(0)$, where $e = |e|$ is the fundamental unit of electric charge and M is the proton mass. The parameter β_i is the CP-violating phase appearing in Fock state i of the light-front wave function for the baryon of interest. Although it has long been recognized that the hadronic matrix element yielding the neutron electric dipole moment must be commensurate in size, up to CP-violating effects, to that of the anomalous magnetic moment [19], ours is the first construction of a general equality based on first principles. A relationship of this kind has also been noted by Feng, Matchev, and Shadmi in their study of the electric-dipole and anomalous-magnetic moments of the muon in supersymmetric models [20], though we find our Eq. (1) to be of more general validity. Indeed, the connection is general and holds for any spin-1/2 state, be it charged lepton, neutrino,[1] quark, or baryon, irrespective of the sources of CP violation. We proceed to examine its implications for constraints on models of CP violation before concluding with a summary and outlook.

II. THE LIGHT-FRONT FOCK REPRESENTATION

The light-front Fock expansion of any hadronic system is constructed by quantizing quantum chromodynamics (QCD) at fixed light-cone time $x^+ = x^0 + x^3$,[2] with $c = \hbar = 1$, and forming the invariant light-cone Hamiltonian $H_{\rm LC}: H_{\rm LC} = P^+P^- - \mathbf{P}_\perp^2$ [12,13,17]. The momentum generators P^+ and $\mathbf{P}_\perp$ are kinematical, so that they are independent of interactions [12]. The generator $P_+ = P^-/2 = i\partial/\partial x^+$ gives rise to light-cone time translations. In principle, solving the $H_{\rm LC}$ eigenvalue problem gives the entire mass spectrum of the color-singlet hadron states in QCD, together with their respective light-front wave functions. In particular, the proton state satisfies $H_{\rm LC}|\psi_p\rangle = M^2|\psi_p\rangle$, where $|\psi_p\rangle$ is an expansion in multiparticle Fock states. The resulting equations can be solved, in principle, using the discretized light-cone quantization (DLCQ) method [21]. A recent example of nonperturbative light-front solutions for a 3 + 1 theory is given in Ref. [22]. The connection to the Bethe-Salpeter formalism is described in Ref. [23], and explicit examples thereof are given in Ref. [24]. In the case of elementary fields such as the electron, one can construct the Fock space in perturbation theory.

The expansion of the proton eigenstate $|\psi_p\rangle$ in QCD on the eigenstates, $\{|n\rangle\}$, of the free light-cone Hamiltonian gives the light-front Fock expansion:

$$\begin{aligned}|\psi_p(P^+, \mathbf{P}_\perp, S_z)\rangle = &\sum_{n,\lambda_i\in n}\int\prod_{i=1}^{n}\left(\frac{\mathrm{d}x_i \mathrm{d}^2\mathbf{k}_{\perp i}}{2\sqrt{x_i}(2\pi)^3}\right)16\pi^3 \\ &\times\delta\left(1-\sum_{i=1}^{n}x_i\right)\delta^{(2)}\left(\sum_{i=1}^{n}\mathbf{k}_{\perp i}\right) \\ &\times\psi_{n/p}^{S_z}(x_i, \mathbf{k}_{\perp i}, \lambda_i)|n; x_iP^+, x_i\mathbf{P}_\perp \\ &+\mathbf{k}_{\perp i}, \lambda_i\rangle,\end{aligned} \tag{2}$$

where we consider a proton with momentum P and spin projection S_z along the $\mathbf{z} \equiv \mathbf{x}^3$ axis. The Fock state n contains n constituents, and we sum over the helicities, $\{\lambda_i\}$, of the constituents as well. The light-cone momentum fractions $x_i = k_i^+/P^+$ and $\mathbf{k}_{\perp i}$ represent the relative momentum coordinates of constituent i in Fock state n, whereas the physical momentum coordinates of constituent i are k_i^+ and $\mathbf{p}_{\perp i} = x_i\mathbf{P}_\perp + \mathbf{k}_{\perp i}$. The label λ_i determines the helicity of a constituent quark or gluon along the $\mathbf{z}$ axis. A free fermion constituent of mass m_i is specified not only by its momentum components k_i^+, $\mathbf{k}_{\perp,i}$ and helicity λ_i, but also by its color c_i and flavor f_i. In writing Eq. (2) we suppress the presence of c_i and f_i in the arguments of the free Fock states and light-front wave functions for notational simplicity. Note, too, that we also implicitly sum over the constituents' colors, $\{c_i\}$, and flavors, $\{f_i\}$. The n-particle states are normalized as

$$\begin{aligned}&\langle n; p_i'^+, \mathbf{p}'_{\perp i}, \lambda_i'|n; p_i^+, \mathbf{p}_{\perp i}, \lambda_i\rangle \\ &\quad=\prod_{i=1}^{n}(16\pi^3 p_i^+\delta(p_i'^+ - p_i^+)\delta^{(2)}(\mathbf{p}'_{\perp i} - \mathbf{p}_{\perp i})\delta_{\lambda_i'\lambda_i}).\end{aligned} \tag{3}$$

The solutions of $H_{LC}|\psi_p\rangle = M^2|\psi_p\rangle$ are independent of P^+ and $\mathbf{P}_\perp$. Thus, given the Fock projections $\langle n; x_i, \mathbf{k}_{\perp i}, \lambda_i|\psi_p(P^+, \mathbf{P}_\perp, S_z)\rangle$, or $\psi_{n/p}^{S_z}(x_i, \mathbf{k}_{\perp i}, \lambda_i)$, the wave function of the proton is determined in any frame [23]. The light-front wave functions $\psi_{n/h}^{S_z}(x_i, \mathbf{k}_{\perp i}, \lambda_i)$ encode all of the bound-state quark and gluon properties of a

[1] The connection is nontrivial only in the case of a Dirac neutrino.

[2] We summarize our conventions in the Appendix.

hadron h, including its momentum, spin, and flavor correlations, in the form of universal process- and frame-independent amplitudes.

III. THE LIGHT-FRONT REPRESENTATION OF THE ELECTROMAGNETIC FORM FACTORS

In the case of a spin-1/2 system, with exact eigenstate $|P, S_z\rangle$, the Dirac and Pauli form factors $F_1(q^2)$ and $F_2(q^2)$, and the electric dipole moment form factor $F_3(q^2)$ are defined by

$$\langle P', S'_z|J^\mu(0)|P, S_z\rangle = \bar{u}(P', \lambda')\Big[F_1(q^2)\gamma^\mu + F_2(q^2) \times \frac{i}{2M}\sigma^{\mu\alpha}q_\alpha + F_3(q^2) \times \frac{-1}{2M}\sigma^{\mu\alpha}\gamma_5 q_\alpha\Big]u(P, \lambda), \quad (4)$$

where $q^\mu = (P' - P)^\mu$ and $u(P, \lambda)$ is the Dirac spinor associated with a spin-1/2 state of momentum P and helicity λ. We employ the standard light-cone frame throughout, so that $q = (q^+, q^-, \mathbf{q}_\perp) = (0, -q^2/P^+, \mathbf{q}_\perp)$ and $P = (P^+, P^-, \mathbf{P}_\perp) = (P^+, M^2/P^+, \mathbf{0}_\perp)$, where $q^2 = -2P \cdot q = -\mathbf{q}_\perp^2$ is the square of the momentum transferred by the photon to the system. We detail other pertinent conventions in the Appendix and note

$$\frac{1}{2P^+}\bar{u}(P', \lambda')\gamma^+ u(P, \lambda) = \delta_{\lambda,\lambda'},$$
$$\frac{1}{2P^+}\bar{u}(P', \lambda')i\sigma^{+1}(\gamma_5)u(P, \lambda) = -\lambda(\lambda)\delta_{\lambda,-\lambda'}, \quad (5)$$
$$\frac{1}{2P^+}\bar{u}(P', \lambda')i\sigma^{+2}(\gamma_5)u(P, \lambda) = -i(\lambda)\delta_{\lambda,-\lambda'}.$$

Using Eq. (5) in conjunction with Eq. (4) we find

$$F_1(q^2) = \left\langle P+q, \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \uparrow \right\rangle = \left\langle P+q, \downarrow \left| \frac{J^+(0)}{2P^+} \right| P, \downarrow \right\rangle, \quad (6)$$

$$\frac{F_2(q^2)}{2M} = \frac{1}{2}\Big[-\frac{1}{q^L}\left\langle P+q, \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \downarrow \right\rangle + \frac{1}{q^R}\left\langle P+q, \downarrow \left| \frac{J^+(0)}{2P^+} \right| P, \uparrow \right\rangle\Big], \quad (7)$$

$$\frac{F_3(q^2)}{2M} = \frac{i}{2}\Big[-\frac{1}{q^L}\left\langle P+q, \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \downarrow \right\rangle - \frac{1}{q^R}\left\langle P+q, \downarrow \left| \frac{J^+(0)}{2P^+} \right| P, \uparrow \right\rangle\Big], \quad (8)$$

where $\uparrow$ and $\downarrow$ denote spin states aligned parallel and antiparallel to the $\mathbf{z}$ axis and $q^{R,L} = q^1 \pm iq^2$. The Dirac and Pauli form factors, for $q^2 \leq 0$, can thus be identified from the helicity-conserving and helicity-flip vector-current matrix elements of the $J^+(0)$ current in the $q^+ = 0$ frame [15]—and we find this true of $F_3(q^2)$ as well. The magnetic and electric dipole moments are defined in the $q^2 \to 0$ limit, namely,

$$\mu = \frac{e}{2M}[F_1(0) + F_2(0)], \qquad d = \frac{e}{M}F_3(0), \quad (9)$$

where e is the charge and M is the mass of the proton if we consider a spin-1/2 baryon system. Recall that $\kappa = (e/2M)F_2(0)$ is the anomalous magnetic moment. For leptons, such as the electron or neutrino, it is convenient to employ the electron mass for M, so that the magnetic moment is given in Bohr magnetons.

Now we turn to the evaluation of the helicity-conserving and helicity-flip vector-current matrix elements in the light-front formalism. In the interaction picture, the current $J^\mu(0)$ is represented as a bilinear product of free fields, so that it has an elementary coupling to the constituent fields [14–16]. The Dirac form factor can then be calculated from the expression

$$F_1(q^2) = \sum_a \int [\mathrm{d}x] \times [\mathrm{d}^2\mathbf{k}_\perp]\sum_j e_j[\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i)], \quad (10)$$

whereas the Pauli and electric dipole form factors are given by

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp]\sum_j e_j \frac{1}{2} \times \Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i)\Big], \quad (11)$$

$$\frac{F_3(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp]\sum_j e_j \frac{i}{2} \times \Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) - \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i)\Big]. \quad (12)$$

The summations are over all contributing Fock states a and struck constituent charges e_j. Here, as earlier, we refrain from including the constituents' color and flavor dependence in the arguments of the light-front wave functions. The phase-space integration is

$$\int[\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \equiv \sum_{\lambda_i,c_i,f_i}\bigg[\prod_{i=1}^{n}\bigg(\iint\frac{\mathrm{d}x_i\mathrm{d}^2\mathbf{k}_{\perp i}}{2(2\pi)^3}\bigg)\bigg]16\pi^3 \times \delta\bigg(1-\sum_{i=1}^{n}x_i\bigg)\delta^{(2)}\bigg(\sum_{i=1}^{n}\mathbf{k}_{\perp i}\bigg), \tag{13}$$

where n denotes the number of constituents in Fock state a and we sum over the possible $\{\lambda_i\}$, $\{c_i\}$, and $\{f_i\}$ in state a. The arguments of the final-state, light-front wave function differentiate between the struck and spectator constituents, namely, we have [14,16]

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1-x_j)\mathbf{q}_\perp \tag{14}$$

for the struck constituent j and

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i\mathbf{q}_\perp \tag{15}$$

for each spectator i, where $i \neq j$. Note that because of the frame choice $q^+ = 0$, only diagonal ($n' = n$) overlaps of the light-front Fock states appear [15].

The simple expressions of Eqs. (11) and (12) rely on the ability to employ the interaction picture for the electromagnetic current and on the assumed simple structure of the vacuum in the light-front formalism. Indeed, the $k^+ > 0$ constraint for massive particles in the light-front formalism removes all $q\bar{q}$ pairs from the physical vacuum. However, gluon modes, which are massless, may possess $k^+ = 0$ and $\mathbf{k}_\perp = 0$ [25] and can contribute, in principle, in color-singlet combinations to the physical vacuum [26]. If these contributions do not enter, then the free, Fock-space vacuum is also an eigenstate of $H_{\rm LC}$ and the relations of Eqs. (11) and (12) follow. As an example where zero modes do occur, and indeed are essential to the description of spontaneous symmetry breaking and the Higgs mechanism, see Ref. [27]. For a discussion of the role of zero modes in the vacuum structure of the (chiral) Schwinger model, see Refs. [28,29]. It is worth noting that an explicit computation of the electron's anomalous magnetic moment, $(g-2)/2$, in light-front perturbation theory yields the expected result [30]: in this case, photon zero modes simply do not appear. The putative gluon zero modes are electrically neutral, so that the electromagnetic coupling of the photon to the constituent fields would be given by the quark charges regardless; the overlap formulas of Eqs. (11) and (12) could miss a contribution, however, when a spectator gluon has zero k^+ and $\mathbf{k}_\perp$.

We now turn to the development of discrete symmetry transformations in the light-front formalism, in order to ascertain the features of the light-front wave functions needed to give rise to a nonzero value of $F_3(q^2)$.

IV. DISCRETE SYMMETRIES ON THE LIGHT FRONT

The development of the transformation properties of the various fermion bilinears under P, T, and C in the light-front formalism can be made in a manner analogous to that of the equal-time formalism [31]. One crucial difference, however, is that we invoke the transformation properties on the perpendicular components of k^μ only, so that we can avoid transformations such as $k^+ \leftrightarrow k^-$, or negative definite values of k^+ or k^-. To be specific, we consider transformations on $\mathbf{k}_\perp$ alone, so that $|\mathbf{k}_\perp|^2$, k^-, and k^+ all remain unchanged. This means that our particles will remain on their energy shell throughout, in analogy to the on-mass-shell condition in the equal-time formalism.

A. Parity

To implement the light-cone parity operation $\mathcal{P}_\perp$ we let the spatial components of any vector d^μ transform as $d^R \to -d^L$, $d^L \to -d^R$, $d^\pm \to d^\pm$. This is equivalent to letting $d^1 \to -d^1$, with all other components transforming into themselves. Note that if we do not flip d^3 we cannot flip the signs of both d^1 and d^2, as this can be realized via a continuous Lorentz transformation from the identity. Flipping the sign of d^1 alone does yield an improper Lorentz transformation, as needed, and we would find analogous results were we to flip simply the sign of d^2 instead. Considering the commutator $[x_i, p_j] = \mathrm{i}\delta_{ij}$ and $\mathbf{L} = \mathbf{r}\times\mathbf{p}$, we find that $\mathcal{P}_\perp$ is a unitary operator and that it flips the spin as well. We thus realize the parity transformation at the operator level via

$$\mathcal{P}_\perp a^\lambda_{p^L,p^R}\mathcal{P}_\perp^\dagger = \eta_a a^{-\lambda}_{-p^R,-p^L}, \qquad \mathcal{P}_\perp b^\lambda_{p^L,p^R}\mathcal{P}_\perp^\dagger = \eta_b b^{-\lambda}_{-p^R,-p^L}, \tag{16}$$

where we suppress, here and throughout, possible internal indices such as color or flavor in the fermion and antifermion annihilation operators, respectively. The fermion field operator $\psi(x)$ on the light front, namely,

$$\psi(x) = \int\frac{dk^+d^2\mathbf{k}_\perp}{\sqrt{2k^+(2\pi)^3}}\{a^\lambda_{k^L,k^R}u(k,\lambda)\exp(-ik\cdot x) + b^{\dagger\lambda}_{k^L,k^R}v(k,\lambda)\exp(ik\cdot x)\}, \tag{17}$$

thus transforms as

$$\begin{aligned}\mathcal{P}_\perp\psi(x)\mathcal{P}_\perp^\dagger &= \int\frac{d\tilde{k}^+d^2\tilde{\mathbf{k}}_\perp}{\sqrt{2\tilde{k}^+(2\pi)^3}}\eta_a\sum_\lambda\{a^{-\lambda}_{\tilde{k}^L,\tilde{k}^R}\gamma^1\gamma_5u(\tilde{k},-\lambda) \\ &\quad\times\exp(-i\tilde{k}\cdot(x^+,x^-,-x^R,-x^L)) \\ &\quad+ b^{\dagger-\lambda}_{\tilde{k}^L,\tilde{k}^R}\gamma^1\gamma_5v(\tilde{k},-\lambda) \\ &\quad\times\exp(i\tilde{k}\cdot(x^+,x^-,-x^R,-x^L))\} \\ &= \eta_a\gamma^1\gamma_5\psi(x^+,x^-,-x^R,-x^L),\end{aligned} \tag{18}$$

where we note $\tilde{k}^\mu \equiv (k^+,k^-,-k^R,-k^L)$, $u(k,\lambda) = \gamma^1\gamma^5u(\tilde{k},\lambda)$, $v(k,\lambda) = \gamma^1\gamma^5v(\tilde{k},\lambda)$, and $\eta_b^* = \eta_a$. With

$$\mathcal{P}_\perp\psi^\dagger(x)\mathcal{P}_\perp^\dagger = \eta_a^*\gamma^1\gamma_5\psi^\dagger(x^+,x^-,-x^R,-x^L), \tag{19}$$

and $|\eta_a|^2 = 1$, we thus conclude that

$$\mathcal{P}_{\perp}\overline{\psi}\psi(x)\mathcal{P}_{\perp}^{\dagger} = \overline{\psi}\psi(x^+, x^-, -x^R, -x^L), \quad (20)$$

$$\mathcal{P}_{\perp}i\overline{\psi}\gamma_5\psi(x)\mathcal{P}_{\perp}^{\dagger} = -i\overline{\psi}\gamma_5\psi(x^+, x^-, -x^R, -x^L), \quad (21)$$

$$\mathcal{P}_{\perp}\overline{\psi}\gamma^{\mu}\psi(x)\mathcal{P}_{\perp}^{\dagger} = \xi^{\mu}\overline{\psi}\gamma^{\mu}\psi(x^+, x^-, -x^R, -x^L), \quad (22)$$

$$\mathcal{P}_{\perp}\overline{\psi}\gamma^{\mu}\gamma_5\psi(x)\mathcal{P}_{\perp}^{\dagger} = -\xi^{\mu}\overline{\psi}\gamma^{\mu}\gamma_5\psi(x^+, x^-, -x^R, -x^L), \quad (23)$$

where $\xi^{\mu} = -1$ for $\mu = 1$ and $\xi^{\mu} = +1$ for $\mu \neq 1$. Repeated indices in μ are not summed. Note that the determined vector and axial-vector transformations are analogous to that of the equal-time case. Moreover, we have

$$\mathcal{P}_{\perp}\overline{\psi}\sigma^{\mu\nu}\psi(x)\mathcal{P}_{\perp}^{\dagger} = \eta^{\mu\nu}\overline{\psi}\sigma^{\mu\nu}\psi(x^+, x^-, -x^R, -x^L), \quad (24)$$

$$\mathcal{P}_{\perp}\overline{\psi}\sigma^{\mu\nu}\gamma_5\psi(x)\mathcal{P}_{\perp}^{\dagger} = -\eta^{\mu\nu}\overline{\psi}\sigma^{\mu\nu}\gamma_5\psi(x^+, x^-, -x^R, -x^L), \quad (25)$$

where $\eta^{\mu\nu} = \xi^{\mu}\xi^{\nu}$ and repeated indices in μ and ν are not summed. These transformations also parallel those found in the equal-time formalism. Applying these transformation properties to the matrix elements which yield F_2 and F_3, in specific Eq. (4), as such are shared by the matrix elements of the Dirac spinors, we see that F_2 is even and F_3 is odd under $\mathcal{P}_{\perp}$. Turning to the explicit forms of Eqs. (7) and (8), we see that since $\lambda \to -\lambda$, $q^R \to -q^L$, and $q^L \to -q^R$ under $\mathcal{P}_{\perp}$, that if

$$-\frac{1}{q^L}\langle P + q, \uparrow |J^+(0)|P, \downarrow\rangle \xrightarrow{\mathcal{P}_{\perp}} \frac{1}{q^R}\langle P + q, \downarrow |J^+(0)|P, \uparrow\rangle, \quad (26)$$

$$\frac{1}{q^R}\langle P + q, \downarrow |J^+(0)|P, \uparrow\rangle \xrightarrow{\mathcal{P}_{\perp}} -\frac{1}{q^L}\langle P + q, \uparrow |J^+(0)|P, \downarrow\rangle, \quad (27)$$

we can conclude here as well that F_2 is even and F_3 is odd under $\mathcal{P}_{\perp}$, precisely as desired. Since the form factors are functions of q^2 only, we note that the matrix element in the left-hand side (LHS) of Eq. (26) must be proportional to q^L, whereas the matrix element in the LHS of Eq. (27) must be proportional to q^R. Indeed, if

$$-\frac{1}{q^L}\langle P + q, \uparrow |J^+(0)|P, \downarrow\rangle = \frac{1}{q^R}\langle P + q, \downarrow |J^+(0)|P, \uparrow\rangle \quad (28)$$

is also satisfied, then $F_3 = 0$ and $\mathcal{P}_{\perp}$ is a "good" symmetry.

Now let us consider the transformation properties of the light-front wave functions in greater detail, as matrix elements of these quantities give rise to $F_3(q^2)$, a P-odd, T-odd observable. To summarize our earlier discussion, the action of $\mathcal{P}_{\perp}$ is such that it transforms the matrix elements entering $F_2(q^2)$ and $F_3(q^2)$ as per Eqs. (26) and (27). At the level of the wave functions themselves, we have

$$\psi_a^{\downarrow}(\mathbf{k}_{\perp i}, x_i, \lambda_i) \xrightarrow{\mathcal{P}_{\perp}} \psi_a^{\uparrow}(\tilde{\mathbf{k}}_{\perp i}, x_i, -\lambda_i), \quad (29)$$

$$\psi_a^{\uparrow}(\mathbf{k}_{\perp i}, x_i, \lambda_i) \xrightarrow{\mathcal{P}_{\perp}} \psi_a^{\downarrow}(\tilde{\mathbf{k}}_{\perp i}, x_i, -\lambda_i), \quad (30)$$

with $\tilde{\mathbf{k}}_{\perp i} = (-k_i^1, k_i^2)$. These transformation properties are consistent with those in Eqs. (26) and (27). We have suppressed the introduction of an overall phase factor as it is without physical relevance. Moreover, if $\psi^{\downarrow}(\mathbf{k}_{\perp i}, x_i, \lambda_i) = \psi^{\uparrow}(\tilde{\mathbf{k}}_{\perp i}, x_i, -\lambda_i)$ then Eq. (28) follows as well and $F_3(q^2)$ vanishes. Thus to realize a nonzero value of $F_3(q^2)$, we must have light-front wave functions which satisfy $\psi^{\downarrow}(\mathbf{k}_{\perp i}, x_i, \lambda_i) \neq \psi^{\uparrow}(\tilde{\mathbf{k}}_{\perp i}, x_i, -\lambda_i)$.

B. Time-reversal

In order to implement the light-cone time-reversal operation $\mathcal{T}_{\perp}$ we let the spatial components of any *momentum* vector transform as $q^R \to -q^L$, $q^L \to -q^R$, so that $q^{\mu} \to (q^+, q^-, -q^1, q^2)$. This implies, ultimately, that the position vector under $\mathcal{T}_{\perp}$ transforms as $x^{\mu} \to (-x^+, -x^-, x^1, -x^2)$, or $x^{\mu} \to (-x^+, -x^-, x^R, x^L)$. We term the transformation time-reversal, since x^0 does flip its sign, even though other coordinates flip sign as well. Our construction of $\mathcal{T}_{\perp}$ is tied to that of $\mathcal{P}_{\perp}$, so that we can conserve $C\mathcal{P}_{\perp}\mathcal{T}_{\perp}$, where C is the charge-conjugation operator in the light-front formalism. Moreover, our choice of $\mathcal{T}_{\perp}$ yields a nonorthochronous operator; the $\mathcal{T}_{\perp}$ transformation should not be connected by a continuous Lorentz transformation to the identity. Considering the commutator $[x_i, p_j] = \mathrm{i}\delta_{ij}$ and $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, we find that $\mathcal{T}_{\perp}$ is antiunitary as expected but that it does *not* flip the spin. We thus realize the time-reversal transformation at the operator level via

$$\mathcal{T}_{\perp}a^{\lambda}_{p^L,p^R}\mathcal{T}_{\perp}^{\dagger} = \tilde{\eta}_a a^{\lambda}_{-p^R,-p^L}, \quad (31)$$

$$\mathcal{T}_{\perp}b^{\lambda}_{p^L,p^R}\mathcal{T}_{\perp}^{\dagger} = \tilde{\eta}_b b^{\lambda}_{-p^R,-p^L}, \quad (32)$$

so that the fermion field operator $\psi(x)$ transforms as

$$\begin{aligned}\mathcal{T}_{\perp}\psi(x)\mathcal{T}_{\perp}^{\dagger} = \int \frac{d\tilde{k}^+ d^2\tilde{\mathbf{k}}_{\perp}}{\sqrt{2\tilde{k}^+(2\pi)^3}}\tilde{\eta}_a\sum_{\lambda}\{&a^{\lambda}_{\tilde{k}^L,\tilde{k}^R}\sigma^{12}u(\tilde{k},\lambda)\\ &\times\exp(-i\tilde{k}\cdot(-x^+, -x^-, x^R, x^L))\\ &+ b^{\dagger\lambda}_{\tilde{k}^L,\tilde{k}^R}\sigma^{12}v(\tilde{k},\lambda)\\ &\times\exp(i\tilde{k}\cdot(-x^+, -x^-, x^R, x^L))\}\\ = \tilde{\eta}_a\sigma^{12}\psi(-x^+, -x^-, x^R, x^L), \quad (33)\end{aligned}$$

where we note that $\tilde{k} \equiv (k^+, k^-, -k^R, -k^L)$, $u(k,\lambda) = \sigma^{12}u(\tilde{k},\lambda)$, $v(k,\lambda) = \sigma^{12}v(\tilde{k},\lambda)$, and $\tilde{\eta}_b^* = -\tilde{\eta}_a$. With

$$\mathcal{T}_\perp \psi^\dagger(x)\mathcal{T}_\perp^\dagger = \tilde{\eta}_a^* \sigma^{12}\psi^\dagger(-x^+, -x^-, x^R, x^L) \quad (34)$$

and $|\tilde{\eta}_a|^2 = 1$, we thus conclude that

$$\mathcal{T}_\perp \bar{\psi}\psi(x)\mathcal{T}_\perp^\dagger = \bar{\psi}\psi(-x^+, -x^-, x^R, x^L), \quad (35)$$

$$\mathcal{T}_\perp i\bar{\psi}\gamma_5\psi(x)\mathcal{T}_\perp^\dagger = -i\bar{\psi}\gamma_5\psi(-x^+, -x^-, x^R, x^L), \quad (36)$$

$$\mathcal{T}_\perp \bar{\psi}\gamma^\mu\psi(x)\mathcal{T}_\perp^\dagger = \xi^\mu\bar{\psi}\gamma^\mu\psi(-x^+, -x^-, x^R, x^L), \quad (37)$$

$$\mathcal{T}_\perp \bar{\psi}\gamma^\mu\gamma_5\psi(x)\mathcal{T}_\perp^\dagger = \xi^\mu\bar{\psi}\gamma^\mu\gamma_5\psi(-x^+, -x^-, x^R, x^L), \quad (38)$$

where $\xi^\mu = -1$ for $\mu = 1$ and $\xi^\mu = +1$ for $\mu \neq 1$. Repeated indices in μ are not summed. The transformations found parallel that of the equal-time case. Moreover,

$$\mathcal{T}_\perp \bar{\psi}\sigma^{\mu\nu}\psi(x)\mathcal{T}_\perp^\dagger = -\eta^{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi(-x^+, -x^-, x^R, x^L), \quad (39)$$

$$\mathcal{T}_\perp \bar{\psi}\sigma^{\mu\nu}\gamma_5\psi(x)\mathcal{T}_\perp^\dagger = -\eta^{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi(-x^+, -x^-, x^R, x^L), \quad (40)$$

where $\eta^{\mu\nu} = \xi^\mu\xi^\nu$ and, once again, we do not sum repeated indices in μ and ν. These transformations also parallel those of the equal-time formalism. Since q^μ and iq^μ yield a ξ^μ and $-\xi^\mu$ under $\mathcal{T}_\perp$, respectively, we thus see upon applying $\mathcal{T}_\perp$ to Eq. (4) that $\mathrm{Re}F_2$ is even and $\mathrm{Re}F_3$ is odd, whereas $\mathrm{Im}F_2$ is odd and $\mathrm{Im}F_3$ is even. Applying these transformation properties to the explicit forms in Eqs. (7) and (8) for $F_2(q^2)$ and $F_3(q^2)$, as such are shared by the matrix elements of Dirac spinors, we see, since $\lambda \to \lambda$, $q^R \to -q^R$, and $q^L \to -q^L$ under $\mathcal{T}_\perp$, and $\mathcal{T}_\perp$ is antiunitary, that if

$$\langle P+q, \uparrow|J^+(0)|P, \downarrow\rangle \xrightarrow{\mathcal{T}_\perp} (\langle P+\tilde{q}, \uparrow|J^+(0)|P, \downarrow\rangle)^* = -\langle P+q, \uparrow|J^+(0)|P, \downarrow\rangle, \quad (41)$$

$$\langle P+q, \downarrow|J^+(0)|P, \uparrow\rangle \xrightarrow{\mathcal{T}_\perp} (\langle P+\tilde{q}, \downarrow|J^+(0)|P, \uparrow\rangle)^* = -\langle P+q, \downarrow|J^+(0)|P, \uparrow\rangle, \quad (42)$$

with $\tilde{q} = (q^+, q^-, \tilde{\mathbf{q}}_\perp)$ and $\tilde{\mathbf{q}}_\perp = (-q^1, q^2)$, then $\mathrm{Re}(F_2)$ and $\mathrm{Im}(F_3)$ are even and $\mathrm{Re}(F_3)$ and $\mathrm{Im}(F_2)$ are odd under $\mathcal{T}_\perp$, precisely as desired. As we shall see, the equalities emerge naturally if a unit of orbital angular momentum distinguishes the spin-up and spin-down light-front wave functions for fixed λ_i, that is, for fixed bachelor quark helicity in a spin-1/2 $q(qq)_0$ wave function—if the light-front wave functions are assumed to be otherwise real. With this, we see that both $\mathrm{Im}(F_2)$ and $\mathrm{Re}(F_3)$ vanish. In order to realize nonzero $\mathcal{T}_\perp$-odd observables, we will have to allow the light-front wave functions to have additional complex contributions.

Now let us consider the transformation properties of the light-front wave functions under $\mathcal{T}_\perp$ in detail. At the level of the light-front wave functions themselves, we have

$$\psi_a^\downarrow(\mathbf{k}_{\perp i}, x_i, \lambda_i) \xrightarrow{\mathcal{T}_\perp} \psi_a^{\downarrow *}(\tilde{\mathbf{k}}_{\perp i}, x_i, \lambda_i), \quad (43)$$

$$\psi_a^\uparrow(\mathbf{k}_{\perp i}, x_i, \lambda_i) \xrightarrow{\mathcal{T}_\perp} \psi_a^{\uparrow *}(\tilde{\mathbf{k}}_{\perp i}, x_i, \lambda_i), \quad (44)$$

with $\tilde{\mathbf{k}}_{\perp i} = (-k_i^1, k_i^2)$. We suppress the introduction of an overall phase factor as it is without physical impact. Under the assumptions which lead to Eqs. (41) and (42), Eqs. (43) and (44) yield

$$\psi_a^{\uparrow *}(\mathbf{k}'_{\perp i}, x_i, \lambda_i)\psi_a^\downarrow(\mathbf{k}_{\perp i}, x_i, \lambda_i) \xrightarrow{\mathcal{T}_\perp} -\psi_a^{\uparrow *}(\mathbf{k}'_{\perp i}, x_i, \lambda_i)\psi_a^\downarrow(\mathbf{k}_{\perp i}, x_i, \lambda_i), \quad (45)$$

$$\psi_a^{\downarrow *}(\mathbf{k}'_{\perp i}, x_i, \lambda_i)\psi_a^\uparrow(\mathbf{k}_{\perp i}, x_i, \lambda_i) \xrightarrow{\mathcal{T}_\perp} -\psi_a^{\downarrow *}(\mathbf{k}'_{\perp i}, x_i, \lambda_i)\psi_a^\uparrow(\mathbf{k}_{\perp i}, x_i, \lambda_i), \quad (46)$$

for any λ_i, and are thus consistent with the transformations of Eqs. (41) and (42). Allowing the light-front wave functions to have additional complex contributions will enable a nonzero value of $F_3(q^2)$, as we shall discuss in Sec. V.

C. Charge conjugation

We realize the light-front, charge-conjugation transformation at the operator level via

$$C a_{p^L,p^R}^\lambda C^\dagger = \eta_a b_{p^L,p^R}^\lambda, \quad (47)$$

$$C b_{p^L,p^R}^\lambda C^\dagger = \eta_b a_{p^L,p^R}^\lambda, \quad (48)$$

precisely as in the equal-time formalism [31]. Indeed, we conclude in this case, as well, that

$$C\psi(x)C^\dagger = -i\gamma^2\psi^*(x). \quad (49)$$

Note that the action of C carries $\psi \to \psi^*$, though C is a unitary operator. Nevertheless, with C, $\mathcal{T}_\perp$, and $\mathcal{P}_\perp$ as we have defined them, all scalar fermion bilinears are invariant under $C\mathcal{P}_\perp\mathcal{T}_\perp$, as they ought be. We note, e.g., that $\bar{\psi}\gamma^\mu\psi$, $\bar{\psi}\sigma^{\mu\nu}\psi$, and $\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi$ all yield -1 under C, so that these operators yield -1, $+1$, and -1, respectively, under the combined action of $C\mathcal{P}_\perp\mathcal{T}_\perp$. If we employ the derivative operator ∂^μ, which transforms with a -1 under $C\mathcal{P}_\perp\mathcal{T}_\perp$, to generate a scalar bilinear from these operators, we do indeed find that the only nonvanishing operator transforms with a $+1$ under $C\mathcal{P}_\perp\mathcal{T}_\perp$.

Let us now turn to the transformation properties of Eq. (4). We note

$$C\bar{u}(k', \lambda')\gamma^\mu u(k, \lambda)C^\dagger = \bar{v}(k, \lambda)\gamma^\mu v(k', \lambda'), \quad (50)$$

$$C\bar{u}(k', \lambda')\sigma^{\mu\nu} u(k, \lambda)C^\dagger = \bar{v}(k, \lambda)\sigma^{\mu\nu} v(k', \lambda'), \quad (51)$$

$$C\,\overline{u}(k',\lambda')\sigma^{\mu\nu}\gamma_5 u(k,\lambda)C^\dagger = \overline{v}(k,\lambda)\sigma^{\mu\nu}\gamma_5 v(k',\lambda'). \quad (52)$$

Since $\overline{v}(k,\lambda)\gamma^\mu v(k',\lambda') = \overline{u}(k',\lambda')\gamma^\mu u(k,\lambda)$ and q^μ transforms with a $+1$ under $\mathcal{C}\mathcal{P}_\perp\mathcal{T}_\perp$, we see that the scalar bilinear formed by contracting Eq. (4) with q^μ does transform with a $+1$ under $\mathcal{C}\mathcal{P}_\perp\mathcal{T}_\perp$, as needed. Writing the analogue of Eq. (4) for an antifermion $\overline{f}$, replacing $F_i(q^2)$ with $\tilde{F}_i(q^2)$, and evaluating the spinor matrix elements, we find

$$\tilde{F}_1(q^2) = \left\langle P+q,\uparrow \left| \frac{J^+(0)}{2P^+} \right| P,\uparrow \right\rangle_{\overline{f}} = \left\langle P+q,\downarrow \left| \frac{J^+(0)}{2P^+} \right| P,\downarrow \right\rangle_{\overline{f}}, \quad (53)$$

$$\frac{\tilde{F}_2(q^2)}{2M} = \frac{1}{2}\left[-\frac{1}{q^L}\left\langle P+q,\uparrow \left| \frac{J^+(0)}{2P^+} \right| P,\downarrow \right\rangle_{\overline{f}} + \frac{1}{q^R}\left\langle P+q,\downarrow \left| \frac{J^+(0)}{2P^+} \right| P,\uparrow \right\rangle_{\overline{f}} \right], \quad (54)$$

and

$$\frac{\tilde{F}_3(q^2)}{2M} = \frac{i}{2}\left[-\frac{1}{q^L}\left\langle P+q,\uparrow \left| \frac{J^+(0)}{2P^+} \right| P,\downarrow \right\rangle_{\overline{f}} - \frac{1}{q^R}\left\langle P+q,\downarrow \left| \frac{J^+(0)}{2P^+} \right| P,\uparrow \right\rangle_{\overline{f}} \right], \quad (55)$$

where

$$\mu = -\frac{e}{2M}[\tilde{F}_1(0) + \tilde{F}_2(0)], \qquad d = -\frac{e}{M}\tilde{F}_3(0), \quad (56)$$

and the electric charge of $\overline{f}$ is given by $-e\tilde{F}_1(0)$. Consequently, we infer the transformation properties

$$\langle P+q,\uparrow|J^+(0)|P,\uparrow\rangle_f \xrightarrow{\mathcal{C}} \langle P+q,\uparrow|J^+(0)|P,\uparrow\rangle_{\overline{f}}, \quad (57)$$

$$\langle P+q,\downarrow|J^+(0)|P,\downarrow\rangle_f \xrightarrow{\mathcal{C}} \langle P+q,\downarrow|J^+(0)|P,\downarrow\rangle_{\overline{f}}, \quad (58)$$

and

$$\langle P+q,\uparrow|J^+(0)|P,\downarrow\rangle_f \xrightarrow{\mathcal{C}} \langle P+q,\uparrow|J^+(0)|P,\downarrow\rangle_{\overline{f}}, \quad (59)$$

$$\langle P+q,\downarrow|J^+(0)|P,\uparrow\rangle_f \xrightarrow{\mathcal{C}} \langle P+q,\downarrow|J^+(0)|P,\uparrow\rangle_{\overline{f}}. \quad (60)$$

The f and $\overline{f}$ subscripts signify that the matrix elements are computed for a composite fermion (f) and antifermion ($\overline{f}$), respectively. At the level of the light-front wave functions themselves, we thus have

$$\psi_f^\downarrow(\mathbf{k}_\perp, x, \lambda) \xrightarrow{\mathcal{C}} \psi_{\overline{f}}^\downarrow(\mathbf{k}_\perp, x, \lambda), \quad (61)$$

$$\psi_f^\uparrow(\mathbf{k}_\perp, x, \lambda) \xrightarrow{\mathcal{C}} \psi_{\overline{f}}^\uparrow(\mathbf{k}_\perp, x, \lambda). \quad (62)$$

We suppress the introduction of an overall phase factor as it is without physical impact.

To conclude this section we consider how the products of the light-front wave functions which yield the electromagnetic form factors, noting Eqs. (10)–(12), behave under $\mathcal{C}\mathcal{P}_\perp\mathcal{T}_\perp$. Using the transformations we have discussed, we have

$$\begin{aligned}\psi_f^{\uparrow *}(\mathbf{k}'_\perp, x, \lambda)\psi_f^\uparrow(\mathbf{k}_\perp, x, \lambda) &\xrightarrow{\mathcal{P}_\perp} \psi_f^{\downarrow *}(\tilde{\mathbf{k}}'_\perp, x, -\lambda)\psi_f^\downarrow(\tilde{\mathbf{k}}_\perp, x, -\lambda) \\ &\xrightarrow{\mathcal{T}_\perp} \psi_f^{\downarrow *}(\mathbf{k}_\perp, x, -\lambda)\psi_f^\downarrow(\mathbf{k}'_\perp, x, -\lambda) \\ &\xrightarrow{\mathcal{C}} \psi_{\overline{f}}^{\downarrow *}(\mathbf{k}_\perp, x, -\lambda)\psi_{\overline{f}}^\downarrow(\mathbf{k}'_\perp, x, -\lambda).\end{aligned} \quad (63)$$

The last, upon integrating over phase space as per Eq. (13), with the change of variable $\mathbf{k}_\perp = \mathbf{k}'_\perp$, yields $\tilde{F}_1(q^2)$, Eq. (53). For $F_2(q^2)$ and $F_3(q^2)$ we consider

$$\begin{aligned}-\frac{1}{q^L}\psi_f^{\uparrow *}(\mathbf{k}'_\perp, x, \lambda)\psi_f^\downarrow(\mathbf{k}_\perp, x, \lambda) &\xrightarrow{\mathcal{P}_\perp} \frac{1}{q^R}\psi_f^{\downarrow *}(\tilde{\mathbf{k}}'_\perp, x, -\lambda)\psi_f^\uparrow(\tilde{\mathbf{k}}_\perp, x, -\lambda) \\ &\xrightarrow{\mathcal{T}_\perp} -\frac{1}{q^R}\psi_f^{\uparrow *}(\mathbf{k}_\perp, x, -\lambda)\psi_f^\downarrow(\mathbf{k}'_\perp, x, -\lambda) \\ &\xrightarrow{\mathcal{C}} -\frac{1}{q^R}\psi_{\overline{f}}^{\uparrow *}(\mathbf{k}_\perp, x, -\lambda)\psi_{\overline{f}}^\downarrow(\mathbf{k}'_\perp, x, -\lambda) \\ &= \frac{1}{q^R}\psi_{\overline{f}}^{\uparrow *}(\mathbf{k}'_\perp, x, -\lambda)\psi_{\overline{f}}^\downarrow(\mathbf{k}_\perp, x, -\lambda)\end{aligned} \quad (64)$$

and

$$\begin{aligned}\frac{1}{q^R}\psi_f^{\downarrow *}(\mathbf{k}'_\perp, x, \lambda)\psi_f^{\uparrow}(\mathbf{k}_\perp, x, \lambda) &\xrightarrow{\mathcal{P}_\perp} -\frac{1}{q^L}\psi_f^{\uparrow *}(\tilde{\mathbf{k}}'_\perp, x, -\lambda)\psi_f^{\downarrow}(\tilde{\mathbf{k}}_\perp, x, -\lambda)\\ &\xrightarrow{\mathcal{T}_\perp} \frac{1}{q^L}\psi_f^{\downarrow *}(\mathbf{k}_\perp, x, -\lambda)\psi_f^{\uparrow}(\mathbf{k}'_\perp, x, -\lambda)\\ &\xrightarrow{\mathcal{C}} \frac{1}{q^L}\psi_{\bar f}^{\downarrow *}(\mathbf{k}_\perp, x, -\lambda)\psi_{\bar f}^{\uparrow}(\mathbf{k}'_\perp, x, -\lambda)\\ &= -\frac{1}{q^L}\psi_{\bar f}^{\downarrow *}(\mathbf{k}'_\perp, x, -\lambda)\psi_{\bar f}^{\uparrow}(\mathbf{k}_\perp, x, -\lambda),\end{aligned} \tag{65}$$

where the equalities arise from making the change of variable $\mathbf{k}_\perp = \mathbf{k}'_\perp$ in the integration over phase space as per Eq. (13). Starting with Eqs. (7) and (8) we find, under $\mathcal{C}\mathcal{P}_\perp\mathcal{T}_\perp$, these last expressions will give rise to $\tilde{F}_2(q^2)$ and $\tilde{F}_3(q^2)$, Eqs. (54) and (55), so that we see explicitly that $F_2(q^2)$ and $F_3(q^2)$—as well as $F_1(q^2)$—yield $+1$ under $\mathcal{C}\mathcal{P}_\perp\mathcal{T}_\perp$, at the level of the light-front wave functions, as consistent with the transformation properties of the original fermion bilinears. This concludes our discussion of discrete symmetry transformations on the light front.

V. LIGHT-FRONT WAVE FUNCTIONS FOR $\mathcal{T}_\perp$-ODD AND $\mathcal{P}_\perp$-ODD OBSERVABLES

In this section we develop simple light-front wave functions of the nucleon which are compatible with a nonzero electric dipole moment. To begin, we consider a quark–-scalar-diquark model of the nucleon, $q(qq)_0$, patterned after the interaction of a fermion and a neutral scalar in Yukawa theory [32]. This model has proved useful in the analysis of single-spin asymmetries in semi-inclusive, deeply inelastic scattering [33]. In this model, the nucleon light-front wave function has two particles: a quark and a scalar diquark, so that the $J^z = +\frac{1}{2}$ nucleon wave function is taken to be of form

$$|\Psi^{\uparrow}_{q(qq)_0}(P^+ = 1, \mathbf{P}_\perp = \mathbf{0}_\perp)\rangle = \int \frac{dx d^2\mathbf{k}_\perp}{\sqrt{x(1-x)}16\pi^3} \times\left[\psi^{\uparrow}_{+1/2}(x,\mathbf{k}_\perp)\left|x,\mathbf{k}_\perp,+\frac{1}{2}\right\rangle + \psi^{\uparrow}_{-1/2}(x,\mathbf{k}_\perp)\left|x,\mathbf{k}_\perp,-\frac{1}{2}\right\rangle\right], \tag{66}$$

where we have labeled the free Fock states and associated light-front wave functions with the relative momentum coordinates, (x, $\mathbf{k}_\perp$), and spin projection along the $\mathbf{z}$-axis, $\lambda/2$, of the bachelor, or unpaired, quark. Computing the $\bar{u}(k', \lambda')u(k, \lambda)$ matrix element, we have

$$\begin{cases}\psi^{\uparrow}_{+1/2}(x,\mathbf{k}_\perp) = f(x)\varphi(x,k_\perp^2),\\ \psi^{\uparrow}_{-1/2}(x,\mathbf{k}_\perp) = -(k^1 + ik^2)g(x)\varphi(x,k_\perp^2),\end{cases} \tag{67}$$

where $f(x)$, $g(x)$, and $\varphi(x, k_\perp^2)$ are real, scalar functions yielding a nucleon wave function normalized to unit probability, namely,

$$\langle\Psi^{\uparrow}_{q(qq)_0}(P^+ = 1, \mathbf{P}_\perp = \mathbf{0}_\perp)|\Psi^{\uparrow}_{q(qq)_0}(P^+ = 1, \mathbf{P}_\perp = \mathbf{0}_\perp)\rangle = 1. \tag{68}$$

Similarly, the $J^z = -\frac{1}{2}$ nucleon wave function is given by

$$|\Psi^{\downarrow}_{q(qq)_0}(P^+ = 1, \mathbf{P}_\perp = \mathbf{0}_\perp)\rangle = \int \frac{dx d^2\mathbf{k}_\perp}{\sqrt{x(1-x)}16\pi^3} \times\left[\psi^{\downarrow}_{+1/2}(x,\mathbf{k}_\perp)\left|x,\mathbf{k}_\perp,+\frac{1}{2}\right\rangle + \psi^{\downarrow}_{-1/2}(x,\mathbf{k}_\perp)\left|x,\mathbf{k}_\perp,-\frac{1}{2}\right\rangle\right], \tag{69}$$

where

$$\begin{cases}\psi^{\downarrow}_{+1/2}(x,\mathbf{k}_\perp) = (k^1 - ik^2)g(x)\varphi(x,k_\perp^2),\\ \psi^{\downarrow}_{-1/2}(x,\mathbf{k}_\perp) = f(x)\varphi(x,k_\perp^2).\end{cases} \tag{70}$$

The structure of Eqs. (67) and (70) is common to that of the electron-photon Fock states of Ref. [15]. The light-front wave functions of Eqs. (66), (67), (69), and (70) satisfy Eq. (28), as well as Eqs. (41) and (42), so that in this model we find $F_3(q^2) = 0$ and $\mathrm{Im}(F_2(q^2)) = 0$.

We can generalize this model, however, so that $\mathcal{T}_\perp$-odd or $\mathcal{P}_\perp$-odd observables no longer vanish. Indeed, if we now include phases, writing

$$\begin{cases}\psi^{\uparrow}_{+1/2}(x,\mathbf{k}_\perp) = f(x)\varphi(x,k_\perp^2)e^{i\alpha_1}e^{+i\beta_1},\\ \psi^{\uparrow}_{-1/2}(x,\mathbf{k}_\perp) = -(k^1 + ik^2)g(x)\varphi(x,k_\perp^2)e^{i\alpha_2}e^{+i\beta_2},\end{cases} \tag{71}$$

$$\begin{cases}\psi^{\downarrow}_{+1/2}(x,\mathbf{k}_\perp) = (k^1 - ik^2)g(x)\varphi(x,k_\perp^2)e^{i\alpha_2}e^{-i\beta_2},\\ \psi^{\downarrow}_{-1/2}(x,\mathbf{k}_\perp) = f(x)\varphi(x,k_\perp^2)e^{i\alpha_1}e^{-i\beta_1},\end{cases} \tag{72}$$

where α_1, α_2, β_1, and β_2 are real constants, $F_3(q^2)$ and $\mathrm{Im}(F_2(q^2))$ can both be nonzero. We regard α_1, α_2, β_1, and β_2 as simple constants and not as functions of $k_\perp^2$ because we implicitly assume that the scale at which CP is broken, M_{CP}, in the fundamental theory is much larger than any we can access experimentally, so that $q^2 \ll M_{CP}^2$. For an explicit example of a mechanism realizing this, see Ref. [34].

In certain exceptional cases, it may be possible to have effective, $k_\perp^2$-dependent phases. Suppose, e.g., two distinct mechanisms of CP violation operate in a single Fock state a. In that event, assuming β_1 and β_2 are small, one could write the nucleon light-front wave function, suppressing all arguments, as $\psi_a = \psi_{1a}\exp(i\beta_1) + \psi_{2a}\exp(i\beta_2) \approx (\psi_{1a} + \psi_{2a}) + i(\psi_{1a}\beta_1 + \psi_{2a}\beta_2) \approx (\psi_{1a} + \psi_{2a})\exp(i\tilde{\beta})$, where $\tilde{\beta} = \tan^{-1}[(\psi_{1a}\beta_1 + \psi_{2a}\beta_2)/(\psi_{1a} + \psi_{2a})]$. Here we see explicitly that if ψ_{1a} and ψ_{2a} differ in their $k_\perp^2$ dependence that $\tilde{\beta}$ will be $k_\perp^2$ dependent even if β_1 and β_2 are not.

Let us consider the impact of the specific phases we have introduced. First, we observe that if $\beta_1 \neq 0$ or $\beta_2 \neq 0$ Eq. (28) no longer holds, so that β_1 and β_2 generate $\mathcal{P}_\perp$-odd effects. Second, if $\alpha_2 - \beta_2 - \alpha_1 - \beta_1 \neq 0$ or $\alpha_2 + \beta_2 - \alpha_1 + \beta_1 \neq 0$, then the equalities of Eqs. (41) and (42) will not follow, and we can recover nonzero $\mathcal{T}_\perp$-odd effects. We evaluate $F_2(q^2)$ and $F_3(q^2)$ with these model wave functions in the next section and determine that $\alpha_1 - \alpha_2 \neq 0$ gives rise to $\mathcal{T}_\perp$-odd and $\mathcal{P}_\perp$-even observables, whereas $\beta_1 \neq 0$ or $\beta_2 \neq 0$ gives rise to $\mathcal{T}_\perp$-odd and $\mathcal{P}_\perp$-odd observables. We remark in passing that α_1 and α_2 can also be introduced to pattern the phases that appear in final-state interactions and produce the Sivers effect [33]. Such pseudo-time-reversal-odd effects are not produced by fundamental sources of CP violation which are our focus here.

VI. RELATING THE ANOMALOUS MAGNETIC AND ELECTRIC DIPOLE MOMENTS

In this section we consider the relationship between $F_2(q^2)$ and $F_3(q^2)$ predicated by the relations of Eq. (7) and (8).

A. Quark–scalar-diquark model

We begin by computing $F_2(q^2)$ and $F_3(q^2)$ using the light-front wave functions of the quark–scalar-diquark model, Eqs. (71) and (72). In the following $\mathcal{A}$ is a function given by

$$\mathcal{A} = \int \frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} e\varphi(x, k_\perp'^2)\varphi(x, k_\perp^2)f(x)g(x) \tag{73}$$

and $\beta = \beta_1 + \beta_2$, where we recall that $\varphi(x, k_\perp^2)$, $f(x)$, and $g(x)$ are real. The bachelor quark is given charge e. From Eq. (11) we have

$$\begin{aligned}\frac{F_2(q^2)}{2M} = \frac{e}{2}\int\frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\Big(&\frac{1}{-q^1 + \mathrm{i}q^2}[\psi^{\uparrow *}(x, \mathbf{k}'_\perp, 1)\psi^{\downarrow}(x, \mathbf{k}_\perp, 1)\\ &+ \psi^{\uparrow *}(x, \mathbf{k}'_\perp, -1)\psi^{\downarrow}(x, \mathbf{k}_\perp, -1)]\\ &+ \frac{1}{q^1 + \mathrm{i}q^2}[\psi^{\downarrow *}(x, \mathbf{k}'_\perp, 1)\psi^{\uparrow}(x, \mathbf{k}_\perp, 1)\\ &+ \psi^{\downarrow *}(x, \mathbf{k}'_\perp, -1)\psi^{\uparrow}(x, \mathbf{k}_\perp, -1)]\Big).\end{aligned} \tag{74}$$

Doing the $d^2\mathbf{k}_\perp$ integral, we note that the terms in $\mathbf{k}_\perp$ will vanish unless $\mathbf{k}_\perp \parallel \mathbf{q}_\perp$, so that we have

$$\begin{aligned}\frac{F_2(q^2)}{2M} = \mathcal{A}\cos\beta[&(1 - x)\exp(i(\alpha_1 - \alpha_2))\\ &+ 2i\sin(\alpha_1 - \alpha_2)].\end{aligned} \tag{75}$$

From (12) we have

$$\begin{aligned}\frac{F_3(q^2)}{2M} = \frac{ie}{2}\int\frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\Big(&\frac{1}{-q^1 + \mathrm{i}q^2}[\psi^{\uparrow *}(x, \mathbf{k}'_\perp, 1)\psi^{\downarrow}(x, \mathbf{k}_\perp, 1)\\ &+ \psi^{\uparrow *}(x, \mathbf{k}'_\perp, -1)\psi^{\downarrow}(x, \mathbf{k}_\perp, -1)]\\ &- \frac{1}{q^1 + \mathrm{i}q^2}[\psi^{\downarrow *}(x, \mathbf{k}'_\perp, 1)\psi^{\uparrow}(x, \mathbf{k}_\perp, 1)\\ &+ \psi^{\downarrow *}(x, \mathbf{k}'_\perp, -1)\psi^{\uparrow}(x, \mathbf{k}_\perp, -1)]\Big).\end{aligned} \tag{76}$$

In doing the $d^2\mathbf{k}_\perp$ integral, we once again note that the terms in $\mathbf{k}_\perp$ will vanish unless $\mathbf{k}_\perp \parallel \mathbf{q}_\perp$, so that we have

$$\begin{aligned}\frac{F_3(q^2)}{2M} = \mathcal{A}\sin\beta[&(1 - x)\exp(i(\alpha_1 - \alpha_2))\\ &+ 2i\sin(\alpha_1 - \alpha_2)].\end{aligned} \tag{77}$$

Comparing Eqs. (75) and (77), we can elucidate the impact of the phases we have introduced. For example, if $\alpha_1 \neq \alpha_2$ with $\beta_1 = \beta_2 = 0$, we can have $\mathrm{Im}(F_2(q^2)) \neq 0$ but $F_3(q^2) = 0$. Alternatively, if $\alpha_1 = \alpha_2$ and $\beta_1, \beta_2 \neq 0$, then $\mathrm{Im}(F_2(q^2)) = 0$ but $F_3(q^2) \neq 0$. Finally, if $\alpha_1 \neq \alpha_2$ and $\beta_1, \beta_2 \neq 0$, then both $\mathrm{Im}(F_2(q^2)) \neq 0$ and $F_3(q^2) \neq 0$. It is remarkable that $F_2(q^2)$ and $F_3(q^2)$ differ only in their explicit dependence in β. We find, in specific, that

$$F_3(q^2) = (\tan\beta)F_2(q^2). \tag{78}$$

We proceed to examine how this relation emerges generally and its consequences for constraints on models of CP violation.

B. General relation

We now consider the relationship between the $F_2(q^2)$ and $F_3(q^2)$ form factors on general grounds. We will realize this by writing the light-front wave function of the nucleon in Fock component a as

$$\begin{cases}\psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) = \phi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i)e^{+i\beta_a/2},\\ \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) = \phi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i)e^{-i\beta_a/2},\end{cases} \tag{79}$$

where we have explicitly pulled out the $\mathcal{P}_\perp$- and $\mathcal{T}_\perp$-violating parameter β_a, which we have allowed to depend on the Fock state a. The remaining function $\phi_a^{\updownarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i)$ explicitly satisfies Eq. (28) and the equalities of Eqs. (41) and (42) in itself. We emphasize that the parametrization of Eq. (79) is a unique and general way of introducing CP-violation, as realized from a local, Lorentz-invariant quantum field theory with a Hermitian Hamiltonian. It is not specific to the standard model. With

Eq. (79) we thus find

$$\frac{F_2(q^2)}{2M} = \sum_a \cos(\beta_a)\Xi_a, \tag{80}$$

$$\frac{F_3(q^2)}{2M} = \sum_a \sin(\beta_a)\Xi_a, \tag{81}$$

where

$$\Xi_a = \int \frac{[\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp]}{16\pi^3}\sum_j e_j \frac{1}{-q^1 + iq^2} \times [\phi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\phi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i)]. \tag{82}$$

Thus for a particular Fock component we can write

$$[F_3(q^2)]_a = \tan\beta_a[F_2(q^2)]_a, \tag{83}$$

where the notation $[F]_a$ denotes the contribution to the form factor from Fock component a. At $q^2 = 0$, this becomes

$$d_a = 2\kappa_a \tan\beta_a, \tag{84}$$

which is the central result of our paper. As β_a is $\mathcal{P}_\perp$- and $\mathcal{T}_\perp$-violating, we can assume it to be small, to write

$$[F_3(q^2)]_a = \beta_a[F_2(q^2)]_a, \tag{85}$$

or

$$d_a = 2\kappa_a\beta_a. \tag{86}$$

We now proceed to consider how such connections can constrain theoretical predictions of $F_3(q^2)$ and hence impact bounds on CP-violating parameters.

VII. CONSEQUENCES FOR MODELS OF *CP* VIOLATION

The relation we have written, namely, Eq. (83), must hold irrespective of the possible sources of CP violation we consider: it holds both in and beyond the standard model. Moreover, it is appropriate to *any* spin-1/2 system, be it nucleon or lepton. That such a relation exists for charged leptons has been recognized by Feng, Matchev, and Shadmi [20] in supersymmetric models; in fact, as we have shown, the anomalous magnetic moment, $g - 2$, and the electric dipole moment of the charged leptons are tied in any model. We have extended this notion to neutral leptons, such as the neutrino, and to composite spin-1/2 systems, such as the nucleon, as well.

In the case of the nucleon, the empirical values of the anomalous magnetic moments are remarkably well known, and we have seen that the essential hadronic matrix elements are also common to the calculation of the electric dipole moment, d. Generally, a diagrammatic calculation of an observable such as d can be interpreted in terms of the contributions to d from states in a Fock expansion. We need only to determine the contributions of the intermediate states of the system, associated with all possible light-cone-time-ordered graphs [23], to which the photon couples. The contribution of a particular Fock state can thus be realized from the sum of many Feynman graphs. In principle, one could compute the light-front wave functions $\psi_{a/N}^{S_z}(x_i, \mathbf{k}_{\perp i}, \lambda_i)$ [21] and evaluate the requisite matrix elements directly, though the former has not yet been realized in QCD. Fortunately, in certain models, the determination of the Fock-state contributions, as well as of their sum, becomes greatly simplified. For example, if we were to make a constituent quark model of the light-front wave functions, such as the $q(qq)_0$ model of Secs. V and VI A , we would be able to simplify our relation still further to write, for small β,

$$F_3(q^2) = \beta F_2(q^2). \tag{87}$$

In such a model we can estimate d directly using the empirical anomalous magnetic moment of the nucleon. Noting $\kappa^n = -1.91$ and $\kappa^p = 1.79$ (in units of μ_N), we thus estimate that

$$d^n \sim -e\beta^n(2\cdot 10^{-14}\mathrm{cm}), \quad d^p \sim +e\beta^p(2\cdot 10^{-14}\mathrm{cm}), \tag{88}$$

where $2 \cdot 10^{-14}$ cm is the proton mass in cm. The current empirical bound on d^n, $|d^n| < 6.3 \cdot 10^{-26}$ e − cm [7], implies, in our simple picture, that $|\beta^n| < 3 \cdot 10^{-12}$. If $\beta^n \sim \beta^p$, then we predict $d^p \sim -d^n$ and thus that the isoscalar electric dipole moment is small, namely, $|d^n + d^p| \ll |d^n - d^p|$, just as the empirical isoscalar anomalous magnetic moment is small, $|\kappa^n + \kappa^p| = 0.12 \ll |\kappa^n - \kappa^p| = 3.70$. For reference, we note the empirical bound on d^p, $|d^p| < 5.4 \cdot 10^{-24}$ e − cm [35]. Although not apparent in this simple model, it is possible to connect the $\mathcal{T}_\perp$- and $\mathcal{P}_\perp$-odd parameter β to fundamental sources of CP violation in a realistic way; we shall now explore this possibility.

As an explicit example, we consider strong-interaction CP violation via a QCD θ-term and adopt the chiral Lagrangian framework of Refs. [36–40] to estimate d^n and d^p. In specific, we assume as in Ref. [37] that Fig. 1(b), realized in terms of the meson and baryon degrees of freedom operative in chiral effective theories at low energies, along with its counterpart containing a CP-violating interaction at the other πNN vertex, drives the value of the neutron's electric dipole moment. As is standard in such assays, we assume the magnitude of the external momentum transfer $|q|$, quark masses, and meson masses all small compared to the nucleon mass M. We wish to connect this language to the Fock-state expansion of the light-front formalism, realized in terms of the fundamental quark and gluon degrees of freedom. The chiral Lagrangian framework allows us to estimate the contribution of the $udd u\bar{u}$ Fock component of the neutron to its electric dipole and anomalous magnetic moment. Comparing the CP-conserving and CP-violating $\pi - N$ loop graphs of

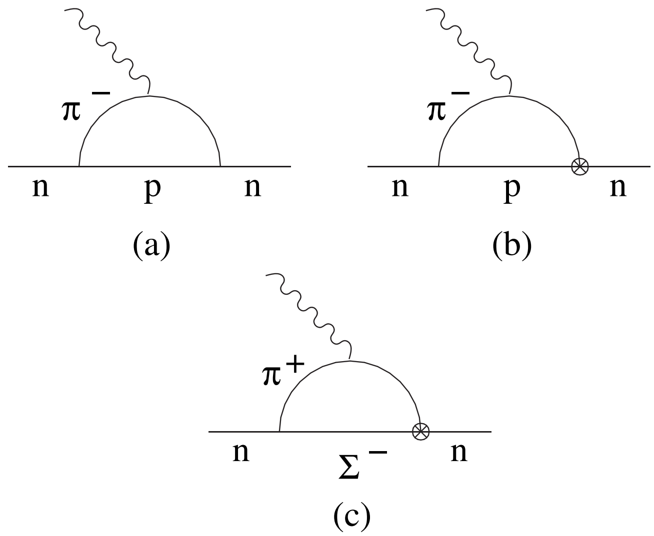

FIG. 1. (a) Feynman diagram for an one-loop contribution to the anomalous magnetic moment of the neutron in heavy-baryon chiral perturbation theory (HBCHPT). (b) Feynman diagram for an one-loop contribution to the electric dipole moment of the neutron from strong-interaction *CP* violation in HBCHPT, where ⊗ denotes a *CP*-violating vertex. (c) Feynman diagram for an one-loop contribution to the electric dipole moment of the neutron from weak interaction *CP* violation in HBCHPT, as per the CKM mechanism of the standard model.

Fig. 1, we estimate

$$|\beta_a| \approx 2\frac{|\bar{g}_{\pi NN}|}{|g_{\pi NN}|}\log(M_N/M_\pi) \approx 4\left(\frac{0.027}{13.4}\right)|\bar{\theta}|, \qquad (89)$$

where $\bar{g}_{\pi NN}$ is the *CP*-violating $g_{\pi NN}$ coupling constant. We use the numerical estimate of Ref. [37] for $\bar{g}_{\pi NN}/g_{\pi NN}$ and include the well-known logarithmic enhancement of the *CP*-violating graphs in β, as such is absent in the *CP*-conserving analog [41,42]. Employing Eq. (1) and assuming the $udd u\bar{u}$ Fock component dominates both the anomalous magnetic and electric dipole moment, we find

$$d^n \sim \bar{\theta} e(2\cdot 10^{-16}\mathrm{cm}), \qquad (90)$$

which is roughly a factor of 2 smaller than the estimate of Crewther *et al.* [37] but just that computed by Ref. [43] employing the QCD sum rule approach. The value of d^n can also be computed within the framework of lattice QCD [44–47]. Both estimates are compatible with a recent computation of d^n employing dynamical light quarks [47]. It is worth noting that the assumptions underlying the two estimates are slightly different. Our estimate also follows if the fractional contributions of the $\pi - N$ loop graph to the electric dipole moment and to the anomalous magnetic moment are the same; they need not be numerically dominant. Indeed, detailed analyses of the magnetic moments show that the $\pi - N$ loop graphs are not numerically dominant [42,48]. In comparison, the d^n estimate of Crewther *et al.* follows from computing the "long-distance" $\pi - N$ loop graph and assuming it numerically dominant. In the chiral, $M_\pi \to 0$, limit, this is seemly, as the d^n estimate contains an explicit factor of $\log(M/M_\pi)$. The numerical dominance of this contribution is less clear for physical values of the π mass, though our own estimate, stemming from a different assumption, is compatible with that of Crewther *et al.* With our assumptions we also conclude that $d^p \sim -d^n$, so that we predict that the electric dipole moment of the nucleon is predominately isovector. Indeed, the isospin structure of the $\pi - N$ loop diagrams precludes any isoscalar contribution; we assume these contributions drive that of the $udd u\bar{u}$ Fock state. Turning to the q^2 dependence of the electric dipole and anomalous magnetic form factors, the structure of our Eq. (83) shows that the q^2 dependence of $F_3(q^2)$ ought track that of $F_2(q^2)$, Fock state by Fock state, as we have argued on general grounds that β should be independent of q^2. Moreover, if we model the contribution of the $dduu\bar{u}$ Fock state through the $\pi - N$ loop graph as per Fig. 1, as we have discussed in the $q^2 = 0$ limit, we observe, under our stated assumptions, that the isospin structure of $F_3(q^2)$ is also isovector. This, too, follows from the isospin structure of the $\pi - N$ loop graphs. Our analysis is at odds with one conclusion of Ref. [40], as its authors find the q^2 dependence of $F_3(q^2)$ unlike that of the other electromagnetic form factors [49,50]. As in the $q^2 = 0$ limit, our prediction does not require the $\pi - N$ loop graphs to dominate the $F_3(q^2)$ form factor.

We can also estimate β through *CP* violation in the weak interaction, as mediated through the CKM matrix. The value of d^n through this mechanism of *CP* violation is much smaller, as it first appears in $\mathcal{O}(G_F^2\alpha_s)$ [51,52]. Here, following Ref. [53], we anticipate that the dominant contribution to the electric dipole moment of the nucleon is mediated by a hadronic loop graph with a $\pi\Sigma$ intermediate state, as illustrated in Fig. 1(c) for the neutron. Note that two diagrams contribute, each with a single *CP*-violating $N\pi\Sigma$ vertex. Since the $ddus\bar{u}$ Fock state makes a negligibly small contribution to the neutron's magnetic moment, we proceed to estimate d^n by summing Eq. (83) over all Fock states for $q^2 = 0$ and assuming that diagrams akin to Fig. 1(c) do drive its numerical value. Using Eq. (88) and writing

$$\beta \approx 2G_F^2\alpha_s(1\ \mathrm{GeV})J_{CP}M_\pi^3, \qquad (91)$$

noting the Jarlskog invariant [54], $J_{CP} \sim 3\cdot 10^{-5}$, and $\alpha_s(1\ \mathrm{GeV}) \approx 0.3$, we find

$$d^n \sim e3.6\cdot 10^{-32}\ \mathrm{cm}, \qquad (92)$$

which is rather comparable to the estimate of Ref. [53], namely,

$$d^n \sim e2\cdot 10^{-32}\ \mathrm{cm}, \qquad (93)$$

which follows from a direct estimate of the chirally enhanced terms. This procedure also implies that $d^p \sim -d^n$, yielding an isovector electric dipole moment. The electric dipole moments have also been computed in chiral perturbation theory in the context of the factorization hypothesis, yielding [55]

$$d^n \approx \pm 5.3 \times 10^{-32}\ \mathrm{e-cm}; \qquad d^p \approx \mp 3.6 \times 10^{-32}\ \mathrm{e-cm}, \tag{94}$$

where the manifest sign of d^n and d^p is not determined. These authors also discuss the connection to the anomalous magnetic moments in the context of their approximations. Note, too, that the dominantly isovector nature of the electric dipole moments is manifest in their results.

VIII. CONCLUSIONS

We have derived exact formulas for the electromagnetic form factors, $F_1(q^2)$, $F_2(q^2)$, and $F_3(q^2)$ of the nucleon, and indeed for all spin-1/2 systems, in the light-front formulation of quantum field theory, thus extending the treatment of Ref. [15] to the analysis of the time-reversal- and parity-odd observable $F_3(q^2)$. To realize this we have developed the light-front representation of discrete symmetry transformations, $\mathcal{T}_\perp$, $\mathcal{P}_\perp$, and C and have shown how $\mathcal{T}_\perp$-odd and $\mathcal{P}_\perp$-odd effects can be represented by the phases of light-front wave functions. The explicit expressions which we have developed for $F_2(q^2)$ and $F_3(q^2)$ have the desired transformation properties under $\mathcal{T}_\perp$, $\mathcal{P}_\perp$, and C. As a result, we find a universal relation between $F_3(q^2)$ and $F_2(q^2)$, Eq. (83), Fock state by Fock state, which follows independently of the mechanism of CP violation at the Lagrangian level.

We have employed our relation to estimate the electric dipole moments of the nucleon through both strong and weak interaction CP violation in the standard model and find results comparable to existing estimates. We find that the relation $d^n \sim -d^p$ emerges on rather general grounds, echoing the isospin structure of the empirical anomalous magnetic moments, $\kappa^n \sim -\kappa^p$.

ACKNOWLEDGMENTS

S. G. thanks the SLAC theory group for hospitality during the completion of this project. Work supported in part by the U.S. Department of Energy under Contract Nos. DE–AC02–76SF00515 and DE–FG02–96ER40989 and the KOSEF (Korea Science and Engineering Foundation).

APPENDIX: CONVENTIONS

We employ the Dirac representation for γ^μ:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \tag{95}$$

where σ^i are Pauli matrices, I is the 2×2 unit matrix, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and $\gamma^\pm \equiv \gamma^0 \pm \gamma^3$. For the light-cone spinors $u(p, \lambda)$ and $v(p, \lambda)$, we use [23]

$$u(p, +1) = \frac{1}{\sqrt{2p^+}} \begin{pmatrix} p^+ + m \\ p^R \\ p^+ - m \\ p^R \end{pmatrix}, \qquad u(p, -1) = \frac{1}{\sqrt{2p^+}} \begin{pmatrix} -p^L \\ p^+ + m \\ p^L \\ -p^+ + m \end{pmatrix} \tag{96}$$

and

$$v(p, +1) = \frac{1}{\sqrt{2p^+}} \begin{pmatrix} -p^L \\ p^+ - m \\ p^L \\ -p^+ - m \end{pmatrix}, \qquad v(p, -1) = \frac{1}{\sqrt{2p^+}} \begin{pmatrix} p^+ - m \\ p^R \\ p^+ + m \\ p^R \end{pmatrix}, \tag{97}$$

where we define $p^R \equiv p^1 + ip^2$, $p^L \equiv p^1 - ip^2$, and $p^\pm \equiv p^0 \pm p^3$. Moreover, we employ the notation $k^\mu = (k^+, k^-, k^L, k^R)$ so that $k \cdot x = (1/2)(k^+x^- + k^-x^+ - k^Lx^R - k^Rx^L) = (1/2)(k^+x^- + k^-x^+) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$.

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