$SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model with two Higgs triplets

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(Received 9 January 2006; published 8 February 2006)

The $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge model with the minimal scalar sector (two Higgs triplets) is studied in detail. One of the vacuum expectation values u is a source of lepton-number violations and a reason for the mixing among the charged gauge bosons—the standard model W and the bilepton (with $L = 2$) gauge bosons as well as among neutral non-Hermitian X^0 and neutral gauge bosons: the photon, the Z, and the new Z'. Because of these mixings, the lepton-number violating interactions exist in both charged and neutral gauge boson sectors. An exact diagonalization of the neutral gauge boson sector is derived and bilepton mass splitting is also given. The lepton-number violation happens only in the neutrino but not in the charged lepton sector. In this model, lepton-number changing ($\Delta L = \pm 2$) processes exist but *only* in the neutrino sector. Constraints on vacuum expectation values of the model are estimated and $u \simeq O(1)$ GeV, $v \simeq v_{\rm weak} = 246$ GeV, and $\omega \simeq O(1)$ TeV.

DOI: 10.1103/PhysRevD.73.035004 PACS numbers: 12.60.Cn, 12.10.Dm, 12.15.Mm, 12.60.Fr

I. INTRODUCTION

The $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ standard model of the strong and electroweak interactions, with the $SU(2)_L \otimes U(1)_Y$ symmetry spontaneously broken down to the $U(1)_Q$ of electromagnetism, is an excellent description of the interactions of elementary particles down to distances in the order of 10^{-16} cm. However, it also leaves many striking features of the physics of our world unexplained. Some of them are the generation number problem, the electric charge quantization, and the neutrino oscillations [1] which confirm that neutrinos are massive and the flavor lepton number is not conserved. It suggests that it is important to point out the complete dynamics of Higgs fields. All this requires new interactions beyond the conventional interactions in the standard model (SM).

A very common alternative to solve some of these problems consists of enlarging the group of gauge symmetry, where the larger group embeds properly the SM. For instance, the SU(5) grand unification model [2] can unify the interactions and predict the electric charge quantization, while the group E_6 can also unify the interactions and might explain the masses of the neutrinos [3]; see also [4]. Nevertheless, such models cannot explain the generation number problem.

A very interesting alternative to explain the origin of generations comes from the cancellation of chiral anomalies [5]. In particular, the models with gauge group $G_{331} = SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, also called 3-3-1 models [6–9], arise as a possible solution to this puzzle, since some such models require the three generations in order to cancel chiral anomalies completely. An additional motivation to study this kind of model comes from the fact that they also can predict the electric charge quantization [10] and the neutrino oscillation [11].

Such 3-3-1 models have been studied extensively over the last decade. In one of them [6], the three known left-handed lepton components for each generation are associated to three $SU(3)_L$ triplets as $(\nu_l, l, l^c)_L$, where l^c_L is related to the right-handed isospin singlet of the charged lepton l in the SM. This model requires that the Higgs sector contains three scalar triplets and one scalar sextet. In the variant model [7,8], three $SU(3)_L$ lepton triplets are of the form $(\nu_l, l, \nu^c_l)_L$, where ν^c_l is related to the right-handed component of the neutrino field ν_l (a model with right-handed neutrinos). The scalar sector of this model requires three Higgs triplets; therefore, hereafter we call this version the 3-3-1 model with three Higgs triplets (331RH3HT). It is interesting to note that, in the 331RH3HT, two Higgs triplets have the same $U(1)_X$ charge with two neutral components at their top and bottom. Allowing these neutral components vacuum expectation values (VEVs), we can reduce the number of Higgs triplets to two. As a result, the dynamics symmetry breaking also affects the lepton number. Hence it follows that the lepton number also is broken spontaneously at a high scale of energy. This kind of model was proposed in Ref. [12] but has not gotten enough attention. In Ref. [13], phenomenology of this model was presented without mixing between charged gauge bosons as well as neutral ones.

Phenomenology of the 3-3-1 model in the version that includes right-handed neutrinos with two Higgs triplets is a subject of this study.

The paper is organized as follows: In Sec. II we recall the idea of constructing the two-Higgs 3-3-1 model. Section III is devoted to Higgs potential. In Sec. IV, masses

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of charged gauge bosons are given and an exact diagonalization of neutral ones and their mixings are presented. Because of the mixings, currents in this model have unusual features in the neutrino sectors which are presented in Sec. V. In Sec. VI, constraints on the parameters of the model and some phenomena are sketched. Finally, in Sec. VI we summarize our main results.

II. THE PARTICLE CONTENT

The particle content in this model, which is anomaly free, is given as follows:

$$\psi_{iL} = \begin{pmatrix} \nu_i \\ e_i \\ \nu_i^c \end{pmatrix}_L \sim \left(3, -\frac{1}{3}\right),$$
$$e_{iR} \sim (1, -1), \qquad i = 1, 2, 3,$$
$$Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ U \end{pmatrix}_L \sim \left(3, \frac{1}{3}\right),$$
$$Q_{\alpha L} = \begin{pmatrix} d_\alpha \\ -u_\alpha \\ D_\alpha \end{pmatrix}_L \sim (3^*, 0), \qquad \alpha = 2, 3,$$
$$u_{iR} \sim \left(1, \frac{2}{3}\right), \qquad d_{iR} \sim \left(1, -\frac{1}{3}\right),$$
$$U_R \sim \left(1, \frac{2}{3}\right), \qquad D_{\alpha R} \sim \left(1, -\frac{1}{3}\right). \tag{1}$$

Here, the values in the parentheses denote quantum numbers based on the $(\mathrm{SU}(3)_L, \mathrm{U}(1)_X)$ symmetry. In this case, the electric charge operator takes a form

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X, \tag{2}$$

where T_a $(a = 1, 2, \ldots, 8)$ and X stand for $\mathrm{SU}(3)_L$ and $\mathrm{U}(1)_X$ charges, respectively. Electric charges of the exotic quarks U and D_α are the same as of the usual quarks, i.e. $q_U = \frac{2}{3}$ and $q_{D_\alpha} = -\frac{1}{3}$.

The $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ gauge group is broken spontaneously via two steps. In the first step, it is embedded in that of the SM via a Higgs scalar triplet

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^0 \end{pmatrix} \sim \left(3, -\frac{1}{3}\right), \tag{3}$$

acquired with VEV given by

$$\langle \chi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ 0 \\ \omega \end{pmatrix}. \tag{4}$$

In the last step, to embed the gauge group of the SM in $\mathrm{U}(1)_Q$, another Higgs scalar triplet

$$\phi = \begin{pmatrix} \phi_1^+ \\ \phi_2^0 \\ \phi_3^+ \end{pmatrix} \sim \left(3, \frac{2}{3}\right) \tag{5}$$

is needed with the VEV as follows:

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}. \tag{6}$$

The Yukawa interactions which induce masses for the fermions can be written in the most general form as

$$\mathcal{L}_Y = (\mathcal{L}_Y^\chi + \mathcal{L}_Y^\phi) + \mathcal{L}_Y^{\mathrm{mix}}, \tag{7}$$

where

$$\begin{aligned} (\mathcal{L}_Y^\chi + \mathcal{L}_Y^\phi) = {} & h'_{11} \overline{Q}_{1L} \chi U_R + h'_{\alpha\beta} \overline{Q}_{\alpha L} \chi^* D_{\beta R} \\ & + h^e_{ij} \overline{\psi}_{iL} \phi e_{jR} + h^\epsilon_{ij} \epsilon_{pmn} (\overline{\psi}^c_{iL})_p (\psi_{jL})_m (\phi)_n \\ & + h^d_{1i} \overline{Q}_{1L} \phi d_{iR} + h^d_{\alpha i} \overline{Q}_{\alpha L} \phi^* u_{iR} + \mathrm{H.c.}, \end{aligned} \tag{8}$$

$$\begin{aligned} \mathcal{L}_Y^{\mathrm{mix}} = {} & h^u_{1i} \overline{Q}_{1L} \chi u_{iR} + h^u_{\alpha i} \overline{Q}_{\alpha L} \chi^* d_{iR} + h''_{1\alpha} \overline{Q}_{1L} \phi D_{\alpha R} \\ & + h''_{\alpha 1} \overline{Q}_{\alpha L} \phi^* U_R + \mathrm{H.c.} \end{aligned} \tag{9}$$

The VEV ω gives mass for the exotic quarks U and D_α, u gives mass for u_1, d_α quarks, while v gives mass for u_α, d_1 and *all* ordinary leptons. Indeed, a consistent mass spectrum was given in Ref. [13]. With this mass pattern for the quarks, it follows that in the model under consideration the *first* family has to be different from the other two. Note that in the usual 331RH3HT, the third family of quarks should be discriminating [14].

As was mentioned above, the VEV ω is responsible for the first step of symmetry breaking, while the second step is due to u and v. Therefore we assume that

$$u, v < \omega. \tag{10}$$

The Yukawa couplings of Eq. (8) possess an extra global symmetry which implies a new conserved charge ($\mathcal{L}$) through the lepton-number (L) by diagonal matrices [15] $L = xT_3 + yT_8 + \mathcal{L}$. Applying L on a lepton triplet, the coefficients will be defined

$$L = \frac{4}{\sqrt{3}} T_8 + \mathcal{L}. \tag{11}$$

Here, the $\mathcal{L}$ charges of the fermion and Higgs multiplets can be obtained by

$$\begin{aligned} & \mathcal{L}\,(\psi_{iL}, Q_{1L}, Q_{\alpha L}, \phi, \chi, e_{iR}, u_{iR}, d_{iR}, U_R, D_{\alpha R}) \\ & = \frac{1}{3}, -\frac{2}{3}, \frac{2}{3}, -\frac{2}{3}, \frac{4}{3}, 1, 0, 0, -2, 2. \end{aligned} \tag{12}$$

It is worth emphasizing that $\mathcal{L}$ is not broken by VEVs v, ω but by u which is behind $L(\chi_1^0) = 2$ (see also [16]). This means that *u is a kind of the lepton-number violating* *parameter*. Moreover, the Yukawa couplings of (9) also violate $\mathcal{L}$ with ± 2 units which confirm that they are very small.

III. HIGGS POTENTIAL

In this model, the most general Higgs potential has a very simple form

$$V(\chi, \phi) = \mu_1^2 \chi^\dagger \chi + \mu_2^2 \phi^\dagger \phi + \lambda_1 (\chi^\dagger \chi)^2 + \lambda_2 (\phi^\dagger \phi)^2 + \lambda_3 (\chi^\dagger \chi)(\phi^\dagger \phi) + \lambda_4 (\chi^\dagger \phi)(\phi^\dagger \chi). \tag{13}$$

Note that there is no trilinear scalar coupling and this makes the Higgs potential much simpler than the previous ones [15,17] and closer to that of the SM. The analysis in Ref. [12] shows that after symmetry breaking there are eight Goldstone bosons and four physical scalar fields. One of two physical neutral scalars is the SM Higgs boson.

To break the symmetry spontaneously, the Higgs vacuums are not $SU(3)_L \otimes U(1)_X$ singlets. Thus, nonzero values of χ and ϕ at the minimum value of $V(\chi, \phi)$ can be easily obtained by

$$\chi^+ \chi = \frac{\lambda_3 \mu_2^2 - 2\lambda_2 \mu_1^2}{4\lambda_1 \lambda_2 - \lambda_3^2} \equiv \frac{u^2 + \omega^2}{2}, \tag{14}$$

$$\phi^+ \phi = \frac{\lambda_3 \mu_1^2 - 2\lambda_1 \mu_2^2}{4\lambda_1 \lambda_2 - \lambda_3^2} \equiv \frac{v^2}{2}. \tag{15}$$

It is worth noting that any other choice of u, ω for the vacuum value of χ satisfying (14) gives the same physics because it is related to (4) by an $SU(3)_L \otimes U(1)_X$ transformation. Thus, in a general case we assume that $u \neq 0$.

IV. GAUGE BOSONS

The covariant derivative of a triplet is given by

$$D_\mu = \partial_\mu - igT_a W_{a\mu} - ig_X T_9 X B_\mu \equiv \partial_\mu - i\mathcal{P}_\mu, \tag{16}$$

where the gauge fields W_a and B transform as the adjoint representations of $SU(3)_L$ and $U(1)_X$, respectively, and the corresponding gauge coupling constants g, g_X. Moreover, $T_9 = \frac{1}{\sqrt{6}}\mathrm{diag}(1, 1, 1)$ is fixed so that the relation $\mathrm{Tr}(T_{a'} T_{b'}) = \frac{1}{2}\delta_{a'b'}$ $(a', b' = 1, 2, \ldots, 9)$ is satisfied. The $\mathcal{P}_\mu$ matrix which appeared in the above covariant derivative is rewritten in a convenient form

$$\mathcal{P}_\mu = \frac{g}{2}\begin{pmatrix} W_{3\mu} + \frac{1}{\sqrt{3}} W_{8\mu} + t\sqrt{\frac{2}{3}} X B_\mu & \sqrt{2} W'^+_\mu & \sqrt{2} X'^0_\mu \\ \sqrt{2} W'^-_\mu & -W_{3\mu} + \frac{1}{\sqrt{3}} W_{8\mu} + t\sqrt{\frac{2}{3}} X B_\mu & \sqrt{2} Y'^-_\mu \\ \sqrt{2} X'^{0*}_\mu & \sqrt{2} Y'^+_\mu & -\frac{2}{\sqrt{3}} W_{8\mu} + t\sqrt{\frac{2}{3}} X B_\mu \end{pmatrix}, \tag{17}$$

where $t \equiv g_X/g$. Let us denote the following combinations

$$W'^\pm_\mu \equiv \frac{W_{1\mu} \mp iW_{2\mu}}{\sqrt{2}}, \qquad Y'^\mp_\mu \equiv \frac{W_{6\mu} \mp iW_{7\mu}}{\sqrt{2}}, \qquad X'^0_\mu \equiv \frac{W_{4\mu} - iW_{5\mu}}{\sqrt{2}}, \tag{18}$$

having defined charges under the generators of the $SU(3)_L$ group. For the sake of convenience in further reading, we note that W_4 and W_5 are pure real and imaginary parts of X'^0_μ and X'^{0*}_μ, respectively,

$$W_{4\mu} = \frac{1}{\sqrt{2}}(X'^0_\mu + X'^{0*}_\mu), \qquad W_{5\mu} = \frac{i}{\sqrt{2}}(X'^0_\mu - X'^{0*}_\mu). \tag{19}$$

The masses of the gauge bosons in this model are followed from

$$\begin{aligned}
\mathcal{L}^{\mathrm{GB}}_{\mathrm{mass}} &= (D_\mu \langle\phi\rangle)^+ (D^\mu \langle\phi\rangle) + (D_\mu \langle\chi\rangle)^+ (D^\mu \langle\chi\rangle) \\
&= \frac{g^2}{4}(u^2 + v^2) W'^-_\mu W'^{+\mu} + \frac{g^2}{4}(\omega^2 + v^2) Y'^-_\mu Y'^{+\mu} + \frac{g^2 u\omega}{4}(W'^-_\mu Y'^{+\mu} + Y'^-_\mu W'^{+\mu}) \\
&\quad + \frac{g^2 v^2}{8}\left(-W_{3\mu} + \frac{1}{\sqrt{3}} W_{8\mu} + t\frac{2}{3}\sqrt{\frac{2}{3}} B_\mu\right)^2 + \frac{g^2 u^2}{8}\left(W_{3\mu} + \frac{1}{\sqrt{3}} W_{8\mu} - t\frac{1}{3}\sqrt{\frac{2}{3}} B_\mu\right)^2 \\
&\quad + \frac{g^2 \omega^2}{8}\left(-\frac{2}{\sqrt{3}} W_{8\mu} - t\frac{1}{3}\sqrt{\frac{2}{3}} B_\mu\right)^2 + \frac{g^2 u\omega}{4\sqrt{2}}\left(W_{3\mu} + \frac{1}{\sqrt{3}} W_{8\mu} - t\frac{1}{3}\sqrt{\frac{2}{3}} B_\mu\right)(X'^{0\mu} + X'^{0*\mu}) \\
&\quad + \frac{g^2 u\omega}{4\sqrt{2}}\left(-\frac{2}{\sqrt{3}} W_{8\mu} - t\frac{1}{3}\sqrt{\frac{2}{3}} B_\mu\right)(X'^{0\mu} + X'^{0*\mu}) + \frac{g^2}{16}(u^2 + \omega^2)\{(X'^0_\mu + X'^{0*}_\mu)^2 + [i(X'^0_\mu - X'^{0*}_\mu)]^2\}.
\end{aligned} \tag{20}$$

The combinations W' and Y' are mixing via

$$\mathcal{L}^{\rm CG}_{\rm mass} = \frac{g^2}{4}(W'^-_\mu, Y'^-_\mu)\begin{pmatrix} u^2+v^2 & u\omega \\ u\omega & \omega^2+v^2 \end{pmatrix}\begin{pmatrix} W'^{+\mu} \\ Y'^{+\mu} \end{pmatrix}.$$

Diagonalizing this mass matrix, we get *physical* charged gauge bosons

$$W^-_\mu = \cos\theta W'^-_\mu + \sin\theta Y'^-_\mu, \quad Y^-_\mu = -\sin\theta W'^-_\mu + \cos\theta Y'^-_\mu, \tag{21}$$

where the mixing angle is defined by

$$\tan\theta = \frac{u}{\omega}. \tag{22}$$

The mass eigenvalues are

$$M^2_W = \frac{g^2 v^2}{4}, \tag{23}$$

$$M^2_Y = \frac{g^2}{4}(u^2+v^2+\omega^2). \tag{24}$$

Because of the constraints in (10), the following remarks are in order:

(1) θ should be very small, and then $W_\mu \simeq W'_\mu$, $Y_\mu \simeq Y'_\mu$.

(2) $v \simeq v_{\rm weak} = 246$ GeV due to identification of W as the W boson in the SM.

Next, from (20), the W_5 gains mass as follows

$$M^2_{W_5} = \frac{g^2}{4}(\omega^2+u^2). \tag{25}$$

Finally, there is a mixing among W_3, W_8, B, W_4 components. In the basis of these elements, the mass matrix is given by

$$M^2 = \frac{g^2}{4}\begin{pmatrix} u^2+v^2 & \frac{u^2-v^2}{\sqrt{3}} & -\frac{2t}{3\sqrt{6}}(u^2+2v^2) & 2u\omega \\ \frac{u^2-v^2}{\sqrt{3}} & \frac{1}{3}(4\omega^2+u^2+v^2) & \frac{\sqrt{2}t}{9}(2\omega^2-u^2+2v^2) & -\frac{2}{\sqrt{3}}u\omega \\ -\frac{2t}{3\sqrt{6}}(u^2+2v^2) & \frac{\sqrt{2}t}{9}(2\omega^2-u^2+2v^2) & \frac{2t^2}{27}(\omega^2+u^2+4v^2) & -\frac{8t}{3\sqrt{6}}u\omega \\ 2u\omega & -\frac{2}{\sqrt{3}}u\omega & -\frac{8t}{3\sqrt{6}}u\omega & u^2+\omega^2 \end{pmatrix}. \tag{26}$$

Note that the mass Lagrangian in this case has the form

$$\mathcal{L}^{\rm NG}_{\rm mass} = \tfrac{1}{2}V^T M^2 V, \qquad V^T \equiv (W_3, W_8, B, W_4). \tag{27}$$

In the limit $u \to 0$, W_4 does not mix with $W_{3\mu}$, $W_{8\mu}$, B_μ. In the general case $u \neq 0$, the mass matrix in (26) contains two *exact eigenvalues* such as

$$M^2_\gamma = 0, \qquad M^2_{W'_4} = \frac{g^2}{4}(\omega^2+u^2). \tag{28}$$

Thus the W'_4 and W_5 components have the same mass, and this conclusion *contradicts the previous analysis in* Ref. [12]. With this result, we should identify the combination of W'_4 and W_5

$$\sqrt{2}X^0_\mu = W'_{4\mu} - iW_{5\mu} \tag{29}$$

as a *physical* neutral *non-Hermitian* gauge boson. The subscript 0 denotes neutrality of gauge boson X. However, in the following, this subscript may be dropped. This boson caries lepton number two, hence it is the bilepton like those in the usual 331RH3HT. From (23), (24), and (28), it follows an interesting relation between the bilepton masses similar to the law of Pythagoras

$$M^2_Y = M^2_X + M^2_W. \tag{30}$$

Thus the charged bilepton Y is slightly heavier than the neutral one X. Let us remind the reader that the similar relation in the 331RH3HT is $|M^2_Y - M^2_X| \leq m^2_W$ [18].

Now we turn to the eigenstate question. The eigenstates corresponding to the two values in (28) are determined as follows:

$$A_\mu = \frac{1}{\sqrt{18+4t^2}}\begin{pmatrix} \sqrt{3}t \\ -t \\ 3\sqrt{2} \\ 0 \end{pmatrix}, \quad W'_{4\mu} = \frac{1}{\sqrt{1+4\tan^2 2\theta}}\begin{pmatrix} \tan 2\theta \\ \sqrt{3}\tan 2\theta \\ 0 \\ 1 \end{pmatrix}. \tag{31}$$

To embed this model in the effective theory at the low energy we follow an appropriate method without Higgs bosons in Refs. [19,20], where the photon field couples with the lepton by strength

$$\mathcal{L}^{\rm EM}_{\rm int} = -\frac{\sqrt{3}g_X}{\sqrt{18+4t^2}}\bar{l}\gamma^\mu l A_\mu. \tag{32}$$

Therefore the coefficient of the electromagnetic coupling constant can be identified as

$$\frac{\sqrt{3}g_X}{\sqrt{18+4t^2}} = e. \tag{33}$$

Using continuation of the gauge coupling constant g of $SU(3)_L$ at the spontaneous symmetry breaking point

$$g = g[SU(2)_L] = \frac{e}{s_W}, \tag{34}$$

from which it follows

$$t = \frac{3\sqrt{2}s_W}{\sqrt{3-4s_W^2}}. \tag{35}$$

The eigenstates are now rewritten as follows:

$$\begin{aligned} A_\mu &= s_W W_{3\mu} + c_W\left(-\frac{t_W}{\sqrt{3}}W_{8\mu} + \sqrt{1-\frac{t_W^2}{3}}B_\mu\right), \\ W'_{4\mu} &= \frac{t_{2\theta}}{\sqrt{1+4t_{2\theta}^2}}W_{3\mu} + \frac{\sqrt{3}t_{2\theta}}{\sqrt{1+4t_{2\theta}^2}}W_{8\mu} \\ &\quad + \frac{1}{\sqrt{1+4t_{2\theta}^2}}W_{4\mu}, \end{aligned} \tag{36}$$

where we have denoted $s_W \equiv \sin\theta_W$, $t_{2\theta} \equiv \tan 2\theta$, and so forth.

The diagonalization of the mass matrix is done via three steps. In the first step, in the base of $(A_\mu, Z_\mu, Z'_\mu, W_{4\mu})$, the two remaining gauge vectors are given by

$$\begin{aligned} Z_\mu &= c_W W_{3\mu} - s_W\left(-\frac{t_W}{\sqrt{3}}W_{8\mu} + \sqrt{1-\frac{t_W^2}{3}}B_\mu\right), \\ Z'_\mu &= \sqrt{1-\frac{t_W^2}{3}}W_{8\mu} + \frac{t_W}{\sqrt{3}}B_\mu. \end{aligned} \tag{37}$$

In this basis, the mass matrix M^2 becomes

$$M'^2 = \frac{g^2}{4} \times \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{u^2+v^2}{c_W^2} & \frac{c_{2W}u^2-v^2}{c_W^2\sqrt{3-4s_W^2}} & \frac{2u\omega}{c_W} \\ 0 & \frac{c_{2W}u^2-v^2}{c_W^2\sqrt{3-4s_W^2}} & \frac{v^2+4c_W^4\omega^2+c_{2W}^2u^2}{c_W^2(3-4s_W^2)} & -\frac{2u\omega}{c_W\sqrt{3-4s_W^2}} \\ 0 & \frac{2u\omega}{c_W} & -\frac{2u\omega}{c_W\sqrt{3-4s_W^2}} & u^2+\omega^2 \end{pmatrix}. \tag{38}$$

Also, in the limit $u \to 0$, $W_{4\mu}$ does not mix with Z_μ, Z'_μ. The eigenstate $W'_{4\mu}$ is now defined by

$$\begin{aligned} W'_{4\mu} &= \frac{t_{2\theta}}{c_W\sqrt{1+4t_{2\theta}^2}}Z_\mu + \frac{\sqrt{4c_W^2-1}t_{2\theta}}{c_W\sqrt{1+4t_{2\theta}^2}}Z'_\mu \\ &\quad + \frac{1}{\sqrt{1+4t_{2\theta}^2}}W_{4\mu}. \end{aligned} \tag{39}$$

We now turn to the second step. To see explicitly that the following basis is orthogonal and normalized, let us put

$$s_{\theta'} \equiv \frac{t_{2\theta}}{c_W\sqrt{1+4t_{2\theta}^2}}, \tag{40}$$

which leads to

$$\begin{aligned} W'_{4\mu} &= s_{\theta'}Z_\mu + c_{\theta'}[t_{\theta'}\sqrt{4c_W^2-1}Z'_\mu \\ &\quad + \sqrt{1-t_{\theta'}^2(4c_W^2-1)}W_{4\mu}]. \end{aligned} \tag{41}$$

Note that the mixing angle in this step θ' is the same order as the mixing angle in the charged gauge boson sector. Taking into account [21] $s_W^2 \simeq 0.231$, from (40) we get $s_{\theta'} \simeq 2.28 s_\theta$. It is now easy to choose two remaining gauge vectors

$$\begin{aligned} \mathcal{Z}_\mu &= c_{\theta'}Z_\mu - s_{\theta'}[t_{\theta'}\sqrt{4c_W^2-1}Z'_\mu \\ &\quad + \sqrt{1-t_{\theta'}^2(4c_W^2-1)}W_{4\mu}], \\ \mathcal{Z}'_\mu &= \sqrt{1-t_{\theta'}^2(4c_W^2-1)}Z'_\mu - t_{\theta'}\sqrt{4c_W^2-1}W_{4\mu}. \end{aligned} \tag{42}$$

Therefore, in the base of $(A_\mu, \mathcal{Z}_\mu, \mathcal{Z}'_\mu, W'_{4\mu})$ the mass matrix M''^2 has a quasidiagonal form

$$M''^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & m_{\mathcal{Z}}^2 & m_{\mathcal{Z}\mathcal{Z}'}^2 & 0 \\ 0 & m_{\mathcal{Z}\mathcal{Z}'}^2 & m_{\mathcal{Z}'}^2 & 0 \\ 0 & 0 & 0 & \frac{g^2}{4}(u^2+\omega^2) \end{pmatrix}, \tag{43}$$

with

$$\begin{aligned} m_{\mathcal{Z}}^2 &= \frac{g^2[(1+3t_{2\theta}^2)u^2 + (1+4t_{2\theta}^2)v^2 - t_{2\theta}^2\omega^2]}{4[c_W^2 + (3-4s_W^2)t_{2\theta}^2]}, \\ m_{\mathcal{Z}\mathcal{Z}'}^2 &= \frac{g^2\sqrt{1+4t_{2\theta}^2}\{[c_{2W} + (3-4s_W^2)t_{2\theta}^2]u^2 - v^2 - (3-4s_W^2)t_{2\theta}^2\omega^2\}}{4\sqrt{3-4s_W^2}[c_W^2 + (3-4s_W^2)t_{2\theta}^2]}, \\ m_{\mathcal{Z}'}^2 &= \frac{g^2\{[c_{2W}^2 + (3-4s_{2W}^2)t_{2\theta}^2]u^2 + v^2 + [4c_W^4 + (1+4c_W^2)(3-4s_W^2)t_{2\theta}^2]\omega^2\}}{4(3-4s_W^2)[c_W^2 + (3-4s_W^2)t_{2\theta}^2]}. \end{aligned} \tag{44}$$

In the last step, it is trivial to diagonalize the mass matrix in (43). The two remaining mass eigenstates are given by

$$Z_\mu^1 = c_\varphi \mathcal{Z}_\mu - s_\varphi \mathcal{Z}'_\mu, \qquad Z_\mu^2 = s_\varphi \mathcal{Z}_\mu + c_\varphi \mathcal{Z}'_\mu, \tag{45}$$

where the mixing angle φ between $\mathcal{Z}$ and $\mathcal{Z}'$ is defined by

$$t_{2\varphi} = \frac{\sqrt{(3-4s_W^2)(1+4t_{2\theta}^2)}\{[c_{2W} + (3-4s_W^2)t_{2\theta}^2]u^2 - v^2 - (3-4s_W^2)t_{2\theta}^2\omega^2\}}{[2s_W^4 - 1 + (8s_W^4 - 2s_W^2 - 3)t_{2\theta}^2]u^2 - [c_{2W} + 2(3-4s_W^2)t_{2\theta}^2]v^2 + [2c_W^4 + (8s_W^4 + 9c_{2W})t_{2\theta}^2]\omega^2}. \quad (46)$$

The physical mass eigenvalues are defined by

$$M_{Z^1}^2 = \frac{c_W^2(u^2+\omega^2) + v^2 - \sqrt{[c_W^2(u^2+\omega^2)+v^2]^2 + (3-4s_W^2)(3u^2\omega^2 - u^2v^2 - v^2\omega^2)}}{2g^{-2}(3-4s_W^2)},$$

$$M_{Z^2}^2 = \frac{c_W^2(u^2+\omega^2) + v^2 + \sqrt{[c_W^2(u^2+\omega^2)+v^2]^2 + (3-4s_W^2)(3u^2\omega^2 - u^2v^2 - v^2\omega^2)}}{2g^{-2}(3-4s_W^2)}.$$

Because of the condition (10), the angle φ has to be very small

$$t_{2\varphi} \simeq -\frac{\sqrt{3-4s_W^2}[v^2 + (11-14s_W^2)u^2]}{2c_W^4\omega^2}. \quad (47)$$

In this approximation, the above physical states have masses

$$M_{Z^1}^2 \simeq \frac{g^2}{4c_W^2}(v^2 - 3u^2), \quad (48)$$

$$M_{Z^2}^2 \simeq \frac{g^2 c_W^2 \omega^2}{3-4s_W^2}. \quad (49)$$

Consequently, Z^1 can be identified as the Z boson in the SM, with Z^2 being the new neutral (Hermitian) gauge boson. It is important to note that in the limit $u \to 0$ the mixing angle φ between Z and Z' is always non-vanishing. This differs from the mixing angle θ between the W boson of the SM and the singly charged bilepton Y. Phenomenology of the mentioned mixing is quite similar to the $W_L - W_R$ mixing in the left-right symmetric model based on the $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$ group (see [20]).

V. CURRENTS

The interaction among fermions with gauge bosons arises in part from

$$i\bar{\psi}\gamma_\mu D^\mu \psi = \text{ kinematic terms} + H^{CC} + H^{NC}. \quad (50)$$

A. Charged currents

Despite neutrality, the gauge bosons X^0, X^{0*} belong to this section by their nature. Because of the mixing among the SM W boson and the charged bilepton Y as well as among $(X^0 + X^{0*})$ with (W_3, W_8, B), the new terms exist as follows:

$$H^{CC} = \frac{g}{\sqrt{2}}(J_W^{\mu -} W_\mu^+ + J_Y^{\mu -} Y_\mu^+ + J_X^{\mu 0*} X_\mu^0 + \text{H.c.}), \quad (51)$$

where

$$J_W^{\mu -} = c_\theta(\bar{\nu}_{iL}\gamma^\mu e_{iL} + \bar{u}_{iL}\gamma^\mu d_{iL}) + s_\theta(\bar{\nu}_{iL}^c\gamma^\mu e_{iL} + \bar{U}_L\gamma^\mu d_{1L} + \bar{u}_{\alpha L}\gamma^\mu D_{\alpha L}), \quad (52)$$

$$J_Y^{\mu -} = c_\theta(\bar{\nu}_{iL}^c\gamma^\mu e_{iL} + \bar{U}_L\gamma^\mu d_{1L} + \bar{u}_{\alpha L}\gamma^\mu D_{\alpha L}) - s_\theta(\bar{\nu}_{iL}\gamma^\mu e_{iL} + \bar{u}_{iL}\gamma^\mu d_{iL}), \quad (53)$$

$$\begin{aligned} J_X^{\mu 0*} = {} & (1-t_{2\theta}^2)(\bar{\nu}_{iL}\gamma^\mu \nu_{iL}^c + \bar{u}_{1L}\gamma^\mu U_L - \bar{D}_{\alpha L}\gamma^\mu d_{\alpha L}) \\ & - t_{2\theta}^2(\bar{\nu}_{iL}^c\gamma^\mu \nu_{iL} + \bar{U}_L\gamma^\mu u_{1L} - \bar{d}_{\alpha L}\gamma^\mu D_{\alpha L}) \\ & + \frac{t_{2\theta}}{\sqrt{1+4t_{2\theta}^2}}(\bar{\nu}_i\gamma^\mu\nu_i + \bar{u}_{1L}\gamma^\mu u_{1L} \\ & - \bar{U}_L\gamma^\mu U_L - \bar{d}_{\alpha L}\gamma^\mu d_{\alpha L} + \bar{D}_{\alpha L}\gamma^\mu D_{\alpha L}). \end{aligned} \quad (54)$$

Comparing with the charged currents in the usual 331RH3HT[8], we get the following discrepancies:

(1) The second term in (52).
(2) The second term in (53).
(3) The second and the third terms in (54).

All mentioned above interactions are lepton-number violating and weak (proportional to $\sin\theta$ or its square $\sin^2\theta$). However, these couplings lead to lepton-number violations only in the neutrino sector.

B. Neutral currents

As before, in this model, a real part of the non-Hermitian neutral X'^0 mixes with the real neutral ones such as Z and Z'. This gives the *unusual* term as follows:

$$H^{NC} = eA^\mu J_\mu^{EM} + \mathcal{L}^{NC} + \mathcal{L}_{\text{unnormal}}^{NC}. \quad (55)$$

Despite the mixing among W_3, W_8, B, W_4, the electromagnetic interactions *remain the same as* in the SM and the usual 331RH3HT, i.e.

$$J_\mu^{EM} = \sum_f q_f \bar{f}\gamma_\mu f, \quad (56)$$

where f runs among all the fermions of the model.

Interactions of the neutral currents with fermions have a common form

$$\mathcal{L}^{\mathrm{NC}} = \frac{g}{2c_W}\bar{f}\gamma^\mu[g_{kV}(f) - g_{kA}(f)\gamma^5]fZ^k_\mu, \qquad k = 1, 2, \tag{57}$$

where

$$g_{1V}(f) = \frac{c_\varphi\{T_3(f_L) - 3t^2_{2\theta}X(f_L) + [(3 - 8s^2_W)t^2_{2\theta} - 2s^2_W]Q(f)\}}{\sqrt{(1 + 4t^2_{2\theta})[1 + (3 - t^2_W)t^2_{2\theta}]}} - \frac{s_\varphi[(4c^2_W - 1)T_3(f_L) + 3c^2_W X(f_L) - (3 - 5s^2_W)Q(f)]}{\sqrt{(4c^2_W - 1)[1 + (3 - t^2_W)t^2_{2\theta}]}}, \tag{58}$$

$$g_{1A}(f) = \frac{c_\varphi[T_3(f_L) - 3t^2_{2\theta}(X - Q)(f_L)]}{\sqrt{(1 + 4t^2_{2\theta})[1 + (3 - t^2_W)t^2_{2\theta}]}} - \frac{s_\varphi[(4c^2_W - 1)T_3(f_L) + 3c^2_W(X - Q)(f_L)]}{\sqrt{(4c^2_W - 1)[1 + (3 - t^2_W)t^2_{2\theta}]}}, \tag{59}$$

TABLE I. The $Z^1_\mu \to \bar{f}f$ couplings.

f	$g_{1V}(f)$	$g_{1A}(f)$
ν_e, ν_μ, ν_τ	$\frac{c_\varphi - s_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}}{2\sqrt{(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$	$\frac{c_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}+s_\varphi}{2\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$
e, μ, τ	$\frac{(3-4c^2_W)[c_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}+s_\varphi]}{2\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$	$-\frac{c_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}+s_\varphi}{2\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$
u	$\frac{c_\varphi\sqrt{4c^2_W-1}[3(1+2t^2_{2\theta})-8s^2_W(1+4t^2_{2\theta})]-s_\varphi(3+2s^2_W)\sqrt{1+4t^2_{2\theta}}}{6\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$	$\frac{c_\varphi\sqrt{4c^2_W-1}(1+2t^2_{2\theta})-s_\varphi c_{2W}\sqrt{1+4t^2_{2\theta}}}{2\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$
d	$\frac{(1-4c^2_W)[c_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}+s_\varphi]}{6\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$	$-\frac{c_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}+s_\varphi}{2\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$
c, t	$\frac{(3-8s^2_W)[c_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}+s_\varphi]}{6\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$	$\frac{c_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}+s_\varphi}{2\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$
s, b	$\frac{c_\varphi\sqrt{4c^2_W-1}[(1-4c^2_W)(1+4t^2_{2\theta})+6t^2_{2\theta}]+s_\varphi(1+2c^2_W)\sqrt{1+4t^2_{2\theta}}}{6\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$	$-\frac{c_\varphi\sqrt{4c^2_W-1}(1+2t^2_{2\theta})-s_\varphi c_{2W}\sqrt{1+4t^2_{2\theta}}}{2\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$
U	$\frac{c_\varphi\sqrt{4c^2_W-1}[3t^2_{2\theta}-4s^2_W(1+4t^2_{2\theta})]+s_\varphi(3-7s^2_W)\sqrt{1+4t^2_{2\theta}}}{3\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$	$\frac{c_\varphi\sqrt{4c^2_W-1}t^2_{2\theta}+s_\varphi c^2_W\sqrt{1+4t^2_{2\theta}}}{\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$
D_2, D_3	$\frac{c_\varphi\sqrt{4c^2_W-1}[2s^2_W(1+4t^2_{2\theta})-3t^2_{2\theta}]-s_\varphi(3-5s^2_W)\sqrt{1+4t^2_{2\theta}}}{3\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$	$-\frac{c_\varphi\sqrt{4c^2_W-1}t^2_{2\theta}+s_\varphi c^2_W\sqrt{1+4t^2_{2\theta}}}{\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$

TABLE II. The $Z^2_\mu \to \bar{f}f$ couplings.

f	$g_{2V}(f)$	$g_{2A}(f)$
ν_e, ν_μ, ν_τ	$\frac{s_\varphi + c_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}}{2\sqrt{(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$	$\frac{s_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}-c_\varphi}{2\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$
e, μ, τ	$\frac{(3-4c^2_W)[s_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}-c_\varphi]}{2\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$	$-\frac{s_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}-c_\varphi}{2\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$
u	$\frac{s_\varphi\sqrt{4c^2_W-1}[3(1+2t^2_{2\theta})-8s^2_W(1+4t^2_{2\theta})]+c_\varphi(3+2s^2_W)\sqrt{1+4t^2_{2\theta}}}{6\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$	$\frac{s_\varphi\sqrt{4c^2_W-1}(1+2t^2_{2\theta})+c_\varphi c_{2W}\sqrt{1+4t^2_{2\theta}}}{2\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$
d	$\frac{(1-4c^2_W)[s_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}-c_\varphi]}{6\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$	$-\frac{s_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}-c_\varphi}{2\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$
c, t	$\frac{(3-8s^2_W)[s_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}-c_\varphi]}{6\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$	$\frac{s_\varphi\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})}-c_\varphi}{2\sqrt{(4c^2_W-1)[1+(3-t^2_W)t^2_{2\theta}]}}$
s, b	$\frac{s_\varphi\sqrt{4c^2_W-1}[(1-4c^2_W)(1+4t^2_{2\theta})+6t^2_{2\theta}]-c_\varphi(1+2c^2_W)\sqrt{1+4t^2_{2\theta}}}{6\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$	$-\frac{s_\varphi\sqrt{4c^2_W-1}(1+2t^2_{2\theta})+c_\varphi c_{2W}\sqrt{1+4t^2_{2\theta}}}{2\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$
U	$\frac{s_\varphi\sqrt{4c^2_W-1}[3t^2_{2\theta}-4s^2_W(1+4t^2_{2\theta})]-c_\varphi(3-7s^2_W)\sqrt{1+4t^2_{2\theta}}}{3\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$	$\frac{s_\varphi\sqrt{4c^2_W-1}t^2_{2\theta}-c_\varphi c^2_W\sqrt{1+4t^2_{2\theta}}}{\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$
D_2, D_3	$\frac{s_\varphi\sqrt{4c^2_W-1}[2s^2_W(1+4t^2_{2\theta})-3t^2_{2\theta}]+c_\varphi(3-5s^2_W)\sqrt{1+4t^2_{2\theta}}}{3\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$	$-\frac{s_\varphi\sqrt{4c^2_W-1}t^2_{2\theta}-c_\varphi c^2_W\sqrt{1+4t^2_{2\theta}}}{\sqrt{(4c^2_W-1)(1+4t^2_{2\theta})[1+(3-t^2_W)t^2_{2\theta}]}}$

$$g_{2V}(f) = g_{1V}(f)(c_\varphi \to s_\varphi, s_\varphi \to -c_\varphi),$$
$$g_{2A}(f) = g_{1A}(f)(c_\varphi \to s_\varphi, s_\varphi \to -c_\varphi). \quad (60)$$

Here $T_3(f_L)$, $X(f_L)$, and $Q(f)$ are, respectively, the third component of the weak isospin, the $U(1)_X$ charge, and the electric charge of the fermion f_L. Note that the isospin for the $SU(2)_L$ fermion singlet (in the bottom of triplets) vanishes: $T_3(f_L) = 0$. The values of $g_{1V}(f)$, $g_{1A}(f)$ and $g_{2V}(f)$, $g_{2A}(f)$ are listed in Tables I and II.

Because of the above-mentioned mixing, the lepton-number violating interactions mediated by neutral gauge bosons Z^1 and Z^2 exist in the *neutrino and the exotic quark sectors*

$$\mathcal{L}^{\rm NC}_{\rm unnormal} = -\frac{g t_{2\theta} g_{kV}(\nu)}{2}(\overline{\nu}_{iL}\gamma^\mu \nu^c_{iL} + \overline{u}_{1L}\gamma^\mu U_L - \overline{D}_{\alpha L}\gamma^\mu d_{\alpha L})Z^k_\mu + \text{H.c.} \quad (61)$$

Again, these interactions are very weak and proportional to $\sin\theta$. From (52)–(54) and (61) we conclude that all lepton-number violating interactions are expressed in the terms dependent only in the mixing angle between the charged gauge bosons.

VI. PHENOMENOLOGY

First of all we should find some constraints on the parameters of the model. There are many ways to get constraints on the mixing angle θ and the charged bilepton mass M_Y. Below we present a simple one. In our model, the W boson has the following *normal main* decay modes:

$$W^- \to l\tilde{\nu}_l (l = e, \mu, \tau),$$
$$\searrow u^c d, u^c s, u^c b, (u \to c), \quad (62)$$

which are the same as in the SM and in the 331RH3HT. Beside the above modes, there are additional ones which are lepton-number violating ($\Delta L = 2$)—the model's specific feature

$$W^- \to l\nu_l (l = e, \mu, \tau). \quad (63)$$

It is easy to compute the tree-level decay widths as follows [22]:

$$\Gamma^{\rm Born}(W \to l\tilde{\nu}_l) = \frac{g^2 c^2_\theta}{8}\frac{M_W}{6\pi}(1-x)\left(1 - \frac{x}{2} - \frac{x^2}{2}\right) \simeq \frac{c^2_\theta \alpha M_W}{12 s^2_W},$$
$$\Gamma^{\rm Born}(W \to l\nu_l) = \frac{g^2 s^2_\theta}{8}\frac{M_W}{6\pi}(1-x)\left(1 - \frac{x}{2} - \frac{x^2}{2}\right) \simeq \frac{s^2_\theta \alpha M_W}{12 s^2_W}, \qquad x \equiv m^2_l/M^2_W,$$
$$\sum_{\rm color}\Gamma^{\rm Born}(W \to u^c_i d_j) = \frac{3g^2 c^2_\theta}{8}\frac{M_W}{6\pi}|V_{ij}|^2[1 - 2(x+\overline{x}) + (x-\overline{x})^2]^{(1/2)}\left[1 - \frac{x+\overline{x}}{2} - \frac{(x-\overline{x})^2}{2}\right] \simeq \frac{c^2_\theta \alpha M_W}{4 s^2_W}|V_{ij}|^2,$$
$$x \equiv m^2_{d_j}/M^2_W, \qquad \overline{x} \equiv m^2_{u^c_i}/M^2_W. \quad (64)$$

QCD radiative corrections modify Eq. (64) by a multiplicative factor [21,22]

$$\delta_{\rm QCQ} = 1 + \alpha_s(M_Z)/\pi + 1.409\alpha_s^2/\pi^2 - 12.77\alpha_s^3/\pi^3 \simeq 1.04, \quad (65)$$

which is estimated from $\alpha_s(M_Z) \simeq 0.12138$. All the state masses can be ignored, the predicted total width for W decay into fermions is

$$\Gamma^{\rm tot}_W = 1.04\frac{\alpha M_W}{2 s^2_W}(1 - s^2_\theta) + \frac{\alpha M_W}{4 s^2_W}. \quad (66)$$

Taking $\alpha(M_Z) \simeq 1/128$, $M_W = 80.425$ GeV, $s^2_W = 0.2312$, and $\Gamma^{\rm tot}_W = 2.124 \pm 0.041$ GeV [21], in Fig. 1, we have plotted $\Gamma^{\rm tot}_W$ as a function of s_θ. From the figure we get an upper limit: $\sin\theta \leq 0.08$.

Since one of the VEVs is close to the those in the SM, $v \simeq v_{\rm weak} = 246$ GeV, only two free VEVs exist in the considering model, namely, u and ω. The bilepton mass limit can be obtained from the "wrong" muon decay

$$\mu^- \to e^- \nu_e \tilde{\nu}_\mu \quad (67)$$

mediated, at the tree level, by both the SM W and the singly charged bilepton Y_μ (see Fig. 2). We remind the reader that in the 331RH3HT, at the lowest order, this decay is medi-

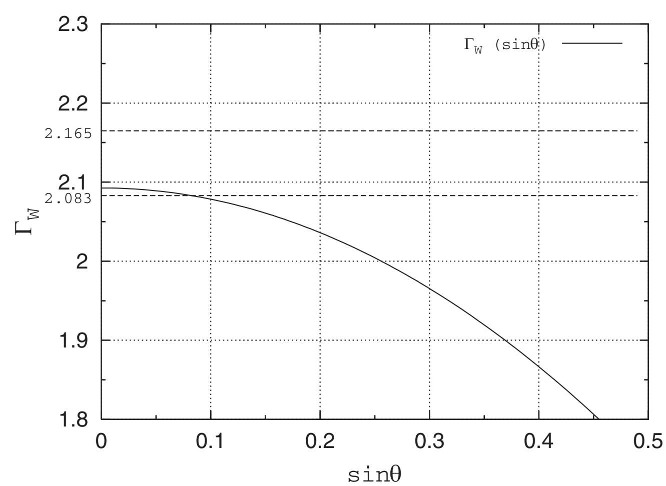

FIG. 1. W width as a function of $\sin\theta$, and the horizontal lines are an upper and a lower limit.

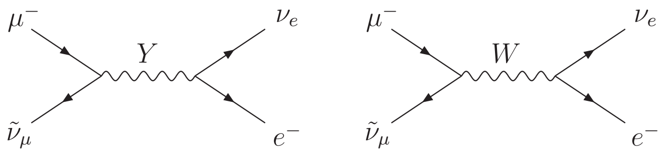

FIG. 2. Feynman diagram for the wrong muon decay $\mu^- \to e^- \nu_e \tilde{\nu}_\mu$.

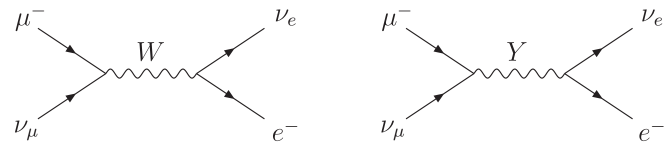

FIG. 3. Feynman diagram for $\mu^- \to e^- \nu_e \tilde{\nu}_\mu$.

ated only by the singly charged bilepton Y. In our case, the second diagram in Fig. 2 gives a main contribution. Taking into account of the famous experimental data [21]

$$R_{\rm muon} \equiv \frac{\Gamma(\mu^- \to e^- \nu_e \tilde{\nu}_\mu)}{\Gamma(\mu^- \to e^- \tilde{\nu}_e \nu_\mu)} < 1.2\% \quad 90\%\text{C.L.} \tag{68}$$

we get the constraint $R_{\rm muon} \simeq \frac{M_W^4}{M_Y^4}$. Therefore, it follows that $M_Y \geq 230$ GeV.

However, the stronger bilepton mass bound has been derived from consideration of an experimental limit on lepton-number violating charged lepton decays [23] of 440 GeV.

In the case of $u \to 0$, analyzing the Z decay width [13], the $Z - Z'$ mixing angle is constrained by $-0.0015 \leq \varphi \leq 0.001$. From atomic parity violation in cesium, bounds for mass of the new exotic Z' and the $Z - Z'$ mixing angles, again in the limit $u \to 0$, are given [13]

$$-0.001\,56 \leq \varphi \leq 0.001\,05, \qquad M_{Z_2} \geq 2.1 \text{ TeV}. \tag{69}$$

These values coincide with the bounds in the usual 331RH3HT [24].

For our purpose we consider the ρ parameter—one of the most important quantities of the SM—having a leading contribution in terms of the T parameter and very useful to get the new physics effects. It is a well-known relation between ρ and T parameter that

$$\rho = 1 + \alpha T. \tag{70}$$

In the usual 3331RH3HT, T gets contribution from the oblique correction and the $Z - Z'$ mixing [18]

$$T_{{\rm RH}N} = T_{ZZ'} + T_{\rm oblique}, \tag{71}$$

where $T_{ZZ'} \simeq \frac{\tan^2\varphi}{\alpha}(\frac{M_{Z_2}^2}{M_{Z_1}^2} - 1)$ is negligible for $M_{Z'}$ less than 1 TeV and $T_{\rm oblique}$ depends on masses of the top quark and the SM Higgs boson. Again at the tree level and the limit (10), from (23) and (48) we get an expression for the ρ parameter in the considering model

$$\rho = \frac{M_W^2}{c_W^2 M_{Z^1}^2} = \frac{v^2}{v^2 - 3u^2} \simeq 1 + \frac{3u^2}{v^2}. \tag{72}$$

Note that formula (72) has only one free parameter u, since v is very close to the VEV in the SM. Neglecting the contribution from the usual 331RH3HT and taking into account the experimental data [21] $\rho = 0.9987 \pm 0.0016$, we get the constraint on u parameter by $\frac{u}{v} \leq 0.01$ which leads to $u \leq 2.46$ GeV. This means that u is much smaller than v, as expected.

It seems that the ρ parameter, at the tree level, in this model, is favorable to be abigger unit and this is similar to the case of the models containing heavy Z' [25].

The interesting new physics compared with other 3-3-1 models is the neutrino physics. Because of lepton-number violating couplings we have the following interesting consequences:

(1) *Processes with* $\Delta L = \pm 2$.—From the charged currents we have the following lepton-number violating $\Delta L = \pm 2$ decays such as

$$\mu^- \to e^- \nu_e \nu_\mu, \qquad \mu^- \to e^- \tilde{\nu}_e \tilde{\nu}_\mu, \qquad (\mu \text{ can be replaced by } \tau), \tag{73}$$

in which both the SM W boson and charged bilepton Y_μ^- are in intermediate states (see Fig. 3). Here the main contribution arises from the first diagram. Note that the wrong muon decay violates only *family* lepton number, i.e. $\Delta L = 0$, but not lepton number at all as in (73). The decay rates are given by

$$R_{\rm rare} \equiv \frac{\Gamma(\mu^- \to e^- \nu_e \nu_\mu)}{\Gamma(\mu^- \to e^- \tilde{\nu}_e \nu_\mu)} = \frac{\Gamma(\mu^- \to e^- \tilde{\nu}_e \tilde{\nu}_\mu)}{\Gamma(\mu^- \to e^- \tilde{\nu}_e \nu_\mu)} \simeq s_\theta^2. \tag{74}$$

Taking $s_\theta = 0.08$, we get $R_{\rm rare} \simeq 6 \times 10^{-3}$. This rate is the same as the wrong muon decay one. Interesting to note that, the family lepton-number violating processes

$$\nu_i \nu_i \to \nu_j \nu_j, \qquad (i \neq j) \tag{75}$$

are mediated not only by the non-Hermitian bilepton X but also by the Hermitian neutral Z^1, Z^2 (see Fig. 4).

The first diagram in Fig. 4 exists also in the 331RH3HT, but the second one does not appear there.

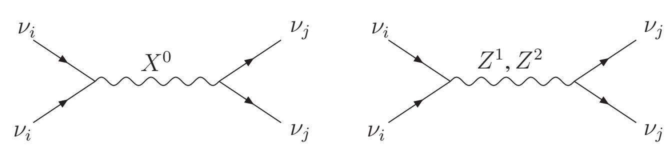

FIG. 4. Feynman diagram for $\nu_i \nu_i \to \nu_j \nu_j (i \neq j = e, \mu, \tau)$.

(2) *Lepton-number violating kaon decays.*—Next, let us consider the lepton-number violating decay [21]

$$K^+ \to \pi^0 + e^+\tilde{\nu}_e < 3 \times 10^{-3} \text{ at } 90\%\text{C.L.} \quad (76)$$

This decay can be explained in the considering model as the subprocess given below

$$\tilde{s} \to \tilde{u} + e^+\tilde{\nu}_e. \quad (77)$$

This process is mediated by the SM W boson and the charged bilepton Y. Amplitude of the considered process is proportional to $\sin\theta$

$$M(\tilde{s} \to \tilde{u} + e^+\tilde{\nu}_e) \simeq \frac{\sin 2\theta}{2M_W^2}\left(1 - \frac{m_W^2}{M_Y^2}\right). \quad (78)$$

Next, let us consider the "normal decay" [21]

$$K^+ \to \pi^0 + e^+\nu_e (4.87 \pm 0.06)\% \quad (79)$$

with amplitude

$$M(\tilde{s} \to \tilde{u} + e^+\nu_e) \simeq \frac{1}{M_W^2}. \quad (80)$$

From (78) and (80) we get

$$R_{\text{kaon}} \equiv \frac{\Gamma(\tilde{s} \to \tilde{u} + e^+\tilde{\nu}_e)}{\Gamma(\tilde{s} \to \tilde{u} + e^+\nu_e)} \simeq \sin^2\theta. \quad (81)$$

In the framework of this model, we derive the following decay modes with rates

$$\begin{aligned} R_{\text{kaon}} &= \frac{\Gamma(K^+ \to \pi^0 + e^+\tilde{\nu}_e)}{\Gamma(K^+ \to \pi^0 + e^+\nu_e)} \\ &\simeq \frac{\Gamma(K^+ \to \pi^0 + \mu^+\tilde{\nu}_\mu)}{\Gamma(K^+ \to \pi^0 + \mu^+\nu_\mu)} \simeq \sin^2\theta \\ &\leq 6 \times 10^{-3}. \end{aligned} \quad (82)$$

Note that the similar lepton-number violating processes exist in the $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$ model (for details, see Ref. [20]).

(3) *Neutrino Majorana masses.*—At the one-loop level neutrinos get Majorana masses via a diagram in which all the charged W, Y and the neutral Z and Z' gauge bosons give contributions. However, the contributions from the above-mentioned fields give the terms diagonal in flavor basis. Fortunately, the Higgs scalar with lepton number two gives interesting terms leading to the neutrino oscillation. This result will be presented elsewhere.

VII. CONCLUSIONS

In this paper we have presented the 3-3-1 model with the minimal scalar sector (only two Higgs triplets). This version belongs to the 3-3-1 model without exotic charges (charges of the exotic quarks are $\frac{2}{3}$ and $-\frac{1}{3}$). The spontaneous symmetry breakdown is achieved with only two Higgs triplets. One of the vacuum expectation values u is a source of lepton-number violations and a reason for the mixing between the charged gauge bosons—the standard model W and the singly charged bilepton gauge bosons as well as between neutral non-Hermitian X^0 and neutral gauge bosons: the photon, the Z, and the new exotic Z'. At the tree level, masses of the charged gauge bosons satisfy the law of Pythagoras $M_Y^2 = M_X^2 + M_W^2$ and in the limit $\omega \gg u, v$, the ρ parameter gets additional contribution dependent only on $\frac{u}{v}$. Thus, this leads to $u \ll v$, and there are three quite different scales for the VEVs of the model: one is very small $u \simeq O(1)$ GeV—a lepton-number violating parameter; the second v is close to the SM one: $v \simeq v_{\text{weak}} = 246$ GeV; and the last is in the range of the new physics scale about $O(1)$ TeV.

In difference with the usual 331RH3HT, in this model the first family of quarks should be distinctive of the two others.

The exact diagonalization of the neutral gauge boson sector is derived. Because of the parameter u, the lepton-number violation happens only in neutrino but not in the charged lepton sector. It is interesting to note that despite the mentioned above mixing, the electromagnetic current remains unchanged. In this model, the lepton-number changing ($\Delta L = \pm 2$) processes exist but only in the neutrino sector. Consequently, neutrinos get Majorana masses at the one-loop level.

It is worth mentioning here the advantage of the considered model: the new mixing angle between the charged gauge bosons θ is connected with one of the VEVs u—the parameter of lepton-number violations. There is no new parameter, but it contains a very simple Higgs sector, hence the significant number of free parameters is reduced.

The model contains new kinds of interactions in the neutrino sector. Hence neutrino physics in this model is very rich and thus deserves further studies.

ACKNOWLEDGMENTS

This work was supported in part by National Council for Natural Sciences of Vietnam.

APPENDIX: MIXING MATRIX OF NEUTRAL GAUGE BOSONS

For the sake of convenience in practical calculations, we use the mixing matrix

$$\begin{pmatrix} W_3 \\ W_8 \\ B \\ W_4 \end{pmatrix} = \mathrm{U} \begin{pmatrix} \mathrm{A} \\ \mathrm{Z}^1 \\ \mathrm{Z}^2 \\ \mathrm{W}_4' \end{pmatrix}, \quad (83)$$

where

$$U = \begin{pmatrix} s_W & c_\varphi c_{\theta'} c_W & s_\varphi c_{\theta'} c_W & s_{\theta'} c_W \\ -\frac{s_W}{\sqrt{3}} & \frac{c_\varphi (s_W^2 - 3c_W^2 s_{\theta'}^2) - s_\varphi \sqrt{(1-4s_{\theta'}^2 c_W^2)(4c_W^2-1)}}{\sqrt{3} c_W c_{\theta'}} & \frac{s_\varphi (s_W^2 - 3c_W^2 s_{\theta'}^2) + c_\varphi \sqrt{(1-4s_{\theta'}^2 c_W^2)(4c_W^2-1)}}{\sqrt{3} c_W c_{\theta'}} & \sqrt{3} s_{\theta'} c_W \\ \frac{\sqrt{4c_W^2-1}}{\sqrt{3}} & -\frac{t_W (c_\varphi \sqrt{4c_W^2-1} + s_\varphi \sqrt{1-4s_{\theta'}^2 c_W^2})}{\sqrt{3} c_{\theta'}} & -\frac{t_W (s_\varphi \sqrt{4c_W^2-1} - c_\varphi \sqrt{1-4s_{\theta'}^2 c_W^2})}{\sqrt{3} c_{\theta'}} & 0 \\ 0 & -t_{\theta'} (c_\varphi \sqrt{1-4s_{\theta'}^2 c_W^2} - s_\varphi \sqrt{4c_W^2-1}) & -t_{\theta'} (s_\varphi \sqrt{1-4s_{\theta'}^2 c_W^2} + c_\varphi \sqrt{4c_W^2-1}) & \sqrt{1-4s_{\theta'}^2 c_W^2} \end{pmatrix}.$$

Here we have denoted $s_{\theta'} = \frac{t_{2\theta}}{c_W \sqrt{1+4t_{2\theta}^2}}$.

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