Time-independent measurements of D^0-$\bar{D}^0$ mixing and relative strong phases using quantum correlations

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(Received 20 July 2005; revised manuscript received 12 January 2006; published 22 February 2006)

Due to the presence of quantum correlations in the $C = -1$ and $C = +1$ $D^0\bar{D}^0$ pairs produced in the reactions $e^+e^- \to D^0\bar{D}^0(n\pi^0)$ and $e^+e^- \to D^0\bar{D}^0\gamma(n\pi^0)$, respectively, the time-integrated $D^0\bar{D}^0$ decay rates are sensitive to interference between amplitudes for indistinguishable final states. The size of this interference is governed by the relevant amplitude ratios and can include contributions from D^0-$\bar{D}^0$ mixing. We present a method for simultaneously measuring the magnitudes and phases of these amplitude ratios and searching for D^0-$\bar{D}^0$ mixing. We make use of fully- and partially-reconstructed $D^0\bar{D}^0$ pairs in both C eigenstates, and we estimate experimental sensitivities based on a plausible charm factory data set. Similar analyses can be applied to coherent $K^0\bar{K}^0$, $B^0\bar{B}^0$, or $B_s^0\bar{B}_s^0$ pairs.

DOI: 10.1103/PhysRevD.73.034024 PACS numbers: 14.40.Lb, 13.20.Fc, 12.15.Mm

I. INTRODUCTION

Studies of the evolution of a K^0 or B^0 into the respective antiparticle, a $\bar{K}^0$ or $\bar{B}^0$ [1], have guided the form and content of the standard model and permitted useful estimates of the masses of the charm [2,3] and top quark [4,5] prior to their direct observation. Neutral flavor oscillation in the D meson system is highly suppressed within the standard model and, thus, with current experimental sensitivity, searches for D^0-$\bar{D}^0$ mixing constitute a search for new physics. In addition, improving constraints on charm mixing is important for elucidating the origin of CP violation in the bottom sector.

The time evolution of the $D^0\bar{D}^0$ system is described by the Schrödinger equation

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\Gamma\right)\begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix}, \tag{1}$$

where the $\mathbf{M}$ and Γ matrices are Hermitian, and CPT invariance requires $M_{11} = M_{22} \equiv M$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$. The off-diagonal elements of these matrices describe the dispersive or long-distance and the absorptive or short-distance contributions to D^0-$\bar{D}^0$ mixing. We define the neutral D meson mass eigenstates to be

$$|D_1\rangle = p|D^0\rangle + q|\bar{D}^0\rangle \tag{2}$$

$$|D_2\rangle = p|D^0\rangle - q|\bar{D}^0\rangle, \tag{3}$$

where $|p|^2 + |q|^2 = 1$, and, following Ref. [6], D_1 is the CP-odd state, and D_2 is the CP-even state, so that $CP|D^0\rangle = -|\bar{D}^0\rangle$. The corresponding eigenvalues of the Hamiltonian are

$$\lambda_{1,2} \equiv m_{1,2} - \frac{i}{2}\Gamma_{1,2} = \left(M - \frac{i}{2}\Gamma\right) \pm \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right), \tag{4}$$

where $m_{1,2}$ and $\Gamma_{1,2}$ are the masses and decay widths, respectively, and

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}. \tag{5}$$

A D^0 can evolve into a $\bar{D}^0$ through on-shell intermediate states, such as K^+K^- with mass $m_{K^+K^-} = m_{D^0}$, or through off-shell intermediate states, such as those that might be present due to new physics. This evolution through the former (latter) states is parametrized by the dimensionless variables $-iy$ (x). We adopt the conventional definitions of these mixing parameters:

$$x \equiv \frac{M_2 - M_1}{\Gamma} \tag{6}$$

$$y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma}, \tag{7}$$

where $\Gamma \equiv (\Gamma_1 + \Gamma_2)/2$. The mixing probability, R_M, is approximately $(x^2 + y^2)/2$ [7]. For hadronic flavored final states, the above time evolution is also governed by the relative magnitudes and phases between Cabibbo-favored (CF) and doubly-Cabibbo-suppressed (DCS) amplitudes, generically denoted by r and δ, respectively.

Standard-model-based predictions for x and y, as well as a variety of non-standard-model expectations, span several orders of magnitude [8]. Several non-standard models predict $|x| > 0.01$. Contributions to x at this level could result from the presence of new particles with masses as high as 100–1000 TeV [9,10]. The standard model short-distance contribution to x is determined by the box diagram in which two virtual quarks and two virtual W bosons are exchanged. Next-to-leading-order calculations show that the short-distance contributions to x and y are expected to be comparable [11]. Long-distance effects are expected to be larger but are difficult to estimate. It is likely that x and y contribute similarly to mixing in the standard model.

Measurement of the phase γ/ϕ_3 of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [12] is challenging and may eventually be limited by experimental constraints on charm mixing [13,14]. Several methods have been proposed using $B^{\mp} \to DK^{\mp}$ decays: the Gronau-London-Wyler (GLW) [15] method, where the D decays to CP eigenstates; the Atwood-Dunietz-Soni (ADS) [16] method, where the D decays to flavor eigenstates; and the Dalitz-plot method [17], where the D decays to a three-body final state. Uncertainties due to D decays contribute to each of these methods. The CLEO-c physics program [18] includes a variety of measurements that will improve the determination of γ/ϕ_3 from the B-factory experiments, *BABAR* and Belle [19,20]. The pertinent components of this program are: improved constraints on charm mixing amplitudes (important for GLW), first measurement of the relative strong phase $\delta_{K\pi}$ between D^0 and $\bar{D}^0$ decay to $K^+\pi^-$ (important for ADS), and studies of charm Dalitz plots tagged by hadronic flavor or CP eigenstates. The total number of charm mesons accumulated at CLEO-c will be much smaller than the samples already accumulated by the B-factories. However, the quantum correlation of the $D^0\bar{D}^0$ system near threshold provides a unique laboratory in which to study charm.

The parameters x and y can be measured in a variety of ways. The most precise constraints are obtained by exploiting the time-dependence of D decays [7]. Previous attempts to measure x and y include: the measurement of the wrong-sign semileptonic branching ratio $D^0 \to K^+\ell^-\bar{\nu}_\ell$ [21–24], which is sensitive to R_M; decay rates to CP eigenstates $D^0 \to K^+K^-$ and $\pi^+\pi^-$ [25–30], which are sensitive to y; the wrong sign $D^0 \to K^+\pi^-$ hadronic branching ratio [31–34], which is sensitive to $x'^2 \equiv (y\sin\delta_{K\pi} + x\cos\delta_{K\pi})^2$ and $y' \equiv y\cos\delta_{K\pi} - x\sin\delta_{K\pi}$; and the decay rate of $D^0 \to K^0_S\pi^+\pi^-$ [35], which determines $\delta_{K^0_S\pi^+\pi^-}$ from a Dalitz-plot analysis and measures x and y.

Time-dependent analyses are not feasible at CLEO-c; however, previous authors have found that the quantum-coherent $D^0\bar{D}^0$ state provides time-integrated sensitivity, through the interference between amplitudes for indistinguishable final states, to y at $\mathcal{O}(1\%)$ and to $\cos\delta_{K\pi}$ at $\mathcal{O}(0.1)$ in 1 fb^{-1} of data at the $\psi(3770)$ [6,18,36]. In this paper, we extend the work of Refs. [6,36,37] and develop a method for simultaneously measuring x, y, r, and δ. Unlike the proposed measurements of Ref. [6], we do not rely on external estimates of the relevant D^0 branching fractions. Our method is a modified version of the double tagging technique originally developed to measure D branching fractions at the $\psi(3770)$ [38–40]. We make use of rates for exclusive $D^0\bar{D}^0$ combinations, where both D final states are specified (known as double tags or DT), as well as inclusive rates, where either the D^0 or $\bar{D}^0$ is identified and the other neutral D decays generically (known as single tags or ST). Although we estimate that CLEO-c will not have sufficient sensitivity to observe standard model charm mixing (see Sec. IV), it should be able to achieve a precision comparable to current experimental results. The analysis presented in this paper can also be applied to coherent $K^0\bar{K}^0$, $B^0\bar{B}^0$, and $B^0_s\bar{B}^0_s$ systems, although with some additional complications.

II. FORMALISM

As shown in Ref. [41], a $D^0\bar{D}^0$ pair produced through a virtual photon in the reaction $e^+e^- \to D^0\bar{D}^0 + m\gamma + n\pi^0$ is in a $C = (-1)^{m+1}$ state. Thus, at the $\psi(3770)$, where no additional fragmentation particles are produced, there is only $C = -1$, while at higher energies above D^*D threshold, we can access both C eigenstates. The DT rates for final states j and k are given by [6,36,37]

$$\Gamma^{C=-1}(j,k) = Q_M|A^{(-)}(j,k)|^2 + R_M|B^{(-)}(j,k)|^2 \quad (8)$$

$$\Gamma^{C=+1}(j,k) = Q'_M|A^{(+)}(j,k)|^2 + R'_M|B^{(+)}(j,k)|^2 + C^{(+)}(j,k),$$

where

$$A^{(\pm)}(j,k) \equiv \langle j|D^0\rangle\langle k|\bar{D}^0\rangle \pm \langle j|\bar{D}^0\rangle\langle k|D^0\rangle \quad (9)$$

$$B^{(\pm)}(j,k) \equiv \frac{p}{q}\langle j|D^0\rangle\langle k|D^0\rangle \pm \frac{q}{p}\langle j|\bar{D}^0\rangle\langle k|\bar{D}^0\rangle \quad (10)$$

$$C^{(+)}(j,k) \equiv 2\Re\left\{A^{(+)*}(j,k)B^{(+)}(j,k)\left[\frac{y}{(1-y^2)^2} + \frac{ix}{(1+x^2)^2}\right]\right\} \quad (11)$$

$$Q_M \equiv \frac{1}{2}\left[\frac{1}{1-y^2} + \frac{1}{1+x^2}\right] \approx 1 - \frac{x^2-y^2}{2} \quad (12)$$

$$R_M \equiv \frac{1}{2}\left[\frac{1}{1-y^2} - \frac{1}{1+x^2}\right] \approx \frac{x^2+y^2}{2} \quad (13)$$

$$Q'_M \equiv \frac{1}{2}\left[\frac{1+y^2}{(1-y^2)^2} + \frac{1-x^2}{(1+x^2)^2}\right] \approx Q_M - x^2 + y^2 \quad (14)$$

$$R'_M \equiv \frac{1}{2}\left[\frac{1+y^2}{(1-y^2)^2} - \frac{1-x^2}{(1+x^2)^2}\right] \approx 3R_M. \quad (15)$$

We parametrize the deviation of q/p from unity by two small CP-violating parameters (magnitude and phase) and find that they only appear in products with x and y; they can only modulate the strength of the mixing signal. Thus, to first order in small parameters, $\Gamma^{C=\pm1}(j,k)$ is insensitive to CP violation in mixing and in the interference between mixing and decay. Therefore, below, we assume $q/p = 1$

and also that CP is conserved in the decay amplitudes (i.e., $|\langle j|D^0\rangle| = |\langle \bar{j}|\bar{D}^0\rangle|$).

As in Refs. [6,36,37], we consider the following categories of D^0 and $\bar{D}^0$ final states:

(i) f or $\bar{f}$: hadronic states that can be reached from either D^0 or $\bar{D}^0$ decay but that are not CP eigenstates. An example is $K^-\pi^+$, which is produced via CF D^0 transitions or DCS $\bar{D}^0$ transitions. We include in this category Cabibbo-suppressed (CS) transitions as well as self-conjugate final states of mixed CP, such as nonresonant $K_S^0\pi^+\pi^-$.

(ii) ℓ^+ or ℓ^-: semileptonic or purely leptonic final states, which, in the absence of mixing, tag unambiguously the flavor of the parent D.

(iii) S_+ or S_-: CP-even and CP-odd eigenstates, respectively.

All D^0 decay modes can be treated uniformly if we enumerate charge-conjugate final states separately, indexed by j and $\bar{j}$. For instance, $K^-\pi^+$ and $K^-\ell^+\nu_\ell$ are labeled by j and $K^+\pi^-$ and $K^+\ell^-\bar{\nu}_\ell$ by $\bar{j}$. CP eigenstates appear in both lists, so there is a $\bar{j}$ for each j. We define the mode-dependent amplitude ratio $\langle j|\bar{D}^0\rangle/\langle j|D^0\rangle \equiv -r_je^{-i\delta_j} = r_je^{-i(\delta_j+\pi)}$, with

$$r_j \equiv \left|\frac{\langle j|\bar{D}^0\rangle}{\langle j|D^0\rangle}\right| \tag{16}$$

$$-\delta_j \equiv \arg\left(\frac{\langle j|\bar{D}^0\rangle}{\langle j|D^0\rangle}\right) = \delta_{\mathrm{strong}} + \delta_{\mathrm{weak}} + \pi, \tag{17}$$

where the phase of π results from our CP convention. Since δ_{weak} in the charm sector is trivial (0 or π), δ_j corresponds to either $-\delta_{\mathrm{strong}}$ (if j is CF) or $\pi - \delta_{\mathrm{strong}}$ (if j is CS). Furthermore, if CP is conserved, then $\langle j|\bar{D}^0\rangle/\langle j|D^0\rangle = \langle \bar{j}|D^0\rangle/\langle \bar{j}|\bar{D}^0\rangle$. To resolve the ambiguity of whether to identify any given final state as j or $\bar{j}$, we choose $0 \le r_j < 1$.

Thus, if we denote decay amplitudes by $A_j \equiv \langle j|D^0\rangle$ and $A_{\bar{j}} \equiv \langle \bar{j}|D^0\rangle$ and use a normalization in which $A_j^2 = \Gamma(D^0 \to j)$, then the total rate for an isolated D^0 or $\bar{D}^0$ is

$$\Gamma_{D^0} = \Gamma_{\bar{D}^0} = \sum_j (A_j^2 + A_{\bar{j}}^2) = \sum_j A_j^2(1 + r_j^2). \tag{18}$$

In our phase convention, CP conjugate amplitudes are given by

$$A_f \equiv \langle f|D^0\rangle = -\langle \bar{f}|\bar{D}^0\rangle \tag{19}$$

$$A_\ell \equiv \langle \ell^+|D^0\rangle = -\langle \ell^-|\bar{D}^0\rangle \tag{20}$$

$$A_{S_\pm} \equiv \langle S_\pm|D^0\rangle = \mp\langle S_\pm|\bar{D}^0\rangle. \tag{21}$$

Table I lists the values of r_j and δ_j for each final state category. The $\mathcal{B}_j$ denote branching fractions for an isolated D^0 or $\bar{D}^0$. Note that since CP eigenstates have $r_{S_\pm} = 1$, $A_{S_\pm}^2$ corresponds to only half the $D^0 \to S_\pm$ rate, which is consistent with Eq. (18). One outcome of neglecting CP violation is that y can be expressed in terms of branching fractions and $z_j \equiv 2\cos\delta_j$:

TABLE I. Values of the amplitude ratio magnitudes r_j and phases δ_j, as well as squares of the decay amplitudes, for each final state category.

j	r_j	δ_j	A_j^2/Γ_{D^0}
f	r_f	δ_f	$\mathcal{B}_f$
ℓ^+	0	—	$\mathcal{B}_\ell$
S_+	1	π	$\mathcal{B}_{S_+}/2$
S_-	1	0	$\mathcal{B}_{S_-}/2$

$$y = -\sum_j A_j^2 r_j z_j = \sum_{S_+}\mathcal{B}_{S_+} - \sum_{S_-}\mathcal{B}_{S_-} - \sum_f \mathcal{B}_f r_f z_f. \tag{22}$$

Using the above definitions, $A^{(\pm)}$, $B^{(\pm)}$, and $C^{(+)}$ become

$$|A^{(\pm)}(j,\bar{k})|^2 = |B^{(\pm)}(j,k)|^2 = A_j^2A_k^2[1 + r_j^2r_k^2 \pm r_jr_kv_{jk}^-] \tag{23}$$

$$|A^{(\pm)}(j,k)|^2 = |B^{(\pm)}(j,\bar{k})|^2 = A_j^2A_k^2[r_j^2 + r_k^2 \pm r_jr_kv_{jk}^+] \tag{24}$$

$$C^{(+)}(j,\bar{k}) = A_j^2A_k^2c_{jk}^+ \tag{25}$$

$$C^{(+)}(j,k) = A_j^2A_k^2c_{jk}^-, \tag{26}$$

where $w_j \equiv 2\sin\delta_j$, $v_{jk}^\pm \equiv (z_jz_k \pm w_jw_k)/2$, and

$$c_{jk}^\pm \equiv \frac{y}{(1-y^2)^2}[(1+r_j^2)r_kz_k + (1+r_k^2)r_jz_j] \pm \frac{x}{(1+x^2)^2} \times [(1-r_j^2)r_kw_k + (1-r_k^2)r_jw_j]. \tag{27}$$

Since x and y have both been constrained to be less than $\mathcal{O}(1\%)$ [42], we keep terms only to leading order in x and y in the remainder of this paper.

In Tables II and III, we give expressions for the DT rates for $D^0\bar{D}^0$ pairs with $C = -1$ and $C = +1$, evaluated using the above formulae and the definitions $y_j' \equiv (z_jy - w_jx)/2$ and $\tilde{y}_j \equiv (z_jy + w_jx)/2$. If both D^0 and $\bar{D}^0$ decay to the same final state, we divide the $|A^{(\pm)}|^2$ and $C^{(+)}$ terms by 2. In Table III, the entries with vanishing rate would be nonzero only the presence of *both* mixing and CP violation [37]. If the $D^0\bar{D}^0$ decay were uncorrelated, these DT rates would be $\Gamma(j,\bar{k}) = \Gamma(\bar{j},k) = \Gamma_{D^0}^2\mathcal{B}_j\mathcal{B}_k(1 + r_j^2r_k^2)$ and $\Gamma(j,k) = \Gamma(\bar{j},\bar{k}) = \Gamma_{D^0}^2\mathcal{B}_j\mathcal{B}_k(r_j^2 + r_k^2)$, with a factor of $1/2$ if $j = k$ in the latter expression.

The ST rate for final state j is obtained by summing all DT rates containing j and is found to be the same for $C = -1$ and $C = +1$:

TABLE II. DT rates normalized by $\Gamma^2_{D^0}$ (*i.e.*, $\Gamma(j,k)/\Gamma^2_{D^0}$), for modes containing f or $\bar{f}$, to leading order in x and y.

	f	$\bar{f}$
	$C = -1$	
f	$\mathcal{B}_f^2 R_M[1 + r_f^2(2 - z_f^2) + r_f^4]$	
$\bar{f}$	$\mathcal{B}_f^2[1 + r_f^2(2 - z_f^2) + r_f^4]$	$\mathcal{B}_f^2 R_M[1 + r_f^2(2 - z_f^2) + r_f^4]$
f'	$\mathcal{B}_f\mathcal{B}_{f'}(r_f^2 + r_{f'}^2 - r_f r_{f'} v^+_{ff'})$	$\mathcal{B}_f\mathcal{B}_{f'}(1 + r_f^2 r_{f'}^2 - r_f r_{f'} v^-_{ff'})$
$\bar{f}'$	$\mathcal{B}_f\mathcal{B}_{f'}(1 + r_f^2 r_{f'}^2 - r_f r_{f'} v^-_{ff'})$	$\mathcal{B}_f\mathcal{B}_{f'}(r_f^2 + r_{f'}^2 - r_f r_{f'} v^+_{ff'})$
ℓ^+	$\mathcal{B}_f\mathcal{B}_\ell r_f^2$	$\mathcal{B}_f\mathcal{B}_\ell$
ℓ^-	$\mathcal{B}_f\mathcal{B}_\ell$	$\mathcal{B}_f\mathcal{B}_\ell r_f^2$
S_+	$\mathcal{B}_f\mathcal{B}_{S_+}[1 + r_f(r_f + z_f)]$	$\mathcal{B}_f\mathcal{B}_{S_+}[1 + r_f(r_f + z_f)]$
S_-	$\mathcal{B}_f\mathcal{B}_{S_-}[1 + r_f(r_f - z_f)]$	$\mathcal{B}_f\mathcal{B}_{S_-}[1 + r_f(r_f - z_f)]$
	$C = +1$	
f	$2\mathcal{B}_f^2 r_f(r_f + y'_f + r_f^2\tilde{y}_f)$	
$\bar{f}$	$\mathcal{B}_f^2[1 - r_f^2(2 - z_f^2) + r_f^4 + 4r_f(\tilde{y}_f + r_f^2 y'_f)]$	$2\mathcal{B}_f^2 r_f(r_f + y'_f + r_f^2\tilde{y}_f)$
f'	$\mathcal{B}_f\mathcal{B}_{f'}(r_f^2 + r_{f'}^2 + r_f r_{f'} v^+_{ff'} + 2c^-_{ff'})$	$\mathcal{B}_f\mathcal{B}_{f'}(1 + r_f^2 r_{f'}^2 + r_f r_{f'} v^-_{ff'} + 2c^+_{ff'})$
$\bar{f}'$	$\mathcal{B}_f\mathcal{B}_{f'}(1 + r_f^2 r_{f'}^2 + r_f r_{f'} v^-_{ff'} + 2c^+_{ff'})$	$\mathcal{B}_f\mathcal{B}_{f'}(r_f^2 + r_{f'}^2 + r_f r_{f'} v^+_{ff'} + 2c^-_{ff'})$
ℓ^+	$\mathcal{B}_f\mathcal{B}_\ell(r_f^2 + 2r_f y'_f)$	$\mathcal{B}_f\mathcal{B}_\ell(1 + 2r_f\tilde{y}_f)$
ℓ^-	$\mathcal{B}_f\mathcal{B}_\ell(1 + 2r_f\tilde{y}_f)$	$\mathcal{B}_f\mathcal{B}_\ell(r_f^2 + 2r_f y'_f)$
S_+	$\mathcal{B}_f\mathcal{B}_{S_+}[1 + r_f(r_f - z_f)](1 - 2y)$	$\mathcal{B}_f\mathcal{B}_{S_+}[1 + r_f(r_f - z_f)](1 - 2y)$
S_-	$\mathcal{B}_f\mathcal{B}_{S_-}[1 + r_f(r_f + z_f)](1 + 2y)$	$\mathcal{B}_f\mathcal{B}_{S_-}[1 + r_f(r_f + z_f)](1 + 2y)$

TABLE III. DT rates normalized by $\Gamma^2_{D^0}$ (i.e., $\Gamma(j,k)/\Gamma^2_{D^0}$), for semileptonic modes and *CP* eigenstates, to leading order in x and y.

	ℓ^+	ℓ^-	S_+	S_-
		$C = -1$		
ℓ^+	$\mathcal{B}_\ell^2 R_M$			
ℓ^-	$\mathcal{B}_\ell^2$	$\mathcal{B}_\ell^2 R_M$		
S_+	$\mathcal{B}_\ell\mathcal{B}_{S_+}$	$\mathcal{B}_\ell\mathcal{B}_{S_+}$	0	
S_-	$\mathcal{B}_\ell\mathcal{B}_{S_-}$	$\mathcal{B}_\ell\mathcal{B}_{S_-}$	$4\mathcal{B}_{S_+}\mathcal{B}_{S_-}$	0
		$C = +1$		
ℓ^+	$3\mathcal{B}_\ell^2 R_M$			
ℓ^-	$\mathcal{B}_\ell^2$	$3\mathcal{B}_\ell^2 R_M$		
S_+	$\mathcal{B}_\ell\mathcal{B}_{S_+}(1 - 2y)$	$\mathcal{B}_\ell\mathcal{B}_{S_+}(1 - 2y)$	$2\mathcal{B}_{S_+}^2(1 - 2y)$	
S_-	$\mathcal{B}_\ell\mathcal{B}_{S_-}(1 + 2y)$	$\mathcal{B}_\ell\mathcal{B}_{S_-}(1 + 2y)$	0	$2\mathcal{B}_{S_-}^2(1 + 2y)$
S'_+	$\mathcal{B}_\ell\mathcal{B}_{S'_+}(1 - 2y)$	$\mathcal{B}_\ell\mathcal{B}_{S'_+}(1 - 2y)$	$4\mathcal{B}_{S_+}\mathcal{B}_{S'_+}(1 - 2y)$	0
S'_-	$\mathcal{B}_\ell\mathcal{B}_{S'_-}(1 + 2y)$	$\mathcal{B}_\ell\mathcal{B}_{S'_-}(1 + 2y)$	0	$4\mathcal{B}_{S_-}\mathcal{B}_{S'_-}(1 + 2y)$

$$\Gamma(j,X) = \sum_k[\Gamma(j,k) + \Gamma(j,\bar{k})] \approx \Gamma^2_{D^0}A_j^2[1 + r_j^2 + r_j z_j y], \tag{28}$$

where we have used $\sum_j A_j^2(1 + r_j^2)/\Gamma_{D^0} = 1$. For self-conjugate final states, the single tag rate is the sum $\Gamma(j,X) + \Gamma(\bar{j},X)$. In uncorrelated $D^0\bar{D}^0$ decay, these ST rates would be $\Gamma(j,X) = \Gamma^2_{D^0}\mathcal{B}_j(1 + r_j^2)$. Table IV shows these ST rates evaluated for the three categories of final states, using the values of r_j and δ_j given in Table I.

TABLE IV. ST rates normalized by $\Gamma^2_{D^0}$ (i.e., $\Gamma(j,X)/\Gamma^2_{D^0}$), to leading order in x and y.

j	$C = +1$ and $C = -1$
f	$\mathcal{B}_f[1 + r_f^2 + r_f z_f y]$
ℓ	$\mathcal{B}_\ell$
$S_\pm$	$2\mathcal{B}_{S_\pm}(1 \mp y)$

TABLE V. Selected ratios of DT rates and double ratios of ST rates to DT rates, evaluated to leading order in r_f^2, x, and y. Rates are represented by the notation $\Gamma_{jk} \equiv \Gamma(j,k)$ and $\Gamma_j \equiv \Gamma(j,X)$.

	$C=-1$	$C=+1$
$(1/4)\cdot(\Gamma_{\ell S_+}\Gamma_{S_-}/\Gamma_{\ell S_-}\Gamma_{S_+} - \Gamma_{\ell S_-}\Gamma_{S_+}/\Gamma_{\ell S_+}\Gamma_{S_-})$	y	$-y$
$(\Gamma_{f\ell^-}/4\Gamma_f)\cdot(\Gamma_{S_-}/\Gamma_{\ell S_-} - \Gamma_{S_+}/\Gamma_{\ell S_+})$	y	$-y$
$(\Gamma_{f\bar{f}}/4\Gamma_f)\cdot(\Gamma_{S_-}/\Gamma_{\bar{f}S_-} - \Gamma_{S_+}/\Gamma_{\bar{f}S_+})$	$y + r_f z_f$	$-(y + r_f z_f)$
$(\Gamma_f\Gamma_{S_+S_-}/4)\cdot(1/\Gamma_{fS_-}\Gamma_{S_+} - 1/\Gamma_{fS_+}\Gamma_{S_-})$	$y + r_f z_f$	0
$(\Gamma_{\bar{f}}/2)\cdot(\Gamma_{S_+S_+}/\Gamma_{\bar{f}S_+}\Gamma_{S_+} - \Gamma_{S_-S_-}/\Gamma_{\bar{f}S_-}\Gamma_{S_-})$	0	$y + r_f z_f$
$\Gamma_{ff}/\Gamma_{f\bar{f}}$	R_M	$2r_f^2 + r_f(z_f y - w_f x)$
$\Gamma_{f\ell^+}/\Gamma_{f\ell^-}$	r_f^2	$r_f^2 + r_f(z_f y - w_f x)$
$\Gamma_{\ell^\pm\ell^\pm}/\Gamma_{\ell^+\ell^-}$	R_M	$3R_M$

The total $D^0\bar{D}^0$ rate is obtained by summing either DT or ST rates:

$$\Gamma_{D^0\bar{D}^0} = \sum_{j,k\geq j}[\Gamma(j,k) + \Gamma(\bar{j},\bar{k})] + \sum_{j,k}\Gamma(j,\bar{k}) = \frac{1}{2}\sum_j[\Gamma(j,X) + \Gamma(\bar{j},X)] \approx \Gamma_{D^0}^2, \tag{29}$$

Thus, for both C eigenvalues, the total $D^0\bar{D}^0$ rate is not modified by the quantum correlation.

For the $C=-1$ configuration, with only one mode of type f, the ST and DT rates depend on only four independent parameters: r_f, z_f, y, and R_M. For $C=+1$, there is one additional parameter, $w_f x$, which appears via y_f' and $\tilde{y}_f$. There is, in principle, sensitivity to x from the $C=+1$ configuration, although no information can be gained if δ_f is 0 or π. In addition, estimates of R_M and y can be combined to obtain x^2. However, in all of these cases, it is impossible, without knowing the sign of δ_f, to determine the sign of x. The mixing and amplitude ratio parameters can be isolated by forming ratios of DT rates and double ratios of ST rates to DT rates. Table V lists a selection of these ratios and functions thereof, evaluated to leading order in r_j^2, x, and y.

III. EFFECT ON BRANCHING FRACTION MEASUREMENTS

If quantum correlations are ignored when using coherent $D^0\bar{D}^0$ pairs to measure D^0 branching fractions, then biases may result. For instance, if a measured branching fraction, denoted by $\tilde{\mathcal{B}}$, is obtained by dividing reconstructed ST yields by the total number of $D^0\bar{D}^0$ pairs ($\Gamma_{D^0\bar{D}^0}$), then $\tilde{\mathcal{B}}$ differs from the desired branching fraction, $\mathcal{B}$, by the factors given in Table IV.

Using a double tag technique pioneered by MARK III [38,39], CLEO-c has recently measured $\mathcal{B}(D^0\to K^-\pi^+)$, $\mathcal{B}(D^0\to K^-\pi^+\pi^0)$, and $\mathcal{B}(D^0\to K^-\pi^+\pi^+\pi^-)$ in a self-normalizing way (*i.e.*, without knowledge of the luminosity or $D^0\bar{D}^0$ production cross section) [40], using $C=-1$ $D^0\bar{D}^0$ pairs from the $\psi(3770)$. Measured ST and DT yields and efficiencies are combined in a least-squares fit [43] to extract the branching fractions and $\Gamma_{D^0\bar{D}^0}$. Quantum correlations were not explicitly accounted for in this analysis, but their effects were included in the systematic uncertainties. Only flavored final states were considered, and DCS contributions to the ST yields were removed, so the observed branching fractions are:

$$\tilde{\mathcal{B}}_f^{C=-1} \approx \frac{\Gamma^{C=-1}(f,\bar{f}')}{\Gamma^{C=-1}(\bar{f}',X)} \approx \mathcal{B}_f\left[1 - r_{f'}z_{f'}\left(y + \frac{r_f z_f}{2}\right) + \frac{r_f r_{f'} w_f w_{f'}}{2}\right] \tag{30}$$

$$\approx \mathcal{B}_f[1 + 2r_f^2 - r_f z_f(y + r_f z_f)] \quad \text{for } f' = f. \tag{31}$$

Similarly, the relationship between the observed $\tilde{\Gamma}_{D^0\bar{D}^0}^{C=-1}$, which is used to obtain the $D^0\bar{D}^0$ cross section, and the desired $\Gamma_{D^0\bar{D}^0}^{C=-1}$ is

$$\tilde{\Gamma}_{D^0\bar{D}^0}^{C=-1} \approx \frac{\Gamma^{C=-1}(f,X)\Gamma^{C=-1}(\bar{f}',X)}{\Gamma^{C=-1}(f,\bar{f}')} \approx \Gamma_{D^0\bar{D}^0}^{C=-1}[1 + r_f r_{f'} v_{ff'}^- + y(r_f z_f + r_{f'} z_{f'})] \tag{32}$$

$$\approx \Gamma_{D^0\bar{D}^0}^{C=-1}[1 - r_f(2 - z_f^2) + 2r_f z_f y] \quad \text{for } f' = f. \tag{33}$$

The differences between the observed and the desired quantities are expected to be $\mathcal{O}(1\%)$.

In principle, any D^0 branching fraction measured in a coherent $D^0\bar{D}^0$ system is subject to such considerations. Analogous caveats pertain to B^0 and B_s^0 branching fractions measured with coherent $B^0\bar{B}^0$ and $B_s^0\bar{B}_s^0$ pairs. However, for $K^0\bar{K}^0$ decays, such as those studied by KLOE, the situation is generally simpler. There, the desired branching fractions [44,45] are for K_S^0 and K_L^0, rather than for K^0 and $\bar{K}^0$, so corrections for the lifetime asymmetry ($y \approx 0.997$) need not be applied.

IV. EXPERIMENTAL SENSITIVITY

The least-squares fit discussed in the previous section can be extended to extract the parameters y, x^2, r_f, z_f, and $w_f x$ ($C = +1$ only), in addition to $\mathcal{B}_j$ and $\Gamma_{D^0\bar{D}^0}$. Efficiency-corrected ST and DT yields are identified with the functions given in Tables II, III, and IV. We estimate uncertainties on the fit parameters based on approximately 3×10^6 $D^0\bar{D}^0$ pairs, using efficiencies and background levels similar to those found at CLEO-c. In the rate ratios in Table V, uncertainties that are correlated by final state, such as tracking efficiency uncertainties, cancel exactly. Therefore, the uncertainties on the mixing and amplitude ratio parameters stem primarily from statistics and from uncorrelated systematic uncertainties.

The decay modes considered are listed in Table VI. There exist, in principle, different r_f, z_f, and w_f parameters for each mode f included in the fit. Therefore, for simplicity, we include only one such mode in the analysis: $D \to K^\pm\pi^\mp$. In practice, adding more hadronic flavored modes does not noticeably improve the precision of y because the limiting statistical uncertainty comes from DT yields involving $S_\pm$. The branching fraction determinations, however, would benefit from having additional modes in the fit.

We omit ST yields for modes with a neutrino or a K_L^0, which typically escapes detection, because they are difficult to measure. In principle, one could reconstruct the remainder of the event inclusively to infer the presence of the missing particle from energy and momentum conservation [46]. The efficiency of this method depends on the hermeticity of the detector.

TABLE VI. Final states included in the fits, along with assumed branching fractions, signal efficiencies, and expected single tag yields for $\Gamma_{D^0\bar{D}^0} = 3 \times 10^6$. ST yields in parentheses are not included in any of the fits.

Final State	Type	$\mathcal{B}$ (%)	ϵ (%)	ST Yield (10^3)
$K^-\pi^+$	f	3.91	66	78
$K^+\pi^-$	$\bar{f}$	3.91	66	78
$K^-e^+\nu_e$	ℓ^+	3.5	62	(65)
$K^+e^-\bar{\nu}_e$	ℓ^-	3.5	62	(65)
K^+K^-	S_+	0.389	59	14
$\pi^+\pi^-$	S_+	0.138	73	6.0
$K_S^0\pi^0\pi^0$	S_+	0.89	15	8.0
$K_L^0\pi^0$	S_+	1.15	62	(43)
$(K_S^0\pi^+\pi^-)_{CP+}$	S_+	1.0	38	27
$(K_L^0\pi^+\pi^-)_{CP+}$	S_+	1.0	76	(46)
$K_S^0\phi$	S_-	0.47	7.7	2.2
$K_S^0\omega$	S_-	1.15	14	9.7
$K_S^0\pi^0$	S_-	1.15	31	21
$K_L^0\pi^0\pi^0$	S_-	0.89	30	(16)
$(K_S^0\pi^+\pi^-)_{CP-}$	S_-	1.0	38	23
$(K_L^0\pi^+\pi^-)_{CP-}$	S_-	1.0	76	(46)

For DT modes with one missing neutrino or K_L^0, this method is more straightforward to implement because the D^0 and $\bar{D}^0$ are both reconstructed exclusively. Therefore, we do include these yields in the fit. In the case of a missing K_L^0, one must veto K_S^0 decays, which would have the opposite CP eigenvalue.

For DT modes with two undetected particles, one can constrain the event kinematically, up to a twofold ambiguity [47]. Background events tend to fail these constraints, so the signal can be isolated effectively. We assume this method is used to measure $\ell^+\ell^-$, $\ell^\pm\ell^\pm$, $K_L^0\ell^\pm$, and $K_L^0K_L^0$ DT yields.

The input ST yields are listed in Table VI, and the input DT yields are derived from products of the ST branching fractions and efficiencies. In most cases, the background is negligible ($\sigma_N^{\text{stat}} \approx \sqrt{N}$). We also include a conservative 1% uncorrelated sytematic uncertainty on each yield measurement. For modes that only have contributions from R_M, which we assume to be zero, we use yield measurements of $0 \pm 1 \pm 1$. Yields for forbidden modes ($S_\pm S_\pm$ for $C = -1$, S_+S_- for $C = +1$) are not included. The fit accounts for the statistical correlations among ST and DT yields.

We perform fits for both C eigenvalues using equal numbers of $D^0\bar{D}^0$ pairs. To improve the precision of these fits, we include constraints to external measurements of $r_{K\pi}^2$ [33,34,42] and branching fractions [42]. Because the resultant precision on $r_{K\pi}^2$ is dominated by the world average (0.00374 ± 0.00018), we do not report fit results for this parameter. The first two columns of Table VII show the expected uncertainties on the mixing and strong phase parameters from these fits. The dramatic difference between the y uncertainties for the two cases stems from the negative correlation between $\mathcal{B}_{S_+}$ and $\mathcal{B}_{S_-}$ introduced by the presence of S_+S_- yields for $C = -1$; these branching fractions are positively correlated for $C = +1$. From the second line of Table V, it can be seen that a negative correlation increases the uncertainty on y. The difference between the x^2 uncertainties for the two cases reflects the factor of 3 accompanying R_M in $\Gamma^{C=+1}(\ell^\pm, \ell^\pm)$.

In reality, $C = +1$ $D^0\bar{D}^0$ pairs (from $e^+e^- \to D^0\bar{D}^0\gamma(n\pi^0)$) are produced above D^*D threshold and are more difficult to identify than $C = -1$ $D^0\bar{D}^0$ pairs produced at the $\psi(3770)$. In particular, one must distinguish $D^0\bar{D}^0\gamma(n\pi^0)$, which gives $C = +1$, from $D^0\bar{D}^0(n\pi^0)$ and $D^0\bar{D}^0\gamma\gamma$, which give $C = -1$. While it is possible to do so for DT modes, where the entire event is reconstructed, a large uncertainty is incurred for ST modes. Therefore, we perform a third fit that combines yields from both C configurations, but with $C = +1$ ST yields omitted. Because of the smaller cross section and efficiencies for $D^0\bar{D}^0\gamma$ (only half the soft photons can be identified), $C = +1$ DT yields above D^*D threshold are an order of magnitude smaller than $C = -1$ yields from an equal luminosity at the $\psi(3770)$. Results for this fit are also shown in Table VII.

TABLE VII. Estimated uncertainties (statistical and systematic, respectively) for different C configurations, with $r_{K\pi}$ and branching fractions constrained to the world averages. We include $C = +1$ ST yields in the second column, but not the third.

Parameter	Value	$\Gamma^{C=-1}_{D^0\bar{D}^0} = 3 \times 10^6$	$\Gamma^{C=+1}_{D^0\bar{D}^0} = 3 \times 10^6$	$\Gamma^{C=-1}_{D^0\bar{D}^0} = 10 \cdot \Gamma^{C=+1}_{D^0\bar{D}^0} = 3 \times 10^6$
y	0	$\pm 0.015 \pm 0.008$	$\pm 0.007 \pm 0.003$	$\pm 0.012 \pm 0.005$
x^2	0	$\pm 0.0006 \pm 0.0006$	$\pm 0.0003 \pm 0.0003$	$\pm 0.0006 \pm 0.0006$
$\cos\delta_{K\pi}$	1	$\pm 0.15 \pm 0.04$	$\pm 0.13 \pm 0.05$	$\pm 0.13 \pm 0.03$
$x\sin\delta_{K\pi}$	0	—	$\pm 0.010 \pm 0.003$	$\pm 0.024 \pm 0.005$

One important source of systematic uncertainty not included above is the purity of the initial $C = \pm 1$ state. The sample composition can be determined from the ratios

$$\frac{\Gamma(S_+, S'_+)\Gamma(S_-, S'_-)}{\Gamma(S_+, S_-)\Gamma(S'_+, S'_-)} = \frac{\Gamma(S_+, S'_+)\Gamma(S_-, S'_-)}{\Gamma(S_+, S'_-)\Gamma(S'_+, S_-)} = \frac{4\Gamma(S_+, S_+)\Gamma(S_-, S_-)}{\Gamma^2(S_+, S_-)} = \left(\frac{\Gamma^{C=+1}_{D^0\bar{D}^0}}{\Gamma^{C=-1}_{D^0\bar{D}^0}}\right)^2, \tag{34}$$

assuming CP is conserved. Thus, if we include the forbidden S_+S_+, S_-S_-, and S_+S_- DT yields that were previously omitted, then we can construct every other ST or DT yield as a sum of $C = -1$ and $C = +1$ contributions, with their relative sizes constrained by Eq. (34). In this way, the systematic uncertainty is absorbed into the statistical uncertainties, and the C content of the sample is self-calibrating. If the fit is performed on pure samples, with either $\Gamma(S_\pm, S_\pm)$ or $\Gamma(S_+, S_-)$ measured to be consistent with zero, then the uncertainties given in Table VII suffer no degradation. On the other hand, if $\Gamma^{C=-1}_{D^0\bar{D}^0} = \Gamma^{C=+1}_{D^0\bar{D}^0}$, then there is an unfortunate cancellation of terms containing z_f or y, and these parameters cannot be determined at all.

To demonstrate the importance of semileptonic modes in this analysis, we consider a variation on the above fits. If no semileptonic yields are measured, then, for $C = -1$ there is only one independent combination of r_f, z_f, and y: $[r_f z_f + y(1 + r_f^2)]/(1 + r_f^2 + r_f z_f y) \approx r_f z_f + y$. The other free parameters would be R_M, $\Gamma_{D^0\bar{D}^0}(1 - y^2)$, $\mathcal{B}_f(1 + r_f^2 + r_f z_f y)/(1 - y^2)$, $\mathcal{B}_{S_+}/(1 + y)$, and $\mathcal{B}_{S_-}/(1 - y)$. Note that we cannot probe the branching fractions directly in such a fit. On the other hand, if only the $\ell^\pm K^0_L$ DT modes are omitted, then y and z_f can be separated, but their uncertainties would be approximately 50% larger than those in Table VII, where the $\ell^\pm K^0_L$ DT modes are included.

The $\ell^\pm\ell^\pm$ and $\ell^+\ell^-$ DT modes improve the uncertainties on x^2 and $\mathcal{B}_\ell$, but not on y and z_f. However, if the self-calibrating fit described above were performed without these modes, then r_f would be strongly coupled to x^2 and z_f [through $\Gamma(f, f)$], resulting in inflated uncertainties. In order to stabilize the fit, it would be necessary to fix the value of x^2.

Experimental constraints on charm mixing are usually presented as a two-dimensional region either in the plane of $x'_{K\pi}$ versus $y'_{K\pi}$ or in the plane of x versus y. In Figs. 1 and 2, we compare current constraints with those projected using the method described in this paper. In Fig. 1, we show the results of the time-dependent analyses of $D^0 \to K^+\pi^-$ from CLEO [31], *BABAR* [32], Belle [33], and FOCUS [34]. The regions for CLEO, *BABAR*, and Belle allow for CP violation in the decay amplitude, in the mixing amplitude, and in the interference between these two processes, while the FOCUS result does not. Several experiments have also measured y directly by comparing the D^0 decay time for the $K^-\pi^+$ final state to that for the CP eigenstates K^+K^- and $\pi^+\pi^-$. The allowed region for y (labeled $\Delta\Gamma$) shown in Figs. 1 and 2 is the average of the results from E791 [25], CLEO [27], *BABAR* [29], and Belle

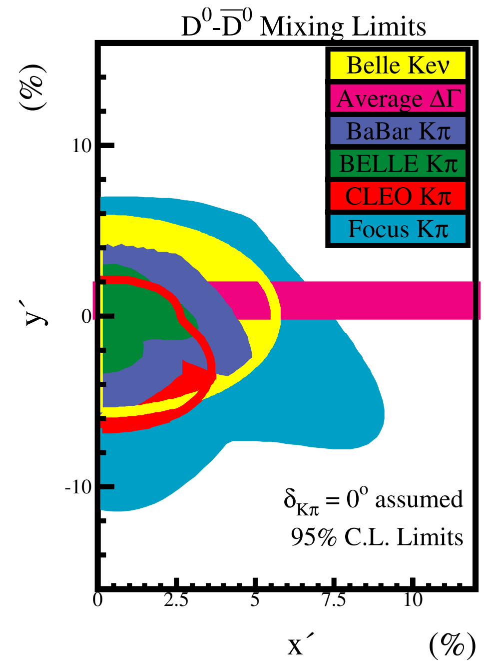

FIG. 1 (color online). Current allowed regions in the plane of $y'_{K\pi}$ versus $x'_{K\pi}$ [24,25,27,29–35], assuming $\delta_{K\pi} = 0$. A nonzero value for $\delta_{K\pi}$ would rotate the $\Delta\Gamma$ confidence region clockwise about the origin by an angle $\delta_{K\pi}$.

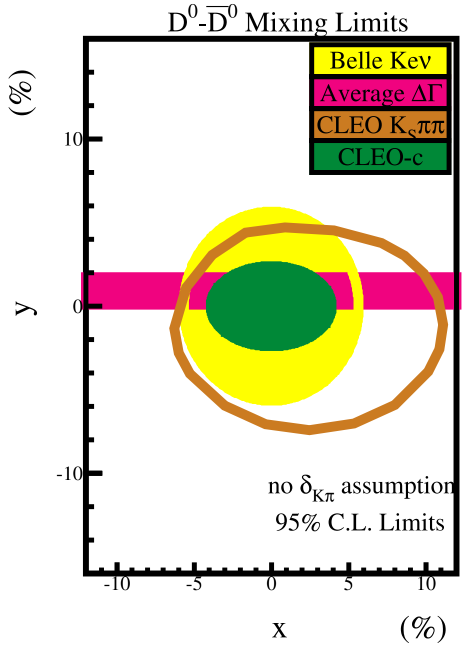

FIG. 2 (color online). Current allowed regions in the plane of y versus x [24,25,27,29,30,35], with our 95% C.L. contour for $\Gamma^{C=-1}_{D^0\bar{D}^0} = 10 \cdot \Gamma^{C=+1}_{D^0\bar{D}^0} = 3 \times 10^6$ superimposed.

[30]. In depicting the y and Dalitz-plot results in Fig. 1, we assume $\delta_{K\pi} = 0$; a nonzero value for $\delta_{K\pi}$ would rotate the $D^0 \to K^-K^+/\pi^+\pi^-$ confidence region clockwise about the origin by an angle $\delta_{K\pi}$. The best limit from semileptonic searches for charm mixing ($D^0 \to \bar{D}^0 \to K^+\ell^-\nu_\ell$), shown in Figs. 1 and 2, is from the Belle experiment [24]. Semileptonic results from E791 [21], *BABAR* [22], and CLEO [23] are not shown.

In Fig. 2, we plot the projected 95% confidence level (C.L.) contour for the fit in the third column of Table VII, along with the results of a time-dependent Dalitz-plot analysis of $D^0 \to K^0_S\pi^+\pi^-$ by CLEO [35], as well as the lifetime and semileptonic results discussed above. We note that our sensitivity to x depends strongly on the value of $\delta_{K\pi}$, while our sensitivity to y does not, and lowering the value of $|\cos\delta_{K\pi}|$ would reduce the area of the contour. In general, our expected upper limits compare favorably with the current best limits on charm mixing.

V. SUMMARY

We have derived ST and DT rate expressions for correlated $D^0\bar{D}^0$ pairs in a definite C eigenstate. Interference between amplitudes for indistinguishable final states enhances some D^0 decays and suppresses others, depending on the D^0-$\bar{D}^0$ mixing parameters and on the magnitudes and phases of various amplitude ratios. By examining different types of final states, which have different interference characteristics, we can extract the mixing and amplitude ratio parameters, in addition to the branching fractions. In contrast to previous measurements of mixing parameters [7], our method is both time-independent and sensitive to x and y at first order, so it is subject to different systematic uncertainties. Also, it is unique to threshold production where the $D^0\bar{D}^0$ initial state is known, unlike with D^0 mesons produced at fixed target experiments or through D^* decays, as at the B-factories and at LEP.

When performing this analysis for $K^0\bar{K}^0$, $B^0\bar{B}^0$, and $B^0_s\bar{B}^0_s$ decays, one should incorporate CP violation and nontrivial weak phases. In principle, given a source of coherent $B^0_s\bar{B}^0_s$ pairs, such as those produced through the $\Upsilon(5S)$, B^0_s-$\bar{B}^0_s$ mixing could be probed in a fashion similar to that presented in this paper for D^0-$\bar{D}^0$ mixing.

ACKNOWLEDGMENTS

We wish to thank David Cinabro and Ed Thorndike for many helpful discussions and for commenting on this manuscript. This work was supported in part by the National Science Foundation under Grant No. PHY-0202078.

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