Conditions for detecting *CP* violation via neutrinoless double beta decay

A. Joniec and M. Zralek

Institute of Physics, University of Silesia, Uniwersytecka 4, 40-007 Katowice, Poland

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Neutrinoless double beta decay data, together with information on the absolute neutrino masses obtained from the future KATRIN experiment and/or astrophysical measurements, provide a chance to find *CP* violation in the lepton sector with Majorana neutrinos. We derive and discuss necessary conditions which make discovery of such *CP* violation possible for the future neutrino oscillation and mass measurements data.

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I. INTRODUCTION

Information on *CP* violation in the lepton sector is very important for building the future theories which go beyond the standard model [1]. As *CP* violation is probably predominantly connected with lepton masses and observed neutrinos are very light, an experimental measurement of the effect is a serious challenge. For three Dirac neutrinos there is one *CP* violating phase (δ) and two additional phases (α_1, α_2) exist for Majorana neutrinos. The charged current state (ν_α) is related to mass states (ν_i) by a unitary transformation

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle \tag{1}$$

where

$$U_{\alpha i} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

c_{ij} and s_{ij} are cosines and sines of the $\theta_{ij}(ij = 12, 13, 23)$ angles. The second matrix in (2) appears only for Majorana neutrinos.

It is commonly believed that *CP* violation owing to the Dirac phase δ will be discovered in the future superbeam or neutrino factory experiments [2,3] where oscillations of neutrinos and antineutrinos will be observed. From the parametrization of the mixing matrix [Eq. (2)] we can see that $\sin\theta_{13}$ and $e^{\pm i\delta}$ always appear in a combination. So, any *CP* breaking effect for Dirac neutrinos will be proportional to $\sin\theta_{13}\sin\delta$ and disappear for $\sin\theta_{13} \to 0$. From the present fits it follows that this mixing angle is small ($\sin^2\theta_{13} < 0.05$ for 99.7% C.L. [4–6]), and the assumption that $\theta_{13} = 0$ agrees with the data equally well. If future, more precise data indicate that θ_{13} is very small, any signal of *CP* symmetry breaking will be difficult to see. It was shown that for $\delta = \pm\frac{\pi}{2}$, effects of *CP* violation will be seen in future experiments if $\sin^2\theta_{13}$ is not smaller than 10^{-4} [7].

If neutrinos are Majorana particles, in addition to the phase δ, two other phases can also be responsible for *CP* symmetry breaking. Many different processes are, in principle, sensitive to these Majorana phases and can generate both *CP*-even and *CP*-odd effects [8,9]. Admittedly, most of them are much beyond an observable limit. The only experiment which could provide evidence for Majorana phases is the search for neutrinoless double beta decay $(\beta\beta)_{0\nu}$. Such a possibility has been discussed many times [10–23] but, to our knowledge, detailed conditions concerning the future experimental results and their necessary precision to discover *CP* violation have not been discussed. The exception is Ref. [24], where authors consider the future anticipated precision of all relevant neutrino experiments, and formulate a very pessimistic "no-go" conclusion. They state that even under a very optimistic assumption about the sensitivity of future experiments it will be impossible to detect neutrino *CP* violation in the $(\beta\beta)_{0\nu}$ decay. We agree with such a statement, but we would like to go a step further. We propose a set of conditions for neutrino masses and mixing angles [best fit values (b.f.v)] altogether with conditions on experimental and theoretical precision for their determination, such that the discovery of *CP* violation arising from Majorana phases in the $(\beta\beta)_{0\nu}$ decay will be possible. We formulate sufficient conditions when *CP* violation could be observed. We should mention that our conditions are completely general. Contrary to Ref. [24] we do not assume from the beginning that the θ_{13} angle vanishes. Similar considerations have been made in [25–27]. Here we concentrate on the degenerate neutrino mass spectrum where *CP* violation has a clear meaning. We investigate in more detail the problem of theoretical determination of the nuclear matrix elements, the mechanism responsible for $(\beta\beta)_{0\nu}$ and the future experimental error of the nuclei decay lifetime.

We found that under a very optimistic assumption on the sensitivity of future experiments considered in Ref. [24], independently of measured b.f.v., it is really impossible to detect CP violation. However, such a possibility is "just around the corner." A little better precision will give one a chance to make a decisive statement about CP Majorana breaking. Even if the required precision for today is estimated to be a very optimistic value, we hope that the problem of lepton CP violation is so important that it is worth keeping in mind.

Another important result of our investigation concerns the θ_{13} and θ_{12} mixing angles. Contrary to neutrino oscillation experiments, a smaller θ_{13} angle gives a better prospect for the CP symmetry breaking measurement. A similar situation takes place for the solar mixing angle θ_{12}. The maximal θ_{12}, $\sin 2\theta_{12} \to 1$, is the best scenario for CP symmetry breaking discovery, contrary to the prospects of finding the neutrino mass bound from $(\beta\beta)_{0\nu}$ decay [28], where $\theta_{12} \to \frac{\pi}{4}$ ruins such a possibility.

In the next section we discuss how CP symmetry breaking could be determined from neutrinoless double beta decay. Then, in Sec. III, we describe the present situation and we predict how precisely all parameters (oscillation mixing angles, effective mass $\langle m_\nu \rangle$ measured in $(\beta\beta)_{0\nu}$, and m_β measured in e.g. tritium beta decay) should be determined in order to discover CP symmetry breaking. Two kinds of presentations are given. The first one is very visual, where correlations between errors are not included. The second involves more sophisticated analyses which show at what confidence level the probes of CP violation could be carried out. Finally, Sec. IV contains our conclusions.

CP SYMMETRY BREAKING AND THE $(\beta\beta)_{0\nu}$ DECAY

The neutrinoless double beta decay $(\beta\beta)_{0\nu}$ of nuclei measures the effective neutrino mass $\langle m_\nu \rangle$ [29]:

$$\langle m_\nu \rangle = \left| \sum_{i=1}^{3} U_{ei}^2 m_i \right| = |c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 e^{2i\phi_2} + s_{13}^2 m_3 e^{2i\phi_3}|, \quad (3)$$

where $\phi_2 = \alpha_2 - \alpha_1$ and $\phi_3 = -\delta - \alpha_1$.

As we will see, the possible precision of future experiments will give one a chance to look for CP violation only for higher neutrino masses ($m_1 \gtrsim 0.1$ eV). For this case the mass spectrum starts to degenerate and we will consider only such a spectrum. Then the effective neutrino mass m_β measured in tritium beta decay, independently of its definition [30], is just equal to neutrino masses

$$m_\beta = \left[\sum_{i=1}^{3} |U_{ei}|^2 m_i^2 \right]^{1/2} = \sum_{i=1}^{3} |U_{ei}|^2 m_i = m_1 \approx m_2 \approx m_3. \quad (4)$$

For Majorana neutrinos, CP symmetry holds if α_i and δ take one of the values $0, \pm\frac{\pi}{2}, \pm\pi$. Then from Eq. (3), four conserving CP values of $\langle m_\nu \rangle$ are obtained:

$$\begin{aligned} \langle m_\nu \rangle_{(1)} &= m_\beta, \qquad \langle m_\nu \rangle_{(2)} = m_\beta \cos 2\theta_{13}, \\ \langle m_\nu \rangle_{(3)} &= m_\beta(\cos^2\theta_{13} |\cos 2\theta_{12}| + \sin^2\theta_{13}), \\ \langle m_\nu \rangle_{(4)} &= m_\beta(\cos^2\theta_{13} |\cos 2\theta_{12}| - \sin^2\theta_{13}). \end{aligned} \quad (5)$$

In all cases, the relation between $\langle m_\nu \rangle$ and m_β is linear with different slopes ($i = 1, 2, 3, 4$),

$$\langle m_\nu \rangle_{(i)} = c_i m_\beta. \quad (6)$$

First we would like to present a very visual method of finding a region of parameters where CP violation can be probed. We will present a result with the correct statistical analysis. Let us assume that θ_{ij} mixing angles are known with definite precision,

$$\sin^2\theta_{ij} \in ((\sin^2\theta_{ij})_{\min}, (\sin^2\theta_{ij})_{\max}) \quad (7)$$

with a central value

$$(\sin^2\theta_{ij})_{\text{best fit}}. \quad (8)$$

For each $c_i (i = 2, 3, 4)$ we can calculate the maximal and minimal values,

$$\begin{aligned} c_2^{\max} &= (\cos 2\theta_{13})_{\max}, \qquad c_2^{\min} = (\cos 2\theta_{13})_{\min}, \\ c_3^{\max} &= (\cos^2\theta_{13})_{\max}(\cos 2\theta_{12})_{\max} + (\sin^2\theta_{13})_{\max}, \\ c_3^{\min} &= (\cos^2\theta_{13})_{\min}(\cos 2\theta_{12})_{\min} + (\sin^2\theta_{13})_{\min}, \\ c_4^{\max} &= (\cos^2\theta_{13})_{\max}(\cos 2\theta_{12})_{\max} - (\sin^2\theta_{13})_{\min}, \\ c_4^{\min} &= (\cos^2\theta_{13})_{\min}(\cos 2\theta_{12})_{\min} - (\sin^2\theta_{13})_{\max}. \end{aligned} \quad (9)$$

We can see that localization of the $\langle m_\nu \rangle_{(i)}$ lines is fully determined by the oscillation parameters, namely θ_{13} and θ_{12} angles.

Let us now assume that in future experiments m_β and $\langle m_\nu \rangle$ masses are determined with precision Δm_β and $\Delta\langle m_\nu \rangle$:

$$\langle m_\nu \rangle_{\exp} \pm \Delta\langle m_\nu \rangle, \quad (10)$$

$$(m_\beta)_{\exp} \pm \Delta m_\beta. \quad (11)$$

Then localization of the rectangle $R = (\Delta m_\beta, \Delta\langle m_\nu \rangle)$ between the lines $c_1 = 1$ and $c_4^{\min}$ (see Fig. 1) determines CP symmetry breaking. If R crosses the error region between the $(c_i^{\min}, c_i^{\max})$ lines $i = 2, 3, 4$, we do not know anything about CP symmetry. But, on the other hand, if R is located outside the c_i error region then there is indication that CP symmetry is broken, as at least one of the angles $\delta, \alpha_1, \alpha_2$ is not equal to its CP conserving value.

Possible localization of the present and prospective $\langle m_\nu \rangle_i = c_i m_\beta$ lines is presented in Figs. 2 and 3, respectively. We can see that localization of R between $c_3^{\max}$ and

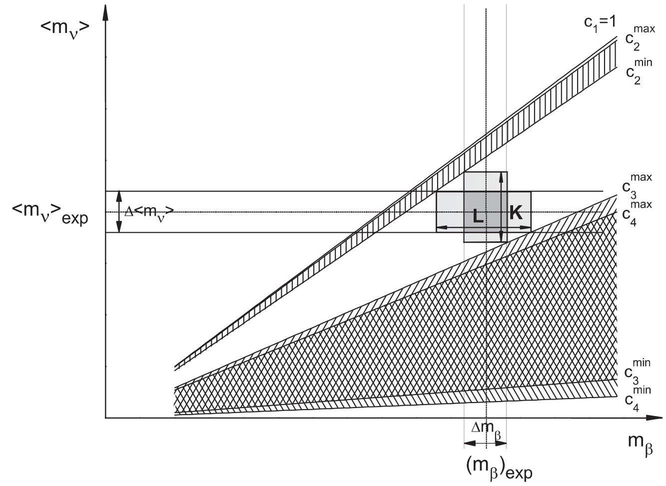

FIG. 1. A localization of the $R = (\Delta m_\beta, \Delta\langle m_\nu\rangle)$ rectangle between $c_2^{\min}$ and $c_3^{\max}$ lines, which indicates that *CP* symmetry is broken.

$c_2^{\min}$ lines is only interesting for a *CP* violation search. If the rectangle R with Δm_β and $\Delta\langle m_\nu\rangle$ sides is fully located between two lines with the $c_3^{\max}$ and $c_2^{\min}$ slopes, then *CP* symmetry is broken (see Fig. 1). So the first conditions for detecting *CP* violation are

$$\Delta m_\beta < L, \qquad \Delta\langle m_\nu\rangle < K. \tag{12}$$

L and K can be found in an easy way:

$$K = (m_\beta)A - (\Delta m_\beta)B, \tag{13}$$

and

$$L = \langle m_\nu\rangle C - \Delta\langle m_\nu\rangle D, \tag{14}$$

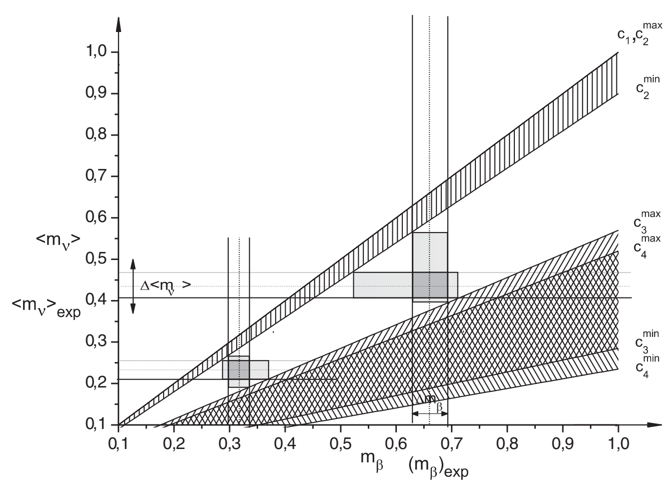

FIG. 2. A localization of the $(c_i^{\min} c_i^{\max})$ regions for the present precision of the θ_{13} and θ_{12} angles. To see *CP* violation, a precision of m_β and $\langle m_\nu\rangle$ measurements should be very good. For smaller m_β (and $\langle m_\nu\rangle$) a region where *CP* violation can be searched for is smaller, so a precision of their measurements should be even better.

where

$$A = c_2^{\min} - c_3^{\max}, \qquad B = \frac{c_2^{\min} + c_3^{\max}}{2}, \qquad C = \frac{A}{c_2^{\min} c_3^{\max}}, \qquad D = \frac{B}{c_2^{\min} c_3^{\max}} \tag{15}$$

for any m_β and $\langle m_\nu\rangle$ values inside the two lines $c_2^{\min}$ and $c_3^{\max}$.

If conditions [Eq. (12)] are satisfied for some central values $(m_\beta)_{\rm exp}$ and $\langle m_\nu\rangle_{\rm exp}$ determined from experiments (and theory), then there are two further possibilities. The rectangle R located at the point $((m_\beta)_{\rm exp}, \langle m_\nu\rangle_{\rm exp})$ can

(1) be fully inside two bounding lines $c_2^{\min}$ and $c_3^{\max}$, or

(2) be located partly on the first or the second line.

In the first case, we can conclude that *CP* symmetry is broken; in the second, the problem is unresolved. The first condition is satisfied if

$$c_3^{\max}\left((m_\beta)_{\rm exp} + \frac{\Delta m_\beta}{2}\right) < \left(\langle m_\nu\rangle_{\rm exp} - \frac{\Delta\langle m_\nu\rangle}{2}\right) \tag{16}$$

and

$$\left(\langle m_\nu\rangle_{\rm exp} + \frac{\Delta\langle m_\nu\rangle}{2}\right) < \left((m_\beta)_{\rm exp} - \frac{\Delta m_\beta}{2}\right)c_2^{\min}. \tag{17}$$

The inequalities given by Eqs. (12), (16), and (17) form the set of necessary conditions for *CP* symmetry breaking. Of course, we are not able to prove in this way that *CP* symmetry holds.

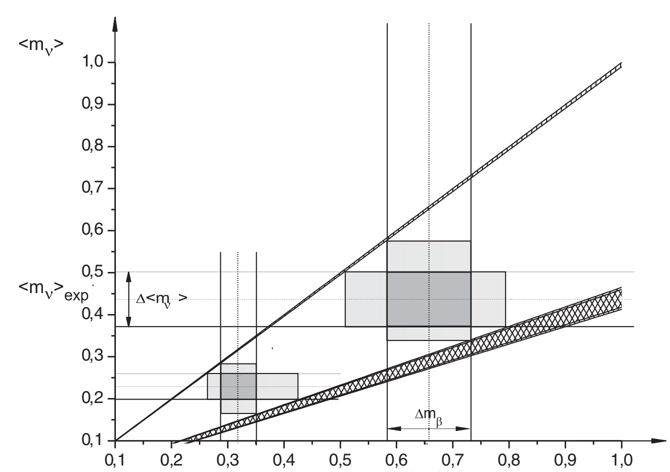

FIG. 3. The *CP* conserving regions (hatched areas) which follow from the future neutrino oscillation experiments. We assume that central values of θ_{13} and θ_{12} are in agreement with present data but their error estimation is supposed to be much better ($\sin^2\theta_{12} = 0.28 \pm 0.01$ and $\sin^2\theta_{13} = 0.005 \pm 0.0001$). The region between $c_2^{\min}$ and $c_3^{\max}$ lines is larger, giving more space for the rectangle $(\Delta m_\beta, \Delta\langle m_\nu\rangle)$ (see text for more details).

Let us parametrize

$$\Delta\langle m_\nu\rangle = 2x\langle m_\nu\rangle, \qquad \Delta m_\beta = 2y m_\beta, \tag{18}$$

where $2x$ is the relative error which measures the theoretical nuclear matrix element uncertainty and experimental decay lifetime of the $\langle m_\nu\rangle$ matrix element. Similarly, $2y$ measures the relative error of the effective mass e.g. from tritium beta decay. As both K and L [in Eqs. (13) and (14)] must be larger than zero, we have two consistency conditions. Both x and y must satisfy the same inequality,

$$x, y \le \frac{1 - \cos 2\theta_{12\,\min} - 3\sin^2\theta_{13\,\max} + \sin^2\theta_{13\,\min}\cos 2\theta_{12\,\min}}{1 + \cos 2\theta_{12\,\min} - \sin^2\theta_{13\,\max} - \sin^2\theta_{13\,\min}\cos 2\theta_{12\,\min}}. \tag{19}$$

These inequalities impose sharp conditions concerning a precision of the m_β and $\langle m_\nu\rangle$ determination. As the right-hand side of Eq. (19) is a decreasing function of $\sin^2\theta_{13}$ and $\cos 2\theta_{12}$, the best circumstances arise for $\sin^2\theta_{13} \to 0$ and $\sin^2\theta_{12} \to \frac{1}{2}$. In this case, lines $c_2^{\min} \to 1$ and $c_3^{\max} \to 0$ give the largest region for localization of $\langle m_\nu\rangle_{\rm exp}$ and $(m_\beta)_{\rm exp}$ where symmetry is broken. As we know, the condition $\theta_{13} \to 0$ ruins the Dirac δ phase determination in oscillation experiments. We can see that both methods, $(\beta\beta)_{0\nu}$ decay and long baseline experiment which could detect δ, are complementary for detecting CP violation [31]. Also, the other condition, the large solar mixing angle ($\theta_{12} \to \frac{\pi}{4}$), is not favorable for Majorana mass determination from the $(\beta\beta)_{0\nu}$ decay [7–18].

The case $\sin^2\theta_{13} = 0$ has been considered in Ref. [24]. Then Eq. (19) gives

$$x < \tan^2\theta_{12} \tag{20}$$

which is exactly the condition given by Eq. (14) in Ref. [24].

From Eqs. (13) and (14) for given relative errors x and Δm_β we can also find the lower limit for the m_β and $\langle m_\nu\rangle$ effective masses for which measurements are still possible,

$$\langle m_\nu\rangle > \frac{\Delta m_\beta}{C - 2xD} \tag{21}$$

and

$$m_\beta > \frac{\Delta m_\beta}{A}\left(B + \frac{2x}{C - 2xD}\right). \tag{22}$$

Now, using the present precision of the neutrino oscillation data and the precision expected in the future, we can estimate how well m_β and $\langle m_\nu\rangle$ should be determined to discover CP symmetry breaking.

III. NUMERICAL RESULTS

Using presently determined θ_{12} and θ_{13} mixing angles [32–36] (with 3σ precision)

$$0.22 \le \sin^2\theta_{12} \le 0.37, \qquad 0 \le \sin^2 2\theta_{13} \le 0.048, \tag{23}$$

from Eq. (19) we obtain

$$x < 0.2. \tag{24}$$

It will be a serious challenge to get such a precision. Let us check it for the isotope of Germanium ^{76}Ge where evidence for the $(\beta\beta)_{0\nu}$ decay is claimed to have been obtained [37]. If we assume that only one standard mechanism, the exchange of Majorana neutrinos with masses m_i, is responsible for the $(\beta\beta)_{0\nu}$ decay, the effective mass $\langle m_\nu\rangle$ is calculated from the decay rate $T(^{76}\mathrm{Ge})$ [38]:

$$T^{-1}(^{76}\mathrm{Ge}) = G|M|^2\langle m_\nu\rangle^2, \tag{25}$$

where G is an accurately calculable phase space integral and M is the calculated nuclear matrix element (NME). Unfortunately, this calculation is a complicated job, and different methods of calculation give different results. For the isotope ^{76}Ge the results differ by 1 order of magnitude. If we parametrize

$$T(^{76}\mathrm{Ge}) = b \times 10^{24}\ \mathrm{y}, \tag{26}$$

then for $\langle m_\nu\rangle = 1$ eV, eleven different results have been obtained [28]:

$$\begin{aligned} b = 1.7(= b_{\min})\ [39],\quad 2.16\ [40],\quad 2.3\ [41],\quad 2.33\ [42],\quad 3.15\ [43],\quad 3.2\ [43],\\ 3.6\ [44],\quad 4.06\ [45],\quad 8.95\ [46],\quad 14.0\ [47],\quad 17.7(= b_{\max})\ [48]. \end{aligned}$$

However, we would like to stress that methods used in Refs. [39–48] are completely independent, different nuclear models are used, and generally models are not calibrated against nuclear properties. If we assume that relative experimental error for $T(^{76}\mathrm{Ge})$ measurements is defined by $2x_T$,

$$\frac{\Delta T(^{76}\mathrm{Ge})}{\langle T(^{76}\mathrm{Ge})\rangle} = 2x_T, \tag{27}$$

then the full relative uncertainty of $\langle m_\nu\rangle$ ($2x = \Delta\langle m_\nu\rangle/\langle m_\nu\rangle$) is given by ($x_T < 1$)

$$x = x(a, x_T) = \frac{\sqrt{(1+x_T)a} - \sqrt{1-x_T}}{\sqrt{(1+x_T)a} + \sqrt{1-x_T}}, \quad (28)$$

where $a = b_{\max}/b_{\min}$. In Fig. 4 we can see the relation between the NME precision (a) and the expected uncertainties for effective neutrino mass $\langle m_\nu \rangle$ ($x = \Delta\langle m_\nu \rangle/2\langle m_\nu \rangle$) for various future experimental errors of the decay lifetime of ^{76}Ge.

We can see that, taking seriously the present discrepancy in the NME determination ($a \approx 10$), we obtain $x \approx 0.52$, much larger than necessary [see Eq. (24)]. The new calculation of NME [49], where the observed $(\beta\beta)_{2\nu}$ decay has been used to fix relevant parameters, has shown the great stability of the final results. For the ^{76}Ge, two methods of calculation, RQRPA and QRPA (see [49] for more details), have given almost the same results, and then

$$a \approx 1.4. \quad (29)$$

With such a precision of the NME determination, we obtain ($x_T \approx 0.3$)

$$x \approx 0.24, \quad (30)$$

still above the present necessary precision [see Eq. (24)], but within reach of the future oscillation experiments.

We should also mention the other uncertainty in the $\langle m_\nu \rangle$ determination—the possible different physical mechanism for the $(\beta\beta)_{0\nu}$ decay. If the lepton number is violated at the TeV scale we can expect the other processes which give equally strong, as light Majorana neutrinos exchange, contributions to $(\beta\beta)_{0\nu}$. Then the relation between the decay lifetime and $\langle m_\nu \rangle$ is not given by Eq. (25). To answer the question, at which scale is the lepton number violated, information from higher energy colliders (e.g. CERN LHC) and other lepton processes is necessary. In Ref. [50] it was shown that a study of two lepton flavor violating processes, $\mu \to e$ conversion and $\mu \to e + \gamma$ decay, will give important insight into the mechanism of the $(\beta\beta)_{0\nu}$ decay.

From Eqs. (21) and (22) we can find conditions for m_β and $\langle m_\nu \rangle$ effective masses for which *CP* symmetry breaking could be seen [see Fig. 2]. For example, if $x \approx 0.15$ with the present 3σ precision of mixing angles in Eq. (23) and for $\Delta m_\beta = 0.03$, 0.02, 0.015 eV, the *CP* symmetry breaking is testable for $\langle m_\nu \rangle > 0.24$, 0.16, 0.12 eV and $m_\beta > 0.32$, 0.21, 0.16 eV, respectively. There is some chance that in future experiments such Δm_β precision can be reached, but the relative error for $\langle m_\nu \rangle$, $x \approx 0.15$ is far beyond the present possibilities.

From Eq. (21) for a given central value of $\langle m_\nu \rangle$, we can find the relation between the x and Δm_β required for probing the *CP* symmetry breaking. Let us assume that a value of $\langle m_\nu \rangle$ is really in an interval given by the Heidelberg group [37],

$$\langle m_\nu \rangle_{\exp} \approx (0.1\text{–}0.9)\ \text{eV}. \quad (31)$$

If $\langle m_\nu \rangle_{\exp} \approx 0.1(0.9)$ eV, Δm_β should be smaller than 0.002, 0.013, 0.026 (0.014, 0.11, 0.24) eV for $x \approx 0.19$, 0.15, and 0.1, respectively, with the central value $m_\beta \approx 0.13(1.2)$ eV.

More careful analysis, taking into account the present precision of the mixing angle determination [33], can give a region in the $(\langle m_\nu \rangle, m_\beta)$ plane where *CP* violation can be probed with various C.L. The regions of relative errors $\Delta m_\nu/m_\nu$ and $\Delta m_\beta/m_\beta$ for which *CP* violation could be seen are presented in Fig. 5. We see that even for 90% C.L. the x parameter should be smaller than $x < 0.15$, so it is completely out of reach with present experimental and theoretical possibilities.

How does a better determination of the θ_{12} and θ_{13} mixing angles affect the x and Δm_β uncertainties? Let us assume that during the next years the precision of experi-

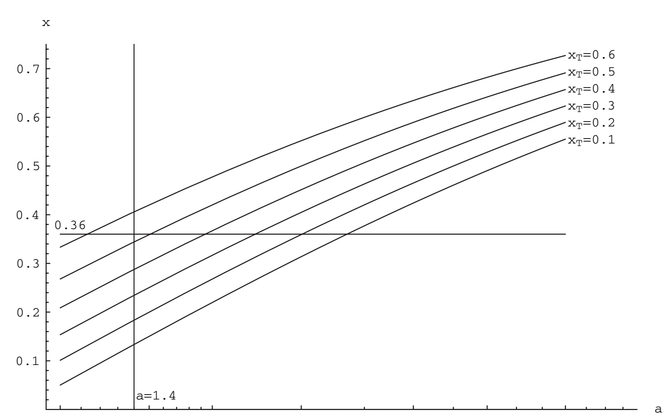

FIG. 4. The lines for the full uncertainty of $\langle m_\nu \rangle$, $x = \Delta\langle m_\nu \rangle/2\langle m_\nu \rangle$ as a function of both theoretical uncertainty in NME calculations $a = b_{\max}/b_{\min}$ (see text) and the experimental relative error (x_T) for the decay lifetime of ^{76}Ge.

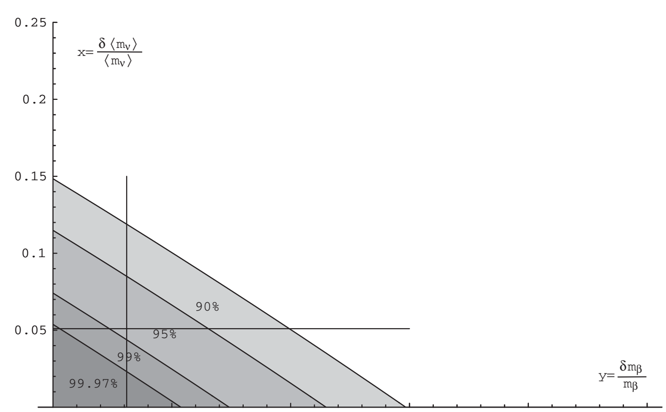

FIG. 5. The regions of the relative error of m_ν versus m_β, where *CP* is violated with confidence level equal to 90%, 95%, 99%, and 99.97% (3σ). To find *CP* symmetry breaking, $\langle m_\nu \rangle$ and m_β should be determined with extremely difficult-to-reach precision.

ments will be strongly improved. Let us also assume that the best values of mixing angles will not change but only the precision will be much better:

(1) The KamLAND and Borexino experiments determine the solar mixing angle with precision $\sin^2\theta_{12} \approx 0.28 \pm 0.01$ [51].
(2) The IHF-Kamioka neutrino experiment or the future neutrino factories [52] will measure the θ_{13} with the precision $\Delta\theta_{13} = 0.01$ (so $\sin^2\theta_{13} = 0.005 \pm 0.0001$).
(3) And assume finally that weak lensing of galaxies by a large scale structure together with cosmic microwave background data measure the sum of neutrino masses $\sum = m_1 + m_2 + m_3$ to an uncertainty of 0.04 eV. So we can expect that each individual mass is known with the precision $\Delta m_\beta = 0.015$ eV [53].

Now from Eq. (19) we get the required precision of Δm_β and $\Delta\langle m_\nu\rangle$,

$$x, y < 0.36. \tag{32}$$

In Fig. 4 we present for this value of x a necessary precision of NME for different relative errors of the $T(^{76}\mathrm{Ge})$ measurements. If the last estimation of NME is confirmed ($a \approx 1.4$) and the decay lifetime of ^{76}Ge is found with $x_T \leq 0.5$, then the necessary precision of $\langle m_\nu\rangle$ will be obtained. Such

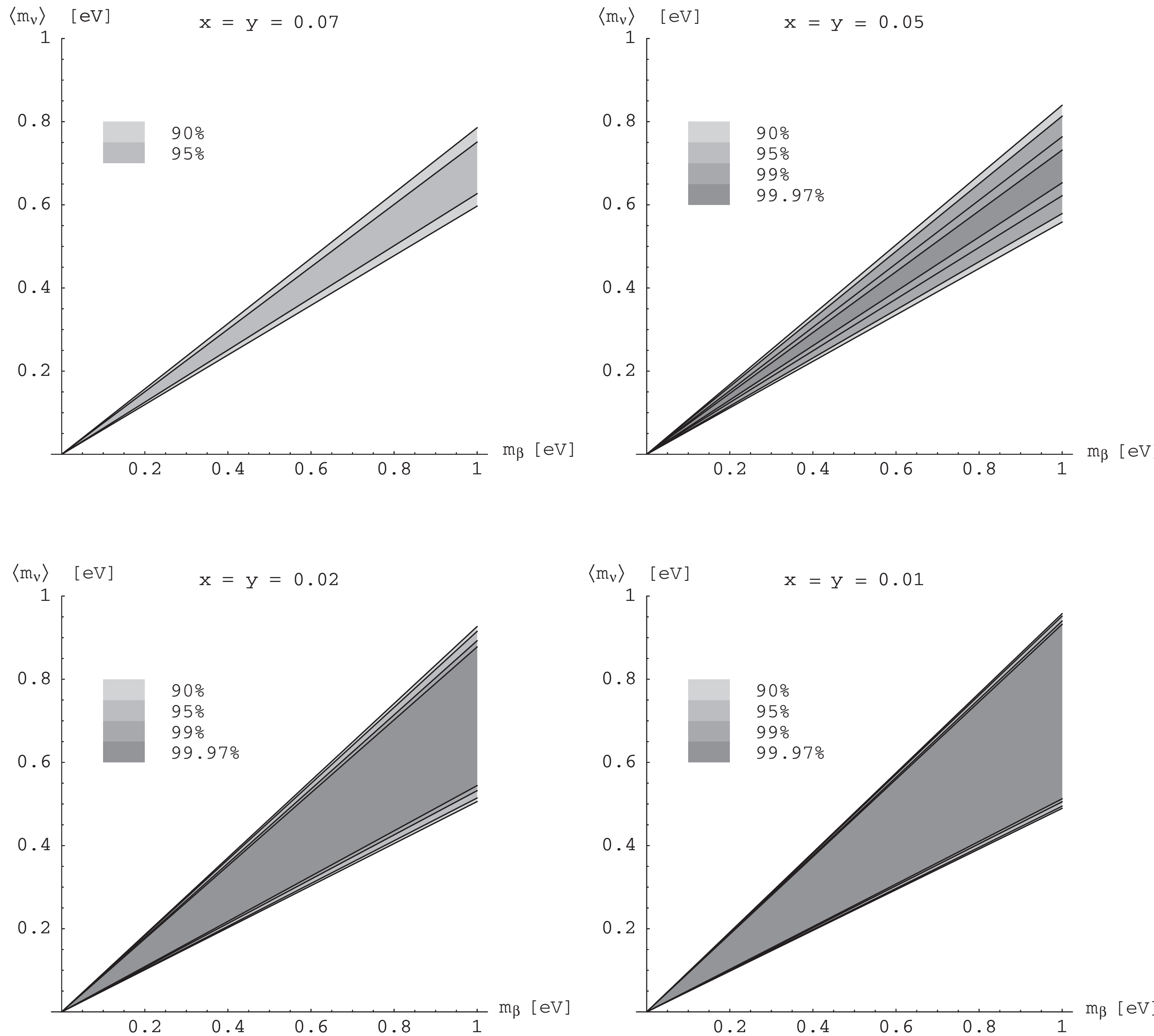

FIG. 6. Regions in the $\langle m_\nu\rangle \Leftrightarrow m_\beta$ plane where CP symmetry is broken with various C.L. for equal relative errors of $\langle m_\nu\rangle$ and m_β ($x = y$).

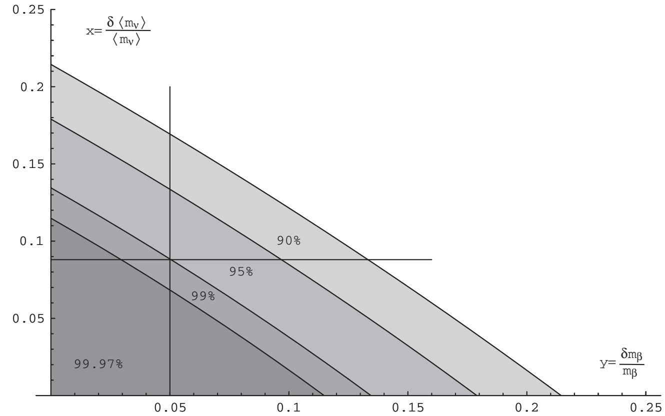

FIG. 7. The regions of relative errors of $\langle m_\nu \rangle$ versus m_β, where *CP* is violated with C.L. equal to 90%, 95%, 99%, and 99.97% (3σ). If $y = 0.05$, then, to determine *CP* violation, at 99% C.L. we have to know the effective Majorana mass with precision $x = 0.09$.

a scenario is not just purely fantasy. A more precise estimation will give a region of $\langle m_\nu \rangle$ and m_β where a probe of *CP* violation could be possible (Fig. 6). We have assumed the same relative uncertainties for $\langle m_\nu \rangle$ and m_β ($x = y$). For $x = y = 0.07$ there is no region where *CP* could be found with C.L. $> 99\%$. This region appears if x and y are smaller. In Fig. 7 a region of x and y relative errors is presented for a given level of C.L. We can see that if we want to probe *CP* violation with C.L. $\approx 90\%$ x must be smaller than $x \leq 0.22$ for very well determined m_β ($y \to 0$) and vice versa, $y \leq 2$ for $x \to 0$. Correlations between quantities give more stringent requirements for relative errors [see Eq. (32)]. We can see from Fig. 4 that to get $x \sim 0.1$, parameter a must be smaller than 1.3 and x_T better than 10%. Knowledge of NME on a 30% level has been postulated recently [54].

IV. CONCLUSIONS

From the estimations presented it follows that a measurement for *CP* violation for Majorana neutrinos in neutrinoless double beta decay could be possible for an almost degenerate spectrum of their masses ($m_\beta > 0.1$ eV). However, several conditions should be satisfied:

(1) Oscillation mixing angles should be measured with better precision e.g. $\Delta(\sin\theta_{13} \approx 0.01)$ and $\Delta(\sin\theta_{12} \approx 0.1)$, which are within the future experimental range (see e.g. [51,52]).

(2) Absolute neutrino masses m_β should be measured with precision $\Delta m_\beta \approx 0.02$ eV with the central value in the range $m_\beta > 0.15$ eV, which is also not a fully fantastic dream [53].

(3) Neutrinoless double beta decay is discovered and the decay lifetime T is measured with precision better than 10%. It is difficult to say at the moment anything about the future precision of T. If we give credit to the last Heidelberg group news about $(\beta\beta)_{0\nu}$ decay of ^{76}Ge, then the error of T is much higher. They derived from the full data taken until May 2003 that [37]

$$T(^{76}\text{Ge}) = (0.69 - 4.18) \times 10^{25}\ \text{y}, \tag{33}$$

with the best value $T(^{76}\text{Ge}) = 1.19 \times 10^{25}$ y, so the relative error $x_T = \frac{\Delta T}{T} \sim 2.9$. To get $x_T < 0.1$ will probably be a very difficult task.

(4) Nuclear matrix elements of decaying isotopes are calculated with much better precision. Future uncertainties for $a = b_{\max}/b_{\min}$ should be smaller than $a < 1.3$. During the last years some improvement in NME calculation has been obtained. The last result, where $a \approx 1.4$ has been presented, is a very good step forward [49]. The model of NME calculation can also be tested via comparison of the results of calculation for three (or more) nuclei with experimental data [55–57]. This test can be accomplished if $(\beta\beta)_{0\nu}$ decay of several nuclei is observed.

(5) There should be independent information about a full mechanism of the $(\beta\beta)_{0\nu}$ decay. We should know that two electrons are produced by two W bosons and the Majorana neutrino exchange virtual process. Any other mechanism should give negligible contribution to the neutrinoless electrons production. The future LHC data and observation of other lepton violating processes give some chance to clarify this issue [50].

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