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Nonlinear realization of supersymmetric AdS space isometries

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The isometries of AdS_5 space and supersymmetric $AdS_5 \otimes S_1$ space are nonlinearly realized on four-dimensional Minkowski space. The resultant effective actions in terms of the Nambu-Goldstone modes are constructed. The dilatonic mode governing the motion of the Minkowski space probe brane into the covolume of supersymmetric AdS_5 space is found to be unstable and the bulk of the AdS_5 space is unable to sustain the brane. No such instability appears in the nonsupersymmetric case.

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I. INTRODUCTION

In recent years, there has been a resurgence of interest in conformal field theories fueled by a deeper understanding of properties of supersymmetric (SUSY) gauge theories. This is particularly the case concerning their connection with theories defined in anti-de Sitter (AdS) space [1,2]. AdS_5 space is defined to be the hyperboloid satisfying the equation

$$\frac{1}{m^2} = (X^0)^2 - (X^1)^2 - (X^2)^2 - (X^3)^2 - (X^4)^2 + (X^5)^2 \tag{1.1}$$

which is embedded in a six-dimensional space with invariant interval

$$ds^2 = dX^{\mathcal{M}} \hat{\eta}_{\mathcal{M}\mathcal{N}} dX^{\mathcal{N}}; \qquad \mathcal{M}, \mathcal{N} = 0, 1, 2, 3, 4, 5 \tag{1.2}$$

and metric tensor $\hat{\eta}_{\mathcal{M}\mathcal{N}}$ with signature $(-1, +1, +1, +1, +1, -1)$. The isometry group of the hyperboloid is $SO(4, 2)$ whose generators, $\hat{M}^{\mathcal{M}\mathcal{N}} = -\hat{M}^{\mathcal{N}\mathcal{M}}$ satisfy the algebra:

$$[\hat{M}_{\mathcal{M}\mathcal{N}}, \hat{M}_{\mathcal{L}\mathcal{R}}] = i(\hat{\eta}_{\mathcal{M}\mathcal{L}}\hat{M}_{\mathcal{N}\mathcal{R}} - \hat{\eta}_{\mathcal{M}\mathcal{R}}\hat{M}_{\mathcal{N}\mathcal{L}} - \hat{\eta}_{\mathcal{N}\mathcal{L}}\hat{M}_{\mathcal{M}\mathcal{R}} + \hat{\eta}_{\mathcal{N}\mathcal{R}}\hat{M}_{\mathcal{N}\mathcal{L}}). \tag{1.3}$$

Alternatively (pseudo)translation generators can be defined as

$$\hat{P}_M = m\hat{M}_{5M}; \qquad M = 0, 1, 2, 3, 4 \tag{1.4}$$

so that the $SO(4, 2)$ algebra takes the form

$$\begin{aligned}
[\hat{M}_{MN}, \hat{M}_{LR}] &= i(\eta_{ML}\hat{M}_{NR} - \eta_{MR}\hat{M}_{NL} - \eta_{NL}\hat{M}_{MR} + \eta_{NR}\hat{M}_{ML}) \\
[\hat{M}_{MN}, \hat{P}_L] &= i(\eta_{ML}\hat{P}_N - \eta_{NL}\hat{P}_M) \\
[\hat{P}_M, \hat{P}_N] &= -im^2\hat{M}_{MN}
\end{aligned} \tag{1.5}$$

where η_{MN} is the five-dimensional Minkowski space metric with signature $(-1, +1, +1, +1, +1)$. Note that, in the limit $m^2 \to 0$, this reduces to the Poincaré algebra of five-dimensional Minkowski space (M_5).

The nonlinear realization of this isometry group which encapsulates the long wavelength dynamical constraints imposed by the spontaneous symmetry breaking when an AdS_4 space is embedded in AdS_5 space was previously constructed [3]. Using coset methods [4,5], an $SO(4, 2)$ invariant action in terms of the Nambu-Goldstone modes, ϕ and v^μ, $\mu = 0, 1, 2, 3$ associated with the spontaneously broken generators $\hat{P}_4$ and $\hat{M}_{\mu 4}$ respectively was secured as

$$\begin{aligned}
S &= -\sigma \int d^4x(\det e) \\
&= -\sigma \int d^dx(\det\bar{e})[\cosh(m\phi)]^4 \\
&\quad \times \left[\cos(\sqrt{v^2}) + v^\nu \frac{\sin(\sqrt{v^2})}{\sqrt{v^2}\cosh(m\phi)}\mathcal{D}_\nu\phi\right]
\end{aligned} \tag{1.6}$$

where σ is the AdS_4 brane tension. Here

$$\bar{e}_\nu{}^\mu(x) = \frac{\sinh(\sqrt{m^2x^2})}{\sqrt{m^2x^2}} P^\mu_{\perp\ \nu}(x) + P^\mu_{\parallel\ \nu}(x) \tag{1.7}$$

is the AdS_4 vierbein and $\mathcal{D}_\mu = \bar{e}_\mu^{-1\nu}\partial_\nu$ is the AdS covariant derivative while

$$P_{T\ \mu\nu}(x) = \eta_{\mu\nu} - \frac{x_\mu x_\nu}{x^2} \qquad P_{L\ \mu\nu}(x) = \frac{x_\mu x_\nu}{x^2} \tag{1.8}$$

are transverse and longitudinal projectors, respectively.

Since this action is independent of $\partial_\mu v$, v^μ can be eliminated via its field equation

$$v^\nu \frac{\tan(\sqrt{v^2})}{\sqrt{v^2}} = \eta^{\mu\nu}\frac{\mathcal{D}_\nu\phi}{\cosh(m\phi)} \tag{1.9}$$

and the $SO(4, 2)$ invariant action can be recast as

$$S = -\sigma \int d^4x(\det\bar{e})[\cosh(m\phi)]^4\sqrt{1 + \frac{\mathcal{D}_\mu\phi\eta^{\mu\nu}\mathcal{D}_\nu\phi}{\cosh^2(m\phi)}}. \tag{1.10}$$

Note that the Nambu-Goldstone mode, ϕ, contains a mass

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term with $m_\phi^2 = 4m^2$ as well as nonderivative interactions. The action constitutes an AdS generalization of the Nambu-Goto action:

$$S_{NG} = -\sigma \int d^4x \sqrt{1 + (\partial_\mu \phi)^2}. \qquad (1.11)$$

Using the factorized form of the AdS_5 vielbein, along with the v^μ field equation, the invariant interval for AdS_5 space reads

$$ds^2 = e^{2A(\phi)} dx^\mu dx^\nu \eta^{\rho\mu} \bar{e}_{\nu\rho}(x) + (d\phi(x))^2 \qquad (1.12)$$

with warp factor $A(\phi) = \ln[\cosh(m\phi)]$. This allows the identification of the Nambu-Goldstone mode ϕ with the covolume coordinate describing the motion of the AdS_4 brane into the remainder of the AdS_5 space.

In this paper, we construct the nonlinear realization of the AdS_5 and SUSY $AdS_5 \otimes S_1$ isometry groups on an embedded four-dimensional Minkowski space. The supersymmetric case turns out to be particularly interesting. Here it is found that the Nambu-Goldstone boson describing the motion of the inserted Minkowski space probe brane into the remainder of the AdS_5 space exhibits an instability which drives the probe brane to $-\infty$. Alternatively, this result can be interpreted as the incompatibility of simultaneous nonlinear realizations of both scale symmetry and supersymmetry or the nonviability of the spectrum containing both the dilaton of spontaneously broken scale symmetry and the Goldstino of spontaneously broken supersymmetry. On the other hand, no such unstable behavior arises when the M_4 probe brane is inserted into nonsupersymmetric AdS_5 space.

II. FOUR-DIMENSIONAL MINKOWSKI SPACE PROBE BRANE IN AdS_5 SPACE

To study the case of the four-dimensional Minkowski space probe brane in AdS_5 space, it proves useful to introduce the AdS_5 coordinates:

$$X^\mu = e^{mx_4} x^\mu \qquad X^4 = \frac{1}{m}\left[\sinh(mx_4) - \frac{m^2x^2}{2} e^{mx_4}\right]$$
$$X^5 = \frac{1}{m}\left[\cosh(mx_4) + \frac{m^2x^2}{2} e^{mx_4}\right] \qquad (2.1)$$

which parametrize the hyperboloid and in terms of which the AdS_5 space invariant interval takes the form

$$ds^2 = e^{2mx_4} dx^\mu \eta_{\mu\nu} dx^\nu + (dx_4)^2. \qquad (2.2)$$

Inserting the Minkowski space probe brane at $x_4 = 0$, the broken generators are identified as $\hat{P}^4 \equiv mD$, $\hat{M}^{\mu 4} \equiv m\mathcal{K}^\mu$. Defining

$$P^\mu = \hat{P}^\mu + m\hat{M}^{\mu 4} \qquad M^{\mu\nu} = \hat{M}^{\mu\nu} \qquad (2.3)$$

the $SO(4, 2)$ algebra, Eq. (1.5), takes the form

$$[P^\mu, P^\nu] = 0; \quad [M^{\mu\nu}, P^\lambda] = i(\eta^{\mu\lambda}P^\nu - \eta^{\nu\lambda}P^\mu)$$
$$[M^{\mu\nu}, M^{\lambda\rho}] = i(\eta^{\mu\lambda}M^{\nu\rho} - \eta^{\mu\rho}M^{\nu\lambda} - \eta^{\nu\lambda}M^{\mu\rho} + \eta^{\nu\rho}M^{\mu\lambda})$$
$$[\mathcal{K}^\mu, \mathcal{K}^\nu] = \frac{i}{m^2}M^{\mu\nu}; \quad [M^{\mu\nu}, \mathcal{K}^\lambda] = i(\eta^{\mu\lambda}\mathcal{K}^\nu - \eta^{\nu\lambda}\mathcal{K}^\mu)$$
$$[P^\mu, \mathcal{K}^\nu] = -i(\eta^{\mu\nu}D - M^{\mu\nu}) \quad [D, P^\mu] = iP^\mu;$$
$$[D, \mathcal{K}^\mu] = \frac{i}{m^2}(P^\mu - m^2\mathcal{K}^\mu); \quad [D, M^{\mu\nu}] = 0. \qquad (2.4)$$

Note that $M^{\mu\nu}$ and P^μ form a Poincaré algebra, while the generators $\mathcal{K}^\mu$, $M^{\mu\nu}$ constitute an $SO(3, 2)$ subalgebra [6–8].

A model independent way of encapsulating the long wavelength dynamical constraints imposed by spontaneous symmetry breakdown is to realize this $SO(4, 2)$ isometry nonlinearly on the Nambu-Goldstone bosons consisting of the dilaton, φ, associated with the broken symmetry generator D and v^μ associated with the $\mathcal{K}^\mu$ spontaneously broken generators. Since the spontaneously broken symmetries are space-time symmetries, the motion in the coset space is accompanied by a motion in space-time. Thus we consider the product of a space-time translation group element with the coset group element and define the group element:

$$\Omega = e^{-ix^\mu P_\mu} e^{i\varphi D} e^{-iv^\mu \mathcal{K}_\mu}. \qquad (2.5)$$

To extract the total variations of the coset coordinates and the corresponding transformation of the space-time point, consider the product $g\Omega$ with g a general group element parametrized by real infinitesimal constants. An explicit calculation then gives

$$g\Omega(x, \phi(x), v(x)) = \Omega(x', \varphi'(x'), v'(x'))h(\theta) \qquad (2.6)$$

with $h = e^{(i/2)\theta^{\mu\nu}(x)M_{\mu\nu}}$ an element of the unbroken (stability) group. Doing this allows the extraction of the forms of x', $\varphi'(x')$, $v^{\mu\prime}(x')$, $\theta^{\mu\nu}(x)$ [7,8].

To construct $SO(4, 2)$ invariants, it is useful to define the algebra valued Maurer-Cartan 1-form $\Omega^{-1}d\Omega$ which, using Eq. (2.6), is seen to have the simple transformation property

$$(\Omega^{-1}d\Omega)'(x') = [h(\Omega^{-1}d\Omega)h^{-1}](x) + (hdh^{-1})(x). \qquad (2.7)$$

Expanding the Maurer-Cartan form in terms of the generators as

$$\Omega^{-1}d\Omega(x) = i[-\omega_P^\mu(x)P_\mu + \omega_D(x)D - \omega_{\mathcal{K}}^\mu(x)\mathcal{K}_\mu + \tfrac{1}{2}\omega_M^{\mu\nu}(x)M_{\mu\nu}] \qquad (2.8)$$

and exploiting the $SO(4, 2)$ algebra along with liberal application of the Baker-Campbell-Haussdorff formula, the various 1-form coefficients $\omega_P^\mu(x) = dx^\nu e_\nu{}^\mu$, $\omega_D(x)$, $\omega_{\mathcal{K}}^\mu(x)$, $\omega_M^{\mu\nu}$ are secured. Here

$$e_\mu{}^\nu = e^\varphi[P^\nu_{\perp\mu}(v) + P^\nu_{\parallel\mu}(v)\cos(\sqrt{m^2/v^2})] - \partial_\mu\varphi v^\nu \frac{\sin(\sqrt{m^2/v^2})}{\sqrt{m^2/v^2}} \tag{2.9}$$

is the AdS_5 vielbein.

Again using the $SO(4,2)$ algebra, this time in the above transformation law (2.7), leads to the invariant combination

$$d^4x' \det e' = d^4x \det e. \tag{2.10}$$

Thus an $SO(4,2)$ invariant action is constructed as

$$\begin{aligned} S &= -\sigma\int d^4x \det e \\ &= -\sigma\int d^4x e^{4\varphi}\Big[\cos(\sqrt{m^2v^2}) \\ &\quad - e^{-\varphi}\partial_\mu\varphi v^\mu \frac{\sin(\sqrt{m^2/v^2})}{\sqrt{m^2/v^2}}\Big] \end{aligned} \tag{2.11}$$

with σ the Minkowski probe brane tension. As previously, the v^μ Nambu-Goldstone field is not an independent dynamical degree of freedom and it can be eliminated using its field equation

$$v^\mu \frac{\tan(\sqrt{m^2/v^2})}{\sqrt{m^2/v^2}} = -e^{-\varphi}\eta^{\mu\nu}\partial_\nu\varphi. \tag{2.12}$$

Substituting back then produces the action

$$S = -\sigma\int d^4x e^{4\varphi}\sqrt{1 + \frac{1}{m^2}e^{-2\varphi}\partial_\mu\varphi\eta^{\mu\nu}\partial_\nu\varphi}. \tag{2.13}$$

After using the v^μ field equation, the invariant interval

$$ds^2 = dx^\mu e_\mu{}^\lambda \eta_{\lambda\rho} e_\nu{}^\rho dx^\nu = e^{2\varphi}dx^\mu\eta_{\mu\nu}dx^\nu + \frac{1}{m^2}(d\varphi)^2 \tag{2.14}$$

is seen to have the same form as the AdS_5 invariant interval obtained previously after identification $\varphi \Leftrightarrow \frac{1}{m}x_4$. Thus the dilaton dynamics describes motion of the brane into the rest of AdS_5 space.

In the above construction, we have chosen a particular combination of the broken generators, referred to as the maximal solvable subgroup basis or parametrization [6], whose nonlinear realization on the Nambu-Goldstone modes has the attractive feature of directly relating the Nambu-Goldstone dilaton to the motion of the brane into the rest of AdS_5 space. An alternate choice of broken generators is given by

$$K^\mu = \frac{1}{m^2}\hat{P}^\mu - \frac{1}{m}\hat{M}^{\mu 4} = \frac{1}{m^2}(P^\mu - 2m^2\mathcal{K}^\mu). \tag{2.15}$$

This choice leads to the four-dimensional conformal algebra

$$\begin{aligned} &[P^\mu, P^\nu] = 0; \quad [M^{\mu\nu}, P^\lambda] = i(\eta^{\mu\lambda}P^\nu - \eta^{\nu\lambda}P^\mu) \\ &[M^{\mu\nu}, M^{\lambda\rho}] = i(\eta^{\mu\lambda}M^{\nu\rho} - \eta^{\mu\rho}M^{\nu\lambda} - \eta^{\nu\lambda}M^{\mu\rho} + \eta^{\nu\rho}M^{\mu\lambda}) \\ &K^\mu, K^\nu] = 0; \quad [M^{\mu\nu}, K^\lambda] = i(\eta^{\mu\lambda}K^\nu - \eta^{\nu\lambda}K^\mu) \\ &[P^\mu, K^\nu] = 2i(\eta^{\mu\nu}D - M^{\mu\nu}) \quad [D, P^\mu] = iP^\mu; \\ &[D, K^\mu] = -iK^\mu; \quad [D, M^{\mu\nu}] = 0. \end{aligned} \tag{2.16}$$

Since the generators K^μ and $\mathcal{K}^\mu$ differ only by the unbroken translation generator P^μ, it follows that the action (2.13) is also invariant under four-dimensional conformal transformations. Moreover, one can subtract the invariant action piece $\sigma\int d^dx e^{\varphi d}$ ensuring a zero vacuum energy and thus producing the conformally invariant action

$$S = -\sigma\int d^dx e^{\varphi d}\Big[\sqrt{1 + \frac{1}{m^2}e^{-2\varphi}\partial_\mu\eta^{\mu\nu}\varphi\partial_\mu\varphi} - 1\Big]. \tag{2.17}$$

Note that the leading term in a momentum expansion is simply

$$S = -\sigma\int d^dx e^{2\varphi}\partial_\mu\varphi\eta^{\mu\nu}\partial_\mu\varphi \tag{2.18}$$

which is the familiar result.

III. FOUR-DIMENSIONAL MINKOWSKI SPACE PROBE BRANE IN SUSY $AdS_5 \otimes S_1$ SPACE

The supersymmetric $AdS_5 \otimes S_1$ isometry algebra includes the generators $\hat{M}^{MN}$, $\hat{P}^M$; $M, N = 0, 1, 2, 3$, of the $SO(4,2)$ isometry algebra, the SUSY fermionic charges $\mathcal{Q}_a$, $\bar{\mathcal{Q}}_b$; $a, b = 1, 2, 3, 4$ and the R charge which is the generator of the $U(1)$ isometry of S_1. This $SU(2,2|1)$ isometry algebra [9,10] is

$$\begin{aligned} &[\hat{M}^{MN}, \hat{M}^{LR}] = i(\eta^{ML}\hat{M}^{NR} - \eta^{MR}\hat{M}^{NL} \\ &\qquad - \eta^{NL}\hat{M}^{MR} + \eta^{NR}\hat{M}^{ML}) \\ &[\hat{M}^{MN}, \hat{P}^L] = i(\eta^{ML}\hat{P}^N - \eta^{NL}\hat{P}^M) \\ &[\hat{P}^M, \hat{P}^N] = -im^2\hat{M}^{MN} \quad [\hat{M}^{MN}, \mathcal{Q}_a] = -\frac{1}{2}(\Sigma^{MN}\mathcal{Q})_a; \\ &[\hat{M}^{MN}, \bar{\mathcal{Q}}_a] = \frac{1}{2}(\bar{\mathcal{Q}}\Sigma^{MN})_a \quad [\hat{P}^M, \mathcal{Q}_a] = -\frac{m}{2}(\Gamma^\mu\mathcal{Q})_a; \\ &[\hat{P}^M, \bar{\mathcal{Q}}_a] = \frac{m}{2}(\bar{\mathcal{Q}}\Gamma^M)_a \quad [R, \mathcal{Q}_a] = \mathcal{Q}_a; \\ &[R, \bar{\mathcal{Q}}_a] = -\bar{\mathcal{Q}}_a \\ &\{\mathcal{Q}_a, \bar{\mathcal{Q}}_b\} = 2\Big(\Gamma^M_{ab}\hat{P}_M - \frac{m}{2}\Sigma^{MN}_{ab}\hat{M}_{MN} - \frac{3}{2}m\delta_{ab}R\Big), \end{aligned} \tag{3.1}$$

where the five-dimensional 4×4 matrices Γ^M satisfy the Clifford algebra

$$\{\Gamma^M, \Gamma^N\} = -2\eta^{MN} \tag{3.2}$$

and the spin matrices are

$$\Sigma^{MN} = \frac{i}{2}[\Gamma^M, \Gamma^N] = -\Sigma^{NM}. \tag{3.3}$$

We choose

$$\Gamma^M = \begin{cases} \gamma^\mu; & M = \mu = 0, 1, 2, 3 \\ i\gamma_5; & M = 4 \end{cases} \tag{3.4}$$

and use a Weyl representation for the γ matrices so that

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_{\alpha\dot\alpha} \\ \bar\sigma^{\mu\ \dot\alpha\alpha} & 0 \end{pmatrix}; \qquad \alpha, \dot\alpha = 1, 2; \qquad \gamma^4 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \tag{3.5}$$

$$\Sigma^{\mu\nu} = \begin{pmatrix} \sigma^{\mu\nu\ \beta}_{\alpha} & 0 \\ 0 & \bar\sigma^{\mu\nu\ \dot\alpha}{}_{\dot\beta} \end{pmatrix}; \qquad \Sigma^{\mu 4} = \begin{pmatrix} 0 & -\sigma^\mu_{\alpha\dot\alpha} \\ \bar\sigma^{\mu\ \dot\alpha\alpha} & 0 \end{pmatrix} = -\Sigma^{4\mu}. \tag{3.6}$$

Embedding a four-dimensional Minkowski space (M_4) probe brane at $x^4 = 0$ breaks the space-time symmetries generated by $\hat P^4 \equiv mD$ and $\hat M^{\mu 4} \equiv m\mathcal{K}^\mu$, as well as the supersymmetries generated by $\mathcal{Q}_a$, $\bar{\mathcal{Q}}_a$ and the R symmetry. Defining $P^\mu = \hat P^\mu + m\hat M^{\mu 4}$ and

$$\mathcal{Q}_a = \begin{pmatrix} Q_\alpha \\ -im\bar S^{\dot\alpha} \end{pmatrix}; \qquad \bar{\mathcal{Q}}_a = (imS^\alpha, \bar Q_{\dot\alpha}) \tag{3.7}$$

the resultant algebra is the SUSY extension of the $P^\mu, M^{\mu\nu}, D, \mathcal{K}^\mu$ algebra given in Eq. (2.4) and reads

$$\begin{gathered}
[P^\mu, P^\nu] = 0; \quad [M^{\mu\nu}, P^\lambda] = i(\eta^{\mu\lambda}P^\nu - \eta^{\nu\lambda}P^\mu) \quad [M^{\mu\nu}, M^{\lambda\rho}] = i(\eta^{\mu\lambda}M^{\nu\rho} - \eta^{\mu\rho}M^{\nu\lambda} - \eta^{\nu\lambda}M^{\mu\rho} + \eta^{\nu\rho}M^{\mu\lambda}) \\
[\mathcal{K}^\mu, \mathcal{K}^\nu] = \frac{i}{m^2}M^{\mu\nu}; \quad [M^{\mu\nu}, \mathcal{K}^\lambda] = i(\eta^{\mu\lambda}\mathcal{K}^\nu - \eta^{\nu\lambda}\mathcal{K}^\mu) \quad [P^\mu, \mathcal{K}^\nu] = -i(\eta^{\mu\nu}D - M^{\mu\nu}); \\
[D, P^\mu] = iP^\mu \quad [D, \mathcal{K}^\mu] = \frac{i}{m^2}(P^\mu - m^2\mathcal{K}^\mu); \quad [D, M^{\mu\nu}] = 0 \quad [R, P^\mu] = 0; \quad [R, \mathcal{K}^\mu] = 0; \\
[R, M^{\mu\nu}] = 0 \quad \{Q_\alpha, Q_\beta\} = 0; \quad \{\bar Q_{\dot\alpha}, \bar Q_{\dot\beta}\} = 0 \quad \{S_\alpha, S_\beta\} = 0; \quad \{\bar S_{\dot\alpha}, \bar S_{\dot\beta}\} = 0 \quad \{Q_\alpha, \bar Q_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu; \\
\{S_\alpha, \bar S_{\dot\alpha}\} = \frac{2}{m^2}\sigma^\mu_{\alpha\dot\alpha}(P_\mu - 2m^2\mathcal{K}_\mu) \quad \{Q_\alpha, S_\beta\} = i(\sigma^{\mu\nu}_{\alpha\beta}M_{\mu\nu} + 2i\epsilon_{\alpha\beta}D + 3\epsilon_{\alpha\beta}R) \\
\{\bar Q_{\dot\alpha}, \bar S_{\dot\beta}\} = -i(\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}M_{\mu\nu} - 2i\epsilon_{\dot\alpha\dot\beta}D + 3\epsilon_{\dot\alpha\dot\beta}R) \quad \{Q_\alpha, \bar S_{\dot\alpha}\} = 0; \quad \{S_\alpha, \bar Q_{\dot\alpha}\} = 0 \quad [P^\mu, Q_\alpha] = 0; \\
[P^\mu, \bar Q_{\dot\alpha}] = 0 \quad [P^\mu, S_\alpha] = i\sigma^\mu_{\alpha\dot\alpha}\bar Q^{\dot\alpha}; \quad [P^\mu, \bar S_{\dot\alpha}] = iQ^\alpha\sigma^\mu_{\alpha\dot\alpha} \\
[M^{\mu\nu}, Q_\alpha] = -\frac{1}{2}\sigma^{\mu\nu\ \beta}_{\alpha}Q_\beta; [M^{\mu\nu}, \bar Q_{\dot\alpha}] = -\frac{1}{2}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}\bar Q^{\dot\beta} \quad [M^{\mu\nu}, S_\alpha] = -\frac{1}{2}\sigma^{\mu\nu}{}_\alpha{}^\beta S_\beta; \quad [M^{\mu\nu}, \bar S_{\dot\alpha}] = -\frac{1}{2}\bar\sigma^{\mu\nu}{}_{\dot\alpha\dot\beta}\bar S^{\dot\beta} \\
[R, Q_\alpha] = Q_\alpha; \quad [R, \bar Q_{\dot\alpha}] = -\bar Q_{\dot\alpha}; \quad [R, S_\alpha] = -S_\alpha; \quad [R, \bar S_{\dot\alpha}] = \bar S_{\dot\alpha} \quad [D, Q_\alpha] = \frac{i}{2}Q_\alpha; \quad [D, \bar Q_{\dot\alpha}] = \frac{i}{2}\bar Q_{\dot\alpha} \\
[D, S_\alpha] = -\frac{i}{2}S_\alpha; \quad [D, \bar S_{\dot\alpha}] = -\frac{i}{2}\bar S_{\dot\alpha} \quad [\mathcal{K}^\mu, Q_\alpha] = -\frac{i}{2}\sigma^\mu_{\alpha\dot\alpha}\bar S^{\dot\alpha}; \quad [\mathcal{K}^\mu, \bar Q_{\dot\alpha}] = -\frac{i}{2}S^\alpha\sigma^\mu_{\alpha\dot\alpha} \\
[\mathcal{K}^\mu, S_\alpha] = \frac{i}{2m^2}\sigma^\mu_{\alpha\dot\alpha}\bar Q^{\dot\alpha}; \quad [\mathcal{K}^\mu, \bar S_{\dot\alpha}] = \frac{i}{2m^2}Q^\alpha\sigma^\mu_{\alpha\dot\alpha}.
\end{gathered} \tag{3.8}$$

Using coset methods, we nonlinearly realize this $SU(2,2|1)$ isometry algebra of the super-$AdS_5 \otimes S_1$ space on the Nambu-Goldstone modes of the broken symmetries. For the case that SUSY is only partially broken, see [11]. These are the dilaton, φ, and v^μ associated with D and $\mathcal{K}^\mu$ respectively, the Goldstinos $\lambda_\alpha, \bar\lambda_{\dot\alpha}$ and $\lambda_{S\alpha}, \bar\lambda_{S\dot\alpha}$ of the spontaneously broken supersymmetries, $Q_\alpha, \bar Q_{\dot\alpha}, S_\alpha, \bar S_{\dot\alpha}$, and the R axion a. Note that all the supersymmetries have been broken and there is no residual unbroken SUSY. The Nambu-Goldstone modes, v^μ, associated with the broken symmetries generated by $\mathcal{K}^\mu$ and the Goldstinos $\lambda_{S\alpha}; \bar\lambda_{S\dot\alpha}$ associated with the supersymmetries $S_\alpha, \bar S_{\dot\alpha}$ are not independent dynamical degrees of freedom but are instead given in terms of the dilaton, φ, the Goldstinos, $\lambda_\alpha, \bar\lambda_{\dot\alpha}$ and the R axion, a. Nonetheless, it is still necessary to include them as auxiliary fields in the coset construction of the $SU(2,2|1)$ invariant action. To implement this construction, we define the product of a space-time translation and coset group elements by

$$\Omega = e^{-ix^\mu P_\mu}e^{i[\lambda^\alpha(x)Q_\alpha + \bar\lambda_{\dot\alpha}\bar Q^{\dot\alpha}]}e^{i\varphi D}e^{iaR}e^{i[\lambda_S^\alpha(x)S_\alpha + \bar\lambda_{S\dot\alpha}\bar S^{\dot\alpha}]} \times e^{-iv^\mu\mathcal{K}_\mu}. \tag{3.9}$$

The covariant building blocks out of which an invariant action can be constructed are secured using the Maurer-Cartan one-form $\Omega^{-1}d\Omega$, where $d = dx^\mu\partial_\mu$. Expanding in terms of the $SU(2,2|1)$ charges via

$$\Omega^{-1}d\Omega = i[-\omega^\mu P_\mu + \omega_Q^\alpha Q_\alpha + \bar{\omega}_{\bar{Q}\dot{\alpha}}\bar{Q}^{\dot{\alpha}} + \omega_D D + \omega_R R - \omega_{\mathcal{K}}^\mu \mathcal{K}_\mu + \omega_S^\alpha S_\alpha + \bar{\omega}_{\bar{S}\dot{\alpha}}\bar{S}^{\dot{\alpha}} + \tfrac{1}{2}\omega_M^{\mu\nu}M_{\mu\nu}], \tag{3.10}$$

the individual covariant one-forms are then extracted as

$$\omega^\mu = \tilde{\omega}^\mu + P^\mu_{\parallel\nu}(v)\tilde{\omega}^\nu[\cos(\sqrt{v^2/m^2}) - 1] - \tilde{\omega}_D \frac{v^\mu}{m^2}\frac{\sin(\sqrt{v^2/m^2})}{\sqrt{v^2/m^2}} \qquad \omega_D = \tilde{\omega}_D\cos(\sqrt{v^2/m^2}) - v_\nu\tilde{\omega}^\nu\frac{\sin(\sqrt{v^2/m^2})}{\sqrt{v^2/m^2}}$$

$$\omega_R = \tilde{\omega}_R \qquad \omega_Q^\alpha = \tilde{\omega}_Q^\alpha\cos\left(\frac{\sqrt{v^2/m^2}}{2}\right) + (\bar{\tilde{\omega}}_{\bar{S}}\bar{\sigma}\cdot v)^\alpha\frac{\sin(\frac{\sqrt{v^2/m^2}}{2})}{m^2\sqrt{v^2/m^2}}$$

$$\bar{\omega}_{\bar{Q}\dot{\alpha}} = \bar{\tilde{\omega}}_{\bar{Q}\dot{\alpha}}\cos\left(\frac{\sqrt{v^2/m^2}}{2}\right) - (\tilde{\omega}_S\sigma\cdot v)_{\dot{\alpha}}\frac{\sin(\frac{\sqrt{v^2/m^2}}{2})}{m^2\sqrt{v^2/m^2}} \qquad \omega_S^\alpha = \tilde{\omega}_S^\alpha\cos\left(\frac{\sqrt{v^2/m^2}}{2}\right) - (\bar{\tilde{\omega}}_{\bar{Q}}\bar{\sigma}\cdot v)^\alpha\frac{\sin(\frac{\sqrt{v^2/m^2}}{2})}{\sqrt{v^2/m^2}}$$

$$\omega_{\bar{S}\dot{\alpha}} = \bar{\tilde{\omega}}_{\bar{S}\dot{\alpha}}\cos\left(\frac{\sqrt{v^2/m^2}}{2}\right) + (\tilde{\omega}_Q\sigma\cdot v)_{\dot{\alpha}}\frac{\sin(\frac{\sqrt{v^2/m^2}}{2})}{\sqrt{v^2/m^2}} \tag{3.11}$$

$$\omega_{\mathcal{K}}^\mu = dv^\mu + \tilde{\omega}_{\mathcal{K}}^\mu + [\cos(\sqrt{v^2/m^2}) - 1]P^\mu_{\perp\ \nu}(v)\tilde{\omega}_{\mathcal{K}}^\nu + \left[\frac{\sin(\sqrt{v^2/m^2})}{\sqrt{v^2/m^2}} - 1\right]P^\mu_{\perp\nu}(v)dv^\nu + m^2[\cos(\sqrt{v^2/m^2}) - 1]$$
$$\times[P^\mu_{\perp\nu}(v)\tilde{\omega}^\nu - P^\mu_{\parallel\nu}(v)\tilde{\omega}^\nu] + \tilde{\omega}_D v^\mu\frac{\sin(\sqrt{v^2/m^2})}{\sqrt{v^2/^2}} - \tilde{\omega}_M^{\mu\nu}v_\nu\frac{\sin(\sqrt{v^2/m^2})}{\sqrt{v^2/m^2}}$$

$$\omega_M^{\mu\nu} = \tilde{\omega}_M^{\mu\nu} - [\cos(\sqrt{v^2/m^2}) - 1]\left(\frac{v^\mu dv^\nu - v^\nu dv^\mu}{v^2}\right) - \frac{1}{2}\frac{\sin(\sqrt{v^2/m^2})}{\sqrt{v^2/m^2}}(\tilde{\omega}^\mu v^\nu - \tilde{\omega}^\nu v^\mu) + \frac{1}{2}[\cos(\sqrt{v^2/m^2}) - 1]$$
$$\times(\tilde{\omega}_M^{\mu\rho}P^\nu_{\parallel\rho}(v) - \tilde{\omega}_M^{\nu\rho}P^\mu_{\parallel\rho}(v)) - (\tilde{\omega}_{\mathcal{K}}^\mu v^\nu - \tilde{\omega}_{\mathcal{K}}^\nu v^\mu)\frac{\sin(\sqrt{v^2/m^2})}{m^2\sqrt{v^2/m^2}}.$$

Here the one-forms denoted by the $\tilde{\omega}$ are defined via the expansion

$$(e^{-i[\lambda_S^\alpha(x)S_\alpha + \bar{\lambda}_{\bar{S}\dot{\alpha}}\bar{S}^{\dot{\alpha}}]}e^{-iaR}e^{-i\varphi D}e^{-i[\lambda^\alpha(x)Q_\alpha + \bar{\lambda}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}]}e^{ix^\mu P_\mu})d(e^{-ix^\mu P_\mu}e^{i[\lambda^\alpha(x)Q_\alpha + \bar{\lambda}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}]}e^{i\varphi D}e^{iaR}e^{i[\lambda_S^\alpha(x)S_\alpha + \bar{\lambda}_{\bar{S}\dot{\alpha}}\bar{S}^{\dot{\alpha}}]})$$
$$= i[-\tilde{\omega}^\mu P_\mu + \tilde{\omega}_Q^\alpha Q_\alpha + \bar{\tilde{\omega}}_{\bar{Q}\dot{\alpha}}\bar{Q}^{\dot{\alpha}} + \tilde{\omega}_D D + \tilde{\omega}_R R - \tilde{\omega}_{\mathcal{K}}^\mu\mathcal{K}_\mu + \tilde{\omega}_S^\alpha S_\alpha + \bar{\tilde{\omega}}_{\bar{S}\dot{\alpha}}\bar{S}^{\dot{\alpha}} + \tfrac{1}{2}\tilde{\omega}_M^{\mu\nu}M_{\mu\nu}], \tag{3.12}$$

which is just the Maurer-Cartan form with $v^\mu = 0$. The individual one-forms with tildes then take the form

$$\tilde{\omega}^\mu = dx^\nu A_\nu{}^\mu e^\varphi\left(1 - \frac{1}{m^2}\lambda_S^2\bar{\lambda}_{\bar{S}}^2\right) + \frac{i}{m^2}(\lambda_S\sigma^\mu d\bar{\lambda} - d\lambda_S\sigma^\mu\bar{\lambda}_S) - \frac{2}{m^2}(\lambda_S\sigma^\mu\bar{\lambda}_S)da + \frac{2}{m^2}e^\varphi[e^{-ia}\lambda_{\bar{S}}^2(d\lambda\sigma^\mu\bar{\lambda}_{\bar{S}})$$
$$+ e^{+ia}\bar{\lambda}_{\bar{S}}^2(\lambda_S\sigma^\mu d\bar{\lambda})] \qquad \tilde{\omega}_D = d\varphi + 2ie^{\varphi - ia}(\lambda_S d\lambda) - 2ie^{\varphi + ia}(\bar{\lambda}_{\bar{S}}d\bar{\lambda}) = \tilde{\omega}^m e^{-\varphi}\tilde{\nabla}_m\varphi$$
$$\tilde{\omega}_R = da + 3e^\varphi[e^{-ia}(\lambda_S d\lambda) + e^{+ia}(\bar{\lambda}_{\bar{S}}d\bar{\lambda})] + 3dx^\nu A_\nu{}^\mu e^\varphi(\lambda_S\sigma_m\bar{\lambda}_{\bar{S}}) \equiv dx^\mu\nabla_\mu a$$
$$\tilde{\omega}_Q^\alpha = e^\varphi(e^{-ia}d\lambda - dx^\mu A_\mu{}^\nu(\bar{\lambda}_{\bar{S}}\bar{\sigma}_\nu)^\alpha) \qquad \bar{\tilde{\omega}}_{\bar{Q}\dot{\alpha}} = e^\varphi(e^{+ia}d\bar{\lambda} + dx^\mu A_\mu{}^\nu(\lambda_S\sigma_\nu)_{\dot{\alpha}})$$
$$\tilde{\omega}_S^\alpha = d\lambda_S^\alpha + \frac{1}{2}d\varphi\lambda_S^\alpha - ida\lambda_S^\alpha - idx^\mu A_\mu{}^\nu e^\varphi(\lambda_S\sigma_\nu\bar{\lambda}_{\bar{S}})\lambda_S^\alpha$$
$$+ \frac{i}{6}(\lambda_S\sigma^\tau\bar{\lambda}_{\bar{S}})\epsilon_{\mu\nu\rho\tau}(\lambda_S\sigma^{\mu\nu})^\alpha e^\varphi dx^\lambda A_\lambda{}^\rho - ie^\varphi[e^{-ia}\lambda_S d\lambda + 2e^{+ia}\bar{\lambda}_{\bar{S}}d\bar{\lambda}]\lambda_S^\alpha - \frac{i}{4}e^{\varphi - ia}(d\lambda\sigma^{\mu\nu}\lambda_S)(\lambda_S\sigma_{\mu\nu})^\alpha \tag{3.13}$$
$$\bar{\tilde{\omega}}_{\bar{S}\dot{\alpha}} = d\bar{\lambda}_{\bar{S}\dot{\alpha}} + \frac{1}{2}d\varphi\bar{\lambda}_{\bar{S}\dot{\alpha}} + ida\bar{\lambda}_{\bar{S}\dot{\alpha}} + idx^\mu A_\mu{}^\nu e^\varphi(\lambda_S\sigma_\nu\bar{\lambda}_{\bar{S}})\bar{\lambda}_{\bar{S}\dot{\alpha}} + \frac{i}{6}(\lambda_S\sigma^\tau\bar{\lambda}_{\bar{S}})\epsilon_{\mu\nu\rho\tau}(\bar{\lambda}_{\bar{S}}\bar{\sigma}^{\mu\nu})_{\dot{\alpha}}e^\varphi dx^\lambda A_\lambda{}^\rho$$
$$+ ie^\varphi[2e^{-ia}\lambda_S d\lambda + e^{+ia}\bar{\lambda}_{\bar{S}}d\bar{\lambda}]\bar{\lambda}_{\bar{S}\dot{\alpha}} + \frac{i}{4}e^{\varphi + ia}(d\bar{\lambda}\bar{\sigma}^{\mu\nu}\bar{\lambda}_{\bar{S}})(\bar{\lambda}_{\bar{S}}\bar{\sigma}_{\mu\nu})_{\dot{\alpha}}$$
$$\tilde{\omega}_{\mathcal{K}}^\mu = 2m^2(dx^\nu A_\nu{}^\mu e^\varphi - \tilde{\omega}^\mu)$$
$$\tilde{\omega}_M^{\mu\nu} = e^\varphi[2(e^{-ia}d\lambda\sigma^{\mu\nu}\lambda_S - e^{+ia}d\bar{\lambda}\bar{\sigma}^{\mu\nu}\bar{\lambda}_{\bar{S}}) + 2\epsilon^{\mu\nu}_{\rho\tau}(\lambda_S\sigma^\tau\bar{\lambda}_{\bar{S}})dx^\lambda A_\lambda{}^\rho],$$

where the Akulov-Volkov vierbein is defined as

$$A_\mu{}^\nu = [\delta_\mu{}^\nu + i(\lambda\sigma^\nu \partial_\mu \bar{\lambda} - \partial_\mu \lambda \sigma^\nu \bar{\lambda})]. \tag{3.14}$$

The vierbein $e_\mu{}^\nu$ relates the coordinate differentials dx^μ to the covariant coordinate differentials ω^ν according to

$$\omega^\nu = dx^\mu e_\mu{}^\nu. \tag{3.15}$$

Since the one-form $\tilde{\omega}^\mu$ can also act as a basis one-form, one can expand

$$\omega^\mu = dx^\nu e_\nu{}^\mu = \tilde{\omega}^\nu N_\nu{}^\mu. \tag{3.16}$$

where using Eq. (3.11), $N_\nu{}^\mu$ is extracted as

$$N_\nu{}^\mu = \delta_\nu{}^\mu + [\cos(\sqrt{v^2/m^2}) - 1] P_{\parallel\nu}{}^\mu(v) - e^{-\varphi}\tilde{\nabla}_\nu \varphi \frac{v^\mu}{m^2} \frac{\sin(\sqrt{v^2/m^2})}{\sqrt{v^2/m^2}}. \tag{3.17}$$

It is also useful to define the vierbein $\tilde{e}_\mu{}^\nu$ as

$$\tilde{\omega}^\mu = dx^\nu \tilde{e}_\nu{}^\mu. \tag{3.18}$$

so that

$$e_\mu{}^\nu = \tilde{e}_\mu{}^\rho N_\rho{}^\nu. \tag{3.19}$$

Using the Akulov-Volkov vierbein, $A_\mu{}^\nu$, the $\tilde{\omega}^\mu$ one-form can be expanded as

$$\tilde{\omega}^\mu = dx^\nu \tilde{e}_\nu{}^\mu = dx^\nu e^\varphi A_\nu{}^\rho T_\rho{}^\mu, \tag{3.20}$$

where $T_\nu{}^\mu$ can be gleaned from Eq. (3.13) as

$$\begin{aligned} T_\nu{}^\mu = {}& \delta_\nu{}^\mu\left(1 - \frac{1}{m^2}\lambda_S^2 \bar{\lambda}_{\bar{S}}^2\right) + \frac{i}{m^2}(\lambda_S \sigma^\mu \mathcal{D}_\nu \bar{\lambda}_{\bar{S}} \\ & - \mathcal{D}_\nu \lambda_S \sigma^\mu \bar{\lambda}_S) e^{-\varphi} - \frac{2}{m^2}(\lambda_S \sigma^\mu \bar{\lambda}_S)\mathcal{D}_\nu a e^{-\varphi} \\ & + \frac{2}{m^2}[\lambda_S^2(\mathcal{D}_\nu \lambda \sigma^\mu \bar{\lambda}_{\bar{S}}) e^{-ia} + \bar{\lambda}_{\bar{S}}^2(\lambda_S \sigma^\mu \mathcal{D}_\nu \bar{\lambda}) e^{+ia}], \end{aligned} \tag{3.21}$$

with $\mathcal{D}_m = A_m^{-1\mu}\partial_\mu$. Using Eqs. (3.19) and (3.20), the vierbein can be written in a product form as

$$e_\mu{}^\nu = e^\varphi A_\mu{}^\rho T_\rho{}^\tau N_\tau{}^\nu. \tag{3.22}$$

Since $d^4x\,\det e$ is invariant and an invariant kinetic energy for the R-axion can be formed by contracting the covariant derivatives with the vierbein, an $SU(2,2|1)$ invariant action is constructed as

$$\begin{aligned} S &= -\sigma \int d^4x \det e(1 + \nabla_\mu a e_\rho^{-1\mu} \eta^{\rho\tau} e_\tau^{-1\nu} \nabla_\nu a) \\ &= -\sigma \int d^4x e^{4\varphi} \det A \det N \det T (1 + e^{-2\varphi} \nabla_\mu a h^{\mu\nu} \nabla_\nu a), \end{aligned} \tag{3.23}$$

where

$$h^{\mu\nu} = N_\rho^{-1\tau} T_\tau^{-1\lambda} A_\lambda^{-1\mu} \eta^{\rho\upsilon} N_\upsilon^{-1\kappa} T_\kappa^{-1\sigma} A_\sigma^{-1\nu}. \tag{3.24}$$

The determinant of N can then be explicitly evaluated giving

$$\det N = \cos(\sqrt{v^2/m^2})\left[1 + e^{-\varphi}\tilde{\nabla}_\mu \varphi \frac{v^\mu}{m^2} \frac{\tan(\sqrt{v^2/m^2})}{\sqrt{v^2/m^2}}\right]. \tag{3.25}$$

Since the action only depends on v^μ and not its derivatives, it is not an independent dynamical degree of freedom. As such it can be eliminated [12] by setting the invariant one-form ω_D to zero. Solving this constraint equation then gives

$$v_\mu \frac{\tan(\sqrt{v^2/m^2})}{\sqrt{v^2/m^2}} = -e^{-\varphi}\tilde{\nabla}_\mu \varphi. \tag{3.26}$$

which, in turn, allows us to write

$$\begin{aligned} N_\nu{}^\mu = {}& \delta_\nu{}^\mu + \left(\frac{1}{\sqrt{1 + \frac{e^{-2\varphi}}{m^2}\tilde{\nabla}_\rho \varphi \eta^{\rho\tau} \tilde{\nabla}_\tau \varphi}} - 1\right) P_{\parallel\nu}{}^\mu(\tilde{\nabla}\varphi) \\ & - \frac{1}{\sqrt{1 + \frac{e^{-2\varphi}}{m^2}\tilde{\nabla}_\rho \varphi \eta^{\rho\tau} \tilde{\nabla}_\tau \varphi}} \frac{e^{-2\varphi}}{m^2} \tilde{\nabla}_\nu \varphi \tilde{\nabla}^\mu \varphi. \end{aligned} \tag{3.27}$$

The superconformal Goldstinos, λ_S and $\bar{\lambda}_{\bar{S}}$, are also not independent dynamical degrees of freedom but can be expressed in terms of derivatives of the Goldstinos λ and $\bar{\lambda}$, and products of these with the Nambu-Goldstone bosons φ and a. The covariant constraint equation is obtained by setting to zero the fermionic one-forms $\omega_Q^\alpha = 0$ and $\bar{\omega}_{\bar{Q}\dot{\alpha}} = 0$. Combining the various one-forms in (3.11) and (3.13), the solution to these covariant constraints begins as

$$\begin{aligned} \lambda_S^\alpha &= -\tfrac{1}{4}(\sigma^\mu \partial_\mu \bar{\lambda})^\alpha + \cdots \\ \bar{\lambda}_{\bar{S}\dot{\alpha}} &= -\tfrac{1}{4}(\partial_\mu \lambda \sigma^\mu)_{\dot{\alpha}} + \cdots. \end{aligned} \tag{3.28}$$

Substituting the above expression for $N_\nu{}^\mu$, the invariant action then takes the form

$$S = -\sigma \int d^4x e^{4\varphi} \det A \det T \sqrt{1 + \frac{e^{-2\varphi}}{m^2}\tilde{\nabla}_\mu \varphi \eta^{\mu\nu} \tilde{\nabla}_\nu \varphi} \times (1 + e^{-2\varphi}\nabla_\mu a h^{\mu\nu} \nabla_\nu a). \tag{3.29}$$

The action is an invariant synthesis of Akulov-Volkov and Nambu-Goto actions. Note that the pure dilatonic part of the action (obtained by setting the Goldstinos and a to zero so that $A_\mu{}^\nu = T_\mu{}^\nu = \delta_\mu^\nu$) reproduces the previous action, Eq. (2.13), of the Minkowski space M_4 probe brane in AdS_5 without SUSY. As such, the dilaton describes the motion of the probe brane into the rest of the AdS_5 space. However, in this case, because of the spontaneous breakdown of the complete SUSY, there is no invariant that can be added to the action to cancel the vacuum energy as one was able to achieve in the nonsupersymmetric Minkowski space probe brane case [cf. Eq. (2.17)]. It follows that the dilaton dynamics feels an $e^{4\varphi}$ potential. This contains a destabilizing term linear in φ which drives the dilaton field $\varphi \to -\infty$. Since the dilaton describes the motion of the probe Minkowski M_4 brane into the remainder of AdS_5 space, it follows that the Minkowski space brane is driven to the infinite boundary of AdS_5 space and the interior of the AdS_5 space cannot sustain the brane. Alternatively expressed, the spectrum cannot include both the Goldstino and the dilaton as Nambu-Goldstone modes.

An alternate combination of broken generators $K^\mu = \frac{1}{m^2}(\hat{P}^\mu - 2m^2\mathcal{K}^\mu)$ can also be defined. This leads to the 4D superconformal algebra

$$
\begin{gathered}
[P^\mu, P^\nu] = 0; \quad [K^\mu, K^\nu] = 0; \quad [P^\mu, K^\nu] = 2i(\eta^{\mu\nu}D - M^{\mu\nu}) \quad [D, P^\mu] = iP^\mu; \quad [D, K^\mu] = -iK^\mu; \\
[D, M^{\mu\nu}] = 0 \quad [M^{\mu\nu}, P^\lambda] = i(\eta^{\mu\lambda}P^\nu - \eta^{\nu\lambda}P^\mu); \quad [M^{\mu\nu}, K^\lambda] = i(\eta^{\mu\lambda}K^\nu - \eta^{\nu\lambda}K^\mu) \\
[M^{\mu\nu}, M^{\lambda\rho}] = i(\eta^{\mu\lambda}M^{\nu\rho} - \eta^{\mu\rho}M^{\nu\lambda} - \eta^{\nu\lambda}M^{\mu\rho} + \eta^{\nu\rho}M^{\mu\lambda}) \quad [R, P^\mu] = 0; \quad [R, K^\mu] = 0; [R, M^{\mu\nu}] = 0 \\
[P^\mu, Q_\alpha] = 0; \quad [P^\mu, \bar{Q}_{\dot\alpha}] = 0 \quad \{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu; \quad \{S_\alpha, \bar{S}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}K_\mu \quad \{Q_\alpha, Q_\beta\} = 0; \\
\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0 \quad [M^{\mu\nu}, Q_\alpha] = -\frac{1}{2}\sigma^{\mu\nu}{}_\alpha{}^\beta Q_\beta; \quad [M^{\mu\nu}, \bar{Q}_{\dot\alpha}] = -\frac{1}{2}\bar{\sigma}^{\mu\nu}{}_{\dot\alpha\dot\beta}\bar{Q}^{\dot\beta} \quad [R, Q_\alpha] = Q_\alpha; \\
[R, \bar{Q}_{\dot\alpha}] = -\bar{Q}_{\dot\alpha} \quad [D, Q_\alpha] = \frac{i}{2}Q_\alpha; \quad [D, \bar{Q}_{\dot\alpha}] = \frac{i}{2}\bar{Q}_{\dot\alpha} \quad [R, S_\alpha] = -S_\alpha; \quad [R, \bar{S}_{\dot\alpha}] = \bar{S}_{\dot\alpha} \\
[D, S_\alpha] = -\frac{i}{2}S_\alpha; \quad [D, \bar{S}_{\dot\alpha}] = -\frac{i}{2}\bar{S}_{\dot\alpha} \quad [K^\mu, Q_\alpha] = i\sigma^\mu_{\alpha\dot\alpha}\bar{S}^{\dot\alpha}; \quad [K^\mu, \bar{Q}_{\dot\alpha}] = iS^\alpha\sigma^\mu_{\alpha\dot\alpha} \quad [P^\mu, S_\alpha] = i\sigma^\mu_{\alpha\dot\alpha}\bar{Q}^{\dot\alpha}; \\
[P^\mu, \bar{S}_{\dot\alpha}] = iQ^\alpha\sigma^\mu_{\alpha\dot\alpha} \quad [M^{\mu\nu}, S_\alpha] = -\frac{1}{2}\sigma^{\mu\nu}{}_\alpha{}^\beta S_\beta; \quad [M^{\mu\nu}, \bar{S}_{\dot\alpha}] = -\frac{1}{2}\bar{\sigma}^{\mu\nu}{}_{\dot\alpha\dot\beta}\bar{S}^{\dot\beta} \quad [K^\mu, S_\alpha] = 0; \\
[K^\mu, \bar{S}_{\dot\alpha}] = 0 \quad \{Q_\alpha, S_\beta\} = i(\sigma^{\mu\nu}_{\alpha\beta}M_{\mu\nu} + 2i\epsilon_{\alpha\beta}D + 3\epsilon_{\alpha\beta}R) \quad \{\bar{Q}_{\dot\alpha}, \bar{S}_{\dot\beta}\} = -i(\bar{\sigma}^{\mu\nu}_{\dot\alpha\dot\beta}M_{\mu\nu} - 2i\epsilon_{\dot\alpha\dot\beta}D + 3\epsilon_{\dot\alpha\dot\beta}R) \\
\{Q_\alpha, \bar{S}_{\dot\alpha}\} = 0; \quad \{S_\alpha, \bar{Q}_{\dot\alpha}\} = 0; \quad \{S_\alpha, S_\beta\} = 0; \quad \{\bar{S}_{\dot\alpha}, \bar{S}_{\dot\beta}\} = 0.
\end{gathered} \tag{3.30}
$$

The spontaneously broken symmetries are R, dilatations (D), special conformal (K^μ), SUSY ($Q_\alpha, \bar{Q}_{\dot\alpha}$) and SUSY conformal ($S_\alpha, \bar{S}_{\dot\alpha}$). Since the generators K^μ and $\mathcal{K}^\mu$ differ only by unbroken translation generator P^μ, the action (3.29) is invariant under superconformal transformations. The leading terms in a momentum expansion are just

$$
S = -\sigma \int d^4x\{e^{4\varphi}\det A - \tfrac{1}{2}\det A e^{2\varphi}\mathcal{D}_\mu\varphi\eta^{\mu\nu}\mathcal{D}_\nu\varphi - \tfrac{1}{2}\det A e^{2\varphi}\mathcal{D}_\mu a\eta^{\mu\nu}\mathcal{D}_\nu a\}. \tag{3.31}
$$

Once again the potential for the dilaton φ is unstable and there is an incompatibility of simultaneous nonlinear realizations of SUSY and scale symmetry in Minkowski space [13]. Note that the origin of this unusual behavior is not simply a consequence of the introduction of a scale due the spontaneously broken SUSY. It has been shown that there is no incompatibility in securing simultaneous nonlinear realization of spontaneously broken scale and chiral symmetries [14] where a scale is also introduced. In that case, the spectrum of the effective Lagrangian admits both a pion and a dilaton.

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