

Virtual gravitons and brane field scattering in the Randall-Sundrum model with a small curvature

A. V. Kisselev*

Institute for High Energy Physics, 142281 Protvino, Russia
(Received 27 November 2005; published 11 January 2006)

The contribution of virtual s -channel Kaluza-Klein (KK) gravitons to high energy scattering of the standard model (SM) fields in the Randall-Sundrum (RS) model with two branes is studied. The small curvature option of the RS model is considered in which the KK gravitons are narrow low-mass spin-2 resonances. The analytical tree-level expression for a process-independent gravity part of the scattering amplitude is derived, accounting for nonzero graviton widths. It is shown that one cannot get a correct result, if a series of graviton resonances is replaced by a continuous mass distribution, in spite of the small graviton mass splitting. Such a replacement appeared to be justified only in the trans-Planckian energy region.

DOI: [10.1103/PhysRevD.73.024007](https://doi.org/10.1103/PhysRevD.73.024007)

PACS numbers: 04.50.+h

I. RS SCENARIO WITH THE SMALL CURVATURE

The models with spacial extra dimensions (ED's) have pretensions to solving theoretical problems which have not yet received a satisfactory answer (such as hierarchy problem, proton lifetime, hierarchy of fermion masses and mixing angles, etc.) and lead to a new phenomenology in the TeV energy region. The multidimensional gravity is strong, and the fundamental Planck scale can be related with the string scale. One manifestation of theories with ED's is the existence of Kaluza-Klein (KK) gravitons and their interactions with the SM fields.

In the present paper, we consider one realization of the ED theory in a slice of the AdS₅ space-time with the following background warped metric:

$$ds^2 = e^{2\kappa(\pi r - |y|)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (1)$$

where $y = r\theta$ ($-\pi \leq \theta \leq \pi$), r being the "radius" of extra dimension, and $\eta_{\mu\nu}$ is the Minkowski metric. The parameter κ defines a 5-dimensional scalar curvature of the AdS₅ space.

We will be interested in the Randall-Sundrum (RS) model [1] which has two 3D branes with equal and opposite tensions located at the point $y = \pi r$ (called the *TeV brane*, or *visible brane*) and point $y = 0$ (referred to as the *Planck brane*). If $k > 0$, then the tension on the TeV brane is negative, whereas the tension on the Planck brane is positive. All the SM fields are constrained to the TeV brane, while the gravity propagates in five dimensions.

The main goal of the paper is to estimate s -channel graviton contribution to the scattering of the *brane* fields in such a scheme.

Let us note that the warp factor in the metric (1) is equal to 1 on the negative tension (visible) brane, and a correct determination of particle masses on this brane is thus achieved [2]. By calculating the zero mode sector of the effective theory, one then obtains the "hierarchy relation",

$$\bar{M}_{\text{Pl}}^2 = \frac{\bar{M}_5^3}{\kappa} (e^{2\pi\kappa r} - 1), \quad (2)$$

with \bar{M}_5 being a 5-dimensional Planck scale.

From the point of view of an observer located on the TeV brane, there exists an infinite number of graviton KK excitations with masses

$$m_n = x_n \kappa, \quad n = 1, 2, \dots, \quad (3)$$

where x_n are zeros of the Bessel function $J_1(x)$, with

$$x_n = \pi(n + \frac{1}{4}) + O(n^{-1}). \quad (4)$$

Note that *all* zeros of $J_1(x)/x$ are simple ones, and that they are *real positive* numbers [3].¹

The interaction Lagrangian on the TeV brane looks like (with the radion field omitted)

$$\mathcal{L} = -\frac{1}{\bar{M}_{\text{Pl}}} T^{\mu\nu} G_{\mu\nu}^{(0)} - \frac{1}{\Lambda_\pi} T^{\mu\nu} \sum_{n=1}^{\infty} G_{\mu\nu}^{(n)}. \quad (5)$$

Here $T^{\mu\nu}$ is the energy-momentum tensor of the matter on the brane, $G_{\mu\nu}^{(n)}$ is the graviton field with the KK number n , and

$$\Lambda_\pi = \bar{M}_5 \left(\frac{\bar{M}_5}{\kappa} \right)^{1/2} \quad (6)$$

is a physical scale on the TeV brane (here and in what follows, we neglect small corrections $O(e^{-\pi\kappa r})$).

To get $m_1 \sim 1$ TeV, the parameters of the model are usually taken to be $\kappa \sim \bar{M}_5 \sim 1$ TeV. Then one obtains a series of massive graviton resonances in the TeV region which interact rather strongly with the SM particles, since $\Lambda_\pi \sim 1$ TeV.

In the present paper we will consider a different scenario which we call "**small curvature option**" [4,5]:

$$\kappa \ll \bar{M}_5 \sim 1 \text{ TeV}. \quad (7)$$

¹The minimal positive zero of the Bessel function $J_\nu(z)$ is approximately equal to $\nu + 1.86\nu^{1/3}$ [3].

*Electronic address: alexandre.kisselev@mail.ihep.ru

In such a scheme, the scale Λ_π appears to be significantly larger than the gravity scale \bar{M}_5 , as one can see from Eq. (6). Namely, we get $\Lambda_\pi = 100(\bar{M}_5/\text{TeV})^{3/2} \times (100 \text{ MeV}/\kappa)^{1/2} \text{ TeV}$. Nevertheless, for both real and virtual graviton production, a magnitude of a scattering amplitude is defined by the 5-dimensional Planck scale \bar{M}_5 , not by Λ_π or κ , separately (see formulas in the next Section).²

Contrary to the case $\kappa \sim \bar{M}_5 \sim 1 \text{ TeV}$, there exists a series of very narrow low-mass spin-2 resonances with an almost continuous mass distribution. For such a case, the following inequalities were derived in Ref. [5]:³

$$10^{-5} \leq \frac{\kappa}{\bar{M}_5} \leq 0.1. \quad (8)$$

Notice, in order the hierarchy relation for the warped metric (2) to be satisfied, we have to put $\kappa r \approx 10$.

It is worth to underline that the AdS₅ space-time differs significantly from a 5-dimensional flat space-time with one large ED even for very small κ (i.e. for the small curvature). Indeed, let us consider the hierarchy relation for d compact ED's of the size R_c :

$$\bar{M}_{\text{Pl}}^2 = (2\pi R_c)^d \bar{M}_{4+d}^{2+d}, \quad (9)$$

where \bar{M}_{4+d} is a gravity scale in the flat space-time with d compact ED's. For $d = 1$, Eq. (9) is a particular case of (2) in the limit $2\pi\kappa r \ll 1$. However, the condition $2\pi\kappa r \ll 1$ means that the ratio \bar{M}_5/κ should be unrealistically large:

$$\frac{\bar{M}_5}{\kappa} \gg \left(\frac{\bar{M}_{\text{Pl}}}{\bar{M}_5}\right)^2. \quad (10)$$

This inequality means, for example, that $\kappa \ll 10^{-22} \text{ eV}$, if $\bar{M}_5 = 1 \text{ TeV}$.

The present astrophysical bounds [8] rule out the possibility $d = 1$ and significantly restrict the parameter space for $d = 2, 3$.⁴ The most stringent constraints come from neutron-star (NS) excess heat due to the trapped cloud of the KK gravitons surrounding the NS (for details, see the second paper in [8]). For instance, one gets $R_c^{-1} > 4.4 \cdot 10^{-12} \text{ GeV}$ (and, correspondingly, $\bar{M}_{4+1} > 1.6 \cdot 10^5 \text{ TeV}$) for $d = 1$.

Fortunately, the above mentioned restrictions can not be directly applied to the AdS₅ space-time, since they were derived in the soft radiation approximation, $\omega \ll T$, where ω is the graviton energy, while T is the temperature of the nuclear medium (for instance, in the NS). Equation (8)

²Contrary to the KK gravitons, the radion will be hardly produced, since the radion coupling to SM fields is equal to $1/(\sqrt{3}\Lambda_\pi)$. The radion-Higgs mixing (if it exists, see [6]) will be also suppressed.

³For the case $\kappa \sim \bar{M}_5 \sim \bar{M}_{\text{Pl}}$, analogous bounds look like $0.01 \leq \kappa/\bar{M}_{\text{Pl}} \leq 0.1$ [7].

⁴If one insists that \bar{M}_{4+1} should be of order of few TeV, the case $d = 1$ is completely excluded, since R_c exceeds the size of the solar system.

means that $\omega \geq m_1 = 38(\bar{M}_5/\text{TeV}) \text{ MeV}$. On the other hand, a typical value of T is equal to 30 MeV. Thus, for $\bar{M}_5 \geq 1 \text{ TeV}$ the condition $\omega \ll T$ is not satisfied, even for $\kappa = 10(\bar{M}_5/\text{TeV}) \text{ MeV}$.

II. VIRTUAL s -CHANNEL GRAVITONS

Let us study the scattering of two SM fields mediated by massive graviton exchanges in the s -channel,

$$a\bar{a} \rightarrow G^{(n)} \rightarrow b\bar{b}, \quad (11)$$

where $a(b) = e^-, \gamma, q, g$, etc. For instance, in hadron collisions virtual graviton effects could be seen in the processes $pp \rightarrow 2 \text{ jets} + X$, $pp \rightarrow \gamma\gamma + X$, and Drell-Yan process $pp \rightarrow l^+l^- + X$. At linear colliders, the promising reactions are $e^+e^- \rightarrow \gamma\gamma$ and $e^+e^- \rightarrow f\bar{f}$. In what follows, the invariant energy of the process, \sqrt{s} , is assumed to be around 1 TeV.⁵ It means that we are working in the following region:

$$\Lambda_\pi \gg \sqrt{s} \sim M_5 \gg \kappa. \quad (12)$$

The matrix element of the process (11) looks like

$$\mathcal{M} = \mathcal{A} S. \quad (13)$$

The first factor in Eq. (13) contains the following contraction of tensors:

$$\mathcal{A} = T_{\mu\nu}^a P^{\mu\nu\alpha\beta} T_{\alpha\beta}^b = T_{\mu\nu}^a T^{b\mu\nu} - \frac{1}{3}(T^a)_\mu^\mu (T^b)_\nu^\nu, \quad (14)$$

where $P^{\mu\nu\alpha\beta}$ is a tensor part of the graviton propagator, while $T_{\mu\nu}^{a(b)}$ is the energy-momentum tensor of the field $a(b)$.

A. Nonzero graviton widths

We will concentrate on the second factor in Eq. (13) which is universal for all types of processes mediated by the s -channel exchanges of the KK gravitons. It is of the form:

$$S(s) = \frac{1}{\Lambda_\pi^2} \sum_{n=1}^{\infty} \frac{1}{s - m_n^2 + im_n \Gamma_n}. \quad (15)$$

Here Γ_n denotes the total width of the graviton with the KK number n and mass m_n .

This sum was estimated in Ref. [4] in a *zero width approximation* (assuming $\Gamma_n = 0$ for all n). It was shown that $S(s)$ is purely imaginary in this limit (if no ultraviolet (UV) cutoff is imposed). Thus, there is no interference of ED effects with SM contributions to the same processes.

The width of the massive graviton is indeed very small if its KK-number n is not too large [9]:

⁵The energy region $\sqrt{s} \gg 1 \text{ TeV}$ will be also briefly considered, see Eqs. (32)–(35).

$$\frac{\Gamma_n}{m_n} = \eta \left(\frac{m_n}{\Lambda_\pi} \right)^2, \quad (16)$$

where $\eta \simeq 0.09$.⁶ However, the main contribution to the sum (15) comes from the region $n \sim \sqrt{s}/\kappa \gg 1$. So, non-zero widths of the gravitons in the RS model with the small curvature should be taken into account. That is why we will study a general case ($\Gamma_n \neq 0$). For comparison, a particular case (all $\Gamma_n = 0$) will be also analyzed (see subsection II C).

It is useful to present $S(s)$ in the form:

$$S(s) = \sum_{n=1}^{\infty} \frac{1}{ax_n^4 - bx_n^2 + c} = \frac{1}{a(\sigma^2 - \rho^2)} \sum_{n=1}^{\infty} \left[\frac{1}{x_n^2 - \sigma^2} - \frac{1}{x_n^2 - \rho^2} \right], \quad (17)$$

with

$$a = i\eta\kappa^4, \quad b = (\kappa\Lambda_\pi)^2, \quad c = s\Lambda_\pi^2. \quad (18)$$

Here

$$\sigma^2 = \frac{s}{\kappa^2} \frac{2}{1 + \sqrt{1 - 4i\eta \frac{s}{\Lambda_\pi^2}}} \quad (19)$$

and

$$\rho^2 = \frac{1}{2i\eta} \left(\frac{\Lambda_\pi}{\kappa} \right)^2 \left[1 + \sqrt{1 - 4i\eta \frac{s}{\Lambda_\pi^2}} \right] \quad (20)$$

are zeros of the quadratic equation $az^2 - bz + c = 0$. In the kinematical region (12), the parameter σ (19) can be approximated as

$$\sigma \simeq \frac{\sqrt{s}}{\kappa} + \frac{i\eta}{2} \left(\frac{\sqrt{s}}{\bar{M}_5} \right)^3, \quad (21)$$

with $|\sigma| \gg 1$.

The sum in Eq. (17) can be calculated analytically by the use of the formula [3]

$$\sum_{n=1}^{\infty} \frac{1}{z_{n,\nu}^2 - z^2} = \frac{1}{2z} \frac{J_{\nu+1}(z)}{J_\nu(z)}, \quad (22)$$

where $z_{n,\nu}$ ($n = 1, 2, \dots$) are zeros of the function $z^{-\nu}J_\nu(z)$. As a result, we obtain:

$$S(s) = -\frac{1}{2\kappa\bar{M}_5^3} \frac{1}{\sqrt{1 - 4i\eta \frac{s}{\Lambda_\pi^2}}} \left[\frac{1}{\sigma} \frac{J_2(\sigma)}{J_1(\sigma)} - \frac{1}{\rho} \frac{J_2(\rho)}{J_1(\rho)} \right]. \quad (23)$$

⁶To estimate Γ_n , masses of the SM particles were neglected with respect to the graviton mass m_n [9]. This approximation is well justified since the sum in n (15) is mainly given by the gravitons with the masses $m_n \sim \sqrt{s} \sim 1$ TeV.

Since $\eta s/\Lambda_\pi^2 \ll 1$, we have the inequality:

$$|\rho| \simeq \frac{1}{\sqrt{\eta}} \frac{\Lambda_\pi}{\kappa} \gg |\sigma|, \quad (24)$$

and Eq. (23) becomes

$$S(s) = -\frac{1}{2\kappa\bar{M}_5^3} \frac{1}{\sigma} \frac{J_2(\sigma)}{J_1(\sigma)}, \quad (25)$$

with σ given by Eq. (21) (here and in what follows, small corrections like $O(\kappa/\sqrt{s})$ are omitted). The function $S(s)$ has no singularities, as all zeros of $J_1(z)$ are real, and $\text{Im}\sigma \neq 0$ at physical s .

By using asymptotic behavior of the Bessel function [3] and formulas (A4) in Appendix A, we obtain from (25):

$$S(s) = -\frac{1}{4\bar{M}_5^3\sqrt{s}} \frac{\sin 2A + i \sinh 2\varepsilon}{\cos^2 A + \sinh^2 \varepsilon}, \quad (26)$$

where

$$A = \frac{\sqrt{s}}{\kappa} + \frac{\pi}{4}, \quad \varepsilon = \frac{\eta}{2} \left(\frac{\sqrt{s}}{\bar{M}_5} \right)^3. \quad (27)$$

Formulas (25) and (26) are **our main result**.

The following inequalities immediately result from (26):

$$-\coth \varepsilon \leq \text{Im}\tilde{S}(s) \leq -\tanh \varepsilon \quad (28)$$

$$|\text{Re}\tilde{S}(s)| \leq \frac{1}{1 + 2\sinh^2 \varepsilon}, \quad (29)$$

$$\left| \frac{\text{Re}\tilde{S}(s)}{\text{Im}\tilde{S}(s)} \right| \leq \frac{1}{\sinh 2\varepsilon}, \quad (30)$$

where the notation $\tilde{S}(s) = [2\bar{M}_5^3\sqrt{s}]S(s)$ is introduced. Note that the upper bound for the ratio $|\text{Re}\tilde{S}(s)/\text{Im}\tilde{S}(s)|$ decreases rapidly with energy, and it becomes as small as 0.08 at $\sqrt{s} = 3\bar{M}_5$. For comparison, this bound is equal to 0.85 at $\sqrt{s} = \bar{M}_5$. The absolute value of $\text{Im}\tilde{S}(s)$ tends to 1 when s grows. For instance, we find $0.98 \leq |\text{Im}\tilde{S}(s)| \leq 1.02$ at $\sqrt{s} = 3\bar{M}_5$.

If $\sqrt{s} \leq \bar{M}_5$, the parameter ε is numerically small, and we obtain that

$$S(s) = -\frac{i}{\eta s^2} \left[1 + \frac{\varepsilon^2}{3} + O(\varepsilon^4) \right] \quad (31)$$

at $\sqrt{s} = z_0\kappa$, where z_0 is some zero of the Bessel function $J_1(z)$. Thus, the value of $S(s)$ at the point $\sqrt{s} = z_0\kappa$ is actually defined by the graviton with the mass $m_n = \sqrt{s}$, while the relative contributions from other KK gravitons are suppressed at least by the factor $\eta^2/12 \approx 7 \cdot 10^{-4}$.

At ultrahigh energies, namely, at

$$\sqrt{\eta s} \gg \Lambda_\pi, \quad (32)$$

our sum can be approximated as follows:

$$S(s) = \sum_{n=1}^{\infty} \frac{1}{ax_n^4 + c}, \quad (33)$$

with the parameters a and c being defined above (18). The sum (33) has the following asymptotics (see Appendix A):

$$S(s)|_{\sqrt{\eta s} \gg \Lambda_\pi} \simeq e^{-iB} \frac{1}{2\sqrt{2}} \frac{1}{\bar{M}_5^3 \sqrt{s}} \left(\frac{\Lambda_\pi^2}{\eta s} \right)^{1/4}, \quad (34)$$

where

$$B = 2\sqrt{2} \left(\sin \frac{\pi}{8} \right) \frac{1}{\kappa} \left(\frac{s \Lambda_\pi^2}{\eta} \right)^{1/4} + \frac{\pi}{8}. \quad (35)$$

Notice, the condition $\sqrt{\eta s} \gg \Lambda_\pi$ means that we are working in the energy region $\sqrt{s} \gg 100(M_5/\text{TeV})^{3/2} \times (100 \text{ MeV}/\kappa)^{1/2} \text{ TeV}$.

B. Continuous mass spectrum of the gravitons

Since $x_n \simeq \pi n$ at large n ,⁷ the mass splitting, $\Delta m_{\text{KK}} \simeq \pi\kappa$, is very small with respect to the energy, and it seems reasonable to approximate a summation in n by integration over graviton mass m_{KK} .

Let us calculate the real and imaginary parts of $S(s)$ separately and compare the results of these calculations with Eq. (26). We start from the following expressions:

$$\text{Re } S(s) = \frac{1}{(\kappa\Lambda_\pi)^2} \sum_{n=1}^{\infty} \frac{\alpha - x_n^2}{(\alpha - x_n^2)^2 + \beta x_n^8}, \quad (36)$$

$$\text{Im } S(s) = -\frac{\eta}{\Lambda_\pi^4} \sum_{n=1}^{\infty} \frac{x_n^4}{(\alpha - x_n^2)^2 + \beta x_n^8}, \quad (37)$$

with $\alpha = s/\kappa^2$ and $\beta = \eta^2(\kappa/\Lambda_\pi)^4$. The usual way of calculating Eqs. (36) and (37) is to replace them by the integrals:

$$\text{Re } S(s) = \frac{1}{\pi} \frac{1}{\sqrt{s}\bar{M}_5^3} \int_{(\pi\kappa)/\sqrt{s}}^{\infty} dz \frac{1 - z^2}{(1 - z^2)^2 + \delta z^8}, \quad (38)$$

$$\text{Im } S(s) = -\frac{\eta}{\pi} \frac{\sqrt{s}}{\kappa\Lambda_\pi^4} \int_{(\pi\kappa)/\sqrt{s}}^{\infty} dz \frac{z^4}{(1 - z^2)^2 + \delta z^8}, \quad (39)$$

where

$$\delta = \left(\frac{\sqrt{\eta s}}{\Lambda_\pi} \right)^4 \ll 1. \quad (40)$$

These integrals are estimated in Appendix B. The result of the calculations is the following:

$$\text{Re } S(s) \simeq \frac{1}{2\bar{M}_5^3 \sqrt{s}} \left[\sqrt{\frac{\eta\kappa s}{2\bar{M}_5^3}} - \frac{2\kappa}{\sqrt{s}} \right], \quad (41)$$

⁷To be more correct, one should use $dx_n/dn \simeq \pi[1 + (3/8)(\pi n)^2]$. However, we can put $x_n = \pi n$ at large values of n which are relevant for our calculations.

$$\text{Im } S(s) \simeq -\frac{1}{2\bar{M}_5^3 \sqrt{s}}. \quad (42)$$

It follows from Eqs. (41) and (42) that the imaginary part dominates the real one.

These expressions for the real and imaginary parts of $S(s)$ do not agree with the ‘‘discrete mass’’ expression (26), although they obey inequalities (28)–(30). Remember that Eq. (26) was derived by the direct calculation of the input sum (15). However, the imaginary parts become practically the same in both cases in the trans-Planckian kinematical region, namely, at $\sqrt{s} > 3\bar{M}_5$, as one can see from (28) and (42). As for the real parts, they are small in comparison with the imaginary parts in this energy region.⁸

Can we approximate the discrete spectrum by the continuous mass distribution, if the mass splitting Δm_{KK} is ‘‘very small’’? First of all, let us note that Δm_{KK} is dimensional and it should be compared with another dimensional quantity. Actually, we may regard a set of narrow graviton resonances to be a continuous mass spectrum (within some interval of n), if only

$$\Delta m_{\text{KK}} < \Gamma_n \quad (43)$$

is satisfied. Let us stress, it is the inequality that allows one to replace a summation in KK number n by integration over graviton mass m_{KK} .⁹

In our case, the relevant values of n , which give the leading contribution to the sought for quantity $S(s)$, are $n \sim \sqrt{s}/(\pi\kappa)$. Then we obtain from (43) and (16):

$$\eta \frac{(\sqrt{s})^3}{\Lambda_\pi^2} > \pi\kappa, \quad (44)$$

or, equivalently,

$$\sqrt{s} \gtrsim 3\bar{M}_5. \quad (45)$$

It is a common belief that in the flat space-time with large ED’s of the size R_c , the mass splitting is so small ($\Delta m_{\text{KK}} = R_c^{-1}$) that the continuous mass approximation is undoubtedly valid.¹⁰ Surprisingly, *it is not a case*. The reason is that the gravitons are extremely narrow resonances, $\Gamma_n \sim m_n^3/\bar{M}_{\text{Pl}}^2$. Accounting for the hierarchy relation for d compact ED’s (9), one finds from (43) that only KK gravitons with unrealistically large masses,

$$m_n^3 > \bar{M}_{\text{Pl}}^{2-(2/d)} \bar{M}_{4+d}^{1+(2/d)}, \quad (46)$$

are continuously distributed for $d \geq 2$. For all that, the

⁸See the comments and numerical estimates after Eqs. (28)–(30).

⁹From the point of view of experimental measurements, the mass splitting must be compared with the experimental resolution Δm_{res} . The spectrum looks continuous when $\Delta m_{\text{KK}} < \Delta m_{\text{res}}$, irrespective of Eq. (43).

¹⁰For instance, $R_c^{-1} \approx 130 \text{ eV}$, for $d = 4$ and $\bar{M}_{4+4} = 1 \text{ TeV}$.

widths of these gravitons remain relatively small, $\Gamma_n \geq \bar{M}_{4+d}(\bar{M}_{4+d}/\bar{M}_{\text{Pl}})^{2/d}$.

In particular, we get the conditions (for $m_n \sim \sqrt{s}$):

$$\begin{aligned} \sqrt{s} &> \bar{M}_{4+1}, & \text{for } d = 1, \\ \sqrt{s} &> (\bar{M}_{\text{Pl}}\bar{M}_{4+2}^2)^{1/3}, & \text{for } d = 2, \\ \sqrt{s} &> (\bar{M}_{\text{Pl}}^2\bar{M}_{4+d})^{1/3}, & \text{for } d \gg 1. \end{aligned} \quad (47)$$

It is not surprising that the first of these inequalities is similar to Eq. (45).

C. Zero width approximation

Now let us consider the limiting case of zero graviton widths (stable KK gravitons). The corresponding formulas can be obtained from the formulas derived above, if one *formally* takes the limit $\eta \rightarrow 0$, $\eta > 0$ in them (remember that all Γ_n are proportional to η). Let us introduce the notation

$$S_0(s) = S(s)|_{\Gamma_n=0}. \quad (48)$$

Then we get from (26):

$$\begin{aligned} S_0(s) = \frac{1}{2\bar{M}_5^3\sqrt{s}} &\left[\mathcal{P} \cot\left(\frac{\sqrt{s}}{\kappa} - \frac{\pi}{4}\right) \right. \\ &\left. - i\pi \sum_{n=0}^{\infty} \delta\left[\frac{\sqrt{s}}{\kappa} - \pi\left(n + \frac{1}{4}\right)\right] \right], \end{aligned} \quad (49)$$

where \mathcal{P} means the principal value. As one can see, neither $S_0(s)$, nor $S(s)|_{\sqrt{s}=\kappa z_0}$ (31) depend on the large mass scale Λ_π .

This result (49) may be also obtained by direct calculations in the zero width approximation, if one uses the asymptotics of x_n at large values of n (4) which are relevant in our case. Indeed, at $\sqrt{s}/\kappa \gg 1$ the quantity $S_0(s)$ is approximated as

$$S_0(s)|_{\sqrt{s}/\pi\kappa \gg 1} = \frac{1}{\pi^2\kappa\bar{M}_5^3} \sum_{n=1}^{\infty} \frac{1}{\frac{s}{(\pi\kappa)^2} - (n + \frac{1}{4})^2 + i0}, \quad (50)$$

and Eq. (49) is then reproduced, as it is shown in Appendix C.

If we replace the summation in n by integration, we find that the imaginary part of $S_0(s)$,

$$\text{Im } S_0(s) = -\frac{\pi\kappa}{2\bar{M}_5^3\sqrt{s}} \sum_{n=1}^{\infty} \delta(\sqrt{s} - m_n), \quad (51)$$

is of the form:

$$\text{Im } S_0(s) = -\frac{1}{2\bar{M}_5^3\sqrt{s}} \int_{m_0}^{\infty} dm \delta(\sqrt{s} - m) = -\frac{1}{2\bar{M}_5^3\sqrt{s}}, \quad (52)$$

where $m_0 = \pi\kappa$. This expression coincides with formula (42) derived in the previous subsection.

The same procedure, when applied to calculating $\text{Re } S_0(s)$, results in

$$\text{Re } S_0(s) = \frac{1}{\pi\bar{M}_5^3} \mathcal{P} \int_{m_0}^{\infty} dm \frac{1}{s - m^2} = -\frac{\kappa}{s\bar{M}_5^3}. \quad (53)$$

The last term in (53) is a particular case of expression (41) in the limit $\eta \rightarrow 0$. As one can see, $S_0(s)$ is actually purely imaginary.¹¹ Note that ‘‘continuous mass spectrum’’ formulas (52) and (53) *are in disagreement* with the imaginary and real parts of the ‘‘discrete mass spectrum’’ expression (49) taken at $\eta \rightarrow 0$ (this discrepancy is discussed in the end of subsection II B).

Let us stress that a more rapid falloff of $S(s)$ at $\sqrt{\eta s} \gg \Lambda_\pi$ (see Eq. (34)) *is completely lost* in the zero width approximation.

III. CONCLUSIONS AND DISCUSSIONS

In the present paper we have estimated the contribution of the virtual s -channel KK gravitons to the scattering of two SM fields. We have considered the small curvature option of the RS model with two branes ($\kappa \ll \bar{M}_5$). In such a scheme, the KK graviton spectrum is a series of rather narrow low-mass resonances. All the SM fields are confined to one of the branes.

We have studied the case when both the colliding energy \sqrt{s} and 5-dimensional Planck scale \bar{M}_5 are equal to one or few TeV (and, consequently, $\sqrt{s} \sim \bar{M}_5 \gg \kappa$). Ultrahigh energy region $\sqrt{s} \gg \bar{M}_5$ is also considered. By taking into account nonzero graviton widths Γ_n (with n being the KK number), we have derived tree-level analytical expression for $S(s)$, the process-independent gravity part of the scattering amplitude (see our main formulas (25) and (26)).

Then we have considered the case when a series of narrow low-mass graviton resonances is replaced by a continuous mass spectrum. Both the real and imaginary parts of $S(s)$ appeared to differ drastically from those derived without such a replacement. Thus, one has to conclude that an accurate estimation of the input sum in n is needed in order to get a correct result, especially at $\sqrt{s} \lesssim \bar{M}_5$. The replacement of the summation in the KK number by integration over graviton mass is well justified if the graviton mass splitting Δm_{KK} is less than Γ_n . In its turn, this condition is actually satisfied only in the trans-Planckian region (when the invariant energy \sqrt{s} is several times larger than the 5-dimensional Planck mass).

Zero width approximation (all $\Gamma_n = 0$) has been also studied. It is shown that some results can not be reproduced in this approximation, as one can see, for instance, from Eq. (34) which does not admit the limit $\Gamma_n \rightarrow 0$.

Possible loop corrections to our results originating from the exchange of s -channel gravitons can be estimated in the

¹¹The real part is suppressed with respect to the imaginary part by the factor $\kappa/(2\sqrt{s})$.

zero width approximation as follows. Each new virtual graviton brings the coupling $1/\Lambda_\pi^2$, while a corresponding summation in n results in the additional dimensional factor $1/\kappa$ (after the substitution $n \rightarrow m_n/(\pi\kappa)$). Using dimensional arguments, we conclude, for example, that m -loop graviton contribution to a vector field scattering amplitude, $\mathcal{M}^{(m)}(s)$, should be proportional to

$$\mathcal{M}^{(m)}(s) \sim \frac{s^2 M_{\text{cut}}^{3m-1}}{(\kappa\Lambda_\pi^2)^{m+1}} = \frac{s^2 M_{\text{cut}}^{3m-1}}{\bar{M}_5^{3m+3}}, \quad (54)$$

where M_{cut} is the UV cutoff in Feynman integrals, and $m \geq 1$. Thus, multiloop effects may become dominating at $M_{\text{cut}} \gtrsim \bar{M}_5$. In such a case, an effective operator analysis will be probably useful (see, for instance, Ref. [4]). Similar arguments are also applied to the flat space-time with the large compact ED's, after replacements $1/\Lambda_\pi^2 \rightarrow 1/\bar{M}_{\text{Pl}}^2$, $1/\kappa \rightarrow R_c^d$. Note that the zero width approximation is very well justified for this case, since $\Gamma_n \sim m_n^3/\bar{M}_{\text{Pl}}^2$ (i.e. gravitons are extremely narrow spin-2 resonances, even for $m_n \sim \sqrt{s}$).

In a general case (nonzero Γ_n), an additional nontrivial dependence on the scale Λ_π appears in the RS scheme, since the graviton widths depend on this mass scale (see Eq. (16)). Both simple dimensional arguments and formulas like Eq. (54) are no longer valid.

Our formula (25) can be also applied to the scattering of the brane particles, induced by t -channel graviton exchanges.¹² Let

$$\frac{\bar{M}_5^3}{\kappa} \gg -t \gg \kappa^2, \quad (55)$$

with t being 4-momentum transfer. Then we obtain from (25):

$$S(t) = -\frac{1}{2\kappa\bar{M}_5^3} \frac{1}{\bar{\sigma}} \frac{I_2(\bar{\sigma})}{I_1(\bar{\sigma})}, \quad (56)$$

where $I_\nu(z) = \exp(-i\nu\pi/2)J_\nu(iz)$ is the modified Bessel function, and

$$\bar{\sigma} \simeq \frac{\sqrt{-t}}{\kappa} - \frac{i\eta}{2} \left(\frac{\sqrt{-t}}{\bar{M}_5} \right)^3. \quad (57)$$

Since $I_2(z)/I_1(z) \rightarrow 1$ at $z \gg 1$ ($-\pi/2 < \arg z < 3\pi/2$), we find from (56) that

$$S(t) = -\frac{1}{2\bar{M}_5^3\sqrt{-t}} \quad (58)$$

in the kinematical region (55). Note that $S(t)$ (58) is *pure real* and it coincides with the imaginary part of $S(s)$ derived in the zero width approximation (52) up to the replacement $s \rightarrow -t$. In more general approach, one

¹²See also Ref. [9] in which t -channel graviton contribution to the scattering amplitude was studied in a large curvature scenario of the RS model.

should sum KK-charged gravi-Reggeons, i.e. Regge trajectories $\alpha_n(t)$ which are numerated by the KK number n [5,10]. In such a case, the amplitude has both real and imaginary parts.

The relations of all cross sections necessary for studying effects induced by tree-level exchange of the KK gravitons with the quantities $S(s)$ and $S(t)$ can be found in the Appendix of Ref. [4].

APPENDIX A

The sum (33) can be written as

$$S(s) = -\frac{i}{\eta(\pi\kappa)^4} \sum_{n=1}^{\infty} \frac{1}{n^4 + C^4}, \quad (A1)$$

where a and c are defined by Eq. (18), and

$$C = e^{-i\pi/8} \frac{1}{\pi\kappa} \left(\frac{s\Lambda_\pi^2}{\eta} \right)^{1/4} = x - iy. \quad (A2)$$

The sum in Eq. (A1) is of the form [11]:

$$\sum_{n=1}^{\infty} \frac{1}{n^4 + C^4} = \frac{\pi}{2\sqrt{2}C^3} \frac{\sinh\sqrt{2}\pi C + \sin\sqrt{2}\pi C}{\cosh\sqrt{2}\pi C - \cos\sqrt{2}\pi C} - \frac{1}{2C^4}. \quad (A3)$$

At large C , the main contribution to (A1) comes from $n \sim |C|$. The disregard of term bx_n^2 in (17) is justified if it is much less than c , that results in the inequality $b\pi^2|C|^2 \ll c$, or, equivalently, $\sqrt{\eta s} \gg \Lambda_\pi$ (32).

By using formulas

$$\begin{aligned} \sin(x - iy) &= \sin x \cosh y - i \cos x \sinh y, \\ \cos(x - iy) &= \cos x \cosh y + i \sin x \sinh y, \end{aligned} \quad (A4)$$

and

$$\begin{aligned} \sinh(x - iy) &= \cosh x \sin y - i \sinh x \cos y, \\ \cosh(x - iy) &= \cosh x \cos y + i \sinh x \sin y, \end{aligned} \quad (A5)$$

and taking into account that $\cosh x \simeq \sinh x \gg \cosh y \simeq \sinh y$ is valid at $x = \cot(\pi/8)y \simeq 2.4y \gg 1$, we obtain formula (34).

APPENDIX B

At small ratio κ/\sqrt{s} , we get from Eq. (36) that

$$\text{Re } S_0(s) = \frac{1}{\pi\sqrt{s}\bar{M}_5^3} \left[I - \frac{\pi\kappa}{\sqrt{s}} \right], \quad (B1)$$

where

$$I = \int_0^\infty dz \frac{1 - z^2}{(1 - z^2)^2 + \delta z^8}, \quad (B2)$$

with δ being defined by Eq. (40).

It is convenient to divide our integral (B2) into two parts:

$$I = I_1 + I_2. \quad (\text{B3})$$

Here

$$I_1 = \int_0^1 du \left[\frac{u(2-u)}{u^2(2-u)^2 + \delta(1-u)^8} - \frac{u(2+u)}{u^2(2+u)^2 + \delta(1+u)^8} \right], \quad (\text{B4})$$

and

$$I_2 = - \int_1^\infty du \frac{u(2+u)}{u^2(2+u)^2 + \delta(1+u)^8}. \quad (\text{B5})$$

Up to higher powers of δ , the integrals (B4) and (B5) are equal to

$$I_1 = \frac{1}{2} \ln 3 + \frac{3\pi}{4} \sqrt{\delta}, \quad I_2 = -\frac{1}{2} \ln 3 + \frac{\pi}{2\sqrt{2}} \sqrt[4]{\delta}, \quad (\text{B6})$$

and we obtain formula (41) of the main text.

As for the imaginary part, it is given by

$$\text{Im } S_0(s) = -\frac{\eta\sqrt{s}}{\pi\kappa\Lambda_\pi^4} J, \quad (\text{B7})$$

where

$$J = \int_0^\infty dz \frac{z^4}{(1-z^2)^2 + \delta z^8}. \quad (\text{B8})$$

As in the previous case, we divide the integral in (B8) into two parts:

$$J = J_1 + J_2, \quad (\text{B9})$$

with

$$J_1 = \int_0^1 du \left[\frac{(1-u)^4}{u^2(2-u)^2 + \delta(1-u)^8} + \frac{(1+u)^4}{u^2(2+u)^2 + \delta(1+u)^8} \right], \quad (\text{B10})$$

and

$$J_2 = \int_1^\infty du \frac{(1+u)^4}{u^2(2+u)^2 + \delta(1+u)^8}. \quad (\text{B11})$$

The leading parts of the integrals (B10) and (B11) are equal to

$$J_1 = \frac{\pi}{2\sqrt{\delta}}, \quad J_2 = \frac{\pi}{2\sqrt{2}} \frac{1}{\sqrt[4]{\delta}}. \quad (\text{B12})$$

Thus, we get

$$J \simeq \frac{\pi}{2\eta} \frac{\Lambda_\pi^2}{s}, \quad (\text{B13})$$

that results in formula (42) in the main text.

APPENDIX C

Let us consider the sum

$$\mathcal{K} = \sum_{n=1}^\infty \frac{1}{u^2 - (n+v)^2}, \quad (\text{C1})$$

with

$$u = \frac{\sqrt{s}}{\pi\kappa} + i0, \quad v = \frac{1}{4} \quad (\text{C2})$$

(remember that $\sqrt{s} \gg \kappa$).

The infinite sum (C1) can be found in Ref. [11]:

$$\mathcal{K} = \frac{1}{2u} [\Psi(v-u) - \Psi(v+u)] - \frac{1}{u^2 - v^2}, \quad (\text{C3})$$

where $\Psi(z)$ is the psi-function. Then one can apply the formula

$$\Psi(-z) = \Psi(z) + \frac{1}{z} + \pi \cot(\pi z), \quad (\text{C4})$$

and use the asymptotic behavior of the function $\Psi(z)$ at large z ($|\arg z| < \pi$),

$$\Psi(z)|_{|z| \gg 1} = \ln z - \frac{1}{2z} + O(z^{-2}) \quad (\text{C5})$$

(both formulas are taken from [12]).

As a result, the asymptotics of \mathcal{K} looks like

$$\mathcal{K}|_{|u| \gg 1} = \frac{\pi}{2u} \cot[\pi(u-v)] + O(u^{-2}), \quad (\text{C6})$$

and we come to Eq. (49) presented in the text.

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
 [2] V.A. Rubakov, Phys. Usp. **44**, 871 (2001); Yu. A. Kubyshin, Lectures given at the XI School ‘‘Particles and Cosmology’’, Baksan, Russia, 2001
 [3] G.N. Watson, *A Treatise on the theory of Bessel functions*

(McMillan, New York, 1922).
 [4] G. F. Giudice, T. Plehn, and A. Strumia, Nucl. Phys. **B706**, 455 (2005).
 [5] A. V. Kisselev and V. A. Petrov, Phys. Rev. D **71**, 124032 (2005).
 [6] G. F. Giudice, R. Rattazzi, and J. D. Wells, Nucl. Phys.

- B595**, 250 (2001).
- [7] H. Davoudiasl, J. L. Hewett, and T. G. Rizzo, *Phys. Rev. D* **63**, 075004 (2001).
- [8] C. Hanhart, D. R. Phillips, S. Reddy, and M. J. Savage, *Nucl. Phys.* **B595**, 335 (2001); S. Hannestad and G. G. Raffelt, *Phys. Rev. D* **67**, 125008 (2003); **69**, 029901(E) (2004).
- [9] A. V. Kisselev, *Eur. Phys. J. C* **42**, 217 (2005).
- [10] A. V. Kisselev and V. A. Petrov, *Eur. Phys. J. C* **36**, 103 (2004); **37**, 241 (2004).
- [11] A. P. Prudnikov, Yu. A. Brychkov, and O. I. Marichev, *Elementary functions, Integrals and series*, Vol. 1 (Gordon and Breach, New York, 1986), translated from Russian.
- [12] *Higher Transcendental Functions*, edited by A. Erdelyi, W. Magnus, F. Oberhettinger, and F. C. Tricomi (McGraw-Hill, New York, 1955), Vol. 1.