# No hair conjecture, non-Abelian hierarchies, and anti-de Sitter spacetime

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We consider globally regular and black holes solutions for the Einstein-Yang-Mills system with negative cosmological constant in $d$-spacetime dimensions. We find that the ADM mass of the spherically symmetric solutions generically diverges for $d > 4$. Solutions with finite mass are found by considering corrections to the YM Lagrangian consisting in higher order terms of the Yang–Mills hierarchy. Such systems can occur in the low energy effective action of string theory. A discussion of the main properties of the solutions and the differences with respect to the four dimensional case is presented. The mass of these configurations is computed by using a counterterm method.

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## I. INTRODUCTION

If we allow for a negative cosmological constant, the solution of the matter-free Einstein equations possessing the maximal number of symmetries is the anti-de Sitter (AdS) spacetime. Recently a considerable amount of interest has been focused on solutions of Einstein equations with this type of asymptotics. This interest is mainly motivated by the proposed correspondence between physical effects associated with gravitating fields propagating in AdS spacetime and those of a conformal field theory on the boundary of AdS spacetime [1,2].

In view of these developments, an examination of the classical solutions of gravitating fields in asymptotically AdS (AAdS) spacetimes seems appropriate. Recently, some authors have discussed the properties of gravitating SU(2) non-Abelian fields with a negative cosmological constant $\Lambda$ [3,4]. Considering the case of four spacetime dimensions, they obtained some surprising results, which are strikingly different from the results found in the asymptotically flat case. For example there are solutions for continuous intervals of the parameter space, rather than discrete points. The asymptotic values of the gauge potentials are arbitrary and there exist solutions supporting magnetic and electric fluxes without a Higgs field. Some of these solutions are stable against spherically symmetric linear perturbations. The literature on AAdS solutions with non-Abelian fields is growing steadily, including stability analyses [5,6], the study of configurations with a Newman-Unti-Tamburino charge [7], topological black holes with non-Abelian hair [8] as well as axially symmetric generalizations [9,10]. The existence of these solutions invalidates the $AdS_4$ version of the no hair conjecture, which states that the black holes are completely characterized by their mass, charge, and angular momentum.

However, all these studies approach the case of a four dimensional AAdS spacetime, and relatively little is known about higher dimensional AAdS solutions with non-Abelian matter fields. Practically all that is known for $d > 4$ is the five dimensional non-Abelian SU(2) solutions discussed in [11]. At the same time gauged supergravity theories playing an important role in AdS/CFT generically contain non-Abelian matter fields in the bulk, although in the literature only Abelian truncations are considered, to date. Thus, the examination of higher dimensional gravitating non-Abelian solutions with $\Lambda < 0$ is a pertinent task.

Higher dimensional asymptotically flat solutions of the Einstein-Yang-Mills (EYM) equations have recently been the subject of several studies. As found in [12], in five spacetime dimensions, the particle spectrum obtained by uplifting the $d = 4$ flat space YM instantons become completely destroyed by gravity, as a result of their scaling behavior. However, by adding higher order[1] terms in the Yang-Mills (YM) hierarchy this obstacle due to scaling is removed and solutions in higher dimensions can be found. Such regular, static, and spherically symmetric solutions in spacetime dimensions $d \leq 8$ were presented in [13], and for $d = 5$, both globally regular and black hole solutions of the EYM system were found in [14]. The properties of these solutions are rather different from the familiar Bartnik-McKinnon (BK) solutions [15] to EYM in $d = 4$, and are somewhat more akin to the gravitating monopole solutions to EYM-Higgs (EYMH) [16]. This is because like in the latter case [16], where the vacuum expectation value (VEV) of the Higgs field features as an additional dimensional constant, here [13] also additional dimensional constants enter with each higher order YM curvature term.[2] They are however quite distinctive in their

[1]The only higher order curvature terms considered in this paper are those constructed from a $2p$-form field such that the Lagrangian contains *velocity square* fields only.

[2]Like the gravitating monopoles, these have gravity decoupling limits except in $d = 5$ (and in $d = 4p + 1$ modulo 4), and in all *odd* spacetime dimensions, the flat space solutions are stabilized by a Pontryagin charge analogous to the magnetic charge of the monopole, provided that the representations of the gauge group are chosen suitably. In all *even* $d$ however, they are like the BK solution in that they are not stabilized by a topological charge and are likewise sphalerons [17]

 

critical behavior. The typical critical features discovered in [13,14] have recently been analyzed and explained in [18].

These results can be systematically extended to all $d \geq 5$ and one finds that no finite mass solutions can exist in EYM theory, unless one modifies the non-Abelian action by adding higher order curvature terms[3] in the YM $2p$-form curvature $F(2p)$, the $p = 1$ case being the usual 2-form YM curvature. Without these higher order YM terms, only vortex-type finite energy solutions [12] exist, describing effective systems in 3 spacelike dimensions, and with a number of codimensions.

It is the purpose of this paper to examine the corresponding situation in higher dimensions, in the presence of a negative cosmological constant. It is interesting to inquire whether the introduction of a negative cosmological constant to these higher dimensional EYM models will lead to some new effects as it does in the $d = 4$ case, due to the different asymptotic structure of the spacetime. In the first place this would lead to our understanding of how the behavior of EYM theory depends on the dimensionality of the spacetime. But such higher dimensional AAdS solutions might be relevant to superstring theory, namely, in the context of solutions to various supergravities containing non-Abelian matter fields. Here, however, we restrict our considerations to the simplest case of systems consisting only of gravitational and YM fields, namely, to a higher dimensional EYM model. In particular, we restrict to Einstein-Hilbert gravity and the first two members of the YM hierarchy, and hence to $d \leq 8$, augmented with negative cosmological constant in $d$ spacetime dimensions. As in [3,4], for the $d = 4$ case, we seek static spherically symmetric solutions in the $d - 1$ spacelike dimensions. We find both globally regular and black hole solutions with finite ADM mass. Unlike in the $d = 4$ case however, we find that for $d > 4$ and a negative cosmological constant, the properties of the AAdS solutions do not differ qualitatively from the asymptotically flat case.

Our strategy is to first consider the usual YM model, namely, the $p = 1$ member of the YM hierarchy, or the square of the 2-form curvature $F(2)$. We present an argument for the absence of solutions with reasonable asymptotics for any spacetime dimension $d \geq 5$. Although the EYM equations in this case present solutions approaching asymptotically the AdS background, the mass generically diverges. This can be seen as a simple version of the no hair theorem, holding for the EYM system in $d > 4$ dimensions. In other words, the Schwarzschild-AdS black hole is the only static, spherically symmetric solution of the EYM system with finite mass. This is presented in Sec. II, where in addition we have considered the special case $d = 3$, extending Deser's analysis [19] for $\Lambda < 0$.

In Sec. III we introduce the higher order YM hierarchy models featuring the terms $F(2p)$, $p \geq 2$. We derive the classical equations subject to our spherically symmetric ansatz, and present a detailed numerical study of both regular and black hole solutions. As is the case with the usual EYM system, where the existence of finite mass regular solutions leads to the violation of the no hair conjecture, here too, these solutions are explicit counterexamples to this conjecture in $\text{AdS}_d$ spacetime.

One may ask about the possible relevance of these higher dimensional configurations within the AdS/CFT correspondence. In Sec. IV we compute the boundary stress tensor and the mass and action of the solutions in a number of spacetime dimensions up to eight. In five and seven dimensions, the counterterm prescription of [20] gives an additional vacuum (Casimir) energy, which agrees with that found in the context of AdS/CFT correspondence. A counterterm based proposal to remove the divergences of a $F(2)$ theory, such that the mass and action be finite, is also presented in Appendix B. We give our conclusions and remarks in the final section.

Everywhere in this paper we employ the notations and conventions of [13].

## II. THE $F(2)$ MODEL

### A. The action principle

We start with the following action principle in $d-$ spacetime dimensions

$$I = \int_{\mathcal{M}} d^d x \sqrt{-g}\left(\frac{1}{16\pi G}(R - 2\Lambda) + \mathcal{L}_m\right) - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^3 x \sqrt{-h} K, \tag{1}$$

where $R$ is the Ricci scalar associated with the spacetime metric $g_{\mu\nu}$, $\Lambda = -(d-1)(d-2)/(2\ell^2)$ is the cosmological constant, and $G$ is the gravitational constant [following [14], we define also $\kappa = 1/(8\pi G)$].

The matter term in the above relation

$$\mathcal{L}_m = -\tfrac{1}{4}\tau_1 \text{tr} F_{\mu\nu}F^{\mu\nu} \tag{2}$$

is the usual $F^2$ non-Abelian action density, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$ being the gauge field strength tensor. Here $\tau_1$ is the coupling constant of the model (in the usual notation $\tau_1 = 1/g^2$).

The last term in (1) is the Hawking-Gibbons surface term [21], where $K$ is the trace of the extrinsic curvature for the boundary $\partial\mathcal{M}$ and $h$ is the induced metric of the boundary. Of course this term does not affect the equations of motion but it is relevant for the discussion of the mass and the action of the solutions in Sec. IV.

The field equations are obtained by varying the action (1) with respect to the field variables $g_{\mu\nu}$, $A_\mu$

[3]In principle higher order terms with the desired scaling can be chosen to consist both of the YM and the Riemann curvatures, but in practice we restrict to the YM hierarchy. The reason will be explained in Sec. II E. Besides, it was found in [13] that the inclusion of Gauss-Bonnet terms does not result in any new qualitative features to the solutions.

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \qquad \nabla_\mu F^{\mu\nu} - i[A_\mu, F^{\mu\nu}] = 0, \tag{3}$$

where the energy momentum tensor is defined by

$$T_{\mu\nu} = \mathrm{tr} F_{\mu\alpha}F_{\nu\beta}g^{\alpha\beta} - \tfrac{1}{4}g_{\mu\nu}\mathrm{tr}F_{\alpha\beta}F^{\alpha\beta}. \tag{4}$$

### B. The general Ansatz

For the case of a $d$-dimensional spacetime, we restrict to static fields that are spherically symmetric in the $d-1$ spacelike dimensions, with a metric ansatz in terms of Schwarzschild coordinates

$$ds^2 = \frac{dr^2}{N(r)} + r^2 d\Omega^2_{d-2} - \sigma^2(r)N(r)dt^2, \tag{5}$$

with $d\Omega_{d-2}$ the $d-2$ dimensional angular volume element and

$$N = 1 - \frac{2m(r)}{\kappa r^{d-3}} + \frac{r^2}{\ell^2}, \tag{6}$$

the function $m(r)$ being related to the local mass-energy density up to some $d$-dependent factor.

As discussed in [13], the choice of gauge group compatible with the symmetries of the line element (5) is somewhat flexible. In [13] the gauge group chosen was SO($d$), in $d$ dimensions. But the gauge field of the static solutions in question took their values in SO($d-1$). Thus in effect, it is possible to choose SO($d$) in the first place. Now for *even* $d$, it is convenient to choose SO($d$) since we can then avail of the chiral representations of the latter, although this is by no means obligatory. Adopting this criterion, namely, to employ chiral representations, also for *odd* $d$, it is convenient to choose the gauge group to be SO($d-1$). We shall therefore denote our representation matrices by $\mathrm{SO}_\pm(\bar{d})$, where $\bar{d} = d$ and $\bar{d} = d-1$ for *even* and *odd* $d$ respectively.

In this unified notation (for odd and even $d$), the spherically symmetric Ansatz for the $\mathrm{SO}_\pm(\bar{d})$-valued gauge fields then reads [13]

$$A_0 = 0, \qquad A_i = \left(\frac{1-w(r)}{r}\right)\Sigma^{(\pm)}_{ij}\hat{x}_j, \qquad \Sigma^{(\pm)}_{ij} = -\frac{1}{4}\left(\frac{1 \pm \Gamma_{\bar{d}+1}}{2}\right)[\Gamma_i, \Gamma_j]. \tag{7}$$

The $\Gamma$'s denote the $\bar{d}$-dimensional gamma matrices and $1, j = 1, 2, \cdots, d-1$ for both cases.

Inserting this ansatz into the action (1), the EYM field equations reduce to

$$0 = (r^{d-4}\sigma N w')' - (d-3)r^{d-6}\sigma(w^2-1)w, \tag{8}$$

$$m' = \frac{\tau_1}{2}r^{d-4}\left(Nw'^2 + (d-3)\frac{(w^2-1)^2}{2r^2}\right), \tag{9}$$

$$\frac{\sigma'}{\sigma} = \frac{\tau_1}{\kappa}\frac{w'^2}{r}, \tag{10}$$

which can also be derived from the reduced action

$$S = \int dr\sigma\left(m' - \frac{\tau_1}{2}r^{d-4}\left(Nw'^2 + (d-3)\frac{w^2-1)^2}{2r^2}\right)\right). \tag{11}$$

For a $F^2$ theory, the constants $\kappa$, $\tau_1$ can always be absorbed by rescaling $r \to cr$, $\Lambda \to \Lambda/c^2$ and $m \to m\kappa c^{d-3}$, with $c = \sqrt{\tau_1/\kappa}$.

The above differential equations have two analytic solutions. One of them with

$$w(r) = \pm 1, \qquad m(r) = M, \qquad \sigma(r) = 1 \tag{12}$$

corresponds to Schwarzschild-AdS spacetime. For $w(r) = 0$ we find a non-Abelian generalization of the magnetic Reissner-Nordstrøm solution with $\sigma(r) = 1$ and

$$m(r) = M_0 + \frac{\tau_1}{2}\log r \quad \text{if } d = 5, \qquad \text{and} \quad m(r) = M_0 + \frac{\tau_1(d-3)}{4(d-5)}r^{d-5} \quad \text{for } d \neq 5, \tag{13}$$

$M_0$ being an arbitrary constant. We can see that, although these solutions are asymptotically AdS, the mass defined in the usual way diverges.

### C. $d = 3$

The (2 + 1) dimensional case is rather special. Three dimensional gravity has provided many important clues about higher dimensional physics. This theory with $\Lambda < 0$ has nontrivial solutions, such as the BTZ black-hole spacetime [22], which provide an important testing ground for quantum gravity and the AdS/CFT correspondence. Many other types of 3$d$ regular and black hole solutions with a negative cosmological constant have also been found by coupling matter fields to gravity in different ways.

However, as proven in [19], there are no $d = 3$ asymptotically flat static solutions of the EYM equations. The arguments in [19] can easily be generalized for the AAdS case. We notice that for $d = 3$, the YM Eq. (8) implies the existence of a first integral $w' = \alpha r/(\sigma N)$ with $\alpha$ an arbitrary real constant. Therefore, assuming $\mathrm{AdS}_3$ asymptotics, $w'$ decays asymptotically as $1/r$ which from (9) implies a divergent value of $m(r)$ as $r \to \infty$. However, similar to the $\Lambda = 0$ case [23], this argument does not exclude the existence of nontrivial solutions of the field equations.

Here we should remark that since for $d = 3$ we are dealing with SO($d-1$) = SO(2), the gauge group is Abelian and we recover Einstein-Maxwell theory with a negative cosmological constant, whose solutions are known in the literature. The corresponding solution with a vanishing electric field was found by Hirschmann and Welch [24] and has a line element[4]

$$ds^2 = \frac{r^2}{r^2 + c^2 \log|r^2/\ell^2 - M|} \frac{dr^2}{(r^2/\ell^2 - M)} + (r^2 + c^2 \log|r^2/\ell^2 - M|)d\varphi^2 - (r^2/\ell^2 - M)dt^2, \tag{14}$$

the magnetic potential being

$$w(r) = w_0 + \frac{c}{\sqrt{2}} \log|r^2/\ell^2 - M|, \tag{15}$$

with $w_0$, $c$ and $M$ arbitrary real constants, the BTZ metric being recovered for $c = 0$ (see also [25,26] for more details on this solution).

One can see that, although the quasilocal mass defined in the usual way diverges as $r \to \infty$, the metric still approaches the $AdS_3$ background. However, a similar problem appears for other $d = 3$ AdS solution, e.g. for the electrically charged BTZ black hole [22], or for a self-interacting scalar field minimally coupled to gravity [27], in which cases it was possible to find a suitable mass definition. We expect the formalism developed in those cases to work also for the Hirschmann-Welch solution (14) and (15), but this lies outside the scope of the present work.

### D. $d = 4$

Four dimensional black hole solutions of the Eqs. (8)–(10) have been found in [3], the globally regular counterparts being discussed in [4]. Differing from the asymptotically flat case, for $\Lambda < 0$ there is a continuum of solutions in terms of the adjustable shooting parameter that specifies the initial conditions at the origin or at the event horizon. As a new feature, the asymptotic value of the gauge function $w_0$ is arbitrary. The spectrum has a finite number of continuous branches, depending on the value of $\Lambda$. When the parameter $\Lambda$ approaches zero, an already-existing branch of solutions collapses to a single point in the moduli space. At the same time new branches of solutions emerge. A fractal structure in the moduli space has been noticed [28]. There are also nontrivial solutions stable against spherically symmetric linear perturbations, corresponding to stable configurations. The solutions are classified by non-Abelian magnetic charge and the ADM mass.

Note also that the $d = 4$ EYM solutions with a negative cosmological constant $\Lambda = -3/\tau_1$ have some relevance in AdS/CFT context. As proven in [29], for this value of the cosmological constant, an arbitrary solution $(g_{\mu\nu}, A_\mu^{(a)})$ of the four dimensional EYM equations gives a solution of the equations of motion of the $d = 11$ supergravity. Based on this observation, an exact Bogomol'nyi-Prasad-Sommerfield monopoles (BPS)-type EYM solution has been constructed in [10]. However, similar to some supersymmetric solutions in Einstein-Maxwell theory with $\Lambda < 0$, this $\Lambda = -3/\tau_1$ configuration presents a naked central singularity.

### E. $d \geq 5$

As discovered by Coleman [30] and Deser [31], there are no flat space static solutions of the YM equations, except for $d = 5$. However, the inclusion of gravity may change this picture, as seen from the famous $d = 4$ asymptotically flat Bartnik-McKinnon solutions [15]. In this case, the repulsive YM force is compensated by the attractive character of the gravity, and as a result we find both regular and black hole unstable configurations (see [32] for a fairly recent survey). As found in [12] the $d = 5$ particlelike solutions are destroyed by gravity, their mass diverging logarithmically, while $w(r)$ presents an infinite number of nodes. The AAdS couterparts of the $d = 5$ asymptotically flat solutions are discussed in [11]. Although approaching asymptotically the $AdS_5$ background, the mass of these configurations also diverges logarithmically.

As conjectured by several authors, this result extends to higher dimensions. Following the approach in [11], we prove in Appendix A the nonexistence of asymptotically flat or AdS solutions with a finite mass in a $F(2)$ EYM model given by (2) for any spacetime dimension $d \geq 5$ (see also the discussion in Sec. III C). Therefore, the $d$-dimensional Schwarzschild-AdS configuration is the only finite mass solution of the Eqs. (8)–(10) and a simple version of the no hair theorem seems to hold for the $F(2)$ (usual) EYM system in $d > 4$.

We should remark that in deriving this result we assumed implicitly that $m(r)$, $\sigma(r)$, $w(r)$ are smooth functions approaching finite values as $r \to \infty$. A divergent asymptotic value of $m(r)$ invalidates the proof presented in Appendix A and also the virial arguments in Sec. III C. Therefore we cannot exclude the existence of spherically symmetric, nontrivial solutions of the field equations for any $d > 4$. However, the mass of these solutions generically diverges, although the spacetimes are still AAdS. The work of Ref. [11] presents an extensive discussion of such AAdS solutions for $d = 5$. Both regular and black hole solutions exist in $d = 5$ for compact intervals of the parameter that specifies the initial conditions at the origin or at the event horizon. Differing from the $\Lambda = 0$ case, the gauge field function $w(r)$ does not oscillate between 1 and $-1$ but approaches asymptotically some finite value $w_0$, the node number being finite. The masses of these solutions behave asymptotically as $(w_0^2 - 1)\log r$, with all $w_0 = \pm 1$ solutions corresponding to pure gauge configurations.

The results we found by solving numerically the Eqs. (8)–(10), for $d = 6, 7, 8$ and several negative values

[4]One can also solve directly the field equations (8)–(10), but the solution takes a much more complicated form for the metric ansatz (5).

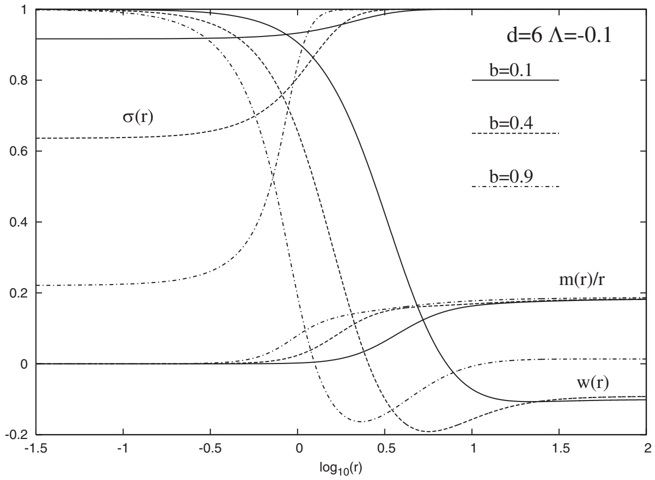


FIG. 1. The functions $\sigma(r)$, $w(r)$, and the ratio $m(r)/r$ are plotted as functions of radius for typical $d = 6$ regular solutions in a $F^2$ EYM theory with $\Lambda = -0.1$ and several values of the parameter $b = -\frac{1}{2}w''(0)$.

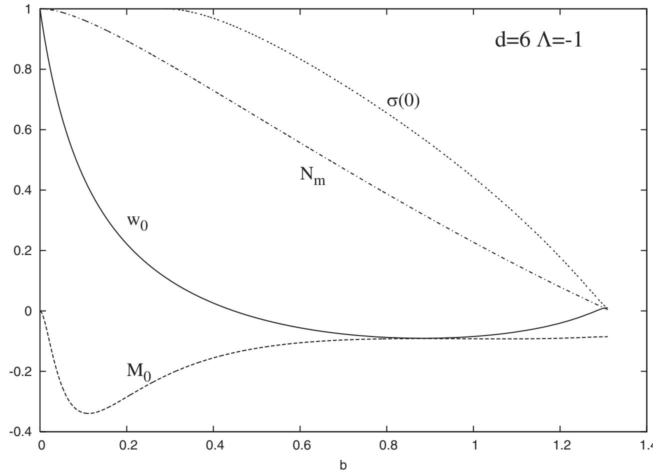


FIG. 2. The value $N_m$ of the minimum of the metric function $N(r)$, the parameter $M_0$ appearing in the asymptotic expansion of $m(r)$, the asymptotic value $w_0$ of the gauge function $w(r)$ as well as the value $\sigma(0)$ of the metric function $\sigma$ at the origin, are shown as functions of $b$ for $d = 6$ solutions of $F^2$ theory with $\Lambda = -1$.

of $\Lambda$ confirm that this is a generic behavior for $d > 4$. The corresponding boundary conditions at the origin (or event horizon) are found by taking $P = 1$ in relations (26) and (27) given in Sec. III. Except for a divergent value of $m(r)$ as $r \to \infty$, according to

$$m(r) = M_0 + \frac{\tau_1(d-3)}{4(d-5)}(w_0^2 - 1)^2 r^{d-5}, \qquad (16)$$

the properties of these solutions are very similar to the more familiar $d = 4$ case. For $d > 5$, the asymptotic value $w_0$ of the gauge field function $w$ is also arbitrary, being fixed by the initial parameters $w''(0)$ or $w(r_h)$ respectively, $w_0 = \pm 1$ corresponding to pure gauge configurations. Solutions for a compact interval of these parameters were found to exist, the general structure being $\Lambda$-dependent. Solutions with nodes in $w(r)$ were also found. Typical $d = 6$ configurations with a regular origin are presented in Fig. 1, for $\Lambda = -0.01$ and three different values of $b = -w''(0)/2$. One can see that the mass function diverges linearly while $\sigma(r)$ and $w(r)$ asymptotically approach some finite values. In Fig. 2 we plot the parameters $M_0$ (appearing in (16), which in Sec. III Cwe argue that it can be taken as the renormalized mass of the solutions), $w_0$, the value $\sigma_0$ of the metric function $\sigma$ at the origin and the minimal value $N_m$ of the metric function $N$ as a function of $b$ for a family of $d = 6$ AAdS solutions with $\Lambda = -1$. This branch ends for some finite value of $b$, where $\sigma(0) \to 0$. Black hole solutions have been found as well, presenting the same general features. Here we also find a continuum of solutions with arbitrary values of $w_0$, the relevant parameter being the value of the gauge potential at the event horizon. Similar to $d = 4$, solutions appear to exist for any value of the event horizon radius.

The drawback of the solutions in $d > 5$ described above is that their ADM masses are divergent, making their physical significance obscure (see, however, the discussion in Appendix B).

One may hope to find a different picture by including some other matter field in the action . Such fields should interact with the YM sector so as to compensate for the scaling behavior of the non-Abelian fields. This excludes the dilaton field, as it can be proven that the latter does not change this nonexistence result. Note that $d = 5$ finite mass spherically symmetric gravitating non-Abelian solutions with a Liouville-type dilaton potential are known to exist [33]; however these solutions are asymptotically neither flat nor AdS.

In the next section, we will remedy this problem of nonexistence by adding higher order YM curvature terms $F(2p)$, $(p > 1)$, to (2). The main role of these terms is to alter the scaling properties of the action density in (1). But logically, such a role can be played also by altering the gravitational part of (1), through the introduction of higher order (in the Riemann curvature) terms $R(p)$, $(p > 1)$. $R(p)$ here denotes the generalized Ricci scalar constructed from the $2p$-form antisymmetrized $p$-fold product of Riemann tensor 2-forms, the $p = 1$ member of which gives the Einstein-Hilbert action, the $p = 2$, the Gauss–Bonnet, etc. Here we have eschewed the possibility of employing additional $R(p)$ terms instead of, or together with $F(2p)$ YM terms, because in the present work we exclude the participation of fields other than gravitational and Yang-Mills. In particular, the exclusion of the dilaton renders the usefulness of higher order (Gauss-Bonnet) gravities trivial from a practical point of view.

To see this, consider the situation where a term scaling as $L^{-2p}$ is needed, i.e. that the term

$$\kappa_p \sqrt{-g} R(p)$$

must be added to the density in (1). Using the metric Ansatz (5), and discarding purely boundary terms in the residual one dimensional (spherically symmetric) Lagrangian, the term $\mathcal{L}_{(p)}$ to be added is

$$\mathcal{L}_{(p)} = \kappa_p \frac{(d-2p)!}{(d-1)!} \sigma r^{d-2p-2} \Big[ r \frac{d}{dr}(1-N)^p + (d-2p-1)(1-N)^p \Big] \tag{17}$$

in spacetime dimension $d$, which vanishes identically in dimensions up to $d = 2p$. This is because in the given dimensions the $p$th member of the gravitational hierarchy becomes a total divergence. But for given $d$ it is necessary to have $2p \geq d$, whence (17) trivializes. The situation would be quite different if the dilaton field were included together with the higher order gravitational terms, preventing the $p$-gravity density becoming a total divergence. But this option is excluded here and we opt to the exclusive use of higher order $p$-YM densities.

Yet other fields occurring in supergravities might be considered, e.g. Kalb-Ramond, or totally antisymmetric tensor fields. But these being Abelian, their effect would be felt only in given dimensions, or, subject to much less stringent symmetries than the spherical. This also is not a flexible option, so we restrict our attention to $p$-YM densities only.

## III. NON-ABELIAN HIERARCHIES

### A. The Lagrangian and field equations

Since finite mass spherically symmetric solutions play a central role in AdS/CFT, it is desirable that the nonexistence result presented in Sec. II E be circumvented.

A simple way to circumvent these arguments and to find nontrivial solutions is to modify the matter Lagrangian by adding higher order terms in the YM hierarchy. As noted in footnote [1], these are constructed exclusively from YM curvature $2p$-forms. For $\Lambda = 0$, asymptotically flat, finite energy solutions of this modified EYM system are constructed in [13,14].

Such terms as we propose to add are predicted by string theory, and hence provide a link with the AdS/CFT correspondence too. But here we are guided predominantly by symmetry considerations and do not claim to be employing terms strictly following from superstring theory. The situation concerning higher order YM curvature terms in the string theory effective action is complex and as yet not fully resolved. While YM terms up to $F^4$ arise from (the non-Abelian version of) the Born–Infeld action [34], it appears that this approach does not yield all the $F^6$ terms [35]. Terms of order $F^6$ and higher can also be obtained by employing the constraints of (maximal) supersymmetry [36]. The results of the various approaches are not identical.

The definition we use for superposed YM hierarchy is

$$\mathcal{L}_m = -\sum_{p=1}^{P} \frac{1}{2(2p)!} \tau_p \sqrt{-g}\, \mathrm{Tr} F(2p)^2, \tag{18}$$

where $F(2p)$ is the $2p$-form $p$-fold totally antisymmetrized product of the SO($d$) YM curvature 2-form $F(2)$

$$F(2p) \equiv F_{\mu_1\mu_2\ldots\mu_{2p}} = F_{[\mu_1\mu_2} F_{\mu_3\mu_4} \ldots F_{\mu_{2p-1}\mu_{2p}]}. \tag{19}$$

Even though the $2p$-form (19) is dual to a total divergence, namely, the divergence of the corresponding Chern-Simons form, the density (18) is never a total divergence since it is the square of one. But the $2p$-form (19) vanishes by (anti)symmetry for $d < 2p$ so that the upper limit in the summation in (18) is $P = \frac{d}{2}$ for even $d$ and $P = \frac{d-1}{2}$ for odd $d$.

We define the $p$-stress tensor pertaining to each term in (18) as

$$T_{\mu\nu}^{(p)} = \mathrm{Tr} F(2p)_{\mu\lambda_1\lambda_2\ldots\lambda_{2p-1}} F(2p)_{\nu}^{\ \lambda_1\lambda_2\ldots\lambda_{2p-1}} - \frac{1}{4p} g_{\mu\nu} \mathrm{Tr} F(2p)_{\lambda_1\lambda_2\ldots\lambda_{2p}} F(2p)^{\lambda_1\lambda_2\ldots\lambda_{2p}}. \tag{20}$$

For the particular spherically symmetric ansatz considered in Sec. II, we express the reduced YM Lagrangian arising from (18) as

$$\mathcal{L}_m = -\sum_{p=1}^{P} L_{\mathrm{YM}}^{(p)} \tag{21}$$

with $L_{\mathrm{YM}}^{(p)}$ given by

$$L_{\mathrm{YM}}^{(p)} = r^{d-2} \sigma \frac{\tau_p}{2\cdot(2p)!} \frac{(d-2)!}{(d-[2p+1])!} \times W^{p-1} \Big[ (2p) N \Big(\frac{1}{r}\frac{dw}{dr}\Big)^2 + (d-[2p+1]) W \Big] \tag{22}$$

having used the shorthand notation

$$W = \Big(\frac{w^2-1}{r^2}\Big)^2. \tag{23}$$

For this general ansatz, we find the field equations

$$m' = \sum_{p=1}^{P} \frac{\tau_p r^{d-2}}{2(2p)!} \frac{(d-3)!}{(d-[2p+1])!} W^{p-1}\left[(2p)N\left(\frac{1}{r}\frac{dw}{dr}\right)^2 + (d-[2p+1])W\right],$$

$$\frac{\sigma'}{\sigma} = \frac{(d-3)!}{\kappa r} w'^2 \sum_{p=1}^{P_2} \frac{\tau_p W^{p-1}}{(d-[2p+1])!(2p-1)!},$$

$$\sum_{p=1}^{P} \frac{d}{dr}\left(\frac{r^{d-4}\sigma\tau_p}{(d-[2p+1])!(2p-1)!} W^{p-1} N w'\right) = \sum_{p=1}^{P} \frac{r^{d-6}\sigma\tau_p 2w(w^2-1)W^{p-1}}{(d-[2p+1])!(2p-1)!}\left(N(p-1)W\frac{w'^2}{r^2} + \frac{d-2p-1}{2}\right), \quad (24)$$

which can also be derived from the reduced action

$$S = \int dr\sigma\left(m' - \sum_{p=1}^{P} \frac{\tau_p r^{d-2}}{2(2p)!} \frac{(d-3)!}{(d-[2p+1])!} W^{p-1}\left[(2p)N(\frac{1}{r}\frac{dw}{dr})^2 + (d-[2p+1])W\right]\right). \quad (25)$$

For $\tau_i = 0$, $i > 2$ the Eqs. (8)–(10) are recovered. Note also the $\sigma$ equation decouples and can be treated separately.

### B. Boundary conditions

The asymptotic solutions to these equations can be systematically constructed in both regions, near the origin (or event horizon) and for $r \gg 1$.

The corresponding expansion as $r \to 0$ is

$$w(r) = 1 - br^2 + O(r^4), \qquad m(r) = \left(\sum_{p=1}^{P} \frac{\tau_p (d-3)!(4b^2)^p}{2(2p)!(d-[2p+1])!}\right) r^{d-1} + O(r^{d+1}),$$
$$\sigma(r) = \sigma_0 + \frac{(d-3)!}{2\kappa}\left(\sum_{p=1}^{P} \frac{\tau_p (4b^2)^p}{(2p-1)!(d-[2p+1])!}\right) r^2 + O(r^4), \quad (26)$$

and contains one essential parameter $b$ [the value of $\sigma(0) \equiv \sigma_0$ can be fixed by rescaling the time coordinate].

For black hole configurations with a regular, nonextremal event horizon at $r = r_h$, the expression near the event horizon is

$$m(r) = m_h + m'(r_h)(r - r_h) + O(r - r_h)^2,$$
$$w(r) = w_h + w'(r_h)(r - r_h) + O(r - r_h)^2, \quad (27)$$
$$\sigma(r) = \sigma_h + \sigma'_h(r - r_h) + O(r - r_h)^2,$$

where

$$m(r_h) = \frac{1}{2}\kappa r_h^{d-3}\left(1 + \frac{r_h^2}{\ell^2}\right), \qquad W_h = \frac{(w_h^2-1)^2}{r_h^4},$$
$$m'(r_h) = \sum_{p=1}^{P} \frac{\tau_p r_h^{d-2}(d-3)!}{2(2p)!(d-2p-2)!} W_h^p,$$
$$N'_h = \frac{d-3}{r_h} + \frac{(d-1)r_h}{\ell^2} - \frac{2m'(r_h)}{\kappa r_h^{d-3}},$$
$$\sigma'_h = \frac{\sigma_h (d-3)!}{\kappa r_h} w_h'^2 \sum_{p=1}^{P} \frac{\tau_p W_h^{p-1}}{(2p-1)!(d-[2p+1])!},$$
$$w'_h = \frac{1}{N'_h} \frac{w_h(w_h^2-1)}{r_h^2} \frac{\sum_{p=1}^{P} \frac{\tau_p W_h^{p-1}}{(2p-1)!(d-2p-2)!}}{\sum_{p=1}^{P} \frac{\tau_p W_h^{p-1}}{(2p-1)!(d-2p-1)!}}, \quad (28)$$

the value of the gauge field on the event horizon being the essential parameter. Here the obvious condition $N'(r_h) > 0$ imposes some limits on the event horizon radius as a function of $w_h$ for given $(\tau_i, \Lambda)$.

Since the field equations are invariant under $w \to -w$, one can take $w(0) = 1$ and $w(r_h) > 0$ without any loss of generality.

For $r \gg 1$ we find for both regular and black hole solutions

$$w(r) = \pm 1 + \frac{w_1}{r^{d-3}} + \ldots,$$
$$m(r) = M - \frac{\tau_1 (d-3)w_1^2}{8\ell^2} \frac{1}{r^{d-3}} + \ldots, \quad (29)$$
$$\sigma(r) = 1 - \frac{w_1^2 (d-3)^2 \tau_1}{2\kappa(d-2)} \frac{1}{r^{d-4}} + \ldots.$$

These boundary conditions are also shared by the asymptotically flat solutions [with a different decay of the mass function $m(r)$, however], $w = \pm 1$ being again the only allowed values of the gauge function as $r \to \infty$. Therefore, we expect to find a qualitatively similar picture in both cases. We will find in Sec. V that the constant $M$ in the above relations is the ADM mass up to a $d$-dependent factor. However, in the discussion of numerical solutions we will refer to $M$ to as the mass of the solutions.

### C. Further relations

The form (25) of the reduced action allow to derive an useful virial relation. To this end, we use the scaling technique proposed in [37,38] for the case of spherically symmetric gravitating systems. Let us assume the existence of a globally regular solution $m(r), \sigma(r), w(r)$ of the field equations (24), with suitable boundary conditions at the origin and at infinity. Then each member of the 1-parameter family

$$m_\lambda(r) \equiv m(\lambda r), \qquad \sigma_\lambda(r) \equiv \sigma(\lambda r), \qquad w_\lambda(r) \equiv w(\lambda r) \tag{30}$$

assumes the same boundary values at $r = 0$ and $r = \infty$, and the action $S_\lambda \equiv S[m_\lambda, \sigma_\lambda, w_\lambda]$ must have a critical point at $\lambda = 1$, i.e. $[dS/d\lambda]_{\lambda=1} = 0$. Therefore, we find the following virial relation satisfied by the finite energy solutions of the field equations (note that following [37], it is possible to write a similar relation for black hole configurations, also)

$$\sum_{p=1}^{P}\int_0^\infty dr\sigma \frac{\tau_p(d-3)!}{2(2p)!d-[2p+1])!}W^{p-1}\Big((d-4p-1)\Big(2pN\frac{w'^2}{r^2}+(d-2p-1)W\Big)+2p\frac{w'^2}{r^2}\Big(\frac{2(d-3)m}{\kappa r^{d-3}}+\frac{2r^2}{\ell^2}\Big)\Big)=0. \tag{31}$$

For $p = 1$, i.e. a $F^2$-theory, the above relation reads

$$\int_0^\infty dr\sigma r^{d-4}\Big((d-5)(Nw'^2+\frac{(w^2-1)^2}{2r^2}\Big)+w'^2\Big(\frac{2m}{\kappa r^{d-3}}(d-3)+\frac{2r^2}{\ell^2})\Big)=0, \tag{32}$$

which clearly shows that no nontrivial gravitating solution with finite mass exists for $d > 4$, since all terms in the integrand are strictly positive quantities.

Therefore, it becomes obvious that new terms in the YM hierarchy should be introduced as the spacetime dimension increases. For a given $d$, the relation $P > [(d+1)/4]$ should be satisfied. As it happens, to go to $5 \leq d < 9$ it is necessary to include at least the second member of the YM hierarchy to provide the requisite scaling (similarly for $9 \leq d < 13$ it is necessary to include the thrid member of the YM hierarchy). In practice we add only the lowest order such term necessary.

We mention here also the Hawking temperature expression of the black hole solutions. For the line element (5), if we treat $t$ as complex, then its imaginary part is a coordinate for a nonsingular Euclidean submanifold if and only if it is periodic with period

$$\beta = \frac{4\pi}{N'(r_h)\sigma(r_h)}. \tag{33}$$

Then continuous Euclidean Green functions must have this period, so by standard arguments the Hawking temperature is (with $k_B = \hbar = 1$)

$$\begin{aligned} T_H &= \frac{\sigma_h}{4\pi r_h}\Big(d-3-\frac{2m'(r_h)}{\kappa r_h^{d-3}}+(d-1)\frac{r_h^2}{\ell^2}\Big) \\ &\leq \frac{\sigma_h}{4\pi r_h}\Big(d-3+(d-1)\frac{r_h^2}{\ell^2}\Big). \end{aligned} \tag{34}$$

Thus the Hawking temperature of such systems appears to be suppressed relative to that of a vacuum black hole of equal horizon area.

In the presence of higher order terms in the YM action, dimensionless quantities are obtained by rescaling

$$\begin{aligned} r \to (\tau_2/\tau_1)^{1/4}, \qquad \Lambda \to (\tau_1/\tau_2)^{1/2}, \\ m(r) \to m(r)\kappa(\tau_1/\tau_2)^{(d-3)/4}, \end{aligned} \tag{35}$$

This reveals the existence of one fundamental parameter which gives the strength of the gravitational interaction

$$\alpha^2 = \frac{\tau_1^{3/2}}{\kappa\tau_2^{1/2}}, \tag{36}$$

and $P - 2$ independent coupling constants

$$\beta_{k-2}^2 = \frac{\tau_k}{\tau_1}\Big(\frac{\tau_1}{\tau_2}\Big)^{k-1}, \tag{37}$$

with $k = 3, P - 2$ (i.e. no such constants appear in a $p = 2$ system).

For the $F(2) + F(4)$ systems in $d = 6, 7, 8$ considered in [13], there exist gravity decoupling solutions at $\alpha = 0$, from which the gravitating solutions branch out to a maximum value of $\alpha_{\max}$ and then decrease. The second limit of $\alpha \to 0$ also exists. For the same system in $d = 5$ studied in [14] on the other hand, there exists no gravity decoupling limit but nonetheless the solution branches out from $\alpha = 0$ by employing a scaling procedure [18], and again $\alpha$ increases to a value $\alpha_{\max}$ and decreases. But in this case it does not reach the $\alpha \to 0$ limit. Rather it stops at a new critical point $\alpha_c$ around which it oscillates [14], which is a new type of critical point identified in [18] and named a "conical" fixed point.

### D. Numerical solutions

In the present work, we restrict our attention to the simplest nontrivial cases with only the two terms $p = 1$ and 2 in the YM hierarchy. However, we have obtained some numerical results also for a $P = 3$ hierarchy with $\beta_1 = 1$, which will be briefly mentioned. For $d = 5$, the solutions we found have some special features which will be discussed separately.

Both regular and black hole solutions of the EYM-hierarchy equations appear to exist for any value of $\Lambda$. Given $(\alpha, d, \Lambda)$, AAdS solutions may exist for a discrete set of shooting parameters $b$ and $w_h$ respectively. We follow the usual approach and, by using a standard ordinary differential equation solver, we evaluate the initial conditions at $r = 10^{-5}$ (or $r_h + 10^{-5}$) for global tolerance $10^{-14}$, adjusting for shooting parameters and integrating towards $r \to \infty$, and looking for AAdS solutions with a finite mass.

Similar to the $p = 1, d = 4$ asymptotically flat case, it can be proven that for all AAdS (or $\Lambda = 0$) solutions, $w(r)$ is confined within the strip $|w(r)| < 1$. This can be proven as follows: we suppose the existence of solutions with $w(r) > 1$ for some interval of $r$. Therefore, $w$ must develop a maximum for some $r_0$, $w'(r_0) = 0$ and $w(r_0) > 1$ with $w''(r_0) < 0$. However, the Eqs. (24) imply that in the region $w > 1$ the only extremum can be a minimum. Therefore, the condition $|w(r)| < 1$ is always fulfilled. As a general feature, all solutions discussed in the rest of this section present only one node in the gauge function $w(r)$. Similar to the $\Lambda = 0$ case, we could not find multinode solutions.[5]

The absence of multinode solutions in this $F(2) + F(4)$ model with $\Lambda = 0$, in the relevant dimensions $5 \leq d \leq 8$ is analytically explained in [18]. We expect that the relevant fixed point analysis yields qualitatively similar results also for $|\Lambda| > 0$. This is borne out by our numerical results.

For any regular solution, the metric functions $m(r)$ and $\sigma(r)$ always increase monotonically with growing $r$ from $m(0) = 0$ and $\sigma(0) = \sigma_0$ at the origin to $m(\infty) = M$ and $\sigma(\infty) = 1$, respectively. The gauge function always interpolates between $w(0) = 1$ and $w(\infty) = -1$ without any local extrema. For black hole configurations, the behavior of the functions $m(r)$, $\sigma(r)$, and $w(r)$ is similar to that for regular solutions. The gauge potential $w(r)$ starts from some finite value $0 < w(r_h) < 1$ at the horizon and monotonically approaches $-1$ at infinity. The metric functions $m(r)$ and $\sigma(r)$ increase also monotonically with $r$. In the asymptotic region, the geometry corresponds to a Schwarzschild-AdS solution. However, although these solutions are static and have vanishing YM charges ($w^2(\infty) = 1$), they are different from the Schwarzschild-AdS black hole, and therefore are not fully characterized by the mass parameter $M$.

[5]Multinode solutions exist if the lowest order YM term is $F(2p)$ with $p \geq 2$, in the appropriate dimensions [18] $d$.

#### *1. Regular solutions* $d = 5$

As $\alpha^2 \to 0$, the YM equations present a nontrivial, finite energy solution in a fixed AdS background. This nongravitating configuration approaches in the $\Lambda = 0$ limit the YM instanton in four dimensional flat space [39].

When $\alpha^2$ increases, this solution gets deformed by gravity and the mass $M$ decreases. At the same time, both the value $\sigma(0)$ and the minimal value $N_m$ of the function $N(r)$ decrease, as indicated in Fig. 3. This branch of solutions exists up to a maximal value $\alpha_{\max}$ of the parameter $\alpha$, which is smaller than the corresponding value in the asymptotically flat case [14]. For example, we find numerically $\alpha^2_{\max} \approx 0.3445$ for $\Lambda = -0.2$ while the corresponding value for $\Lambda = -0.01$ is $\alpha^2_{\max} \approx 0.5322$. (Without a cosmological term, this branch extends up to $\alpha^2_{\max} \approx 0.5648$.)

Similar to the EYM theory with $\Lambda = 0$ [14], we found always another branch of solutions on the interval $\alpha^2 \in [\alpha^2_{\mathrm{cr}(1)}, \alpha^2_{\max}]$ with $\alpha^2_{\mathrm{cr}(1)}$ depending again on the value of $\Lambda$ (e.g. $\alpha^2_{\mathrm{cr}(1)} \approx 0.2876$ for $\Lambda = -0.2$). On this second branch of solutions, both $\sigma(0)$ and $N_m$ continue to decrease but stay finite. However, a third branch of solutions exists for $\alpha^2 \in [\alpha^2_{\mathrm{cr}(1)}, \alpha^2_{\mathrm{cr}(2)}]$, on which the two quantities decrease further. A fourth branch of solutions has also been found, with a corresponding $\alpha^2_{\mathrm{cr}(3)}$ close to $\alpha^2_{\mathrm{cr}(2)}$. Further branches of solutions, exhibiting more oscillations very likely exist but their study is a difficult numerical problem. Along this succession of branches, the main observation is that the value $\sigma(0)$ decreases much faster than that of $N_m$ as illustrated in Fig. 3. Also, the mass parameters do not increase significantly along these secondary branches. However, the shooting parameter $b$ increases to very large values. The pattern strongly suggests that after a finite (or more likely infinite) number of oscillations of $\sigma(0)$, the

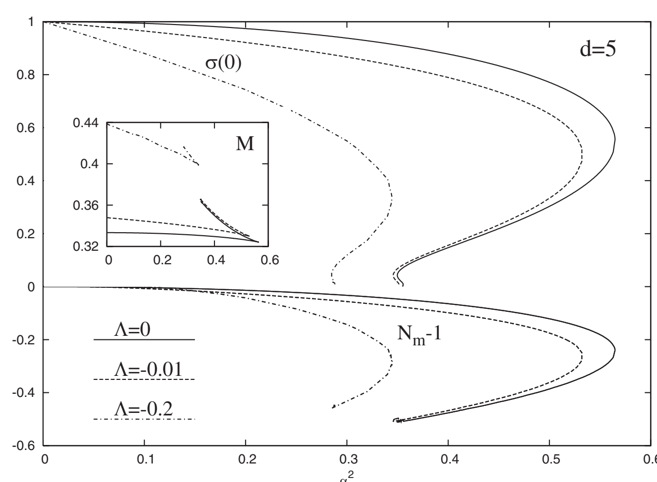


FIG. 3. The value $N_m$ of the minimum of the metric function $N(r)$, the mass parameter $M$ as well as the value of the metric function $\sigma$ at the origin, $\sigma(0)$, are shown for $d = 5$ solutions as functions of $\alpha^2$ and several values of $\Lambda$.

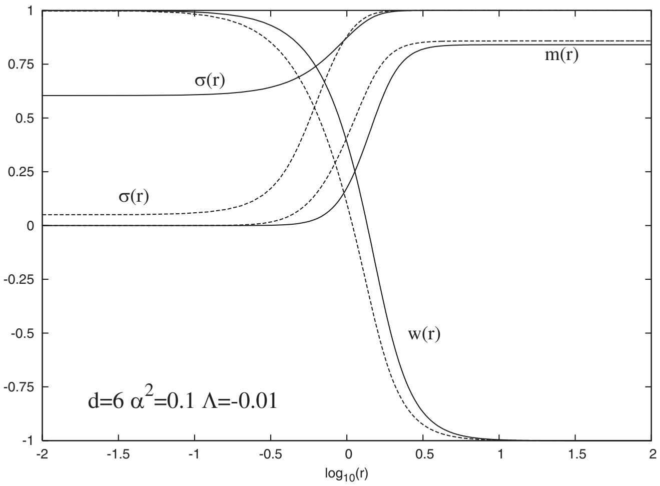


FIG. 4. The profiles of the functions $m(r)$, $\sigma(r)$, and $w(r)$ are plotted as functions of radius for typical $d = 6$ regular solutions in a EYM theory with $p = 1, 2$ terms and $\alpha^2 = 0.1$, $\Lambda = -0.01$. Here and in Fig. 14 the continuous line corresponds to an upper branch solution, the dotted line denoting lower branch profiles.

solution terminates into a singular solution with $\sigma(0) = 0$ and a finite value of $N(0)$.

This is the behavior observed in [14] for the EYM theory with $\Lambda = 0$. The inclusion of a negative cosmological constant does not seem to qualitatively change the properties of the system, but leads to different values of the critical parameters. As in the $\Lambda = 0$ case, the dominant term at the gravity decoupling limit $\alpha \to 0$ is the $F(2)$ term, the energy being given by the action of the usual instanton [39], while as $\alpha \to \alpha_{cr}$ the dominant term is $F(4)$. The mechanism for this effect is explained in [18] (for $\Lambda = 0$) and is supported by our numerical results here. The typical $d = 5$ globally regular solutions look very similar to the $d = 6, 8$ profiles presented in Figs. 4 and 5.

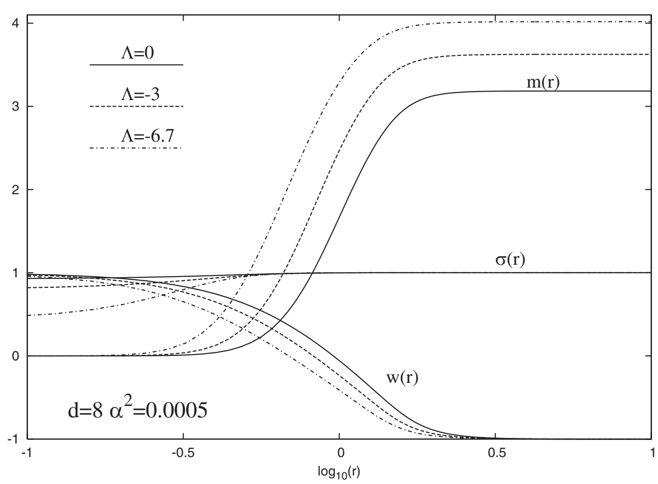


FIG. 5. Typical globally regular solutions of $d = 8$ EYM theory with $\alpha^2 = 0.0005$ and $p = 1, 2$ terms are plotted as functions of radius for several values of $\Lambda$.

### 2. *Regular solutions* $d > 5$

The solutions in this case resemble again the $\Lambda = 0$ situation. In the presence of suitable higher order term in the hierarchy, the YM equations admit finite energy solutions in a fixed $AdS_d$ A $p = 1$, $AdS_4$ exact solution was found in [40], its higher order generalization (specifically for $p = 2$, $AdS_8$) for $d > 4$ being discussed in [41]. According to the standard arguments, this AdS soliton can be generalized in the presence of gravity, provided that the dimensionless coupling constant $\alpha$ is small enough. Therefore the gravitating solutions exist up to a maximal value $\alpha_{max}$ of the gravitational coupling constant. This value $\alpha_{max}$ for a given $d$ depends on the value of $\Lambda$. For example in $d = 6$, $\alpha_{max}(\Lambda = 0) = 0.126\,75$; $\alpha_{max}(\Lambda = -1) = 0.070\,422$; in 8 dimensions we find $\alpha_{max}(\Lambda = 0) = 0.002\,193$ while $\alpha_{max}(\Lambda = -5) = 6.69 \times 10^{-4}$.

When $\alpha$ increases, the mass of the gravitating solutions decreases while the function $N(r)$ develops a local minimum $N_m$ which becomes deeper while gravity becomes stronger and the value $\sigma(0)$ decreases from one. At the same time, the value of the shooting parameter $b$ increases with $\alpha$. Our numerical analysis for $d \leq 10$ indicates that a second branch of regular solutions always exists, starting at $\alpha_{max}$. Along this second branch the values $\sigma(0)$ and $N_m$ decrease monotonically with $\alpha$, while $b$ and $M$ still increase. The mass of a second branch solution is always larger than the corresponding mass (for the same value of $\alpha$) on the first branch. For $d > 6$, the numerical analysis suggests that this second branch persists up to $\alpha^2 \simeq 0$ and that in this limit $\sigma(0)$ approaches a very small value. As far as our numerical analysis indicates, the value $N_m$ tends to a finite value in this limit so that there occurs no horizon. Therefore, two regular solutions seems always to exist for any $\alpha < \alpha_d$.

The case $d = 6$ is special, since the numerical procedure fails to give reliable results for second branch solutions, starting with some $\alpha_d$, whose value is $\Lambda$-dependent [for example we found $\alpha_d(\Lambda = -1) = 0.0435$ while $\alpha_d(\Lambda = 0) = 0.0573$]. The quantity $\sigma(0)$ reaches a very small value as $\alpha \to \alpha_d$. The minimal value of $N(r)$ remains finite so that no horizon is approached. We expect that a different parametrization of the metric and variables would allow us to continue this second branch to $\alpha \to 0$ in this case, too.

The behaviors just described qualitatively duplicate those of the $\Lambda = 0$ case [13], and are analyzed in [18]. Likewise, the solutions are dominated by the $F(2)$ terms in the gravity decoupling limit $\alpha \to 0$, while at the other end (on the second mass-branch) it is the $F(4)$ terms that dominate.

Typical $d = 6, 8$ solutions are presented in Figs. 4 and 5, respectively. In Figs. 5–8 we plot some relevant quantities for $d = 6, 7, 8$ and several values of $\Lambda$. One can see that the qualitative behavior of the functions $m, \sigma, w$ does not change by changing the value of $\Lambda$.

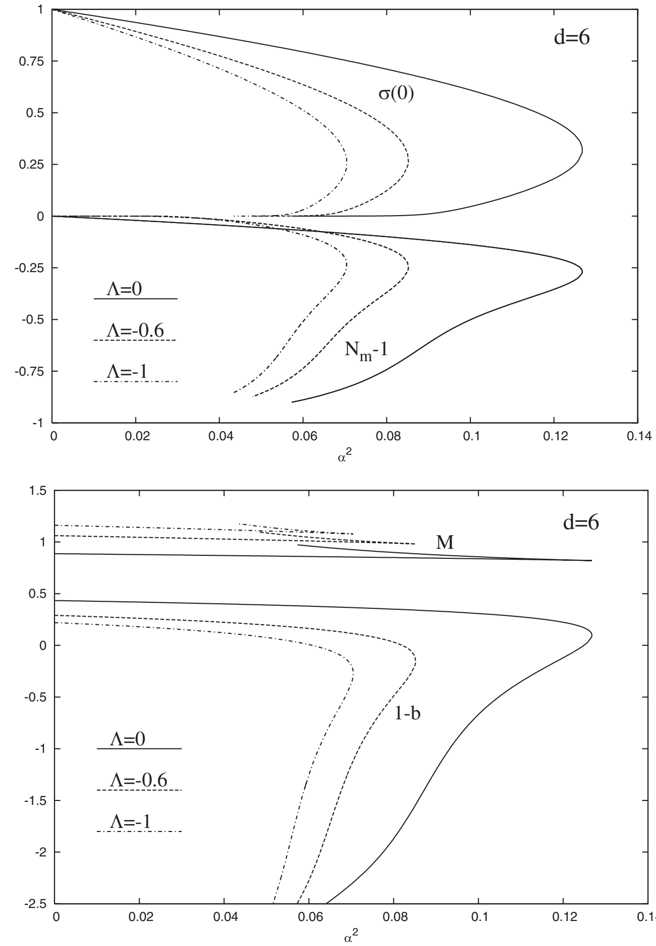


FIG. 6. The value $N_m$ of the minimum of the metric function $N(r)$ and the value of the metric function $\sigma$ at the origin $\sigma(0)$ (a), and, the parameters $M$ and $b$ (b), are shown for $d = 6$ solutions as functions of $\alpha^2$ and several values of $\Lambda$.

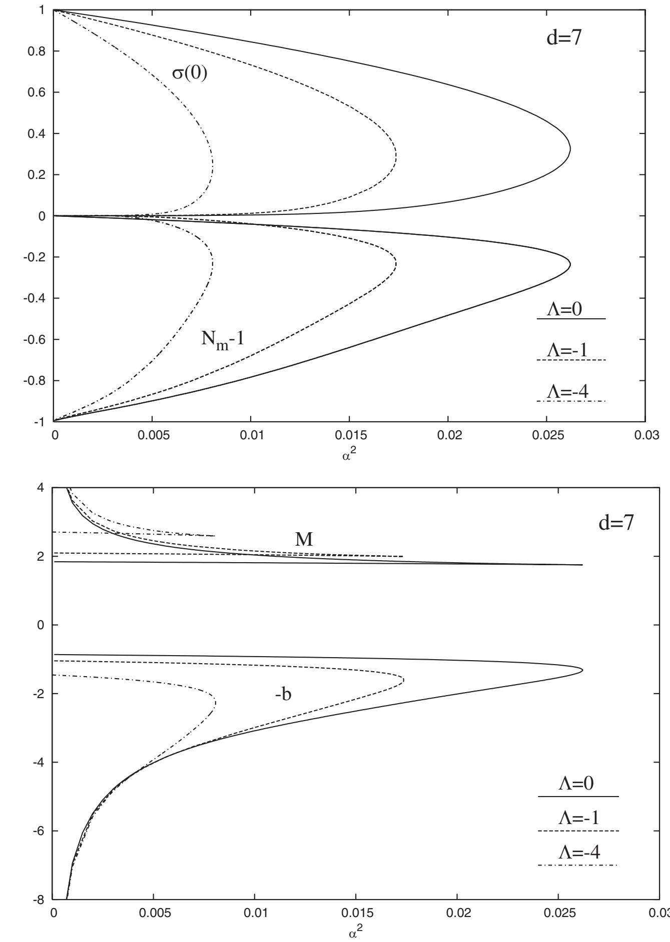


FIG. 7. The value $N_m$ of the minimum of the metric function $N(r)$ and the value of the metric function $\sigma$ at the origin $\sigma(0)$ (a), and, the parameters $M$ and $b$ (b), are shown for $d = 7$ solutions as functions of $\alpha^2$ and several values of $\Lambda$.

The results we found by including the $p = 3$ term in the YM-hierarchy for $d = 9, 10$ follows the same pattern. Although the picture gets more complicated by the existence of one more coupling parameters, two branches of solution are always found to exist. We noticed also the existence of a maximal value of $\alpha$ which is $(\Lambda, \beta_1)$ dependent.

### 3. Black hole solutions $d = 5$

According to the standard arguments, one can expect to find black hole generalizations for any regular configurations, at least for small values of the horizon radius $r_h$. For completeness we discuss here the basic features of the AAdS black hole solutions.

Again, the case $d = 5$ is special. The properties of these AAdS solutions are rather similar to the five dimensional asymptotically flat black hole configurations discussed in [14]. First, black hole solutions seem to exist for all values of $\alpha$ for which regular solutions could be constructed. Also, solutions exist only for a limited region of the $(r_h, \alpha)$ space.

The typical behavior of solutions as a function of $r_h$ is presented in Fig. 9, for a small value of $\alpha$ compared to the maximal value $\alpha_{\max}$ of the regular solutions, in which case we notice the existence of only one $r_h = 0$ regular configuration. Starting from this regular solution and increasing the event horizon radius, we find a first branch of solutions which extends to a maximal value $r_{h(\max)}$. As seen in Figs. 9 and 10, the value of $r_{h(\max)}$ depends on $\Lambda, \alpha$. The Hawking temperature decreases on this branch, while the mass parameter increases; however, the variation of mass and $\sigma(r_h)$ is relatively small. Extending backwards in $r_h$, we find a second branch of solutions for $r_h < r_{h(\max)}$. This second branch stops at some critical value $r_{h(\mathrm{cr})}$, where the numerical iteration fails to converge. The value of $\sigma(r_h)$ on this branch decreases drastically, as shown in Fig. 9. Also, the Hawking temperature after initially increasing, strongly decreases for values near $r_{h(\mathrm{cr})}$, ap-

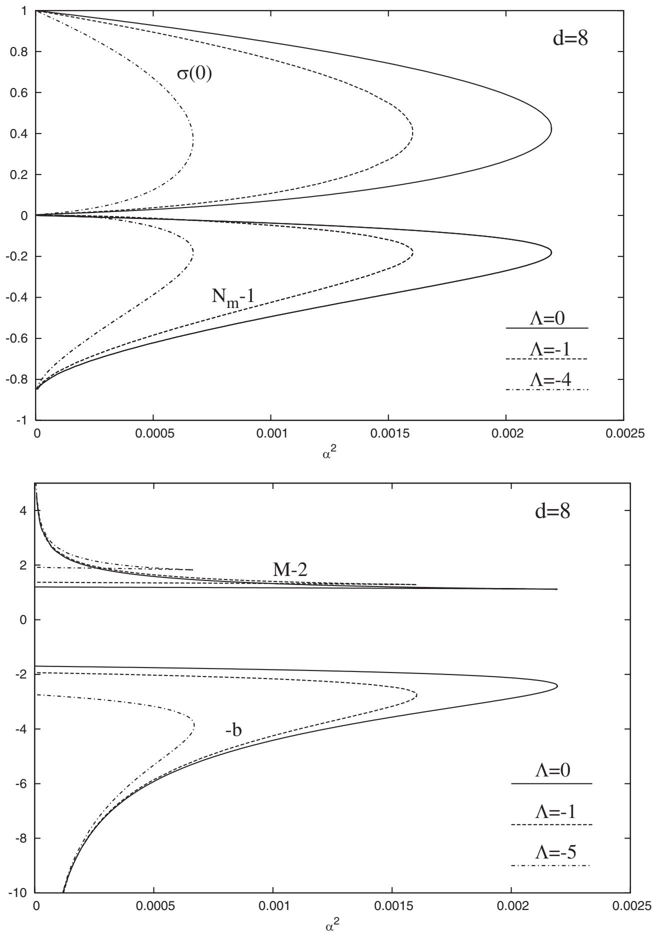


FIG. 8. The value $N_m$ of the minimum of the metric function $N(r)$ and the value of the metric function $\sigma$ at the origin $\sigma(0)$ (a), and the parameters $M$ and $b$ (b), are shown for $d = 8$ solutions as functions of $\alpha^2$ and several values of $\Lambda$.

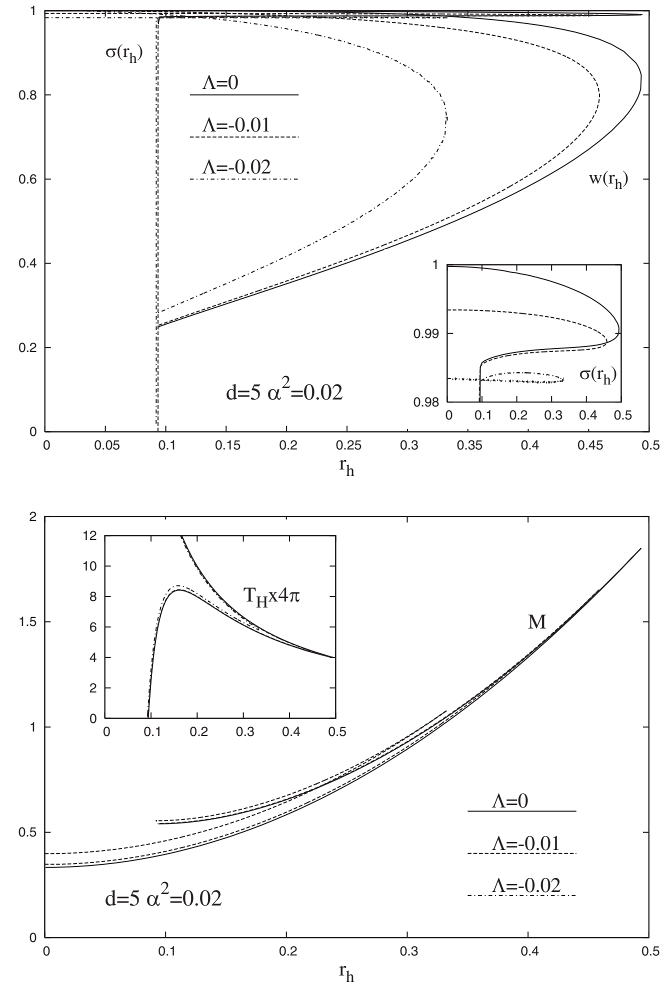


FIG. 9. In (a) we plot the value of the gauge field function at the horizon $w(r_h)$ and $\sigma(r_h)$, the value of the metric function $\sigma$ at the horizon (the magnified profiles of $\sigma(r_h)$ are displayed in the window, to help distinguish these from the profiles of $w(r_h)$). In (b), the mass parameter $M$ and the Hawking temperature $T_H$ are presented. All profiles as functions of the event horizon radius $r_h$ for the $p = 1, 2$ black hole solutions in five dimensions with $\alpha^2 = 0.02$ and several values of $\Lambda$.

proaching a very small value, while the increase of the total mass is still very small. Similar to the $\Lambda = 0$ case [14], higher branches of solutions on which the value $\sigma(r_h)$ continues to decrease further to zero are likely to exist. However, the extension of these branches in $r_h$ will be very small, which makes their study difficult. An approach to this problem with different parametrization appears to be necessary.

However we find, that the global picture is changed by considering large enough values of $\alpha$. In this case, more than one regular configuration exists for a given value of $\alpha$. This situation is illustrated in Fig. 10, for solutions with $\alpha^2 = 0.5$. Two regular solutions exist for this particular value of $\alpha$ and we find two black hole branches connecting these $r_h = 0$ configurations. Again, the mass of the second branch solutions is always larger than the corresponding mass on the first branch.

Preliminary numerical results indicate an even more complicated picture for solutions with $\alpha$ near $\alpha_{\max}$. In this case the configurations combine features of both types of solutions discussed above. Several branches of black hole solutions are found for the same $\alpha$. These branches start from regular configurations and are possibly disconnected.

#### *4. Black hole solutions* $d > 5$

Although predicted in [14], no discussion of the $\Lambda = 0$, $d > 5$ black holes is presented in literature.

Again, black hole counterparts appear to exist for any regular solution. However, solutions with the right asymptotics are found for a limited region of the $(r_h, \alpha)$ space only. We plot in Figs. 11–13 some results we found for $d = 6, 7$ and $d = 8$. Starting for a given $\alpha_0 < \alpha_{\max}$ from a $r_h = 0$ first branch regular solution, we found the existence of a branch of black hole solutions extending up to a maximal

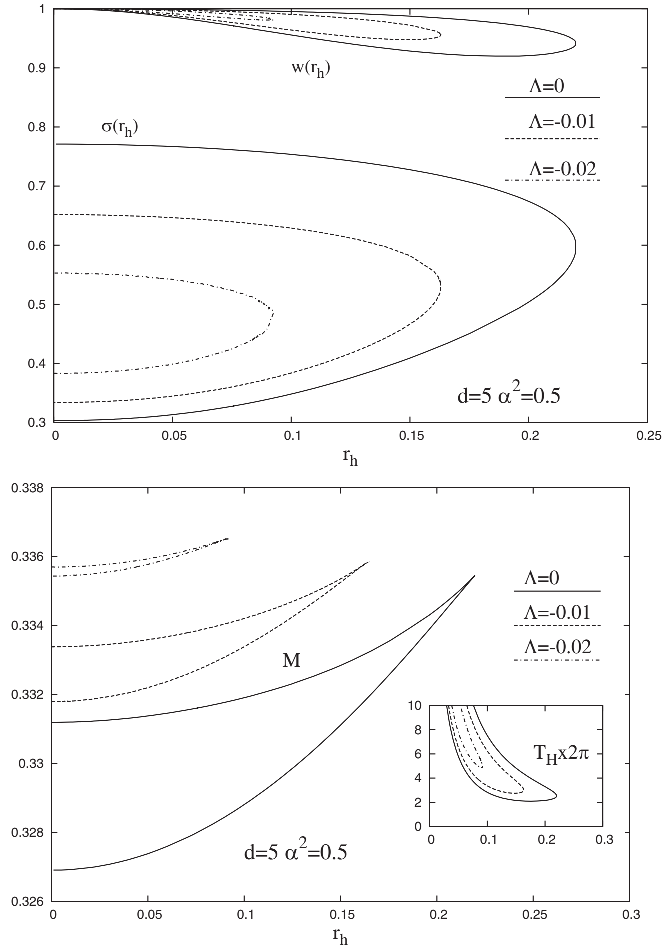


FIG. 10. The value of the gauge field function at the horizon $w(r_h)$ and $\sigma(r_h)$, the value of the metric function $\sigma$ at the horizon, (a), as well as the mass parameter $M$ and the Hawking temperature $T_H$ (b) are shown as functions of the event horizon radius $r_h$ for the $p = 1, 2$ black hole solutions in five dimensions with $\alpha^2 = 0.5$ and several values of $\Lambda$.

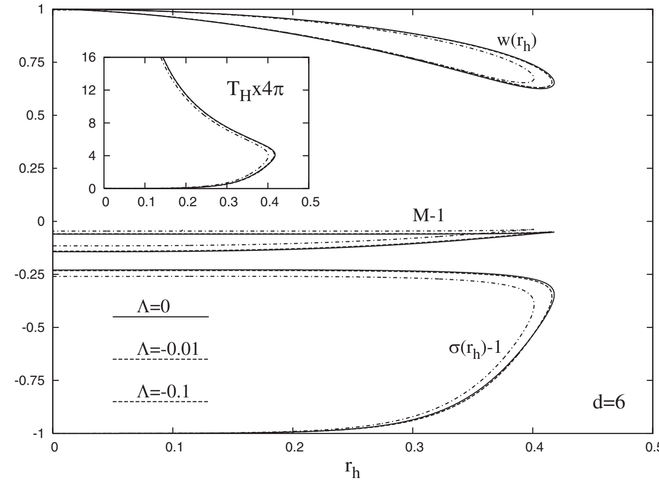


FIG. 11. The value of the gauge field function at the horizon $w(r_h)$ and $\sigma(r_h)$, the value of the metric function $\sigma$ at the horizon, as well as the mass parameter $M$ and the Hawking temperature $T_H$ are shown as functions of the event horizon radius $r_h$ for the $p = 1, 2$ black hole solutions in six dimensions with $\alpha^2 = 0.066$ and several values of $\Lambda$.

value of the event horizon radius $r_h = r_h^{\max}$. When $r_h$ increases, the mass and the Hawking temperature increase while the value $\sigma(r_h)$ decreases from its value at the regular solution. A second branch of black hole solutions seems to appear always at $r_h^{\max}$, extending backwards to a zero event horizon radius. This limiting solution corresponds to the second branch of the regular solution at this value of $\alpha = \alpha_0$. Like in those $d = 5$ cases where there are two regular solutions for a given $\alpha$, say $\alpha_0$, here too these two regular solutions are the $r_h \to 0$ limits of the corresponding black hole "loop," connecting the two regular solutions. Along this second branch the values $\sigma(r_h)$ of the metric function $\sigma$ on the event horizon decrease monotonically with $r_h$, while the Hawking temperature strongly increases. The mass of the solution of the second branch is larger than the corresponding one on the first branch, as illustrated by Figs. 10–12. It is interesting to note here, from the point of view of numerics, that a black hole loop corresponding to the two $r_h \to 0$ limits of two regular solutions, say at $\alpha_0$, can be constructed numerically even when the numerical process for the higher mass branch of the regular solutions for this $\alpha_0$ runs into difficulties.

The introduction of a negative cosmological constant does not appear to change this picture qualitatively. However, we notice a smaller value of $r_h^{\max}$ with increasing $|\Lambda|$ and a larger value of $M$ for the same $\alpha$.

In Fig. 14, we present the profiles of the metric functions $N$ and $\sigma$ and gauge function $w(r)$ for the same values of

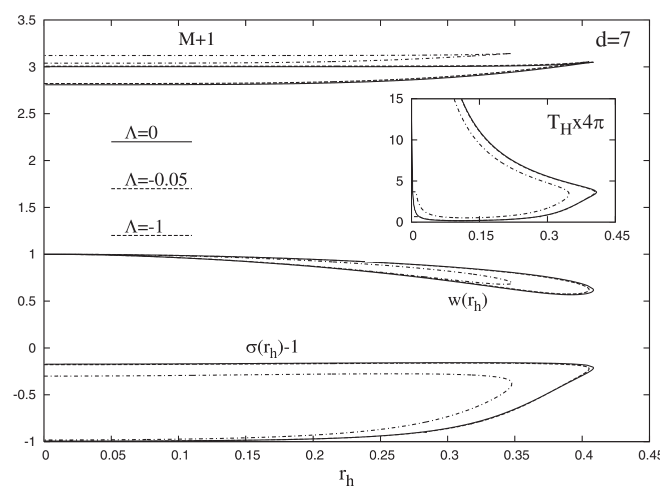


FIG. 12. The value of the gauge field function at the horizon $w(r_h)$ and $\sigma(r_h)$, the value of the metric function $\sigma$ at the horizon, as well as the mass parameter $M$ and the Hawking temperature $T_H$ are shown as functions of the event horizon radius $r_h$ for the $p = 1, 2$ black hole solutions in seven dimensions with $\alpha^2 = 0.011$ and several values of $\Lambda$.

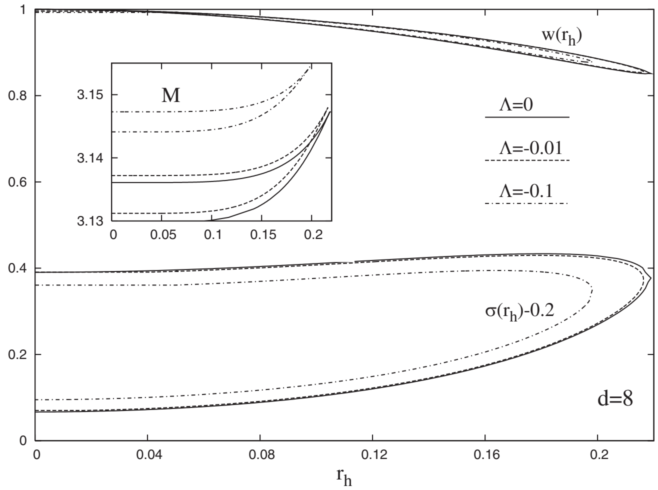


FIG. 13. The value of the gauge field function at the horizon $w(r_h)$ and $\sigma(r_h)$, the value of the metric function $\sigma$ at the horizon, as well as the mass parameter $M$ are shown as functions of the event horizon radius $r_h$ for the $p = 1, 2$ black hole solutions in eight dimensions with $\alpha^2 = 0.002$ and several values of $\Lambda$.

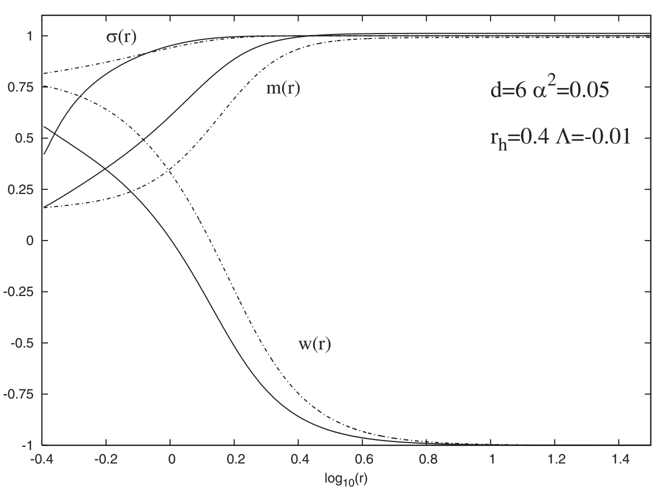


FIG. 14. The profiles of the functions $m(r)$, $\sigma(r)$, and $w(r)$ are plotted as functions of the radius for typical $d = 6$ black hole solutions in a EYM theory with $p = 1, 2$ terms and $\alpha^2 = 0.05$, $\Lambda = -0.01$.

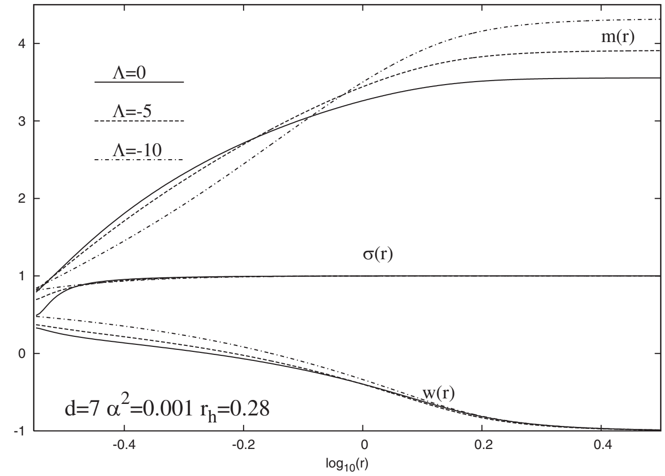


FIG. 15. Typical black hole solutions of $p = 1, 2$ EYM theory in seven dimensions with $\alpha^2 = 0.001$, $r_h = 0.28$ are plotted as functions of the radius for several values of $\Lambda$.

$\alpha^2, \Lambda$ on the first and second branch for some $d = 6$ solutions. The dependence of these functions on the value of $\Lambda$ is illustrated in Fig. 15 for several $d = 7$ black hole solutions.

## IV. A COMPUTATION OF MASS AND ACTION

### A. The counterterm method

It is well known that the generalization of Komar's formula for AAdS spacetimes is not straightforward and requires the further subtraction of a background configuration in order to render a finite result for the mass. This problem was addressed for the first time in the 1980s, with different approaches (see for instance Refs. [42,43]). Another formalism to define conserved charges in AAdS spacetimes was proposed in [44] and uses conformal techniques to construct conserved quantities yielding the results obtained by Hamiltonian methods. Other more recent approaches to the same problem are presented in Ref. [45].

As expected, these different methods yield the same total mass for the spherically symmetric AAdS configurations considered in Sec. III

$$M_{\rm ADM} = \frac{(d-2)\Omega_{d-2}}{8\pi G} M, \tag{38}$$

where $\Omega_{d-2} = 2\pi^{(d-1)/2}/\Gamma((d-1)/2)$ is the area of a unit $(d-2)$-dimensional sphere, and $M$ is defined in the second member of (29).

A procedure leading (for odd dimensions) to a different result has been proposed by Balasubramanian and Kraus [20]. This technique was inspired by AdS/CFT correspondence and consists in adding suitable counterterms $I_{\rm ct}$ to the action of the theory in order to ensure the finiteness of the boundary stress tensor derived by the quasilocal energy definition [46]. These counterterms are built up with curvature invariants of a boundary $\partial\mathcal{M}$ (which is sent to infinity after the integration) and thus obviously they do not alter the bulk equations of motion. Unlike background subtraction, the counterterm approach does no require the identification of a reference spacetime. Given the potential relevance of the EYM solutions in an AdS/CFT context, we present here a computation of the boundary stress tensor and of the total mass, by using the counterterm prescription.

The following counterterms are sufficient to cancel divergences in a pure gravity theory for $d \leq 9$, with several exceptions (see e.g. [47])

$$I_{ct} = -\frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^{d-1}x\sqrt{-h}\Big[\frac{d-2}{\ell} + \frac{\ell\Theta(d-4)}{2(d-3)}\mathcal{R} + \frac{\ell^3\Theta(d-6)}{2(d-5)(d-3)^2}\Big(\mathcal{R}_{AB}\mathcal{R}^{AB} - \frac{d-1}{4(d-2)}\mathcal{R}^2\Big) + \frac{\ell^5\Theta(d-8)}{(d-2)^3(d-4)(d-6)}\Big(\frac{3d+2}{4(d-1)}\mathcal{R}R^{AB}\mathcal{R}_{AB} - \frac{d(d+2)}{16(d-1)^2}\mathcal{R}^3 - 2\mathcal{R}^{AB}\mathcal{R}^{CD}\mathcal{R}_{ABCD} - \frac{d}{4(d-1)}\nabla_A\mathcal{R}\nabla^A\mathcal{R} + \nabla^C\mathcal{R}^{AB}\nabla_C\mathcal{R}_{AB}\Big)\Big]. \tag{39}$$

Here $\mathcal{R}_{ABCD}$, $\mathcal{R}_{AB}$ are the Riemann and Ricci tensors, $\mathcal{R}$ is the Ricci scalar for the boundary metric $h$ and $\Theta(x)$ is the step function, which is equal to 1 for $x \geq 0$ and zero otherwise; $A, B, \ldots$ indicate the intrinsic coordinates of the boundary.

Using these counterterms one can construct a nondivergent boundary stress tensor, which is given by the variation of the total action at the boundary with respect to $h_{AB}$. Its explicit expression, restricting for simplicity to $d < 7$, is

$$T_{AB} = \frac{1}{8\pi G}\Big(K_{AB} - Kh_{AB} - \frac{d-2}{\ell}h_{AB} + \frac{\ell}{d-3}E_{AB}\Big), \tag{40}$$

where $K_{AB} = -\frac{1}{2}(\nabla_A n_B + \nabla_B n_A)$ is the extrinsic curvature defined in terms of the normal $n_A$ to the boundary, $K$ is its trace, and $E_{AB}$ is the Einstein tensor of the intrinsic metric $h_{AB}$. (The corresponding form of (40) for $d = 7, 8$ is given e.g. in [48]).

The result we find in this way for $T_{AB}$ is given by

$$T_A^B = \frac{1}{8\pi G\ell}\Big(M + \sum_p (-1)^p \ell^{d-3}\frac{\Gamma(\frac{2p-1}{2})}{2\sqrt{\pi}\Gamma(p+1)}\delta_{2p,d-1}\Big) \times ((d-1)u_A u^B + \delta_A^B)\frac{1}{r^{d-1}} + O\Big(\frac{1}{r^d}\Big), \tag{41}$$

where $u_A = \delta_A^t$ and $p$ is an integer. We can use this approach to assign a mass-energy to an AAdS geometry by writing the boundary metric in an ADM form

$$h_{AB}dx^A dx^B = -N_\Sigma^2 dt^2 + \sigma_{ab}(dx^a + N_\sigma^a dt)(dx^b + N_\sigma^b dt) \tag{42}$$

and the definition of the energy in this context is

$$E = \int_{\partial\Sigma} d^{d-1}x\sqrt{\sigma}N_\Sigma\epsilon. \tag{43}$$

Here $\epsilon = u^\mu u^\nu T_{\mu\nu}$ is the proper energy density while $u^\mu$ is a timelike unit normal to $\Sigma$.

If there are matter fields on $\mathcal{M}$, additional counterterms may be needed to regulate the action. This is the case of $F(2)$ theory, discussed in Appendix B. The counterterm action depends in this case not only on the boundary metric but also on the boundary value of the gauge field.

However, we find that for $P > 1$ EYM solutions with $\Lambda < 0$ in $d = 5, 6, 7, 8$ dimensions, the prescription (39) removes all divergences. The use of higher order terms in the YM curvature, namely $F(2p)$ forms with $p > 1$ introduced in Sec. III, results in this regularizing of the masses. The crucial point here is that these solutions approach asymptotically a Schwarzschild-AdS background, and the YM asymptotic parameter $w_1$ appears only in the next to leading order of the $T_{AB}$ expression.

The mass-energy of solutions computed in this way is

$$E = \frac{(d-2)\Omega_{d-2}}{8\pi G}M + E_0 \tag{44}$$

where, for $3 < d < 9$

$$E_0 = \frac{(d-2)\Omega_{d-2}}{16\pi G}\Big(\frac{3}{4}\ell^2\delta_{5,d} - \frac{5}{8}\ell^4\delta_{d,7}\Big). \tag{45}$$

The additional term $E_0$ appearing in $E$ for $d = 5, (7)$ is the mass of pure global $AdS_{5,(7)}$ and is usually interpreted as the energy dual to the Casimir energy of the CFT defined on a four (six) dimensional Einstein universe [20].

The metric restricted to the boundary $h_{AB}$ diverges due to an infinite conformal factor $r^2/\ell^2$. The background metric upon which the dual field theory resides is

$$\gamma_{AB} = \lim_{r\to\infty}\frac{\ell^2}{r^2}h_{AB}. \tag{46}$$

For the asymptotically $AdS_d$ solutions considered here, the $(d-1)$ dimensional boundary is the Einstein universe, with the line element

$$\gamma_{AB}dx^A dx^B = -dt^2 + \ell^2 d\Omega_{d-2}^2. \tag{47}$$

In light of the AdS/CFT correspondence, Balasubramanian and Kraus have interpreted Eq. (40) as $\langle\tau^{AB}\rangle = \frac{2}{\sqrt{-\gamma}}\frac{\delta S_{\text{eff}}}{\delta\gamma_{AB}}$, where $\langle\tau^{AB}\rangle$ is the expectation value of the CFT stress tensor. Then, the divergences which appear are simply the standard ultraviolet divergences of a quantum field theory and we can cancel them by adding local counterterms to the action. Corresponding to the boundary metric (47), the stress-energy tensor $\tau_{AB}$ for the dual theory can be calculated using the following relation [49]

$$\sqrt{-\gamma}\gamma^{AB}\langle\tau_{BC}\rangle = \lim_{r\to\infty}\sqrt{-h}\,h^{AB}T_{BC}. \tag{48}$$

### B. Action and entropy

The above approach can be used to compute the Euclidean action and to prove in a rigorous way that the entropy of the EYM black hole solutions is one quarter of the event horizon area, as expected. Here we start by constructing the path integral [21]

$$Z = \int D[g]D[\Psi]e^{-iI[g,\Psi]} \tag{49}$$

by integrating over all metrics and matter fields between some given initial and final hypersurfaces, $\Psi$ corresponding here to the SU(2) potentials. By analytically continuing the time coordinate $t \to i\tau$, the path integral formally converges, and in the leading order one obtains

$$Z \simeq e^{-I_{\rm cl}} \tag{50}$$

where $I_{\rm cl}$ is the classical action evaluated on the equations of motion of the gravity/matter system. The physical interpretation of this formalism is that the class of regular stationary metrics forms an ensemble of thermodynamic systems at equilibrium temperature $T$ (see e.g. [50]). $Z$ has the interpretation of partition function and we can define the free energy of the system $F = -\beta^{-1}\log Z$. Therefore

$$\log Z = -\beta F = S - \beta E, \tag{51}$$

or

$$S = \beta E - I_{cl}, \tag{52}$$

straightforwardly follows.

To compute $I_{\rm cl}$, we make use of the Einstein equations, replacing the $R - 2\Lambda$ volume term with $2R_t^t - 16\pi G T_t^t$. For our purely magnetic ansatz, the term $T_t^t$ exactly cancels the matter field Lagrangian in the bulk action. The divergent contribution given by the surface integral term at infinity in $R_t^t$ is also canceled by $I_{\rm surface} + I_{\rm ct}$ and for $3 < d < 9$ we arrive at the simple finite expression

$$I_{\rm cl} = \frac{\beta\Omega_{d-2}}{8\pi G}\left(M - \frac{r_h^{d-2}}{\ell^2} + \frac{3}{8}\ell^2\delta_{d,5} - \frac{5}{16}\ell^4\delta_{d,7}\right). \tag{53}$$

Replacing $I_{\rm cl}$ now in (52) (where $E$ is the mass-energy computed in Sec. IV A), we find

$$S = \tfrac{1}{4}\Omega_{d-2}r_h^{d-2}, \tag{54}$$

which is one quarter of the event horizon area, as expected.

From the AdS/CFT correspondence, we expect the non-Abelian hairy black holes to be described by some thermal states in a dual theory formulated in a Einstein universe background. The spherically symmetric solitons will correspond to zero-temperature states in the same theory. The existence of these hairy configurations suggest that there should be some observables in the dual CFT that encode the hair information.

## V. CONCLUSIONS

Motivated by recent results in EYM theory in four dimensional AAdS spacetime, we studied higher dimensional spherically symmetric solutions with non-Abelian fields in the presence of a negative cosmological constant. Since the mass-energy of the AAdS configurations plays a central role in its application to the AdS/CFT correspondence, we emphasized this aspect of the solutions we found. The mass-energy of the usual EYM solutions (both asymptotically flat and AAdS), defined according to the standard prescription, always diverges in spacetime dimensions $d > 4$. One of the tasks performed in this work was a demonstration of this fact in the AAdS case. The main properties of these higher dimensional $F(2)$ solutions resemble the $d = 4$ case, a continuum of solutions with arbitrary asymptotic values of the gauge function $w(r)$ being found.

Then we focused on two possible approaches of dealing with the divergences of the mass and action. One of these involved the regularization of the mass-energy using a counterterm mechanism. In this approach, it turns out that the counterterm action depends not only on the boundary metric, but also on the boundary values of the gauge fields. However, the masses of the solutions defined in this way may take negative values, leading us to the second approach.

The other, which formed the main thrust of the work, was to augment the action density of the system with higher order curvature terms, consisting of $2p$-form curvatures $F(2p)$. These terms were added to the usual YM system constructed from $F(2)$. It resulted in EYM solutions supporting finite mass-energy in all spacetime dimensions $5 \le d \le 2p$.

Concerning the construction of regular finite energy classical AAdS solutions in higher dimensions, we restricted ourselves to systems consisting exclusively of gravitational and non-Abelian gauge fields. The salient features of the resulting solutions are captured in this framework, the addition of other (string theory inspired) matter terms are being deferred to later work. With the asymptotically flat versions of the higher-$p$ EYM systems having been studied in [13,14,18], our task here involved the introduction of a negative cosmological constant. The most succinct way of listing our conclusions is:

(i) The qualitative properties of the regular AAdS solutions in spacetime dimensions $d = 5, 6, 7, 8$ are the same as the corresponding asymptotically flat ones, namely
   (1) A one parameter family of solutions parametrized by the (dimensionless) gravitational coupling constant $\alpha$ start at $\alpha = 0$ (the gravity decoupling limit) and exist up to a maximum $\alpha_{\rm max}$, after which $\alpha$ decreases again and ends at critical value.
   (2) For $d = 6, 7, 8$ the value of $\alpha$ in the second, not that of gravity decoupling, endpoint becomes very small and stops. In the $d = 8$ case it actually vanishes. For $d = 5$ the value of $\alpha$ in the second endpoint reaches a finite critical value, where it does not stop, but oscillates around this critical value.

(3) As long as the physically important $F(2)$ term is present in the YM sector, there exist no multinode solutions.

(ii) Black hole counterparts appear to exist for any regular solution. The qualitative properties of the AAdS black hole solutions are similar to the asymptotically flat case:

(1) Different from the four dimensional theory, the event horizon radius presents a maximal value. This maximal value is a function of the gravitational coupling constant $\alpha$.

(2) For $d = 6, 7, 8$ the black hole solutions form a loop connecting the two regular solutions with the same value of $\alpha$. The solutions of the five dimensional theory are somehow special, presenting a complicated branch structure which depends on $\alpha$.

Axially symmetric generalizations of these solutions are likely to exist. We expect them to share the basic properties of the spherically symmetric configurations.

## ACKNOWLEDGMENTS

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## APPENDIX A: A NONEXISTENCE PROOF FOR $p = 1$, $d > 5$ FINITE MASS SOLUTIONS WITH $\Lambda \leq 0$

Following the notation used in [11], we introduce a new variable

$$z = 2\log r, \tag{A1}$$

and rewrite the basic Eqs. (8) and (9) in the form

$$\frac{dm}{dz} = \frac{\tau_1}{4} e^{(d-5)z/2}\left(4N\left(\frac{dw}{dz}\right)^2 + \frac{(d-3)}{2}(w^2-1)^2\right), \tag{A2}$$

$$N\frac{d^2w}{dz^2} + \left(\frac{(d-5)N}{2} + (d-3)e^{-(d-3)z/2}\frac{m}{\kappa} + \frac{e^z}{\ell^2} - \frac{\tau_1 e^{-z}}{4\kappa}(d-3)(w^2-1)^2\right)\frac{dw}{dz} = \frac{(d-3)}{4}w(w^2-1), \tag{A3}$$

where the metric function $\sigma$ has been eliminated by using (10). The function $N$ is given by (6), namely, as $N = 1 - \frac{2}{\kappa}e^{-(d-3)z/2}m(z) + e^z/\ell^2$.

To devise a proof for the nonexistence of finite mass solutions of the above system, it is convenient to introduce the function

$$E = \frac{1}{2}N\left(\frac{dw}{dz}\right)^2 - \frac{(d-3)}{16}(w^2-1)^2, \tag{A4}$$

which, from (A2) and (A3) satisfies the equation

$$\frac{dE}{dz} = -\left(\frac{dw}{dz}\right)^2\left(\frac{\tau_1}{\kappa}e^{-z}E + \frac{e^z}{\ell^2} + \frac{m}{\kappa}(d-3)e^{-(d-3)z/2}\right). \tag{A5}$$

The relations in Appendix B of Ref. [11] are recovered for $d = 5$, where a nonexistence proof is presented for particlelike solutions with finite mass (the extension to the black hole case is considered in [51]). Therefore, in what follows we will take $d > 5$ only.

The approximate form of the function $E$ at the origin (or event horizon) and infinity is found by taking $P = 1$ in relations (26), (27), and (29) given in Sec. III and replacing in (A4). It is obvious that $E(r_h) < 0$, since $N(r_h) = 0$; at the same time, the corresponding expression as $r \to 0$ (i.e. $z \to -\infty$) is

$$E \simeq \tfrac{1}{4}(5-d)b^2e^{2z} + \ldots, \tag{A6}$$

and we find $E \to -0$ in this limit.

Also, the relations (26) and (27) give $m(r=0) > 0$, $m(r_h) > 0$, which, together with the Eq. (A2) implies that the mass function $m(r)$ is positive definite.

Besides, by replacing in (A4) the asymptotic expressions (29) as $r \to \infty$ it follows that

$$E \simeq \frac{w_1^2(d-3)^2}{8\ell^2}e^{-(d-4)z/2} + \frac{1}{8}w_1^2(d-3)(d-5)e^{-(d-3)z} + \ldots, \tag{A7}$$

i.e. $E \to +0$ as $z \to \infty$, for finite mass solutions.

Therefore, if the solution is regular everywhere, $E$ must vanish at some finite point $z_0$, and $dE/dz \geq 0$ there, with $E > 0$ for $z > z_0$ (when there are several positive roots of $E$, we take the largest one). However, another point should exist $z_1 > z_0$ such that $dE/dz = 0$ i.e. the function $E$ should present a positive maximum for some value of $z$. Now we integrate the Eq. (A5) between $z_0$ and $z_1$ and find

$$E(z_1) = -\int_{r_0}^{r_1}\left(\frac{dw}{dz}\right)^2\left(\frac{\tau_1}{\kappa}e^{-z}E + \frac{e^z}{\ell^2} + \frac{m}{\kappa}(d-3)e^{-(d-3)z/2}\right)dz < 0, \tag{A8}$$

which contradicts $E(z_1) > 0$. Therefore $E(z)$ should vanish identically and one finds no $d > 5$ finite mass, spherically symmetric EYM configurations in a $F(2)$ theory. Note that this argument does not exclude the existence of configuration with a diverging mass functions as $r \to \infty$.

## APPENDIX B: A MATTER COUNTERTERM PROPOSAL

In this section we comment on the issue of mass definition of AAdS solutions in a $F(2)^2$ (i.e. $p = 1$) theory, if we do not exercise the option of employing higher order YM curvature terms. As proven in Sec. II, although the spacetime is still AAdS, the mass function $m(r)$ of these solutions generically diverges as $r^{d-5}$ (or as $\log r$ for $d = 5$). AAdS solutions with a diverging ADM mass have been considered recently by some authors, mainly for a scalar field in the bulk (see e.g. [52–57]. In this case it might be possible to relax the standard asymptotic conditions without loosing the original symmetries, but modifying the charges in order to take into account the presence of matter fields.

Similar to the case of scalar field, for $d > 5$ it is still possible to obtain a finite mass of EYM solutions in a $F^2$ theory by allowing $I_{\rm ct}$ to depend not only on the boundary metric $h_{AB}$, but also on the gauge field strength tensor. This means that the quasilocal stress-energy tensor (40) also acquires a contribution coming from the matter fields.

We find that by adding a counterterm of the form

$$I_{\rm ct}^{(m)} = -\frac{1}{(d-5)}\int_{\partial M} d^{d-1}x\sqrt{-h}\,{\rm tr}F_{AB}F^{AB} \qquad \text{(B1)}$$

to the expression (39), the divergence disappears. This yields a supplementary contribution to (40)

$$T_{AB} = -\frac{1}{8\pi G}\frac{1}{d-5}h_{AB}\,{\rm tr}F_{CD}F^{CD}. \qquad \text{(B2)}$$

The mass of the $d > 5$ solutions computed in this way is

$$E = \frac{(d-2)\Omega_{d-2}}{8\pi G}M_0 + E_0 \qquad \text{(B3)}$$

where $M_0$ is the constant appearing in the asymptotic expansion (16). It can also be proven that this prescription leads to a finite action and the entropy-area relation is satisfied. However, as seen in Fig. 2, the parameter $M_0$ of the $p = 1$ solutions takes negative values, pointing to some pathological properties.

It would be nice to have a rigorous derivation of the matter counterterm expression, possibly along the lines of Ref. [58]. Also, there remains the issue of the $d = 5$ solutions, whose logarithmic divergences require a different approach.


[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); Int. J. Theor. Phys. **38**, 1113 (1999 ).
[2] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[3] E. Winstanley, Classical Quantum Gravity **16**, 1963 (1999).
[4] J. Bjoraker and Y. Hosotani, Phys. Rev. D **62**, 043513 (2000).
[5] O. Sarbach and E. Winstanley, Classical Quantum Gravity **18**, 2125 (2001).
[6] P. Breitenlohner, D. Maison, and G. Lavrelashvili, Classical Quantum Gravity **21**, 1667 (2004).
[7] E. Radu, Phys. Rev. D **67**, 084030 (2003).
[8] J. J. Van der Bij and E. Radu, Phys. Lett. B **536**, 107 (2002).
[9] E. Radu, Phys. Lett. B **548**, 224 (2002); Phys. Rev. D **65**, 044005 (2002).
[10] E. Radu and E. Winstanley, Phys. Rev. D **70**, 084023 (2004).
[11] N. Okuyama and K. i. Maeda, Phys. Rev. D **67**, 104012 (2003).
[12] M. S. Volkov, Phys. Lett. B **524**, 369 (2002).
[13] Y. Brihaye, A. Chakrabarti, and D. H. Tchrakian, Classical Quantum Gravity **20**, 2765 (2003).
[14] Y. Brihaye, A. Chakrabarti, B. Hartmann, and D. H. Tchrakian, Phys. Lett. B **561**, 161 (2003).
[15] R. Bartnik and J. McKinnon, Phys. Rev. Lett. **61**, 141 (1988).
[16] P. Breitenlohner, P. Forgacs, and D. Maison, Nucl. Phys. **B383**, 357 (1992); **B442**, 126 (1995)
[17] Y. Brihaye and D.H. Tchrakian, J. Phys. A **38**, 3679 (2005).
[18] P. Breitenlohner, D. Maison, and D. H. Tchrakian, gr-qc/0508027.
[19] S. Deser, Classical Quantum Gravity **1**, L1 (1984).
[20] V. Balasubramanian and P. Kraus, Commun. Math. Phys. **208**, 413 (1999).
[21] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2752 (1977).
[22] M. Banados, C. Teitelboim, and J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992).
[23] D. Maison, private communication.
[24] E. W. Hirschmann and D. L. Welch, Phys. Rev. D **53**, 5579 (1996).
[25] M. Cataldo and P. Salgado, Phys. Rev. D **54**, 2971 (1996).
[26] Y. Kiem and D. Park, Phys. Rev. D **55**, 6112 (1997).
[27] M. Henneaux, C. Martinez, R. Troncoso, and J. Zanelli, Phys. Rev. D **65**, 104007 (2002).
[28] Y. Hosotani, J. Math. Phys. (N.Y.) **43**, 597 (2002).
[29] C. N. Pope, Classical Quantum Gravity **2**, L77 (1985).
[30] S. Coleman, in *Int. School of Subnuclear Physics, Ettore Majorana, Erice, Sicily, Jul 11-31, 1975* [Erice Subnuclear Physics 297 (1975)].
[31] S. Deser, Phys. Lett. B **64**, 463 (1976).

[32] M. S. Volkov and D. V. Gal'tsov, Phys. Rep. **319**, 1 (1999).
[33] A. H. Chamseddine and M. S. Volkov, J. High Energy Phys. 04 (2001) 023; G. Bertoldi, J. High Energy Phys. 10 (2002) 042.
[34] A. A. Tseytlin in *Born–Infeld Action, Supersymmetry and String Theory* edited by M. Shifman, Yuri Golfand Memorial Vol. (World Scientific, Singapore, 2000).
[35] E. A. Bergshoeff, M. de Roo, and A. Sevrin, Fortschr. Phys. **49**, 433 (2001); Nucl. Phys. B, Proc. Suppl. **102**, 50 ( 2001).
[36] M. Cederwall, B. E. W. Nilsson, and D. Tsimpis, J. High Energy Phys. 06 (2001) 034.
[37] M. Heusler, Helv. Phys. Acta **69**, 501 (1996).
[38] M. Heusler and N. Straumann, Classical Quantum Gravity **9**, 2177 (1992).
[39] A. A. Belavin, A. M. Polyakov, A. S. Shvarts, and Y. S. Tyupkin, Phys. Lett. B **59**, 85 (1975).
[40] H. Boutaleb-Joutei, A. Chakrabarti, and A. Comtet, Phys. Rev. D **20**, 1884 (1979).
[41] Y. Brihaye, A. Chakrabarti, and D. H. Tchrakian, J. Math. Phys. (N.Y.) **41**, 5490 (2000).
[42] L. F. Abbott and S. Deser, Nucl. Phys. **B195**, 76 (1982).
[43] M. Henneaux and C. Teitelboim, Commun. Math. Phys. **98**, 391 (1985).
[44] A. Ashtekar and A. Magnon, Classical Quantum Gravity **1**, L39 (1984); A. Ashtekar and S. Das, Classical Quantum Gravity **17**, L17 (2000).
[45] R. Aros, M. Contreras, R. Olea, R. Troncoso, and J. Zanelli, Phys. Rev. Lett. **84**, 1647 (2000); R. Aros, M. Contreras, R. Olea, R. Troncoso, and J. Zanelli, Phys. Rev. D **62**, 044002 (2000); E. W. Mielke, Phys. Rev. D **63**, 044018 (2001).
[46] J. D. Brown and J. W. York, Phys. Rev. D **47**, 1407 (1993).
[47] M. M. Taylor-Robinson, hep-th/0002125; K. Skenderis, Int. J. Mod. Phys. A **16**, 740 (2001); S. de Haro, S. N. Solodukhin, and K. Skenderis, Commun. Math. Phys. **217**, 595 (2001).
[48] S. Das and R. B. Mann, J. High Energy Phys. 08 (2000) 033.
[49] R. C. Myers, Phys. Rev. D **60**, 046002 (1999).
[50] R. B. Mann, Found. Phys. **33**, 65 (2003).
[51] Y. Brihaye, B. Hartmann, and E. Radu, Phys. Rev. D **72**, 104008 (2005).
[52] T. Hertog and K. Maeda, J. High Energy Phys. 07 (2004) 051.
[53] T. Hertog and K. Maeda, Phys. Rev. D **71**, 024001 (2005).
[54] M. Henneaux, C. Martinez, R. Troncoso, and J. Zanelli, Phys. Rev. D **70**, 044034 (2004).
[55] E. Radu and D. H. Tchrakian, Classical Quantum Gravity **22**, 879 (2005).
[56] S. Hollands, A. Ishibashi, and D. Marolf, Classical Quantum Gravity **22**, 2881 (2005).
[57] J. T. Liu and W. A. Sabra, Phys. Rev. D **72**, 064021 (2005).
[58] A. Batrachenko, J. T. Liu, R. McNees, W. A. Sabra, and W. Y. Wen, J. High Energy Phys. 05 (2005) 034.