Modular inflation and the orthogonal axion as the curvaton

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We study a particular supersymmetric realization of the Peccei-Quinn symmetry which provides a suitable candidate for the curvaton field. The class of models considered also solves the μ problem, while generating the Peccei-Quinn scale dynamically. The curvaton candidate is a pseudo-Nambu-Goldstone boson corresponding to an angular degree of freedom orthogonal to the axion field. Its order parameter increases substantially following a phase transition during inflation. This results in a drastic amplification of the curvaton perturbations. Consequently, the mechanism is able to accommodate low-scale inflation with Hubble parameter at the TeV scale. Hence, we investigate modular inflation using a string axion field as the inflaton with inflation scale determined by gravity mediated soft supersymmetry breaking. We find that modular inflation with the orthogonal axion as curvaton can indeed account for the observations for natural values of the parameters.

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I. INTRODUCTION

The past decade experienced a plethora of precise cosmological observations. These observations have confirmed the basic predictions of inflation rendering the latter an essential extension of the hot big bang model. Precision cosmology upgraded inflation model building at the next level, that of the design of complex realistic models (in contrast to early single-field fine-tuned inflationary models). These models utilize and reflect the rich content of particle theory. A first such attempt was the well-known hybrid inflation model [1–3], which couples the inflaton field with the Higgs field of a grand unified theory (GUT). In this way, hybrid inflation dispenses with a number of tuning problems, which plagued most simplistic single-field inflation models.

In the same spirit, curvaton models [4–6] (see also [7]) use a second field to generate curvature perturbations. This curvaton field is not an *ad hoc* degree of freedom introduced by hand, but, instead, it is [8–10] a realistic field already present in simple extensions of the standard model (SM). In the context of the curvaton, inflation model building is substantially liberated [11,12] allowing for more realistic and less fine-tuned models.

A particular advantage of inflation model building in the context of the curvaton mechanism is that it is possible to construct [11,13] models with inflationary energy scale much lower than the GUT scale. In this spirit, we investigate, in this paper, the possibility of using a string axion field as the inflaton.

The nature and origin of the inflaton field are still open questions in inflation model building. Typically, what is required is a light field with suppressed interactions with other fields including those of the SM. By "light," we mean a field whose effective mass is smaller than the Hubble parameter H_* at the time when the cosmological scales exit the horizon during inflation. This guarantees that inflation lasts long enough to encompass the cosmological scales. The interactions of this field have to be suppressed in order not to lift the flatness of the scalar potential along the inflationary trajectory.

However, the case of slow-roll inflation (i.e. the case with inflaton mass $\ll H_*$) suffers from the fact that, typically, supergravity (SUGRA) introduces [2,14] corrections to the inflaton mass of order H_* during the inflationary period. To keep the inflaton mass under control, one may use as inflaton a pseudo-Nambu-Goldstone boson (PNGB) field, since the flatness of the potential of such a field is protected by a global U(1) symmetry. Promising such candidates are [15] the string axions, which are the imaginary parts of string moduli fields with the flatness of their potential lifted only by (soft) supersymmetry (SUSY) breaking. This results in inflaton masses of order H_*. Hence, such modular inflation is of the fast-roll type [16]. Fast-roll inflation lasts only a limited number of e-foldings, which, however, can be enough to solve the horizon and flatness problems.

The inflationary energy scale, in this model, is much lower than the GUT scale. As a result, the perturbations of the inflaton field are not sufficiently large to account for the observations. Consequently, a curvaton field is necessary to provide the observed curvature perturbation. However, even the curvaton cannot [17] generically help us to reduce the inflationary scale to energies low enough for modular inflation. This is possible only in certain curvaton models which amplify [13,18] additionally the curvaton perturbations. We describe in detail such a model belonging to a class of SUSY realizations of the Peccei-Quinn (PQ) symmetry [19], which also solves the strong CP and μ problems. We use as curvaton an angular degree of freedom orthogonal to the QCD axion.

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Our paper is structured as follows. In Sec. II, we present a brief outline of modular inflation. In Sec. III, we analyze the amplification mechanism for the curvature perturbations due to a PNGB curvaton with varying order parameter. In Sec. IV, we investigate whether it is possible to employ in the role of such a PNGB curvaton an angular degree of freedom orthogonal to the QCD axion ("orthogonal axion") in SUSY theories. We show that the simplest constructions using two PQ superfields cannot work because the orthogonal axion is not appropriately light during inflation. In Sec. V, we construct in detail an appropriate class of PQ models involving three SM singlet superfields, only two of which carry PQ charges. In Sec. VI, we study in detail the characteristics of the scalar potential in the above class of curvaton models. In Sec. VII, we focus on curvaton physics and derive a number of important constraints necessary for the model to be a successful curvaton model. In Sec. VIII, we quantify our findings in a concrete example of this class of models. We find that this concrete model can indeed work for natural values of the model parameters. Finally, in Sec. IX, we discuss our results and present our conclusions. Throughout the paper, we use natural units, where $c = \hbar = 1$ and the Newton's gravitational constant is $G = 8\pi M_{\rm P}^{-2}$ with $M_{\rm P} \simeq 2.44 \times 10^{18}$ GeV being the reduced Planck mass.

II. MODULAR INFLATION

String theory, in general, contains a number of moduli fields Φ_i, whose tree-level Kähler potential, in 4-dimensional effective SUGRA, is of the form

$$K = -M_{\rm P}^2 \sum_i \ln[(\Phi_i + \Phi_i^*)/M_{\rm P}]. \tag{1}$$

Hence, the Kähler potential is flat in the directions of the imaginary parts of the moduli $\mathrm{Im}(\Phi_i)$. The same is true for the F-term scalar potential V_F, despite the fact that the superpotential may receive nonperturbative contributions (e.g. from gaugino condensation) of the form

$$\Delta W(\Phi_i) = \Lambda^3 \exp(-\beta_i \Phi_i / M_{\rm P}), \tag{2}$$

since V_F turns out to be independent of the phase of ΔW. The mass parameter Λ in Eq. (2) is usually taken to be the string scale and the β_i's are model-dependent coefficients of order unity. The $\mathrm{Im}(\Phi_i)$ fields are periodic (by modular invariance)

$$\mathrm{Im}\,(\Phi_i) \equiv \mathrm{Im}(\Phi_i) + 2\pi f_i, \tag{3}$$

where $f_i \sim M_{\rm P}$. This is why the $\mathrm{Im}(\Phi_i)$ fields are also called string axions.

In compactified heterotic string theory, these axions correspond [20], in fact, to the massless modes of the second-rank antisymmetric tensor field B:

$$B = b_{\mu\nu} dx^\mu \wedge dx^\nu + b_I \omega^I_{\alpha\beta^*} dy^\alpha \wedge dy^{\beta *}, \tag{4}$$

where x^μ are the coordinates in the usual 4-dimensional space, y^α the complex coordinates in the compactified extra-dimensional space, and $\omega^I_{\alpha\beta^*}$ harmonic (1,1) forms parametrizing the geometry of the compactified space [21]. The usual 4-dimensional components $b_{\mu\nu}$ of B correspond to the so-called model-independent axion, while the extra-dimensional components b_I of B correspond to the so-called model-dependent axions.

The flatness of the string axion potential is lifted only by soft SUSY breaking, which tilts the vacuum manifold by an amount determined by the SUSY-breaking scale. One can consider [15] that it is this potential that provides the vacuum energy density necessary for inflation. Assuming gravity mediated soft SUSY breaking, the inflationary potential V_* at the time when the cosmological scales exit the horizon is of intermediate scale:

$$V_*^{1/4} \sim \sqrt{m_{3/2} M_{\rm P}} \sim 10^{10.5}\ \mathrm{GeV} \tag{5}$$

for which $H_* \sim m_{3/2}$, where $m_{3/2} \sim 1$ TeV stands for the gravitino mass. The inflationary potential is of the form

$$V(s) = V_{\rm m} - \tfrac{1}{2} m_s^2 s^2 + \cdots, \tag{6}$$

where the ellipsis denotes terms which are expected to stabilize the potential at $s \sim M_{\rm P}$ with s being the canonically normalized string axion. Therefore, in the above formula, we have

$$V_{\rm m} \sim (m_{3/2} M_{\rm P})^2 \quad \text{and} \quad m_s \sim m_{3/2}. \tag{7}$$

This inflation model results in fast-roll inflation, where

$$s = s_{\rm i} \exp(F_s \Delta N) \quad \text{with } F_s \equiv \frac{3}{2}(\sqrt{1 + 4c/9} - 1), \qquad c \equiv \left(\frac{m_s}{H_*}\right)^2 \sim 1. \tag{8}$$

Here, ΔN is the number of the elapsed e-foldings and $s_{\rm i}$ the initial value of the inflaton field s. From the above, one can obtain the inflation potential N e-foldings before the end of inflation as

$$V(N) \simeq V_{\rm m}(1 - e^{-2F_s N}). \tag{9}$$

Even though fast-roll, modular inflation keeps the Hubble parameter H rather rigid. Indeed, it can be easily shown that

$$\epsilon = \frac{1}{2} c^2 \left(\frac{s}{M_{\rm P}}\right)^2 \ll 1, \tag{10}$$

because $c \sim 1$ and $s \ll M_{\rm P}$ during inflation with ϵ being one of the so-called slow-roll parameters defined as

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \tag{11}$$

where the dot denotes derivative with respect to the cosmic time.

For modular inflation, the initial conditions for the inflaton field are determined by the quantum fluctuations which send the field off the top of the potential hill. Hence, we expect that the initial value for the inflaton is

$$s_{\rm i} \simeq \frac{H_{\rm m}}{2\pi}, \tag{12}$$

where $H_{\rm m} \simeq \sqrt{V_{\rm m}}/\sqrt{3}M_{\rm P}$.

Using the above and considering that the final value of s is close to its vacuum expectation value (VEV) $s_{\rm VEV} \sim M_{\rm P}$, we can estimate, through the use of Eq. (8), the total number of e-foldings as

$$N_{\rm tot} \simeq \frac{1}{F_s} \ln\left(\frac{M_{\rm P}}{m_{3/2}}\right), \tag{13}$$

where we took into account that

$$H_{\rm m} \sim m_{3/2}. \tag{14}$$

Inflation at such a low energy scale as in Eq. (7) can provide the required amplitude for the curvature perturbations only through the use of a special kind of curvaton field, whose perturbations are amplified during inflation. In the following, we describe the mechanism for such amplification.

III. AMPLIFYING THE CURVATON PERTURBATIONS

We discuss here the case of an axionlike curvaton, i.e. a PNGB. Examples of such a curvaton can be found in Refs. [9,10]. However, in contrast to those works, we consider a PNGB curvaton whose order parameter has [13,18] a different (larger) expectation value in the vacuum than during inflation and, in particular, when the cosmological scales exit the horizon. Thus, the potential for the real canonically normalized curvaton field σ is

$$V(\sigma) = (v\tilde{m}_\sigma)^2\left[1 - \cos\left(\frac{\sigma}{v}\right)\right] \Rightarrow V(|\sigma| \ll v) \simeq \frac{1}{2}\tilde{m}_\sigma^2\sigma^2, \tag{15}$$

where $v = v(t)$ is the order parameter of the PNGB (determined by the values of the radial fields in the model) with t being the cosmic time and $\tilde{m}_\sigma = \tilde{m}_\sigma(v)$ is the mass of the curvaton at a given moment. In the true vacuum, we have $v = v_0$ and $\tilde{m}_\sigma = m_\sigma$ with v_0 and m_σ being the order parameter and the mass of the PNGB curvaton in the vacuum, respectively.

Note that, in principle, the PNGB does not need to have an exact sinusoidal potential. Instead, one could substitute $[1 - \cos(\sigma/v)]$ by $f(\sigma/v)$, where f is a periodic function of period 2π with a global minimum at the origin and $f(0) = 0$. Then, generically, the second (approximate) equality in Eq. (15) continues to be valid if σ is close enough to the global minimum [23].

A. The amplification factor

In this section, we will demonstrate that the curvaton perturbations can be amplified by the nontrivial evolution of its order parameter v if the curvaton is a PNGB. This mechanism was first presented in Ref. [13] (see also Ref. [18]).

We begin by using the fact that, on a foliage of spacetime corresponding to spatially flat hypersurfaces, the curvature perturbation attributed to each of the universe components (labeled by the index i) is given by [25]

$$\zeta_i \equiv -H\frac{\delta\rho_i}{\dot{\rho}_i}, \tag{16}$$

where ρ_i and $\delta\rho_i$ are, respectively, the energy density and its perturbation of the component in question.

The total curvature perturbation $\zeta(t)$, which is also given by Eq. (16) with ρ_i and $\delta\rho_i$ replaced, respectively, by the total energy density of the universe $\rho = \sum_i\rho_i$ and its perturbation $\delta\rho$, may be calculated as follows. Using the fact that $\delta\rho = \sum_i\delta\rho_i$ and the continuity equation $\dot{\rho}_i = -3H(\rho_i + p_i)$, where p_i is the pressure of the ith component of the universe, it is easy to find that

$$\zeta = \sum_i \frac{\rho_i + p_i}{\rho + p}\zeta_i, \tag{17}$$

where $p = \sum_i p_i$ is the total pressure. Now, since in the curvaton scenario, all contributions to the curvature perturbation other than the curvaton's are negligible, we find that

$$\zeta = \zeta_\sigma\left(\frac{1 + w_\sigma}{1 + w}\right)_{\rm dec} \frac{\rho_\sigma}{\rho}\bigg|_{\rm dec}, \tag{18}$$

where $\zeta \simeq 2 \times 10^{-5}$ is the curvature perturbation observed by the cosmic microwave background explorer [26], w_σ and w are the curvaton and the overall barotropic parameters, respectively (with $w_i \equiv p_i/\rho_i$) and ζ_σ is the partial curvature perturbation of the curvaton. The right-hand side (RHS) of this equation is evaluated at the time when the curvaton decays and this is indicated by the subscript "dec." This decay occurs after the end of inflation in which case Eq. (18) gives

$$\zeta \sim \Omega_{\rm dec}\zeta_\sigma, \tag{19}$$

where $\Omega_{\rm dec}$ is the ratio of the curvaton energy density to the total energy density of the universe at the time of the decay of the curvaton:

$$\Omega_{\rm dec} \equiv \frac{\rho_\sigma}{\rho}\bigg|_{\rm dec} \leq 1. \tag{20}$$

From the bound [27] on the possible non-Gaussian component of the curvature perturbation from the recent cosmic microwave background radiation (CMBR) data obtained by the Wilkinson microwave anisotropy probe (WMAP) satellite, one finds [25] that, at 95% confidence level (C.L.),

$$10^{-2} \lesssim \Omega_{\rm dec} \leq 1. \tag{21}$$

The partial curvature perturbation of the curvaton when the latter oscillates in a quadratic potential [cf. Eq. (15)] is given [24] by

$$\zeta_\sigma = \frac{2}{3}\frac{\delta\sigma}{\sigma}\bigg|_{\rm dec} \sim \frac{\delta\sigma}{\sigma}\bigg|_{\rm osc}, \tag{22}$$

where "osc" denotes the time when the curvaton oscillations begin.

In this paper, we assume that the Hubble parameter during inflation is comparable to the (tachyonic) masses that the radial fields which determine the value of the order parameter of the PNGB acquire after inflation. This means that the evolution of the curvaton's order parameter v ceases at (or soon after) the end of inflation. Therefore, at the end of inflation, $v \to v_0$ and the mass of the curvaton assumes its vacuum value m_σ. Hence, in the following, *we assume that the curvaton mass has already assumed its vacuum value before the onset of the curvaton oscillations*. Consequently, the curvaton oscillations begin when

$$H_{\rm osc} \simeq \frac{1}{\sqrt{3}} m_\sigma. \tag{23}$$

Before the oscillations begin, the phase corresponding to the curvaton degree of freedom is overdamped and remains frozen. More precisely, this means that

$$\theta_{\rm osc} \simeq \theta_*, \qquad \delta\theta_{\rm osc} \simeq \delta\theta_*, \tag{24}$$

where the subscript star denotes the values of quantities at the time when the cosmological scales exit the horizon during inflation,

$$\theta \equiv \frac{\sigma}{v} \tag{25}$$

with $\theta \in (-\pi, \pi]$ and $\delta\theta$ is its perturbation. Hence, for the curvaton partial perturbation, we find

$$\frac{\delta\sigma}{\sigma}\bigg|_{\rm osc} = \frac{\delta\theta}{\theta}\bigg|_{\rm osc} \simeq \frac{\delta\theta}{\theta}\bigg|_{*} = \frac{\delta\sigma}{\sigma}\bigg|_{*}. \tag{26}$$

Now, for the perturbation of the curvaton, we have

$$\delta\sigma_* = \frac{H_*}{2\pi}. \tag{27}$$

We assume that the order parameter of the PNGB during inflation is smaller compared to its value in the vacuum by a factor

$$\varepsilon \equiv \frac{v_*}{v_0} \ll 1. \tag{28}$$

Combining Eqs. (24)–(28), we find that

$$\delta\sigma_{\rm osc} \simeq \frac{H_*}{2\pi\varepsilon}, \tag{29}$$

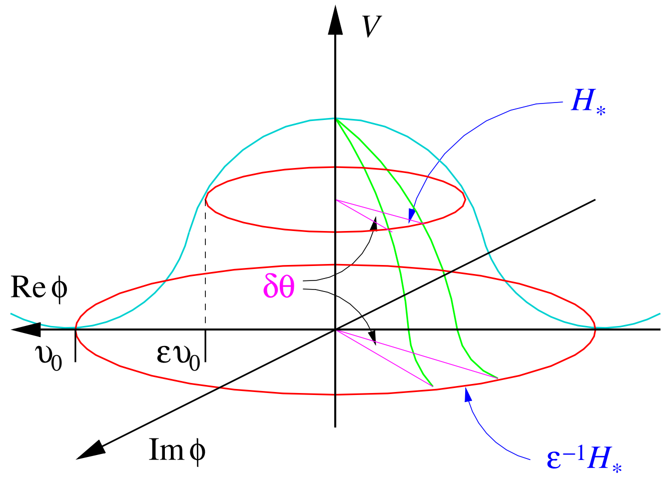

FIG. 1 (color online). Schematic representation of the amplification of the PNGB curvaton perturbation when the order parameter v increases from the value it has when the cosmological scales exit the horizon $v_* = \varepsilon v_0$ to its vacuum value v_0. V is the potential of a fiducial complex field ϕ parametrizing the growth of the order parameter such that $v/\sqrt{2} = |\phi|$. The phase θ of ϕ corresponds to the PNGB degree of freedom σ (i.e. $\theta = \sigma/v$). The perturbation in σ at horizon crossing has amplitude $\delta\sigma_* \sim H_*$, which corresponds to a phase perturbation of magnitude $\delta\theta = \delta\sigma_*/v_*$. As the order parameter grows, $\delta\theta$ remains constant (the phase perturbation is frozen on superhorizon scales) but the amplitude of the curvaton perturbation is increased up to $\delta\sigma \sim \varepsilon^{-1}H_*$.

which means that after the end of inflation, when v assumes its vacuum value, *the curvaton perturbation is amplified by a factor* ε^{-1} (see Fig. 1). From Eqs. (19) and (22), we have

$$\sigma_{\rm osc} \sim \frac{\Omega_{\rm dec}}{\zeta}\,\delta\sigma_{\rm osc}. \tag{30}$$

Using Eq. (29), we can recast the above as

$$\sigma_{\rm osc} \sim \frac{H_*\Omega_{\rm dec}}{\pi\varepsilon\zeta}. \tag{31}$$

We may obtain a lower bound on ε as follows:

$$\frac{\delta\sigma_*}{\sigma_*} \leq 1 \Rightarrow \varepsilon \geq \varepsilon_{\rm min} \equiv \frac{H_*}{2\pi v_0}, \tag{32}$$

where we have used Eqs. (26) and (29) and that $\sigma_{\rm osc} \lesssim v_0$.

B. The bounds on the inflationary scale

As shown in Ref. [13], in the case when the curvaton oscillations begin after the order parameter of the PNGB has attained its vacuum value, we have

$$H_* \sim \Omega_{\rm dec}^{-(2/5)}\left(\frac{H_*}{\min\{m_\sigma, \Gamma_{\rm inf}\}}\right)^{1/5}(\pi\varepsilon\zeta)^{4/5} \times \left(\frac{\max\{H_{\rm dom}, \Gamma_\sigma\}}{H_{\rm BBN}}\right)^{1/5}(M_{\rm P}^3 T_{\rm BBN}^2)^{1/5}, \qquad (33)$$

or equivalently (using $V_*^{1/4} \simeq \sqrt{3^{1/2} H_* M_{\rm P}}$)

$$V_*^{1/4} \sim \Omega_{\rm dec}^{-(1/5)}\left(\frac{H_*}{\min\{m_\sigma, \Gamma_{\rm inf}\}}\right)^{1/10}(\pi\varepsilon\zeta)^{2/5} \times \left(\frac{\max\{H_{\rm dom}, \Gamma_\sigma\}}{H_{\rm BBN}}\right)^{1/10}(M_{\rm P}^4 T_{\rm BBN})^{1/5}, \qquad (34)$$

where $\Gamma_{\rm inf}$ and Γ_σ are the decay rates of the inflaton and the curvaton fields, respectively, $H_{\rm dom}$ is the Hubble parameter at the time when the curvaton energy density dominates the universe (if the curvaton does not decay earlier), and $H_{\rm BBN} = (\pi/3)\sqrt{g_{\rm BBN}/10}(T_{\rm BBN}^2/M_{\rm P})$ and $T_{\rm BBN} \approx 1$ MeV are, respectively, the Hubble parameter and the cosmic temperature at the time of big bang nucleosynthesis (BBN) with $g_{\rm BBN} = 10.75$ being the effective number of relativistic degrees of freedom at that time.

Now, we require that the curvaton field decays before BBN, i.e. $\Gamma_\sigma > H_{\rm BBN}$. We also have $\Gamma_{\rm inf} \leq H_*$. Hence, Eqs. (33) and (34) provide the following bounds:

$$H_* > \Omega_{\rm dec}^{-(2/5)}(\pi\varepsilon\zeta)^{4/5}(M_{\rm P}^3 T_{\rm BBN}^2)^{1/5} \sim \left(\frac{\varepsilon^2}{\Omega_{\rm dec}}\right)^{2/5} \times 10^7 \text{ GeV} \qquad (35)$$

and

$$V_*^{1/4} > \Omega_{\rm dec}^{-(1/5)}(\pi\varepsilon\zeta)^{2/5}(M_{\rm P}^4 T_{\rm BBN})^{1/5} \sim \left(\frac{\varepsilon^2}{\Omega_{\rm dec}}\right)^{1/5} \times 10^{12} \text{ GeV}. \qquad (36)$$

Furthermore, we also note that, generically,

$$\Gamma_\sigma \geq \frac{m_\sigma^3}{M_{\rm P}^2}, \qquad (37)$$

where the equality corresponds to gravitational decay. The above can be shown [13] to imply that

$$H_* \geq \Omega_{\rm dec}^{-1}(\pi\varepsilon\zeta)^2 M_{\rm P}\left(\frac{m_\sigma}{H_*}\right)\left(\max\left\{1, \frac{m_\sigma}{\Gamma_{\rm inf}}\right\}\right)^{1/2}, \qquad (38)$$

which results in the bounds

$$H_* \geq \Omega_{\rm dec}^{-1}(\pi\varepsilon\zeta)^2 M_{\rm P}\left(\frac{m_\sigma}{H_*}\right) \sim \left(\frac{\varepsilon^2}{\Omega_{\rm dec}}\right)\left(\frac{m_\sigma}{H_*}\right) \times 10^{10} \text{ GeV} \qquad (39)$$

and

$$V_*^{1/4} \geq \Omega_{\rm dec}^{-(1/2)}(\pi\varepsilon\zeta)M_{\rm P}\left(\frac{m_\sigma}{H_*}\right)^{1/2} \sim \left(\frac{\varepsilon^2}{\Omega_{\rm dec}}\right)^{1/2}\left(\frac{m_\sigma}{H_*}\right)^{1/2} \times 10^{14} \text{ GeV}. \qquad (40)$$

These bounds may be relaxed if ε is small enough. For a PNGB curvaton, in particular, we may have $m_\sigma \ll H_*$, which also relaxes the bounds in Eqs. (39) and (40). However, in our case, $m_\sigma \sim H_*$ (see below). Comparing the bounds in Eqs. (35) and (36) with those in Eqs. (39) and (40), respectively, we find that the first set of bounds is more stringent if

$$\varepsilon < \frac{1}{\pi\zeta\Omega_{\rm dec}^{1/2}}\left(\frac{T_{\rm BBN}}{M_{\rm P}}\right)^{1/3}\left(\frac{H_*}{m_\sigma}\right)^{5/6} \sim 10^{-3}\Omega_{\rm dec}^{-(1/2)}\left(\frac{H_*}{m_\sigma}\right)^{5/6}. \qquad (41)$$

Thus, for $\varepsilon \ll 1$, the second set of bounds is typically less stringent than the first one.

From Eqs. (32) and (36) and after a little algebra, it is easy to get

$$V_*^{1/4} \geq \left(\frac{M_{\rm P}}{v_0}\right)^2 10^{-13} \text{ GeV} \Rightarrow H_* \geq \left(\frac{M_{\rm P}}{v_0}\right)^4 10^{-44} \text{ GeV}, \qquad (42)$$

which means that, in principle, *the larger v_0 is the smaller $V_*^{1/4}$ can become*.

C. Scale invariance requirement

The evolution of the order parameter $v(t)$ during inflation is subject to an important constraint which has to do with preserving the scale invariance of the spectrum of the curvature perturbations.

The amplitude of the curvature perturbation is determined by the magnitude of the perturbation in the curvaton field, which, in this scenario, apart from the scale of H_* is also determined by the amplification factor ε^{-1}. The latter is determined by the value of the order parameter v_* when the curvaton quantum fluctuations exit the horizon during inflation. A strong variation of $v(t)$ at that time results in a strong dependence of $\varepsilon(k)$ on the comoving momentum scale k, which would reflect itself on the perturbation spectrum threatening significant departure from scale invariance.

In Ref. [13], it was shown that, in order for this to be avoided, the rate of change of the order parameter must be constrained as

$$\left|\left(\frac{\dot{v}}{v}\right)_*\right| \ll H_*. \qquad (43)$$

From the above, it is evident that, in order not to violate the observational constraints regarding the scale invariance of the curvature perturbation spectrum, *the order parameter must either remain constant or, at most, have a very slow variation when the cosmological scales exit the horizon*

[28]. However, this cannot remain so indefinitely because we need $v_0 \gg v_*$ to have substantial amplification of the perturbation (i.e. $\varepsilon \ll 1$). Consequently, v has to increase dramatically at some point *after* the exit of the cosmological scales from the inflationary horizon. This requirement is crucial for model building.

In this paper, we will show that the evolution of v begins at a phase transition during inflation, as presented in Ref. [18]. Initially, the growth of v is very slow, but later, near the end of inflation, v grows substantially until it reaches its vacuum value v_0.

IV. CAN WE CONSTRUCT A PQ MODEL WITH A PNGB CURVATON?

We will now address the question whether we can construct a PQ model [19] which, in addition to the standard axion, contains another axionlike field with the right properties to play the role of a PNGB curvaton as discussed in the previous section. It is well known that, in the SUGRA extension of the minimal supersymmetric standard model (MSSM), there exist certain D- and F-flat directions in field space which can generate intermediate scales

$$M_{\rm I} \sim (m_{3/2}M_{\rm P}^n)^{1/(n+1)}, \tag{44}$$

where n is a positive integer. It seems natural to try to identify $M_{\rm I}$ with the symmetry breaking scale $f_{\rm a}$ of the PQ symmetry $\mathrm{U(1)_{PQ}}$, such that a μ term is generated with $\mu \sim f_{\rm a}^{n+1}/M_{\rm P}^n \sim m_{3/2}$ [29]. This would simultaneously resolve the strong CP and μ problems of MSSM.

The resolution of the μ problem forces us to consider nonrenormalizable superpotential terms such as

$$\lambda P^{n+1}h_1h_2/M_{\rm P}^n, \tag{45}$$

where λ is a dimensionless parameter, P is a SM singlet superfield and h_1, h_2 are the electroweak Higgs doublets. The fact that the PQ symmetry carries QCD anomalies implies that the combination h_1h_2 must have a nonzero PQ charge as one can easily deduce from the Yukawa couplings of the quarks. The field P must then necessarily carry a nonzero PQ charge. So, if it acquires a VEV of order $M_{\rm I}$, the PQ symmetry breaks spontaneously and a μ term of the right magnitude is generated via the superpotential term in Eq. (45).

However, the field P has no self-couplings due to its nonzero PQ charge. Moreover, its couplings to the MSSM fields involve at least two of them since there are no SM singlets in MSSM. As a consequence, before the electroweak symmetry breaking, P has a flat potential. To lift the flatness of its potential and generate an intermediate VEV for P of the order of $M_{\rm I}$ in Eq. (44), we must introduce [30–32] a second SM singlet superfield Q with nonzero PQ charge having a coupling of the type

$$\xi P^{n+3-k}Q^k/M_{\rm P}^n, \tag{46}$$

where ξ is a dimensionless coupling constant and k is a positive integer smaller than $n+3$. The superpotential term in Eq. (46) determines the PQ charge of Q.

After soft SUSY breaking, we obtain the following scalar potential for the spontaneous breaking of the PQ symmetry:

$$V = |\xi|^2[(n+3-k)^2|Q|^2 + k^2|P|^2]\left|\frac{P^{n+2-k}Q^{k-1}}{M_{\rm P}^n}\right|^2 + m_P^2|P|^2 + m_Q^2|Q|^2 + \left[A\xi\frac{P^{n+3-k}Q^k}{M_{\rm P}^n} + \text{H.c.}\right], \tag{47}$$

where m_P^2, $m_Q^2 \sim m_{3/2}^2$ and can have either sign, while A is a complex parameter with magnitude of order $m_{3/2}$. For large enough $|A||\xi|$, this potential possesses nontrivial (local) minima at

$$|P|, |Q| \sim (m_{3/2}M_{\rm P}^n)^{1/(n+1)} \tag{48}$$

even if m_P^2, m_Q^2 are positive since the last term in the RHS of Eq. (47) can be adequately negative. We see that here the PQ scale $f_{\rm a}$ is generated dynamically and is not inserted by hand as in the PQ schemes with renormalizable interactions.

In order to implement our scenario, we need a valley of local minima of the potential (with respect to the direction perpendicular to the valley) which has negative inclination. Along this valley, the fields $|P|$ and $|Q|$ must take values which are much smaller than their vacuum values. If the system happens to slowly roll down this valley during the relevant part of inflation, the order parameter v of the PNGB remains, during inflation, much smaller than its vacuum value v_0 and our amplification mechanism for the curvaton perturbations may work. This can be achieved only if one of the masses-squared m_P^2, m_Q^2 is negative. Let us assume, for definiteness, that $m_P^2 < 0$ and $m_Q^2 > 0$. Note, however, that the following discussion applies equally well to the opposite case too, where $m_P^2 > 0$ and $m_Q^2 < 0$. In the case under consideration, the scalar potential is [32] unbounded below on the P axis (i.e. for $Q = 0$) unless $k = 1$ since, for $k > 1$, all the terms in the RHS of Eq. (47) vanish on the P axis except the negative mass term of P. So, we restrict ourselves to the case $k = 1$.

During inflation, m_P^2, m_Q^2 acquire [2,14,33] SUGRA corrections of order H^2 which are assumed to be positive. Also, A receives SUGRA corrections of order H. As already mentioned, the Hubble parameter during inflation is, in our case, of order $m_{3/2}$. So, in the initial stages of inflation, the effective mass-squared of P after SUGRA corrections, which we will call $\bar{m}_P^2$, can be positive. In this case, the origin in field space becomes a local minimum and the system may be initially trapped there. As H gradually decreases during inflation, $\bar{m}_P^2$ becomes negative and the system starts slowly rolling down in the P direction.

For nonzero P, the last term in the RHS of Eq. (47) yields a linear term in Q (recall that $k = 1$) and, thus, Q is also shifted from zero. More precisely, we get

$$|Q| \sim \frac{|P|^{n+1}}{m_{3/2}M_{\rm P}^n}|P| \ll |P|. \tag{49}$$

The last term in the RHS of Eq. (47) then yields

$$-2|\bar{A}||\xi|\frac{|P|^{n+2}|Q|}{M_{\rm P}^n}\cos\left[\frac{(n+2)\phi_P}{\sqrt{2}|P|}+\frac{\phi_Q}{\sqrt{2}|Q|}\right], \tag{50}$$

where ϕ_P and ϕ_Q are canonically normalized real fields corresponding to the phases of P and Q, respectively, and $\bar{A}$ is the effective A after SUGRA corrections. Note that, in deriving Eq. (50), the fields P and Q were appropriately rephased so that the product $\bar{A}\xi$ is negative. This is a convenient choice since, in this case, the expression in the above equation is minimized when the argument of the cosine vanishes. The orthogonal axion direction, which we would like to use as a PNGB curvaton, corresponds to the canonically normalized real field

$$\frac{(n+2)|Q|\phi_P + |P|\phi_Q}{\sqrt{(n+2)^2|Q|^2+|P|^2}}. \tag{51}$$

Its mass-squared can be evaluated from the term in Eq. (50), which, in view of Eq. (49), yields

$$\frac{|\bar{A}||\xi||P|^{n+2}}{M_{\rm P}^n|Q|} \sim m_{3/2}^2. \tag{52}$$

So, the mass of the orthogonal axion during inflation is of order $m_{3/2}$, which is comparable to the inflationary Hubble parameter. Consequently, this field does not qualify as a curvaton, because it cannot obtain a superhorizon spectrum of perturbations. In conclusion, we have seen that our scenario cannot be realized within extensions of the MSSM with a PQ symmetry which contain only two SM singlet superfields.

The addition of a third SM singlet superfield S, however, can drastically change the situation allowing the implementation of our mechanism. We could keep the masses-squared of P and Q positive and include a superpotential term of the type in Eq. (46) with any value of the integer k between unity and $n+2$. In this case, as already mentioned, the potential for P and Q (with $S=0$) can possess nontrivial minima at $|P|$ and $|Q|$ given by Eq. (48). Now, we introduce an extra superpotential term of the type

$$\xi_q P^{n+3-p-q}Q^pS^q/M_{\rm P}^n, \tag{53}$$

where p, q are non-negative integers with $p+q \leq n+3$ and $q \geq 3$. This term determines the PQ charge of S. Of course, we should keep in mind that all possible terms involving P, Q, and S that satisfy the global symmetries (including R symmetries) of the terms in Eqs. (46) and (53) should be present in the superpotential. We assume that the term in Eq. (53) is the term of this type with the smallest power of S. We take the mass-squared of S negative. Then, for small values of S, we obtain a valley of minima with negative inclination at almost constant values of $|P|$ and $|Q|$ given by Eq. (48).

If the system slowly rolls down this valley during inflation, the A-term corresponding to the coupling in Eq. (46) generates a mass term for the canonically normalized field

$$\frac{(n+3-k)|Q|\phi_P + k|P|\phi_Q}{\sqrt{(n+3-k)^2|Q|^2+k^2|P|^2}} \tag{54}$$

with mass-squared of order $m_{3/2}^2$. This is easily shown by repeating the argument which led to Eqs. (51) and (52). The A-term corresponding to the coupling in Eq. (53) yields

$$-2|\bar{A}||\xi_q|\frac{|P|^{n+3-p-q}|Q|^p|S|^q}{M_{\rm P}^n} \times\cos\left[\frac{(n+3-p-q)\phi_P}{\sqrt{2}|P|}+\frac{p\phi_Q}{\sqrt{2}|Q|}+\frac{q\phi_S}{\sqrt{2}|S|}\right], \tag{55}$$

where ϕ_S is a canonically normalized real scalar field corresponding to the phase of S and $\bar{A}\xi_q$ was taken negative by rephasing S. This generates a "mass term" for the canonically normalized field

$$\frac{(n+3-p-q)|Q||S|\phi_P + p|P||S|\phi_Q + q|P||Q|\phi_S}{\sqrt{(n+3-p-q)^2|Q|^2|S|^2+p^2|P|^2|S|^2+q^2|P|^2|Q|^2}} \tag{56}$$

with "mass-squared"

$$\frac{q^2|\bar{A}||\xi_q||P|^{n+3-p-q}|Q|^p|S|^{q-2}}{M_{\rm P}^n} \sim m_{3/2}^2\left(\frac{|S|}{|P|}\right)^{q-2} \tag{57}$$

for $|S| \ll |P| \sim |Q|$ (we use quotation marks to indicate that the "mass term" and "mass-squared" do not necessarily correspond to a mass eigenstate). We see that, for $q \geq 3$, this "mass-squared" is suppressed relative to $m_{3/2}^2$. Superpotential terms with higher powers of S obviously give even more suppressed "masses-squared." So, the linear combination of ϕ_P, ϕ_Q, and ϕ_S which is orthogonal to the combination in Eq. (54) and the axion direction has mass-squared much smaller than $m_{3/2}^2$ during inflation and can be used as PNGB curvaton. It is though important to make sure that at least one of the three terms $S^{n+3}/M_{\rm P}^n$, $S^{n+2}P/M_{\rm P}^n$, and $S^{n+2}Q/M_{\rm P}^n$ is allowed in the superpotential, since otherwise the potential will be unbounded below on the S axis. In the next section, we will present a concrete class of models of this category.

V. PQ MODELS WITH AN AXIONLIKE CURVATON

We consider a class of simple extensions of MSSM which are based on the SM gauge group $G_{\rm SM}$, but also possess two continuous global U(1) symmetries, namely, a

PQ symmetry $U(1)_{PQ}$ and a R symmetry $U(1)_R$, and a discrete Z_2^P symmetry. In addition to the usual MSSM left-handed superfields h_1, h_2 [Higgs $SU(2)_L$ doublets], l_i [$SU(2)_L$ doublet leptons], e_i^c [$SU(2)_L$ singlet charged leptons], q_i [$SU(2)_L$ doublet quarks], and u_i^c, d_i^c [$SU(2)_L$ singlet antiquarks] with $i = 1, 2, 3$ being the family index, the models also contain the SM singlet left-handed superfields P, Q, and S. The charges of the superfields under $U(1)_{PQ}$ and $U(1)_R$ are

$$\begin{aligned} &\mathrm{PQ:} \quad P(-2), Q(2), S(0), h_1, h_2(n+1), \\ &\mathrm{R:} \quad P\left(\frac{n+3}{2}\right), Q\left(\frac{n-1}{2}\right), S\left(\frac{n+1}{2}\right), h_1, h_2(0) \end{aligned} \tag{58}$$

with the "matter" (quark and lepton) superfields having $\mathrm{PQ} = -(n+1)/2$, $\mathrm{R} = (n+1)(n+3)/4$. The integer n is taken to be of the form

$$n = 4l + 1, \tag{59}$$

where $l = 0, 1, 2, \ldots$ is any non-negative integer providing a numbering of the models in this class (see Sec. VI B), and the charges are normalized so that they take their absolutely smallest possible integer values. Finally, under the Z_2^P symmetry, P changes sign.

The most general superpotential compatible with these symmetries is

$$\begin{aligned} W = {} & y_{eij}(l_i h_1)e_j^c + y_{uij}(q_i h_2)u_j^c + y_{dij}(q_i h_1)d_j^c \\ & + \lambda P^{n+1}(h_1 h_2)/M_{\mathrm{P}}^n + \sum_{k=0}^{(n+3)/4} \lambda_k S^{n+3-4k}(PQ)^{2k}/M_{\mathrm{P}}^n, \end{aligned} \tag{60}$$

where y_{eij}, y_{uij}, y_{dij} are the usual Yukawa coupling constants, λ, λ_k are complex dimensionless parameters, (XY) indicates the $SU(2)_L$ invariant product $\epsilon_{ab}X_a Y_b$ with ϵ denoting the 2×2 antisymmetric matrix with $\epsilon_{12} = 1$, and summation over the family indices is implied. The R charge of W is $(n+1)(n+3)/2$. Baryon number is automatically conserved to all orders in perturbation theory as a consequence of the R symmetry. The reason is [34] that the R charge of any combination of three color triplet or antitriplet superfields exceeds the R charge of W and there are no superfields with negative R charge to compensate. Note that the Z_2 subgroup of $U(1)_{PQ}$ coincides with the discrete matter parity symmetry (denoted by Z_2^{mp}), which changes the sign of all matter superfields. It is obvious that the superpotential in Eq. (60) also conserves lepton number, which is a consequence of both the R and the PQ symmetry.

Note that the superpotential in Eq. (60) is of the same type as the superpotential which has been considered in Ref. [10]. The main difference is that here the discrete Z_{n+3} symmetry of Ref. [10] is replaced by a much more powerful continuous U(1) R symmetry and, also, that an extra discrete Z_2^P symmetry is added. Moreover, in contrast to Ref. [10], we include here all the superpotential terms which are compatible with the symmetries of the model. In this sense, the superpotential in Eq. (60) is a completely natural superpotential. It should be emphasized that continuous global symmetries such as the $U(1)_R$ or the $U(1)_{PQ}$ symmetry used here, rather than being imposed, can arise in a natural manner from an underlying superstring theory. Indeed, as it was pointed out in Ref. [35], discrete symmetries (including R symmetries) that typically arise after compactification could effectively behave as if they are continuous.

To see that the above superpotential has the most general form allowed by the symmetries, observe that, due to matter parity, any term in W must contain an even number of matter superfields. Actually, we may have either no or two matter fields since higher combinations carry R charge larger than that of W. The possible combinations of two matter fields are of the type ll, $e^c e^c$, le^c, qu^c, and qd^c (from color conservation) and have the R charge of W. So, we can multiply them only by superfields of zero R charge, i.e. h_1, h_2, and Q in the case $n = 1$. However, multiplying ll, $e^c e^c$ just by Q's, we cannot compensate their nonzero weak hypercharge. We need to multiply them at least once by h_1 or h_2. Indeed, the weak hypercharge of ll ($e^c e^c$) is compensated if we take the combination llh_2h_2 ($e^c e^c h_1 h_1 h_1 h_1$) whose PQ charge is $n + 1(3(n+1))$, which could only be canceled by including P's, the only superfields with negative PQ charge. This is though not allowed by R symmetry. So, the only matter field bilinears which are allowed are qu^c, qd^c, and le^c, which conserve lepton number. Actually, to cancel all the SM quantum numbers and the PQ charge, we must take the combinations $(qh_2)u^c$, $(qh_1)d^c$, and $(lh_1)e^c$. No further superfields can be included in these combinations since, by $U(1)_{PQ}$, we could only include the combination PQ, which has though positive R charge. In conclusion, we see that the only superpotential terms involving matter superfields which can be present are the usual Yukawa couplings.

We still have to consider superpotential terms with no matter superfields. If such terms involve h_1, h_2, these superfield must enter through the combination $(h_1 h_2)$ which is neutral under both $SU(2)_L$ and $U(1)_Y$. To cancel the PQ charge, we must then take $P^{n+1}(h_1 h_2)$. To retain the $U(1)_R$ symmetry, we could multiply this only by the combination PQ, which has $\mathrm{PQ} = 0$. However, the R charge of $P^{n+1}(h_1 h_2)$ is already equal to the R charge of W and PQ has positive R charge. So, $P^{n+1}(h_1 h_2)$ is the only allowed term which involves Higgs but no matter superfields. We are left to consider combinations which involve only SM singlet superfields. The PQ symmetry allows only the combinations $S^p(PQ)^q$, where p and q are non-negative integers. The R charge of PQ (S) is $n+1$ ($(n+1)/2$), which implies that the term S^{n+3} can be present in the superpotential. Note that the combination PQ has the same R charge as S^2. Consequently, the terms $S^{n+1}(PQ)$,

$S^{n-1}(PQ)^2, \ldots, (PQ)^{(n+3)/2}$ are also allowed by $U(1)_R$. The Z_2^P symmetry, however, forbids all the odd powers of PQ and we arrive at the terms in the sum in the RHS of Eq. (60).

The PQ symmetry is anomalous (as it should). In particular, one can show that the QCD instantons break it explicitly to its $Z_{\mathcal{N}}$ subgroup with $\mathcal{N} = 6(n+1)$. This subgroup contains the Z_2^{mp} (matter parity), which thus always remains unbroken by instanton effects. On the contrary, the R symmetry is nonanomalous since the fermionic components of all the color triplet or antitriplet superfields (actually, all the matter fermions) have zero R charge. The soft SUSY-breaking terms (especially, the A-type terms), however, break $U(1)_R$ to its $Z_{\mathcal{M}}$ subgroup, where $\mathcal{M} = (n+1)(n+3)/2$.

VI. THE SCALAR POTENTIAL

The part of the superpotential in Eq. (60) which is relevant for the PQ breaking is the sum in the RHS of this equation. This sum contains at least two terms. For $n = 1$, in particular, it consists of just two terms. The resulting scalar potential after including soft SUSY-breaking effects is

$$V = |F_P|^2 + |F_Q|^2 + |F_S|^2 + V_{\mathrm{soft}}, \tag{61}$$

where

$$F_P = \sum_{k=1}^{(n+3)/4} 2k\lambda_k \frac{S^{n+3-4k}(PQ)^{2k-1}Q}{M_{\mathrm{P}}^n} \equiv FQ, \tag{62}$$

$$F_Q = \sum_{k=1}^{(n+3)/4} 2k\lambda_k \frac{S^{n+3-4k}(PQ)^{2k-1}P}{M_{\mathrm{P}}^n} \equiv FP \tag{63}$$

and

$$F_S = \sum_{k=0}^{(n-1)/4} (n+3-4k)\lambda_k \frac{S^{n+2-4k}(PQ)^{2k}}{M_{\mathrm{P}}^n}, \tag{64}$$

are the F-terms and

$$V_{\mathrm{soft}} = m_P^2|P|^2 + m_Q^2|Q|^2 + m_S^2|S|^2 + \left[A \sum_{k=0}^{(n+3)/4} \lambda_k \frac{S^{n+3-4k}(PQ)^{2k}}{M_{\mathrm{P}}^n} + \mathrm{H.c.}\right] \tag{65}$$

the soft SUSY-breaking terms. Here, the soft SUSY-breaking masses-squared m_P^2, m_Q^2, and m_S^2 are of the order of the $m_{3/2}^2$ and can have either sign. Also, for simplicity, we assumed universal soft SUSY-breaking A-terms with the magnitude of the complex parameter A being of the order of $m_{3/2}$. Note that the sums in the RHS of Eqs. (62)–(64) contain at least one term. Actually, these sums consists of just one term for $n = 1$. On the other hand, the sum in Eq. (65) contains at least two terms with the minimum number of terms corresponding to $n = 1$.

As $|P|$, $|Q|$, $|S| \to \infty$, the potential is generally dominated by the positive F-terms and thus V is bounded below independently of the sign of the soft mass terms. However, on the P or Q axis, i.e. for $Q = S = 0$ or $P = S = 0$, respectively, the F-terms vanish identically together with the A-terms and the potential is given by just the mass term of P or Q, respectively. So, to have V bounded below on these two axes also, we must restrict the masses-squared of P and Q to be positive. For simplicity, we will take these two soft masses-squared to be equal, i.e. we will put

$$m_P^2 = m_Q^2 \equiv m^2. \tag{66}$$

The potential on the S axis (i.e. for $P = Q = 0$) is

$$V = (n+3)^2|\lambda_0|^2 \frac{|S|^{2(n+2)}}{M_{\mathrm{P}}^{2n}} + m_S^2|S|^2 + \left(A\lambda_0 \frac{S^{n+3}}{M_{\mathrm{P}}^n} + \mathrm{H.c.}\right) \tag{67}$$

with the first term in the RHS of this equation originating from the F-term F_S. We see that, for $|S| \to \infty$, the potential is dominated by the positive F-term and thus V on the S axis is bounded below no matter what the sign of the mass-squared of S is. So, this sign can be chosen at will. For reasons which will become clear later, however, we take

$$m_S^2 < 0. \tag{68}$$

Therefore, the origin in field space ($P = Q = S = 0$) is a saddle point of the potential with positive curvature in the P and Q directions and negative in the S direction.

Using Eqs. (61)–(65), one can readily show that, for m_P^2, $m_Q^2 > 0$, the potential V in Eq. (61) has a valley of local minima with respect to $|P|$ and $|Q|$ which lies on the S axis (i.e. at $P = Q = 0$). The potential right on the bottom line of this valley can be found from Eq. (67) by choosing the phase θ_S of S so that the sum of the terms in the parentheses in the RHS of Eq. (67) is minimized. In this case, this equation takes the form

$$V = (n+3)^2|\lambda_0|^2 \frac{|S|^{2(n+2)}}{M_{\mathrm{P}}^{2n}} + m_S^2|S|^2 - 2|A||\lambda_0| \frac{|S|^{n+3}}{M_{\mathrm{P}}^n}, \tag{69}$$

which has a minimum at

$$\frac{|S|^{n+1}}{M_{\mathrm{P}}^n} = \frac{|A| + \sqrt{|A|^2 - 4(n+2)m_S^2}}{2(n+2)(n+3)|\lambda_0|}, \tag{70}$$

where $|S| \sim (m_{3/2}M_{\mathrm{P}}^n)^{1/(n+1)}$. So, as $|S|$ increases from zero, the depth of the valley increases (i.e. the valley has initially negative inclination) until $|S|$ reaches the value in Eq. (70), where the maximal depth is achieved. As $|S|$ increases further, the bottom line of the valley rises and, for $|S| \to \infty$, tends to infinity. We will call this valley, which starts from the trivial saddle point at the origin, the trivial valley.

A. The nontrivial minima of the potential

The extrema of the full potential V in Eq. (61) are given by the equations

$$\frac{\partial V}{\partial P} = F_P^* \frac{\partial F_P}{\partial P} + F_Q^* \frac{\partial F_Q}{\partial P} + F_S^* \frac{\partial F_S}{\partial P} + m^2 P^* + AF_P = 0, \tag{71}$$

$$\frac{\partial V}{\partial Q} = F_P^* \frac{\partial F_P}{\partial Q} + F_Q^* \frac{\partial F_Q}{\partial Q} + F_S^* \frac{\partial F_S}{\partial Q} + m^2 Q^* + AF_Q = 0, \tag{72}$$

and

$$\frac{\partial V}{\partial S} = F_P^* \frac{\partial F_P}{\partial S} + F_Q^* \frac{\partial F_Q}{\partial S} + F_S^* \frac{\partial F_S}{\partial S} + m_S^2 S^* + AF_S = 0. \tag{73}$$

Multiplying Eqs. (71) and (72) by P and Q respectively, subtracting and using Eqs. (62)–(64), we obtain

$$(|F|^2 + m^2)(|P|^2 - |Q|^2) = 0, \tag{74}$$

which implies that $|P| = |Q|$. So, the complex fields P and Q have exactly the same magnitude in any extremum of the full potential V. This is actually true also in the extrema of V with respect to P and Q only for fixed S since we have not used Eq. (73). In the trivial minimum in Eq. (70) which lies on the trivial valley, $|P| = |Q| = 0$. However, the potential V possesses nontrivial minima also, where

$$|P| = |Q|, |S| \sim (m_{3/2} M_{\rm P}^n)^{1/(n+1)}. \tag{75}$$

It is important to note that the continuous global symmetry of the model is not exactly $U(1)_{PQ} \times U(1)_R$. The reason is that there are elements of $U(1)_R$ which are indistinguishable from elements of $U(1)_{PQ}$ since they have the same action on all the superfields of the model. Actually, the only elements of $U(1)_R$ which can, in principle, be identified with elements of $U(1)_{PQ}$ are the ones belonging to its $Z_{\mathcal{M}}$ subgroup with $\mathcal{M} = (n+1)(n+3)/2$ generated by the element

$$e^{i[(2\pi)/\mathcal{M}]} \in U(1)_R. \tag{76}$$

This is so because this $Z_{\mathcal{M}}$ is the maximal ordinary (i.e. non-R) symmetry group contained in $U(1)_R$. One can show that the element

$$e^{-i[(8\pi)/(n+1)]} \in U(1)_R \tag{77}$$

has the same action on all the superfields as the element

$$e^{i[(4\pi)/(n+1)]} \in U(1)_{PQ} \tag{78}$$

and, thus, the $Z_{(n+1)/2}$ subgroup of $Z_{\mathcal{M}}$ which is generated by it is identical with the $Z_{(n+1)/2}$ subgroup of $U(1)_{PQ}$ generated by the element in Eq. (78). So, the continuous global symmetry of the model is actually $U(1)_{PQ} \times (U(1)_R/Z_{(n+1)/2})$.

As already explained, $U(1)_R$ is a symmetry of the model only in the limit of exact SUSY. In the scalar potential V which includes the soft SUSY-breaking terms also, it is explicitly broken to its discrete subgroup $Z_{\mathcal{M}}$. Thus, the group of global symmetries of V (except Z_2^P) is $U(1)_{PQ} \times (Z_{\mathcal{M}}/Z_{(n+1)/2})$. The $Z_{\mathcal{M}}$ symmetry can be factorized as follows:

$$Z_{\mathcal{M}} = Z_{n+3} \times Z_{(n+1)/2}, \tag{79}$$

where Z_{n+3} is generated by

$$e^{i[(2\pi)/(n+3)]} \in U(1)_R. \tag{80}$$

So, the global symmetry of V takes the simple form $U(1)_{PQ} \times Z_{n+3} \times Z_2^P$.

In any nontrivial minimum (actually, for any fixed nonzero values of P, Q and S), this symmetry is spontaneously broken to the $Z_2^{\rm mp}$ subgroup of $U(1)_{PQ}$. In case there was, after inflation, a phase transition from the origin in field space ($P = Q = S = 0$) to a nontrivial minimum, we would encounter copious production of axionic strings [36] as well as domain walls. As we perform a full rotation around such a string, the phase of P or Q changes, respectively, by -2π or 2π. There are two types of walls separating vacua which are related either by the group element in Eq. (80) or the generator of Z_2^P. Note that $n + 3$ walls of the former type can terminate together on the same line.

As explained in the previous section, after the onset of instantons at the QCD transition, $U(1)_{PQ}$ is explicitly broken to $Z_{\mathcal{N}}$ with $\mathcal{N} = 6(n+1)$ which is generated by the element

$$e^{i\{(2\pi)/[6(n+1)]\}} \in U(1)_{PQ}. \tag{81}$$

So, after instantons, the axionic strings become [36] boundaries of $3(n+1)$ axionic walls [37] which separate vacua related by the group element in Eq. (81). We see that, if, after inflation, a transition from the origin in field space to a nontrivial minimum takes place, a rich system of domain walls is produced leading to a cosmological catastrophe. So, it is clear that such a transition should be avoided in our model.

B. The shifted valley of minima

For $|S| \ll |P| \sim |Q|$, the F-terms in Eqs. (62)–(64) can be approximated as follows:

$$F_P = \frac{n+3}{2} \lambda_{(n+3)/4} \frac{(PQ)^{(n+1)/2} Q}{M_{\rm P}^n} + \frac{n-1}{2} \lambda_{(n-1)/4} \frac{S^4 (PQ)^{(n-3)/2} Q}{M_{\rm P}^n} + \cdots, \tag{82}$$

$$F_Q = \frac{n+3}{2}\lambda_{(n+3)/4}\frac{(PQ)^{(n+1)/2}P}{M_P^n} + \frac{n-1}{2}\lambda_{(n-1)/4}\frac{S^4(PQ)^{(n-3)/2}P}{M_P^n} + \cdots, \quad (83)$$

$$F_S = 4\lambda_{(n-1)/4}\frac{S^3(PQ)^{(n-1)/2}}{M_P^n} + \cdots, \quad (84)$$

while the expression in the brackets in the RHS of Eq. (65) takes the form

$$\left(A\lambda_{(n+3)/4}\frac{(PQ)^{(n+3)/2}}{M_P^n} + A\lambda_{(n-1)/4}\frac{S^4(PQ)^{(n-1)/2}}{M_P^n} + \text{H.c.}\right) + \cdots, \quad (85)$$

where the ellipses in these equations represent terms of higher order in S. Note that the second term in the RHS of Eqs. (82) and (83) exists only for $n \geq 5$ ($l \geq 1$), while the second term in Eq. (85) exists for all values of n in Eq. (59). Using these relations, we can expand, in this regime, the potential V in Eq. (61):

$$V = \frac{(n+3)^2}{4}|\lambda_{(n+3)/4}|^2\frac{|PQ|^{n+1}(|P|^2+|Q|^2)}{M_P^{2n}} + m_P^2|P|^2 + m_Q^2|Q|^2 - 2|A||\lambda_{(n+3)/4}|\frac{(|P||Q|)^{(n+3)/2}}{M_P^n} \times \cos\left[\frac{n+3}{2}(\theta_P+\theta_Q)\right] + \cdots \equiv V_{(0)} + \cdots, \quad (86)$$

where θ_P and θ_Q are the phases of the complex scalar fields P and Q, respectively. Here we assumed, without loss of generality, that the product $A\lambda_{(n+3)/4}$ is real and negative, which can be readily achieved by redefining the phase of the product of fields PQ.

The leading order part $V_{(0)}$ of the potential V in Eq. (86) (consisting of the explicitly displayed terms in the RHS of this equation) is minimized with respect to the phases θ_P and θ_Q for

$$\frac{n+3}{2}(\theta_P+\theta_Q) = 0 \text{ modulo } 2\pi. \quad (87)$$

Under this restriction on the phases, the extrema of $V_{(0)}$ with respect to $|P|$ and $|Q|$ are given by the conditions

$$\frac{\partial V_{(0)}}{\partial|P|} = \frac{(n+3)^2}{4}|\lambda_{(n+3)/4}|^2[(n+3)|P|^2+(n+1)|Q|^2] \times \frac{|P|^n|Q|^{n+1}}{M_P^{2n}} + 2m_P^2|P| - 2|A|\frac{(n+3)}{2} \times |\lambda_{(n+3)/4}|\frac{|P|^{(n+1)/2}|Q|^{(n+3)/2}}{M_P^n} = 0 \quad (88)$$

and

$$\frac{\partial V_{(0)}}{\partial|Q|} = \frac{(n+3)^2}{4}|\lambda_{(n+3)/4}|^2[(n+1)|P|^2+(n+3)|Q|^2] \times \frac{|P|^{n+1}|Q|^n}{M_P^{2n}} + 2m_Q^2|Q| - 2|A|\frac{(n+3)}{2} \times |\lambda_{(n+3)/4}|\frac{|P|^{(n+3)/2}|Q|^{(n+1)/2}}{M_P^n} = 0. \quad (89)$$

Multiplying Eqs. (88) and (89) by $|P|$ and $|Q|$ respectively and subtracting, we obtain

$$\frac{(n+3)^2}{4}|\lambda_{(n+3)/4}|^2(|P|^2-|Q|^2)\frac{|P|^{n+1}|Q|^{n+1}}{M_P^{2n}} + (m_P^2|P|^2 - m_Q^2|Q|^2) = 0, \quad (90)$$

which, for $m_P^2 = m_Q^2 \equiv m^2 > 0$ [see Eq. (66)], implies that $|P| = |Q|$. Substituting $|P|$ for $|Q|$ in any of the Eqs. (88) and (89), we then obtain $|P| = 0$ or

$$\frac{(n+3)^2}{4}|\lambda_{(n+3)/4}|^2(2n+4)\frac{|P|^{2(n+1)}}{M_P^{2n}} + 2m^2 - 2|A|\frac{(n+3)}{2}|\lambda_{(n+3)/4}|\frac{|P|^{n+1}}{M_P^n} = 0. \quad (91)$$

This equation has two real and positive solutions for

$$|A|^2 > 4(n+2)m^2,$$

which are given by

$$\left(\frac{|P|^{n+1}}{M_P^n}\right)_\pm \equiv x_\pm = \frac{|A| \pm \sqrt{|A|^2 - 4(n+2)m^2}}{(n+2)(n+3)|\lambda_{(n+3)/4}|}. \quad (92)$$

Obviously, $x = 0$ and $x = x_+$ correspond to (local) minima of $V_{(0)}$, while $x = x_-$ corresponds to a local maximum (here $x \equiv |P|^{n+1}/M_P^n$). So, $V_{(0)}$ has a trivial minimum at $|P| = |Q| = 0$ and a "shifted" minimum at $|P| = |Q| \sim (m_{3/2}M_P^n)^{1/(n+1)}$, which is the absolute minimum for $|A| > (n+3)m$. Note that the presence of the term $\lambda_{(n+3)/4}(PQ)^{(n+3)/2}/M_P^n$ in the superpotential of Eq. (60) is vital to the existence of the shifted minimum. In view of the Z_2^P symmetry, however, this superpotential term can only exist if $(n+3)/2$ is an even positive integer, which implies the restriction in Eq. (59).

The trivial minimum of $V_{(0)}$ cannot be consistent with our starting hypothesis that $|S| \ll |P| \sim |Q|$ and should, thus, be discarded. However, as we have already shown, it happens to exist as a minimum of the full potential V in Eq. (61) with respect to $|P|$ and $|Q|$ for all values of $|S|$ and constitutes the trivial valley of minima.

To see how the shifted minimum of $V_{(0)}$ evolves as $|S|$ increases from zero, we consider the dominant S-dependent part of the potential V for $|S| \ll |P| \sim |Q|$, which is given by

$$V_{(1)} = m_S^2|S|^2 + \left[A\lambda_{(n-1)/4}\frac{S^4(PQ)^{(n-1)/2}}{M_{\rm P}^n} + \frac{(n-1)(n+3)}{4}\lambda_{(n-1)/4}\lambda^*_{(n+3)/4}S^4(P^*Q^*)^2 \times (|P|^2+|Q|^2)\frac{(|P||Q|)^{n-3}}{M_{\rm P}^{2n}} + {\rm H.c.}\right], \quad (93)$$

where the first term in the brackets corresponds to the second term in Eq. (85), while the second term in the brackets originates from the interference of the two explicitly displayed terms in the RHS of Eq. (82) plus the interference of the two explicitly displayed terms in the RHS of Eq. (83). So, the first term in the brackets exists for all values of n, while the second only for $n \geq 5$. Note that $V_{(1)}$ contains the next-to-leading and the next-to-next-to-leading order parts of V in the expansion of Eq. (86), which are quadratic and quartic in S, respectively. Actually, for $|P|$ and $|Q|$ at the shifted minimum of $V_{(0)}$, all the terms in $V_{(0)}$ are of the same order of magnitude, while the first (mass) term in the RHS of Eq. (93) is suppressed by $(|S|/|P|)^2$ and the terms in the brackets by $(|S|/|P|)^4$.

The minimization conditions for V with respect to $|P|$ and $|Q|$ coincide to leading order with Eqs. (88) and (89). The dominant corrections to these conditions for $|S| \ll |P| \sim |Q|$ originate from the next-to-next-to-leading order terms in $V_{(1)}$ and are, thus, suppressed by $(|S|/|P|)^4$. So, for $|S| \ll |P| \sim |Q|$, the shifted minimum of $V_{(0)}$ is also a minimum of V with respect to $|P|$ and $|Q|$ at a practically S-independent position. As a consequence, for small values of $|S|$, we obtain a shifted valley of minima of V at almost constant values of $|P|$ and $|Q|$. This valley has obviously negative inclination for nonzero and small values of $|S|$, due to the negative mass term of S. It starts from the shifted saddle point of V which lies at $|S| = 0$ and $|P|$, $|Q|$ equal to their values at the shifted minimum of $V_{(0)}$. Let us note, in passing, that a shifted valley of minima was first used in Ref. [38] as an inflationary trajectory in order to avoid the overproduction of doubly charged [39] magnetic monopoles at the end of hybrid inflation in a SUSY Pati-Salam [40] GUT model.

C. The PNGB curvaton

The dominant S-dependent part of V can be expressed in terms of the phases of P, Q, and S as follows:

$$V_{(1)} = m_S^2|S|^2 - 2|A||\lambda_{(n-1)/4}|\frac{|S|^4(|P||Q|)^{(n-1)/2}}{M_{\rm P}^n} \times \cos\left(4\theta_S + \frac{n-1}{2}(\theta_P+\theta_Q)\right) + \frac{(n-1)(n+3)}{2} \times |\lambda_{(n-1)/4}||\lambda_{(n+3)/4}||S|^4(|P|^2+|Q|^2)\frac{(|P||Q|)^{n-1}}{M_{\rm P}^{2n}} \times \cos(4\theta_S - 2(\theta_P+\theta_Q)), \quad (94)$$

where we assumed that $A\lambda_{(n-1)/4}$ is real and negative, which can be readily arranged by redefining the phase of S. Note that all the other $A\lambda_k$'s (except $A\lambda_{(n+3)/4}$ and $A\lambda_{(n-1)/4}$) remain in general complex since there is no extra field-rephasing freedom left. From the preceding discussion, we see that, on the shifted valley of minima of V, the second and third term in the RHS of Eq. (94) are of the same order of magnitude. For $n = 1$, the third term vanishes. Moreover, using Eq. (92), one can show that, for $n \geq 5$, the coefficient of the cosine in the second term is always greater in absolute value than the coefficient of the cosine in the third term. Under these circumstances, $V_{(1)}$ is minimized with respect to the phases θ_P, θ_Q, θ_S of the fields by taking

$$4\theta_S + \frac{n-1}{2}(\theta_P+\theta_Q) = 0 \text{ modulo } 2\pi. \quad (95)$$

This together with Eq. (87) implies that

$$4\theta_S - 2(\theta_P+\theta_Q) = 0 \text{ modulo } 2\pi, \quad (96)$$

which maximizes the third term in the RHS of Eq. (94).

We can define the real canonically normalized fields corresponding to the phases of P, Q, S as follows:

$$\phi_P \equiv \sqrt{2}|P|\theta_P, \qquad \phi_Q \equiv \sqrt{2}|Q|\theta_Q, \qquad \phi_S \equiv \sqrt{2}|S|\theta_S. \quad (97)$$

On the shifted valley, the last term in the RHS of Eq. (86) generates a "mass-squared" for the real canonically normalized field $\phi_{PQ} \equiv (\phi_P + \phi_Q)/\sqrt{2}$ given by

$$m_{PQ}^2 = |A|\frac{(n+3)^2}{2}|\lambda_{(n+3)/4}|x_+, \quad (98)$$

where $x_+ \equiv (|P|^{n+1}/M_{\rm P}^n)_+$ is given by Eq. (92) and m_{PQ}^2 is of order $m_{3/2}^2$. Also, the second and third term in the RHS of Eq. (94) generate on this valley "masses-squared," respectively, for the combinations

$$\phi_{S1} = \frac{4\sqrt{2}|P|\phi_S + (n-1)|S|\phi_{PQ}}{\sqrt{32|P|^2 + (n-1)^2|S|^2}}, \qquad \phi_{S2} = \frac{\sqrt{2}|P|\phi_S - |S|\phi_{PQ}}{\sqrt{2|P|^2+|S|^2}} \quad (99)$$

given, respectively, by

$$m_{S1}^2 = 16|A||\lambda_{(n-1)/4}|x_+\frac{|S|^2}{|Q|^2}\left(1 + \frac{(n-1)^2}{32}\frac{|S|^2}{|Q|^2}\right),$$
$$m_{S2}^2 = -8(n-1)(n+3)|\lambda_{(n-1)/4}||\lambda_{(n+3)/4}|x_+^2\frac{|S|^2}{|Q|^2} \times \left(1 + \frac{|S|^2}{2|Q|^2}\right), \quad (100)$$

which are suppressed by $|S|^2/|Q|^2$ relative to $m_{3/2}^2$. Note that the field $a = (\phi_P - \phi_Q)/\sqrt{2}$ remains massless to all orders in perturbation theory since it corresponds to the axion. So, we obtain a 2-dimensional subspace of massive fields spanned by ϕ_{PQ} and ϕ_S which are orthogonal to the massless axion direction. The quadratic form obtained by summing the three "mass terms" corresponding to the masses-squared in Eqs. (98) and (100) must then be diagonalized to find the two mass eigenstates and eigenvalues. This task is particularly simple in the limit $|S| \ll |P|$. To lowest order in $|S|/|P|$, the eigenstates coincide with ϕ_{PQ} and ϕ_S, which we will also call σ as it will be our PNGB curvaton (see below). The masses-squared of these fields on the shifted valley are equal to m_{PQ}^2 in Eq. (98) and

$$\tilde{m}_\sigma^2 = \frac{8}{n+2}|\lambda_{(n-1)/4}|x_+ \frac{|S|^2}{|P|^2}[(n+5)|A| - (n-1) \times \sqrt{|A|^2 - 4(n+2)m^2}], \tag{101}$$

respectively. Obviously, $\tilde{m}_\sigma^2$ is positive and suppressed by $|S|^2/|P|^2$ relative to m_{PQ}^2, which is of order $m_{3/2}^2$. So, the field $\phi_S \equiv \sigma$ is a light PNGB when the system rolls down the shifted valley of the potential with $|S| \ll |P|$, while ϕ_{PQ} is a massive field. Inclusion of higher order corrections in the potential V along the shifted valley does not change this situation.

The discrete Z_2^P symmetry of the model, under which P changes sign, is very important for the PNGB nature of ϕ_S. Without this symmetry, the next-to-leading order term in the RHS of Eqs. (82) and (83) would be proportional to S^2 rather than S^4, the leading order term in the RHS of Eq. (84) would be linear in S rather than cubic, and the next-to-leading order term in Eq. (84) would contain S^2 instead of S^4. As a consequence, the terms in the dominant S-dependent part of V for $|S| \ll |P| \sim |Q|$ which depend on the phases of the fields would be quadratic in S [compare with Eqs. (93) and (94)] and, thus, the mass of ϕ_S on the shifted valley would be of order $m_{3/2}$.

D. Cosmological evolution

SUGRA corrections [2,14,33] during inflation will add to A a term proportional to the Hubble parameter during inflation, which is of order $m_{3/2}$ in our case. To simplify the discussion, we take these corrections to be universal. Also, the masses-squared m_P^2, m_Q^2, and m_S^2 will acquire corrections proportional to H^2. We assume that these corrections are positive and, for simplicity, we also take them universal at least for m_P^2 and m_Q^2. So, nothing changes in the above discussion and formulas after including the SUGRA corrections during inflation except that A, m^2, and m_S^2 must now be replaced by their effective values

$$\bar{A} = A + c_A H, \qquad \bar{m}^2 = m^2 + c_{PQ}H^2, \qquad \bar{m}_S^2 = m_S^2 + c_S H^2, \tag{102}$$

respectively. Here, c_A is a complex parameter of order unity, while c_{PQ} and c_S are real and positive parameters again of order unity.

The effective parameters $\bar{A}$ and $\bar{m}$ are of the same order of magnitude as A and m, i.e. they are of order $m_{3/2}$. The mass-squared of S, which is taken negative, receives positive corrections from SUGRA, which are of the same order of magnitude as m_S^2. We can arrange the parameters so that the effective mass-squared of S is positive during the initial stages of inflation. In this case, the shifted saddle point of V becomes a local minimum of the effective potential and the system may be initially trapped in this minimum during inflation. As H decreases gradually during inflation, the SUGRA corrections become smaller and, at some moment of time, this minimum may turn into a saddle point. The system then slowly rolls down the shifted valley which has a very small slope given by the small effective mass-squared of S [41]. During this slow roll, ϕ_S is an effectively massless PNGB which can act as curvaton.

After the end of inflation, the system keeps rolling down the shifted valley and eventually ends up in damped oscillations about a nontrivial minimum of V where all the fields P, Q, and S acquire nonzero values of order $(m_{3/2}M_{\rm P}^n)^{1/(n+1)}$ and the PQ symmetry is broken. Note that, if, in the initial stages of inflation, the system happened to be trapped in the trivial saddle point of the potential V at $P = Q = S = 0$ (which, of course, is a local minimum of the effective potential in this case), it would later enter into the trivial valley along the S axis rather than the shifted one. So, it would end up in the minimum of Eq. (70), where $P = Q = 0$. This is obviously highly undesirable since, in this case, the PQ symmetry remains unbroken and no μ term is generated. It is also important that, in our case, the fields P, Q, and S have nonzero values during inflation after the time when the cosmological scales exit the horizon as they lie on the shifted valley. So, the global symmetry of the model is already broken to $Z_2^{\rm mp}$ during the relevant part of inflation. Consequently, neither domain walls nor axionic strings [36] are generated as the system settles in a nontrivial minimum of V. Also, no axionic walls [37] appear at the QCD transition since the spontaneous breaking of $Z_{\mathcal{N}}$ to $Z_2^{\rm mp}$ takes place before the relevant part of inflation. Therefore, no cosmological catastrophe is encountered.

VII. CURVATON PHYSICS

A. The curvaton potential

From Eqs. (61), (84), and (94), we find that the dominant part of the scalar potential which is relevant to our curvaton candidate, in the case when $|S| \ll |P| \sim |Q|$, is given by

$$V_{\rm curv} = \bar{m}_S^2|S|^2 - 2|\bar{A}||\lambda_{(n-1)/4}|\frac{|S|^4(|P||Q|)^{(n-1)/2}}{M_{\rm P}^n} \times \cos\left[4\theta_S + \frac{n-1}{2}(\theta_P+\theta_Q)\right] + \frac{(n-1)(n+3)}{2} \times |\lambda_{(n-1)/4}||\lambda_{(n+3)/4}||S|^4(|P|^2+|Q|^2)\frac{(|P||Q|)^{n-1}}{M_{\rm P}^{2n}} \times \cos[4\theta_S - 2(\theta_P+\theta_Q)] + 16|\lambda_{(n-1)/4}|^2 \times \frac{(|P||Q|)^{n-1}}{M_{\rm P}^{2n}}|S|^6 + \cdots, \quad (103)$$

where we included the SUGRA corrections during inflation and the ellipsis denotes terms of higher order in $|S|$, which are, therefore, subdominant. In the above, according to Eq. (102), we have

$$\bar{m}_S^2 = c_S H^2 - |m_S^2|, \quad (104)$$

where $c_S \sim +1$ and $|m_S^2| \sim m_{3/2}^2$.

The potential $V_{\rm curv}$ is simplified by considering that the $|P|$ and $|Q|$ fields have already assumed their minimum value on the shifted valley as given by Eq. (92) with A and m replaced by $\bar{A}$ and $\bar{m}$, respectively. Furthermore, we can set the phases θ_P and θ_Q equal to zero since we are only interested in the curvaton field direction, which practically corresponds to θ_S for $|S| \ll |P|$. Then, in view also of Eq. (97), the above potential becomes

$$V_{\rm curv} \simeq \bar{m}_S^2|S|^2 - \kappa\frac{|\bar{A}|^2|S|^4}{|P|_{\rm val}^2}\cos\left(2\sqrt{2}\frac{\phi_S}{|S|}\right) + 16|\lambda_{(n-1)/4}|^2\frac{|P|_{\rm val}^{2(n-1)}}{M_{\rm P}^{2n}}|S|^6 + \cdots, \quad (105)$$

where the value $|P|_{\rm val}$ of $|P|$ on the shifted valley is given by $|P|_{\rm val}^{n+1}/M_{\rm P}^n \equiv x_+$ [with x_+ from Eq. (92), where A and m are replaced by $\bar{A}$ and $\bar{m}$, respectively] and

$$\kappa \equiv \left|\frac{\lambda_{(n-1)/4}}{\lambda_{(n+3)/4}}\right|\frac{(1+Z)[(n+5)-(n-1)Z]}{(n+2)^2(n+3)} \quad (106)$$

with Z being

$$Z \equiv \sqrt{1 - 4(n+2)\left(\frac{\bar{m}}{|\bar{A}|}\right)^2}. \quad (107)$$

Note that $|P|_{\rm val}$ is practically constant when the cosmological scales exit the inflationary horizon since the shifted valley is almost $|S|$-independent for $|S| \ll |P|$ as shown in Sec. VI B and the Hubble parameter is very slowly varying [see Eq. (10)].

From Eq. (107), it is evident that $0 < Z < 1$ and, therefore, κ is positive [42]. Hence, setting

$$\sigma \simeq \phi_S \quad \text{and} \quad v \simeq \frac{1}{2\sqrt{2}}|S|, \quad (108)$$

we obtain the curvaton potential for $v \ll v_0$ (the value of v in the vacuum) as

$$V(\sigma) \simeq 64\kappa|\bar{A}|^2\frac{v^4}{|P|_{\rm val}^2}\left[1 - \cos\left(\frac{\sigma}{v}\right)\right]. \quad (109)$$

Considering that the value of v_0 is given by the value $|S|_0$ of $|S|$ in the vacuum, for which $|S|_0 \sim |P|_{\rm val}$, we find

$$V(\sigma) \sim \kappa|\bar{A}|^2\frac{v^4}{v_0^2}\left[1 - \cos\left(\frac{\sigma}{v}\right)\right], \quad (110)$$

where [cf. Eq. (44)]

$$v_0 \sim M_{\rm I} \sim (m_{3/2}M_{\rm P}^n)^{1/(n+1)}. \quad (111)$$

Comparing the above with Eq. (15), we see that the mass of the curvaton is given by

$$\tilde{m}_\sigma \sim \sqrt{\kappa}|\bar{A}|\left(\frac{v}{v_0}\right), \quad (112)$$

which agrees with Eq. (101). Therefore, when the cosmological scales exit the horizon, we have

$$\tilde{m}_\sigma \sim \varepsilon m_{3/2}, \quad (113)$$

where we used Eq. (28) and considered that $|\bar{A}| \sim m_{3/2}$ and $\kappa \sim 1$. Hence, because, in the modular inflation model that we are considering, we have $H_* \sim m_{3/2}$, we find that, since $\varepsilon \ll 1$, σ is, as required, effectively massless when the cosmological scales exit the horizon during inflation.

The order parameter v for our curvaton field is determined by the value of $|S|$, for which the potential in Eq. (105), when taking $\sigma \to 0$, becomes

$$V(|S|) \simeq \bar{m}_S^2|S|^2 - \kappa\frac{|\bar{A}|^2|S|^4}{|P|_{\rm val}^2} + \kappa_S^2|\bar{A}|^2\frac{|S|^6}{|P|_{\rm val}^4} + \cdots, \quad (114)$$

where

$$\kappa_S \equiv \left|\frac{\lambda_{(n-1)/4}}{\lambda_{(n+3)/4}}\right|\frac{4(1+Z)}{(n+2)(n+3)} \quad (115)$$

and we have used Eq. (92). The minimum of the above potential occurs at

$$|S| \simeq \frac{1}{\kappa_S}\sqrt{\frac{2\kappa}{3}}|P|_{\rm val} \sim M_{\rm I}, \quad (116)$$

which, after the end of inflation, becomes equal to $|S|_0$ as $\bar{A}$ and $\bar{m}$ are replaced by A and m, respectively.

B. The required ε

Let us now calculate the value of ε required so that our curvaton scenario works. First, we note that, in our case, the curvaton assumes a random value at the phase transition at which, during inflation, the system leaves the shifted saddle point of the potential and starts slowly rolling down the shifted valley (see Sec. VI D). This value typically is

$\sigma \sim v$. After the end of inflation and before the onset of the oscillations, the phase θ corresponding to the curvaton degree of freedom is overdamped and remains frozen. Hence, we expect that, at the onset of the oscillations, we have

$$\sigma_{\rm osc} \sim \theta v_0, \tag{117}$$

where, typically, $\theta \simeq \theta_S \sim 1$ and we took into account that the order parameter assumes its vacuum value very soon after the end of inflation. Combining Eqs. (31) and (117), we find

$$\varepsilon \sim \frac{\Omega_{\rm dec}}{\pi\zeta\theta}\left(\frac{m_{3/2}}{M_{\rm P}}\right)^{n/(n+1)}, \tag{118}$$

where we also used Eq. (111) and that $H_* \sim m_{3/2}$. The ε above is always larger than $\varepsilon_{\rm min}$, where

$$\varepsilon_{\rm min} \sim \left(\frac{m_{3/2}}{M_{\rm P}}\right)^{n/(n+1)}, \tag{119}$$

which is derived from Eq. (32) with $H_* \sim m_{3/2}$.

Let us now enforce the constraint in Eq. (35), which, for $H_* \sim m_{3/2}$, reads

$$\varepsilon < \frac{\Omega_{\rm dec}^{1/2}}{\pi\zeta}\left(\frac{M_{\rm P}}{T_{\rm BBN}}\right)^{1/2}\left(\frac{m_{3/2}}{M_{\rm P}}\right)^{5/4} \sim 10^{-4}\Omega_{\rm dec}^{1/2}. \tag{120}$$

From Eqs. (118) and (120), it is easy to find that the above bound can be satisfied only if n is large enough:

$$n > \frac{8 + \log(\Omega_{\rm dec}^{1/2}/\theta)}{7 - \log(\Omega_{\rm dec}^{1/2}/\theta)}. \tag{121}$$

In view of Eq. (21), we see that, for $\theta \sim 1$, we have $n \geq 1$.

An upper bound on n can be obtained by requiring that the curvaton decays before BBN. The interaction of σ with ordinary particles is governed by the effective μ term in Eq. (45), which results [10] into the following decay rate of σ into two Higgs particles:

$$\Gamma_\sigma \sim \frac{m_\sigma^3}{v_0^2}. \tag{122}$$

Demanding that $\Gamma_\sigma \geq H_{\rm BBN}$ results in the bound

$$\Gamma_\sigma \sim 10^{-[(30n)/(n+1)]}\left(\frac{m_\sigma}{\rm TeV}\right)^3 {\rm TeV} \geq H_{\rm BBN} \sim 10^{-27}\ {\rm TeV}$$
$$\Rightarrow m_\sigma \gtrsim 10^{(n-9)/(n+1)}\ {\rm TeV}, \tag{123}$$

where we used Eq. (111). The above requirement is always satisfied if $m_\sigma \gtrsim 10$ TeV. However, in the opposite case where $m_\sigma < 10$ TeV, we see that it yields an upper bound on n:

$$n \leq \frac{9 + \log(m_\sigma/{\rm TeV})}{1 - \log(m_\sigma/{\rm TeV})}, \tag{124}$$

which, roughly, demands that $n \leq 9$ for $m_\sigma \lesssim 1$ TeV.

C. The reheating of the universe

We will now proceed further by first discussing the reheating of the universe. This requires that we consider separately the cases when the curvaton decays before or after it dominates the universe.

1. Curvaton decay before domination ($\Omega_{\rm dec} \ll 1$)

During the radiation era and after the onset of curvaton oscillations, for the curvaton energy density fraction, we have $\rho_\sigma/\rho \propto a(t) \propto H^{-1/2}$, where $a(t)$ is the scale factor of the universe. Hence, in this case, we find that

$$\Omega_{\rm dec} \sim \left(\frac{\min\{m_\sigma, \Gamma_{\rm inf}\}}{\Gamma_\sigma}\right)^{1/2}\left(\frac{\sigma_{\rm osc}}{M_{\rm P}}\right)^2, \tag{125}$$

where we have used Eq. (31) and

$$\frac{\rho_\sigma}{\rho}\bigg|_{\rm osc} \sim \left(\frac{\sigma_{\rm osc}}{M_{\rm P}}\right)^2, \tag{126}$$

which is derived from the fact that $\rho_\sigma|_{\rm osc} \simeq \frac{1}{2}m_\sigma^2\sigma_{\rm osc}^2$ and $\rho_{\rm osc} \simeq m_\sigma^2 M_{\rm P}^2$. Using Eq. (122) into Eq. (125) and also Eqs. (31) and (111), we obtain

$$\varepsilon \sim \frac{g^{1/2}\Omega_{\rm dec}^{1/2}}{\pi\zeta}\left(\frac{m_{3/2}}{M_{\rm P}}\right)^{(1/2)[(n+2)/(n+1)]}. \tag{127}$$

Here, we have also used that $\Gamma_{\rm inf} < H_* \sim m_{3/2} \sim m_\sigma$ and

$$\Gamma_{\rm inf} \sim g^2 m_{3/2} \tag{128}$$

with g being the dimensionless coupling constant of the inflaton to its decay products and the mass of the inflaton field s taken to be $m_s \lesssim H_* \sim m_{3/2}$.

In principle, g can be as low as $m_s/M_{\rm P}$ if the inflaton decays gravitationally. However, since reheating has to occur before BBN, g has to lie in the range

$$10^{-14} \sim 10\frac{m_{3/2}}{M_{\rm P}} < g < 1, \tag{129}$$

where we used the fact that the reheat temperature $T_{\rm reh}$, in this case, is

$$T_{\rm reh} \sim \sqrt{\Gamma_{\rm inf} M_{\rm P}} \sim g\sqrt{m_{3/2}M_{\rm P}}. \tag{130}$$

Combining Eqs. (118) and (127), we find the relation

$$\frac{g}{\Omega_{\rm dec}} \sim \frac{1}{\theta^2}\left(\frac{m_{3/2}}{M_{\rm P}}\right)^{(n-2)/(n+1)}, \tag{131}$$

which results in

$$n \simeq \frac{30 - \log g + 2\log(\Omega_{\rm dec}^{1/2}/\theta)}{15 + \log g - 2\log(\Omega_{\rm dec}^{1/2}/\theta)}. \tag{132}$$

In view of Eqs. (21) and (129), we see that, for $\theta \sim 1$, the allowed range for n is

$$2 \leq n \leq 44. \tag{133}$$

The lower bound in the above is tighter than the bound in Eq. (121). In fact, comparing Eqs. (121) and (132), it is easy to obtain the bound

$$g\Omega_{\rm dec}^{1/2} \leq 10^6\theta, \tag{134}$$

which is easily satisfied provided that the angle θ is not extremely small.

2. *Curvaton decay after domination* ($\Omega_{\rm dec} \approx 1$)

In this case, the curvaton dominates the energy density of the universe when $H = H_{\rm dom}$, where $H_{\rm dom}$ is given by

$$H_{\rm dom} \sim \left(\frac{\sigma_{\rm osc}}{M_{\rm P}}\right)^4 \min\{m_\sigma, \Gamma_{\rm inf}\}. \tag{135}$$

Now, using Eqs. (117), (122), and (128), it can be shown that the requirement $\Gamma_\sigma < H_{\rm dom}$ results in the bound

$$g > \frac{1}{\theta^2}\left(\frac{m_{3/2}}{M_{\rm P}}\right)^{(n-2)/(n+1)}, \tag{136}$$

which yields

$$n > \frac{30 - \log g - 2\log\theta}{15 + \log g + 2\log\theta}. \tag{137}$$

This provides a lower bound on g for given n and θ, reminiscent of Eq. (132) with $\Omega_{\rm dec} \approx 1$. For $\theta \sim 1$, the above bound implies

$$n \geq 2. \tag{138}$$

This time the hot big bang begins after the decay of the curvaton, which suggests that the reheat temperature is now given by

$$T_{\rm REH} \sim \sqrt{\Gamma_\sigma M_{\rm P}} \sim m_{3/2}\left(\frac{m_{3/2}}{M_{\rm P}}\right)^{(1/2)[(n-1)/(n+1)]}. \tag{139}$$

It can be easily checked that the above is higher that $T_{\rm BBN}$ when $n \leq 9$, in agreement with Eq. (124).

D. Avoiding axion overproduction

There is a stringent upper bound on the PQ scale originating from the requirement that the generated axions do not overclose the universe. The typical mass of the axion is [43]

$$m_{\rm a} \sim 10^{-5}\left(\frac{10^{12}\ {\rm GeV}}{f_{\rm a}}\right)\ {\rm eV}, \tag{140}$$

where, for the PQ scale (axion decay constant), we have $f_{\rm a} \approx v_0$ [44]. The above mass implies that the onset of axion oscillations takes place when the energy density of the universe is

$$\rho_{\rm axosc}^{1/4} \sim \sqrt{m_{\rm a} M_{\rm P}} \sim 10^2\left(\frac{10^{12}{\rm GeV}}{v_0}\right)^{1/2}\ {\rm GeV}, \tag{141}$$

where the subscript "axosc" indicates the time at which axion oscillations begin. In contrast, the energy density of the universe when the curvaton decays is

$$\rho_{\rm dec}^{1/4} \sim \sqrt{\Gamma_\sigma M_{\rm P}} \sim 10\left(\frac{10^{12}\ {\rm GeV}}{v_0}\right)\ {\rm GeV}, \tag{142}$$

where we have used Eq. (122) and also that $m_\sigma \sim m_{3/2}$. Now, from Eq. (111), we have

$$v_0 \geq \sqrt{m_{3/2} M_{\rm P}}, \tag{143}$$

which suggests that, in all cases,

$$\rho_{\rm dec} < \rho_{\rm axosc}. \tag{144}$$

Hence, the axion oscillations always start before the curvaton decays. Thus, at the onset of the axion oscillations, the ratio of the axion energy density to the energy density of the universe is

$$\frac{\rho_{\rm a}}{\rho}\bigg|_{\rm axosc} \sim \theta_{\rm a}^2\left(\frac{v_0}{M_{\rm P}}\right)^2 \sim \theta_{\rm a}^2\left(\frac{m_{3/2}}{M_{\rm P}}\right)^{2/(n+1)}, \tag{145}$$

where we have considered that the amplitude of the axion field at the onset of its oscillations is $\sim\theta_{\rm a} f_{\rm a}$ with $\theta_{\rm a}$ being the initial misalignment angle.

Let us first investigate the case when the curvaton decays before it dominates the universe. In this case, the axion oscillates in a radiation background until the time $t_{\rm eq}$ of equal matter and radiation energy densities, denoted hereafter by the subscript "eq." Since the energy density of the oscillating axion scales like pressureless matter with the expansion of the universe, we have that, until $t_{\rm eq}$,

$$\frac{\rho_{\rm a}}{\rho} \propto a, \tag{146}$$

where the scale factor of the universe $a \propto 1/T$. In view of the above, the requirement that axions do not overclose the universe translates into requiring that

$$\frac{\rho_{\rm a}}{\rho}\bigg|_{\rm eq} \leq 1, \tag{147}$$

since, by definition, $\rho_{\rm eq} \sim \rho_m$, where ρ_m is the energy density of matter. From Eqs. (145)–(147), we obtain the bound

$$\theta_{\rm a} \leq \left(\frac{10^{12}\ {\rm GeV}}{v_0}\right)^{3/4}, \tag{148}$$

where we used that $T_{\rm eq} \approx 2.8 \times 10^{-9}$ GeV. Hence, an initial misalignment angle of order unity is possible only if $v_0 \leq 10^{12}$ GeV. In view of Eq. (111), this, in turn, is possible only if $n \leq 1$.

Thus, if $n > 1$ and we insist on $\theta_{\rm a} \sim 1$, the only way that axion overproduction can be avoided is by considering that the curvaton *does* dominate the universe before decaying. In this case, reheating due to curvaton decay dilutes the axion energy density because of a dramatic production of entropy. The entropy ratio at curvaton decay is easily estimated as

$$\frac{\mathcal{S}_{\rm after}}{\mathcal{S}_{\rm before}} \sim \left(\frac{\rho_\sigma}{\rho_\gamma}\right)_{\rm REH}^{3/4} \sim \left(\frac{H_{\rm dom}}{\Gamma_\sigma}\right)^{1/2} \sim \frac{g\theta^2 v_0^3}{m_{3/2}M_{\rm P}^2} \sim g\theta^2\left(\frac{M_{\rm P}}{m_{3/2}}\right)^{(n-2)/(n+1)}, \quad (149)$$

where the subscript "REH" denotes the time of the curvaton decay, ρ_γ is the energy density of the background radiation due to the decay of the inflaton, and we have used Eqs. (111), (122), (128), and (135) considering also that $\sigma_{\rm osc} \sim \theta v_0$. The exact calculation multiplies [46] the result by a factor of $1.83 g_\star^{1/4} \sim 1$, where $g_\star \sim 10$–10^2 is the effective number of relativistic degrees of freedom. As expected, the entropy production depends on the value of the coupling constant g. Hence, avoiding axion overproduction, which could overclose the universe, is expected to set another lower bound on g, more stringent than the one in Eq. (136) [47].

The fact that the case of curvaton domination requires a larger value of g [cf. Eq. (136)] is to be expected because larger g implies that the inflaton decays earlier and, therefore, the energy density fraction ρ_σ/ρ grows substantially, allowing the curvaton to dominate the universe before its decay. The higher g is the more dominant the curvaton will be at its decay and, hence, the more diluted the axion energy density will become after the decay of the curvaton.

A crucial further requirement for the dilution of the axion energy density is [30,48–50] that the entropy release occurs after the onset of axion oscillations. Hence, the requirement is

$$\rho_{\rm axosc} \gg T_{\rm REH}^4. \quad (150)$$

E. The evolution of the order parameter

Let us now concentrate on the evolution of the order parameter v, which has to be such as to achieve the required value for ε. The order parameter is determined by the rolling $|S|$. When the cosmological scales exit the horizon, the field $|S|$ has to be slowly rolling because we need the order parameter to vary slowly enough not to destabilize the approximate scale invariance of the perturbation spectrum [cf. Sec. III C]. Therefore, the Klein-Gordon equation for $|S|$ takes the form

$$3H|\dot{S}| + \bar{m}_S^2|S| \simeq 0. \quad (151)$$

Using Eq. (104), the rate of growth of the order parameter in this case can be easily found to be

$$\frac{\dot{v}}{v} = \frac{|\dot{S}|}{|S|} = \frac{1}{3}c_S\left(\frac{|m_S^2|}{c_S H^2} - 1\right)H. \quad (152)$$

The amplification factor ε^{-1} can be found as follows. Using Eq. (9), we can write $|S|$ as a function of the number N of the remaining e-foldings of inflation. Starting from Eq. (151) and after a little algebra, we obtain

$$\frac{3}{c_S}\frac{d\ln|S|}{dN} = \frac{e^{-2F_sN_{\rm x}} - e^{-2F_sN}}{1 - e^{-2F_sN}}, \quad (153)$$

where $N_{\rm x}$ corresponds to the phase transition which changes the sign of $\bar{m}_S^2$. Here, we used the fact that, by definition,

$$|m_S^2| \equiv c_S H_{\rm x}^2 \simeq c_S H_{\rm m}^2(1 - e^{-2F_sN_{\rm x}}), \quad (154)$$

where $H_{\rm x} \equiv H(N_{\rm x})$ and $H_{\rm m} = \sqrt{V_{\rm m}}/\sqrt{3}M_{\rm P}$ with $V_{\rm m}$ being the scale of the inflaton potential as given in Eq. (7). Integrating Eq. (153), we get

$$\frac{6}{c_S}\ln\left(\frac{|S|_*}{|S|_{\rm x}}\right) = (1 - e^{-2F_sN_{\rm x}})F_s^{-1}\ln\left(\frac{e^{2F_sN_{\rm x}} - 1}{e^{2F_sN_*} - 1}\right) - 2(N_{\rm x} - N_*), \quad (155)$$

where $|S|_* \equiv |S|(N_*)$ and $|S|_{\rm x} \equiv |S|(N_{\rm x})$.

Now, it is straightforward to check that, if $2F_sN_* \gg 1$, the RHS of the above equation tends to zero, which yields $|S|_* \approx |S|_{\rm x}$. This is also understood by observing that, in this case, $H^2(N) \simeq H_{\rm m}^2(1 - e^{-2F_sN}) \approx H_{\rm m}^2$, which means that $\delta(H^2)/H^2 \approx 2F_s\delta N e^{-2F_sN} \ll 1$, where $\delta(H^2) = H_{\rm x}^2 - H_*^2$ and $\delta N = N_{\rm x} - N_* > 0$. Thus, in the e-folding interval δN, the effective mass-squared of S hardly changes: $\bar{m}_S^2(N_*) \approx \bar{m}_S^2(N_{\rm x}) \equiv 0$, i.e. the mass is very close to zero. This implies that $|S|$ remains frozen and, thus, $|S|_* \approx |S|_{\rm x}$. Moreover, the displacement of $|S|$ from the origin at the phase transition is determined by its quantum fluctuations, which means that

$$|S|_{\rm x} \sim \frac{H_{\rm x}}{2\pi}. \quad (156)$$

For $2F_sN_* \gg 1$, $H_{\rm x} \approx H_*(\approx H_{\rm m})$ and, thus, $|S|_* \sim H_*/2\pi$. So, under these circumstances, we have, for the amplification factor, $\varepsilon = \varepsilon_{\rm min}$, which is defined in Sec. III A.

It is easily seen that, generally,

$$\varepsilon = \frac{|S|_*}{|S|_0} \sim \frac{|S|_*}{H_*}\frac{H_*}{v_0} \Rightarrow |S|_* \sim \frac{\varepsilon}{\varepsilon_{\rm min}}H_*. \quad (157)$$

Note that, since the required ε is always much bigger than $\varepsilon_{\rm min}$, as we saw from Eqs. (118) and (119), $|S|_* \gg H_*$. Furthermore, it is obvious from the above discussion that, for $\varepsilon > \varepsilon_{\rm min}$, we need to have

$$F_s \lesssim \frac{1}{2N_{\rm x}}. \quad (158)$$

Finally, when $2F_sN_{\rm x} \ll 1$, Eq. (155) reduces to

$$\frac{3}{c_S}\ln\left(\frac{|S|_*}{|S|_{\rm x}}\right) \simeq N_{\rm x}\left[\ln\left(\frac{N_{\rm x}}{N_*}\right) - 1\right] + N_*. \quad (159)$$

In view of Eq. (152), the requirement in Eq. (43) takes the form

$$\frac{c_S}{3}(H_x^2 - H_*^2) \ll H_*^2, \tag{160}$$

where we took into account Eq. (154). This inequality can be recast as

$$\frac{c_S}{3} e^{-2F_s N_*}\left(\frac{1 - e^{-2F_s(N_x - N_*)}}{1 - e^{-2F_s N_*}}\right) \ll 1. \tag{161}$$

When $2F_s N_x \ll 1$, the above reduces to

$$\frac{c_S}{3}\frac{N_x - N_*}{N_*} \ll 1. \tag{162}$$

This means that, in this case, the cosmological scales must exit the horizon not much later than the phase transition which changes the sign of $\bar{m}_S^2$.

Another issue to be addressed concerns the requirement that $|S|$ *does* slow roll at the time when the cosmological scales exit the horizon, in contrast to the case when the slope of the potential is so small that the motion of $|S|$ is dominated by its quantum fluctuations. Indeed, since $|\bar{m}_S^2(H_*)| \ll H_*^2$, $|S|$ is effectively massless and, hence, it obtains a superhorizon spectrum of perturbations of order $H_*/2\pi$, much like σ. In order for its quantum fluctuations not to dominate its motion, $|S|$ has to be outside the quantum diffusion zone. The condition for this to occur is

$$\left|\frac{\partial V}{\partial |S|}\right|_* = 2|\bar{m}_S^2(H_*)||S|_* \geq H_*^3. \tag{163}$$

Using Eqs. (104) and (154) and working as before, the above constraint is recast as

$$\ln\left(\frac{|S|_*}{|S|_x}\right) \geq 2F_s N_* - \ln\left(\frac{c_S}{\pi}\right) + \ln\left(\frac{1 - e^{-2F_s N_*}}{1 - e^{-2F_s(N_x - N_*)}}\right) + \frac{1}{2}\ln\left(\frac{1 - e^{-2F_s N_*}}{1 - e^{-2F_s N_x}}\right), \tag{164}$$

where we have also used Eq. (156). If $2F_s N_x \ll 1$, the above equation becomes

$$\ln\left(\frac{|S|_*}{|S|_x}\right) + \ln\left(\frac{c_S}{\pi}\right) + \ln\left[\frac{N_x^{1/2}(N_x - N_*)}{N_*^{3/2}}\right] \geq 0. \tag{165}$$

VIII. A CONCRETE EXAMPLE: $n = 5$ AND $\theta \sim 1$

From the bounds on n in Eqs. (124), (133), and (138) and also in view of Eq. (59), we see that not many choices for n are allowed. In fact, we can only accept the cases corresponding to $l = 1, 2$ (i.e. $n = 5, 9$) with the latter choice being marginal as far as the BBN constraint in Eq. (124) is concerned. Hence, to illustrate the above, we present an example taking $n = 5$ [i.e. $l = 1$ in Eq. (59)] and considering that the orthogonal axion assumes a random value after the phase transition, i.e. $\theta \sim 1$.

The bound in Eq. (124) suggests that this case is acceptable provided that

$$m_\sigma \gtrsim 220\ \text{GeV}. \tag{166}$$

The superpotential for the SM singlet superfields is comprised only of the terms [cf. Eq. (60)]

$$W_{\text{singlet}} = [\lambda_0 S^8 + \lambda_1 S^4(PQ)^2 + \lambda_2(PQ)^4]/M_P^5. \tag{167}$$

The resulting scalar potential is shown in Fig. 2.

Using Eq. (118), we obtain the value of the amplification factor necessary for the model to work:

$$\varepsilon \sim 10^{-8.5}\Omega_{\text{dec}}. \tag{168}$$

Now, let us assume, at first, that $\Omega_{\text{dec}} < 1$, i.e. the curvaton decays before domination. Then, Eq. (127) gives

$$\varepsilon \sim 10^{-4.75} g^{1/2}\Omega_{\text{dec}}^{1/2}, \tag{169}$$

which means that

$$g \sim 10^{-7.5}\Omega_{\text{dec}}. \tag{170}$$

This value lies well within the range given in Eq. (129).

With these values, it is straightforward to show that all the relevant constraints are satisfied. For example, the constraint in Eq. (33), in the case when $\Omega_{\text{dec}} < 1$, becomes

$$g\Omega_{\text{dec}} \sim (\varepsilon\zeta)^2\left(\frac{M_P}{m_{3/2}}\right)^{(n+2)/(n+1)}, \tag{171}$$

which, for $n = 5$ and the value of g found above, can be easily checked to hold true. According to Eq. (130), the

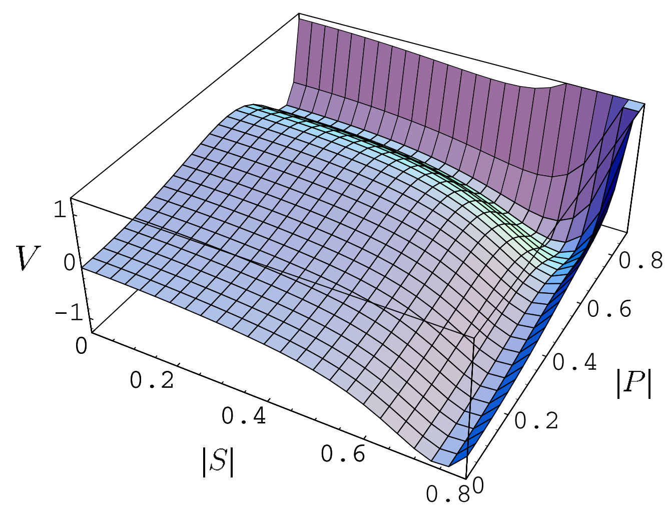

FIG. 2 (color online). Plot of the scalar potential $V(|S|, |P|)$ which is defined in Eqs. (61)–(65) in units of M_I^{14}/M_P^{10} with respect to $|S|$ and $|P|$, which are measured in units of M_I. Here, we have taken $n = 5$ and $M_I \equiv (mM_P^5)^{1/6}$. We have also chosen $m_P^2 = m_Q^2 = -m_S^2 \equiv m^2$, $A = -9m$, $\lambda_0 = \lambda_1 = \lambda_2 = 1$, $|P| = |Q|$, and $\theta_S = \theta_P = \theta_Q = 0$ so that the potential is minimized according to Eqs. (87) and (95). The shifted valley is clearly visible. It starts at the value $|P| \approx 0.814 M_I$ when $|S| = 0$ and arcs down towards the $|S|$ axis, passing through the nontrivial minimum at $|S|_0 \approx 0.683 M_I$ and $|P|_0 \approx 0.772 M_I$. The trivial valley at $|P| = 0$ can also be discerned.

universe reheats at temperature

$$T_{\rm reh} \sim 10^3 \Omega_{\rm dec} \text{ GeV}, \tag{172}$$

which is far from challenging the gravitino bound.

However, the PQ symmetry breaking scale is found from Eq. (111) to be

$$v_0 \sim 10^{15.5} \text{ GeV}. \tag{173}$$

Such a high PQ scale results in axion overproduction unless the original axion misalignment angle $\theta_{\rm a}$ is much less than unity. Indeed, in this case, Eq. (148) suggests that

$$\theta_{\rm a} \lesssim 10^{-3} \tag{174}$$

should be enough to avoid axion overproduction. The above constraint is much less stringent than the constraint on the present value of $\theta_{\rm a}$ from *CP* violation in strong interactions (coming from experimental bounds on the electric dipole moment of the neutron): $\theta_{\rm a} < 10^{-9}$, but it still undermines the motivation for the PQ symmetry, which is meant to explain how we can get the present value of $\theta_{\rm a}$ so small starting with a natural value of the initial $\theta_{\rm a} \sim 1$. This is why it is preferable to consider the case when the curvaton decays *after* domination, where the axion energy density can be efficiently diluted by the entropy production from the curvaton decay.

When the curvaton decays after domination, we have $\Omega_{\rm dec} \approx 1$. Then, Eq. (168) becomes

$$\varepsilon \sim 10^{-8.5}. \tag{175}$$

Also, from Eq. (136), we obtain the bound

$$g > 10^{-7.5}. \tag{176}$$

As a result of the above, the reheat temperature after the end of inflation is found, from Eq. (130), to be

$$T_{\rm reh} \sim g\sqrt{m_{3/2} M_{\rm P}} > 10^3 \text{ GeV}. \tag{177}$$

Satisfying the gravitino bound ($T_{\rm reh} \lesssim 10^9$ GeV) sets a weak upper bound on g: $g \lesssim 0.03$. However, this bound is relaxed by the dilution of the gravitinos due to the entropy production by the curvaton decay. Indeed, in this case, according to Eq. (139), the hot big bang begins at the temperature

$$T_{\rm REH} \sim 10 \text{ MeV}, \tag{178}$$

which is close to the BBN bound, but does not violate it.

Now, from Eq. (149), we find the entropy production ratio to be

$$\frac{\mathcal{S}_{\rm after}}{\mathcal{S}_{\rm before}} \sim 10^{7.5} g. \tag{179}$$

The relic abundance of axions can be calculated by applying the formulas of Ref. [43], where we take the QCD scale $\Lambda_{\rm QCD} = 200$ MeV and ignore the uncertainties for simplicity. Comparing this relic abundance with the best-fit value of the cold dark matter (CDM) abundance in the universe from the measurements of WMAP [51], we find that, for v_0 as large as the one shown in Eq. (173), the entropy of the universe must be increased by a factor of order 10^5 at least. In the present model, this is actually an overestimate since, as it turns out, the axions are generated, in the viable cases, after the domination of the oscillating curvaton field. Axion production in a universe dominated by a coherently oscillating scalar field has been studied in Ref. [49]. In such a universe, axion oscillations begin at a smaller temperature than in a radiation dominated universe. Consequently, the initial ratio of the axion number density to the photon number density is smaller. This then implies that, in this case, less entropy production is required. Taking this effect carefully into account, we obtain the bound

$$g > 10^{-4.5}, \tag{180}$$

which is more stringent than the bound in Eq. (176), as expected. A large coupling between the inflaton field and its decay products can be realized if the VEV of the s modulus corresponds to a point of enhanced symmetry. Note that, in this case, there is no moduli problem because s decays much earlier than BBN.

Moreover, we have to make sure that the entropy production occurs after the onset of the axion oscillations. The latter, according to Eq. (141), occurs at energy density given by

$$\rho_{\rm axosc}^{1/4} \sim 1 \text{ GeV}, \tag{181}$$

where we used Eq. (173). Comparing the above equation with Eq. (178), we see that the requirement in Eq. (150) is satisfied. Thus, we conclude that axion overproduction can be avoided, due to entropy release at curvaton decay.

It should be noted, however, that this mechanism of diluting the axions by the entropy produced when the curvaton decays after dominating the universe may lead [30] to a cosmological disaster. Generally, a sizable fraction of the curvaton's decay products consists of sparticles, which eventually turn into stable lightest sparticles (LSPs) in models (such as ours) with an unbroken matter parity symmetry. The freeze-out temperature of the LSPs is typically much higher than the reheat temperature in Eq. (178). Actually, it is even higher than the energy density scale corresponding to the onset of axion oscillations [see Eq. (181)]. Consequently, the LSPs freeze-out immediately after their production and can, subsequently, overclose the universe leading to a cosmological catastrophe.

In the present model, the PNGB curvaton can decay into a pair of squarks, sleptons, charginos, or neutralinos with a decay width which can be comparable to the width of its main decay channel to a pair of Higgs particles [see Eq. (122)]. We should, however, observe that our curvaton, being a SM singlet, couples to the charginos and neutralinos only via their Higgsino component. Taking μ to be much greater than the soft masses of the bino and wino, which is often encountered in various realizations of MSSM, we can ensure that the lighter charginos and neutralinos are predominantly gauginos. We can further choose all the squark, slepton, and heavier chargino and neutralino masses to exceed half the curvaton mass [see Eq. (166)] so that all the curvaton decay channels involving these particles are kinematically blocked. Thus, the only decay channels allowed are to a pair of lighter charginos or neutralinos. The corresponding rates can be easily suppressed by reducing the Higgsino component of these sparticles. Indeed, assuming that their Higgsino components are about 1%, we obtain a suppression factor of order 10^{-8}. In view of the fact that these sparticles have comparable masses to the Higgs particles, we then conclude that only a fraction of about 10^{-8} of the energy density of the universe soon after the curvaton decay consists of LSPs. This fraction is enhanced by a factor of about 10^7 until the time $t_{\rm eq}$ of equal matter and radiation energy densities when the cosmic temperature $T_{\rm eq} \sim 10^{-9}$ GeV and remains essentially constant thereafter. So, the cosmological disaster from the possible overproduction of LSPs at the curvaton decay can be avoided. Moreover, the LSPs can contribute to the CDM in the universe.

Using Eq. (119) with $n = 5$, we find

$$\varepsilon_{\rm min} \sim 10^{-12.5}. \tag{182}$$

Therefore, Eqs. (157) and (175) suggest that

$$|S|_* \sim 10^4 H_*. \tag{183}$$

The above can, in principle, be used in Eqs. (155) and (164) to constrain the parameters of the underlying model (e.g. F_s).

A useful quantity to calculate in order to evaluate Eqs. (155) and (164) is the number of e-foldings which corresponds to the cosmological scales N_*. The cosmological scales range from a few times the size of the horizon today $\sim H_0^{-1}$ down to scales $\sim 10^{-6} H_0^{-1}$ corresponding to masses of order $10^6 M_\odot$ with $M_\odot$ being the solar mass. Typically, this spans about 13 e-foldings of inflation. For the estimate of N_*, we will choose a scale roughly in the middle of this range. More precisely, we will take the scale that reenters the horizon at the time when structure formation begins, i.e. at the time $t_{\rm eq}$ of equal matter and radiation energy densities. Then it is straightforward to obtain

$$\frac{\exp(N_*)}{H_*^{1/3} t_{\rm eq}^{1/2}} \sim \left(\frac{\Gamma_\sigma \Gamma_{\rm inf}}{H_{\rm dom}}\right)^{1/6} \sim \frac{(m_\sigma^3 M_{\rm P}^4)^{1/6}}{v_0}, \tag{184}$$

where we have used Eqs. (122) and (135) with $\sigma_{\rm osc} \sim v_0$. Note that, remarkably, the above is independent from g. Putting $H_* \sim m_\sigma \sim m_{3/2}$, we obtain

$$N_* \simeq 38, \tag{185}$$

where we have also taken Eq. (173) into account. The number of e-foldings that corresponds to the decoupling of matter and radiation (when the CMBR is emitted) is roughly $N_* + 1.5$, while the one which corresponds to the present horizon is about $N_* + 9$.

Using the above, let us attempt to investigate first the case when

$$2F_s N_{\rm x} \ll 1. \tag{186}$$

Substituting Eq. (159) in Eq. (162), we obtain

$$\frac{(22.1 - \ln\frac{N_{\rm x}}{N_*})(\frac{N_{\rm x}}{N_*} - 1)}{76[\frac{N_{\rm x}}{N_*}(\ln\frac{N_{\rm x}}{N_*} - 1) + 1]} \ll 1, \tag{187}$$

where we also used Eqs. (156), (183), and (185), and the fact that $H^2 \simeq H_{\rm m}^2(1 - e^{-2F_s N})$, which, in this case, is $\approx 2F_s N H_{\rm m}^2$. In order to find how small the left-hand side (LHS) of this inequality should actually be, we must observe that the contribution to the spectral index of density perturbations n_s originating from the evolution of v during inflation is $-2H_*^{-1}(\dot{v}/v)_*$, which is negative. Moreover, one can easily check that, in the present example, all the other contributions [52] to the spectral tilt for the curvaton are negligible. This is due to the fact that $\varepsilon \ll 1$ [see Eq. (175)] and, as it turns out, also $c \ll 1$ (see below). Using Eq. (152), one can further show that, under these circumstances, $dn_s/d\ln k$ is very small and, thus, no running of the spectral index is predicted in our model. For fixed n_s, the recent results of WMAP imply [51] that $n_s = 0.96 \pm 0.02$. Therefore, at 95% C.L., $n_s \geq 0.92$. It is then obvious that the LHS of the inequality in Eq. (187) should not exceed about 0.04. This requirement is met provided that

$$\frac{N_{\rm x}}{N_*} \gtrsim 500, \tag{188}$$

which, in view of Eq. (185), implies that

$$N_{\rm x} \gtrsim 1.9 \times 10^4. \tag{189}$$

From Eq. (159), we then obtain that

$$c_S \lesssim 2.4 \times 10^{-4}. \tag{190}$$

It can be checked that, with these values, the requirement in Eq. (165) is well satisfied. From Eqs. (186) and (189), one obtains

$$F_s \ll 2.63 \times 10^{-5} \Rightarrow c \ll 7.89 \times 10^{-5}, \tag{191}$$

where we also used Eq. (8). Such a small c implies that modular inflation is not really of the fast-roll type and may last for a large number of e-foldings. Indeed, according to Eq. (13), $N_{\rm tot} \gg 10^6$. In view of Eqs. (8) and (154), Eqs. (190) and (191) suggest that

$$|m_S| \lesssim 1.55 \times 10^{-2} H_* \quad \text{and} \quad m_s \ll 8.88 \times 10^{-3} H_* \tag{192}$$

with $H_* \lesssim 4.47 \times 10^{-2} H_{\rm m}$. The above values for c and c_S are plausible (requiring only mildly tuned masses) but not very pleasing, since we would prefer $c \sim c_S \sim 1$. (Note that larger values of $N_{\rm x}$ result in more severe tuning of c and c_S.) The small values obtained may be due to the condition in Eq. (186), which we imposed to simplify the problem.

Therefore, let us consider, now, that

$$2F_s N_* \ll 1 \quad \text{and} \quad 2F_s N_{\rm x} \gg 1. \tag{193}$$

In this limit, Eq. (161) takes the form

$$-\frac{22.1 + \ln(2F_s N_*)}{76\ln(2F_s N_*)} \ll 1. \tag{194}$$

We find that the LHS of this inequality remains smaller than 0.04 provided that $2F_s N_* \lesssim 0.004$, which yields

$$F_s \lesssim 5.26 \times 10^{-5} \Rightarrow c \lesssim 1.58 \times 10^{-4}. \tag{195}$$

This is less fine-tuned than the values in Eq. (191). In order to estimate the lower bound on $N_{\rm x}$ corresponding to the upper limit on F_s, we approximate Eq. (161) for $2F_s N_* \ll 1$, but any value of $2F_s N_{\rm x}$:

$$\frac{[22.1 - \ln(\frac{1-e^{-2F_s N_{\rm x}}}{2F_s N_*})](1 - e^{-2F_s N_{\rm x}})}{76[(1 - e^{-2F_s N_{\rm x}})\ln(\frac{e^{2F_s N_{\rm x}}-1}{2F_s N_*}) - 2F_s N_{\rm x}]} \ll 1. \tag{196}$$

For $2F_s N_* \simeq 0.004$, the LHS of this inequality is kept smaller than 0.04 even if $2F_s N_{\rm x}$ becomes as low as 4, which yields

$$N_{\rm x} \gtrsim 3.8 \times 10^4. \tag{197}$$

Saturating the bounds in Eqs. (195) and (19), i.e. taking

$$F_s \simeq 5.26 \times 10^{-5} (c \simeq 1.58 \times 10^{-4}) \quad \text{and} \quad N_{\rm x} \simeq 3.8 \times 10^4, \tag{198}$$

we find from Eq. (155) that

$$c_S \simeq 4.92 \times 10^{-4}, \tag{199}$$

which is a little more natural than the values in Eq. (190). It can be checked that, with the values in Eq. (198), the requirement in Eq. (164) is well satisfied. Using Eq. (13), it is easy to see that, in this case, the total number of e-foldings of inflation is

$$N_{\rm tot} \simeq 6.6 \times 10^5. \tag{200}$$

In view of Eqs. (8) and (154), Eq. (198) suggests that

$$|m_S| \simeq 2.22 \times 10^{-2} H_* \quad \text{and} \quad m_s \simeq 1.26 \times 10^{-2} H_* \tag{201}$$

with $H_* \simeq 6.32 \times 10^{-2} H_{\rm m}$, which are not very different from the upper bounds in Eq. (192). After some investigation, it can be realized that not much improvement can be made on the results.

In conclusion, we see that, in the $n = 5$ case with $\theta \sim 1$, our model can work, typically, for values

$$c_S, c \lesssim \mathcal{O}(10^{-4}) \tag{202}$$

or, equivalently, for masses

$$|m_S|, m_s \lesssim \mathcal{O}(10^{-2}) H_*, \tag{203}$$

where $H_* \sim m_{3/2} \sim 1$ TeV. Such values are quite natural and imply only a mild tuning on the masses of the rolling field $|S|$ and the inflaton modulus. This is necessary because the effective mass $|\bar{m}_S|$ should remain small enough during the relevant part of inflation for $|S|$ to be slowly rolling and the constraint in Eq. (43) to be satisfied. The condition for this is that c_S or, equivalently, $|m_S|$ be small [53]. Yet, a substantial variation of $|S|$ from the phase transition until the time when the cosmological scales exit the horizon is necessary for obtaining the required value of the amplification factor ε^{-1} (recall that the required ε is always much bigger than $\varepsilon_{\rm min}$). This is achieved with a small mass for the inflaton modulus, which leads to a large number of e-foldings. So, modular inflation cannot be of the fast-roll type in this case. The above findings are similar to the ones in Ref. [18], despite the fact that there the example studied considered the case when the curvaton decayed before domination with $n = 2$. This suggests that the above results are quite robust.

One may wonder why, since both the inflaton s and the field $|S|$ turn out to be light when the cosmological scales exit the inflationary horizon, we cannot use those fields to generate the observed curvature perturbation. The reason is that, in contrast to the PNGB curvaton, the perturbations of those fields are not amplified. Hence, their contribution to the overall curvature perturbation is insignificant. Indeed, for the inflaton, we have

$$\zeta_s \simeq \frac{1}{5\sqrt{3}\pi} \frac{V_*^{3/2}}{|V'|_* M_{\rm P}^3} \simeq \frac{3}{5\pi}\left(\frac{H_*}{m_s}\right)^3 \frac{m_s}{M_{\rm P}} e^{F_s N_*}, \tag{204}$$

where the prime denotes derivative with respect to the inflaton s and we have used Eqs. (6) and (8). For the above discussed values in Eqs. (185), (198), and (201) and for $H_* \sim m_{3/2} \sim 1$ TeV, Eq. (204) gives $\zeta_s \sim 10^{-12}$, which is much smaller than the observed value $\zeta \simeq 2 \times 10^{-5}$. Similarly, for $|S|$, we have

$$\zeta_{|S|} \simeq \frac{2}{3} \frac{\delta|\bar{S}|}{|\bar{S}|}\bigg|_* \sim \frac{H_*}{v_0} \sim \varepsilon \zeta_\sigma, \tag{205}$$

where $|\bar{S}| \equiv ||S| - |S|_0|$, $\delta|\bar{S}|$ the perturbation in $|\bar{S}|$ and we have used the fact that $|S|_0 \sim v_0 \gg |S|_*$ with $|S|_*$ given by Eq. (183). For the values discussed above, $\zeta_{|S|} \sim 10^{-13} \ll \zeta$, where we considered that $\zeta \approx \zeta_\sigma$.

IX. DISCUSSION AND CONCLUSIONS

In this paper, we have studied modular inflation, which uses a string axion as the inflaton field. The inflationary scale ($\sim 10^{10.5}$ GeV) is determined by the scale of gravity mediated soft SUSY breaking. Such low-scale inflation, even though it can still solve the flatness and horizon problems of the standard hot big bang cosmology, cannot generate the observed curvature perturbation (necessary to explain the CMBR anisotropy and structure formation) from the quantum fluctuations of the inflaton field. However, we have shown that this type of modular inflation *can* generate the appropriate amplitude of superhorizon curvature perturbation to account for the observations through the use of a suitable curvaton field.

The curvaton field that we have used is a PNGB which is an angular degree of freedom orthogonal to the QCD axion field (that we called the orthogonal axion) in a class of SUSY PQ models. We considered models that generate the PQ scale dynamically (by using flaton fields), while they also solve the μ problem of MSSM. In these models, one needs more than one SM singlet superfields to break the global $U(1)_{PQ}$ symmetry (with the exception of its matter parity subgroup). Hence, apart from the axion, there is at least one other angular degree of freedom (the orthogonal axion) which may be kept appropriately light during inflation and can be responsible for the curvature perturbation in the universe. This could be achieved if the potential possesses a valley of minima with a negative inclination and the system happens to slowly roll down this valley during the relevant part of inflation with some of the SM singlet fields acquiring values much smaller than their vacuum values. Under these circumstances, the orthogonal axion may be kept light during inflation and its perturbation from inflation may be later amplified as the SM singlets acquire their vacuum values accounting for the observed curvature perturbation in the universe.

Following this promising idea, we have attempted to construct appropriate curvaton models using two SM singlet superfields P and Q, charged under the PQ symmetry. However, we have shown that it is not possible to construct suitable curvaton models by using only two SM singlet superfields, because, in this case, the orthogonal axion mode cannot avoid being massive during the relevant part of inflation. Hence, we studied PQ models which involve a third superfield S and specified the general conditions under which these models can contain a suitable PNGB curvaton. Actually, we have shown that they possess a shifted valley of minima at almost constant values of $|P|$ and $|Q|$ of the order of their vacuum values. The value of $|S|$, however, which parametrizes the valley, can be kept much smaller than its vacuum value during the relevant part of inflation. Also, there exists an orthogonal axion which remains light during inflation and can serve as curvaton.

For definiteness, we considered a concrete class of models of this category where the superfield S has vanishing PQ charge (see also Ref. [10]). These models are simple extensions of MSSM based on the SM gauge group and possessing, besides the global anomalous PQ symmetry, a global U(1) R symmetry and a discrete Z_2 symmetry. The superpotential of the models includes all the terms satisfying these symmetries. The baryon and lepton numbers are automatically conserved to all orders in perturbation theory as a consequence of the R (and the PQ) symmetry.

To study the cosmology, we have also taken into account that the SM singlet fields in these models (P, Q, and S) are expected to receive SUGRA corrections to their soft SUSY-breaking masses-squared (as well as their soft SUSY-breaking trilinear A-terms) of order set by the Hubble parameter. We consider the corrections to the masses-squared to be positive in all cases. Since our inflation model has H_* of order the electroweak scale, which is also the scale of the soft masses, these SUGRA corrections do not seriously affect the physics with the exception of the S field, whose soft mass-squared is taken to be negative. This is intentional in order to facilitate a phase transition during inflation which sends $|S|$ rolling away from the origin along the shifted valley and reaching its true minimum by the end of inflation (cf. Ref. [18]). As a result, the order parameter of our PNGB curvaton field increases substantially after the cosmological scales exit the inflationary horizon. This amplifies the curvaton perturbation according to the mechanism presented in Ref. [13] and the observed value of the curvature perturbation in the universe can be achieved despite the low inflation energy scale.

We have investigated in detail the above scenario and showed that the requirements for a successful curvaton put important constraints both on the choice of model and also on the model parameters. Indeed, we have shown that only a few members of our class of PQ models are eligible for successful curvaton. We then have concentrated on a particular such model and, using natural values for the model parameters, we have studied analytically its performance as curvaton model. We found that the model can indeed work successfully in the context of modular inflation with only a mild tuning of the inflaton's mass and the mass of S: $|m_S|, m_s \lesssim 0.01 H_*$. The bound on $|m_S|$ comes from the requirement that $|S|$ be slowly rolling when the cosmological scales exit the horizon, otherwise the scale invariance of the spectrum of curvature perturbations will be destabilized. The bound on m_s, on the other hand, originates from the large variation of $|S|$ between the phase transition and the exit of the cosmological scales from the horizon, which is necessary for obtaining the correct amplification of the curvaton perturbation. This bound implies that modular inflation cannot be of the fast-roll type and may last for a large number of e-foldings.

In this model, the PQ scale turns out to be quite large (it is comparable to the scale of grand unification). It is actually well above the standard cosmological bound from the requirement that the universe is not overclosed by axion overproduction. However, overclosure of the universe can be avoided, in this model, by diluting the primordial axions through the entropy release by the decay of the curvaton field. For this to be effective, the inflaton modulus has to decay early enough so that the curvaton may well dominate the radiation background. Consequently, we need a comparatively large decay coupling constant for the inflaton, which is possible if the VEV of the inflaton corresponds to an enhanced symmetry point.

After the curvaton decays, the hot big bang begins. This occurs not long before BBN. So, baryogenesis has to take place soon after the time of the curvaton decay at the latest. Moreover, it is clear that, in the present case where the curvaton decays after dominating the energy density of the universe, baryons must be generated through the decay of the curvaton (or of some of its decay products), since otherwise there will be [25] an unacceptably large baryon isocurvature perturbation. It is also obvious that leptogenesis [54] (or electroweak baryogenesis) cannot work here since the reheat temperature is very low for the nonperturbative electroweak sphaleron effects to operate. Therefore, almost the only viable option is that the observed baryon asymmetry of the universe is directly generated by the decay of the curvaton (or of its decay products). One possibility is that the curvaton has suitable baryon (and lepton) number violating decay channels via nonrenormalizable Lagrangian operators of higher order with decay widths comparable to its main decay width to Higgs particles. This, of course, requires an appropriate extension of our model, which, as it stands, has exact baryon (and lepton) number conservation. It may be possible [55] to obtain the required Lagrangian operators by embedding our model in a larger scheme with (large) extra dimensions. This baryogenesis issue deserves further study, which we postpone for the future.

The curvaton model analyzed in this paper can be considered to accommodate [56] low-scale inflationary models other than modular inflation of the type considered here. However, modular inflation due to string axions is one of the better theoretically motivated low-scale inflation models in the literature.

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