Precision measurement of the mean curvature

Lloyd Knox*

Department of Physics, University of California, One Shields Avenue, Davis, California 95616, USA
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Very small mean curvature is a robust prediction of inflation worth rigorous checking. Since current constraints are derived from determinations of the angular-diameter distance to the CMB last-scattering surface, which is also affected by dark energy, they are limited by our understanding of the dark energy. Measurements of luminosity or angular-diameter distances to redshifts in the matter-dominated era can greatly reduce this uncertainty. With a 1% measurement of the distance to $z = 3$, combined with the CMB data expected from Planck, one can achieve $\sigma(\Omega_k h^2) \sim 10^{-3}$. A nonzero detection at this level would be evidence against inflation or for unusually large curvature fluctuations on super-Hubble scales.

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I. INTRODUCTION.

One of the great triumphs of inflation has been the inference from CMB observations that $\Omega_{\rm tot} \equiv \rho/\rho_c \simeq 1$ [1,2]. Here ρ is the mean total density of the Universe today and ρ_c is the critical density that is a function of the expansion rate. As always in general relativity, matter properties determine metric properties. In particular, from $\rho = \rho_c$ it follows that the mean curvature is zero.

The triumphant verification of the zero mean curvature prediction was especially rewarding for inflation theorists given the decades of strong observational evidence that the density of matter (both baryonic and dark) is only about $1/3$ of the critical density; i.e., $\Omega_m \simeq 0.3$. For example, the ratio of dark matter mass to baryonic mass (inferred to be about 6 from x-ray observations of the hot baryons in the dark matter-dominated potential wells of galaxy clusters) combined with nucleosynthesis determinations of $\Omega_b \simeq 0.05$ [3] lead to $\Omega_m \simeq 0.3$ [4].

We can understand the difference between $\Omega_{\rm tot} \simeq 1$ and Ω_m as due to an additional component, called "dark energy," that is causing the expansion of the Universe to accelerate [5,6]. The dark energy, and our lack of understanding of it, is actually what currently limits the precision with which the mean curvature is determined.

Even if we make the strong assumption that the dark energy is a cosmological constant, it is not possible to separately determine the cosmological constant and the mean curvature from CMB data alone [7,8]. However, with this assumption, measurements to distances in the low-redshift (dark energy dominated) era, for example, as inferred from supernovae, can be used to break the CMB parameter degeneracy and thereby allow simultaneous determination of the cosmological constant and the mean curvature [9,10].

If we assume the dark energy is a cosmological constant then current constraints from Wilkinson Microwave Anisotropy Probe (WMAP) data alone are $\Omega_{\rm tot} = 1.09^{+0.06}_{-0.13}$ [11]. The main source of uncertainty here is due to the uncertain value of the cosmological constant. Including the power spectrum of galaxies from the Sloan Digital Sky Survey and luminosity distances to SNe Ia improves the determination of the cosmological constant, tightening up the curvature constraint somewhat to $\Omega_{\rm tot} = 1.054^{+0.048}_{-0.041}$ [11]. The best constraint on the mean curvature (once again assuming the dark energy is a cosmological constant) comes from the distance determination to $z \sim 0.35$ made possible by the detection of the acoustic oscillation feature in the galaxy correlation function [12]; they find $\Omega_{\rm tot} = 1.01 \pm 0.009$. Again with the assumption of a cosmological constant, supernova data alone can be used to constrain the mean curvature [13].

While the determination of the curvature to ± 0.01 is a remarkable achievement, the assumption of the dark energy as a cosmological constant is a very strong one. Indeed, whether the dark energy is a cosmological constant or something else is perhaps one of the most important questions in fundamental physics today. Given the low level of our theoretical understanding of the dark energy [14], we cannot draw robust conclusions if they depend on the assumption that the dark energy is a cosmological constant. Dropping this assumption would greatly weaken all of the above constraints on the mean curvature.

Very small mean curvature is a highly robust prediction of inflation. During inflation the Universe is in a nearly time-translation invariant state. Perfect time-translation invariance would mean inflation lasts forever. With inflation lasting forever, the mean curvature is sent to zero. The near time-translation invariance is responsible for the near scale-invariance of the power spectrum of curvature fluctuations produced during inflation. The near scale-invariance of the power spectrum[15] is evidence that inflation indeed lasted a long time and therefore that the mean curvature is very close to zero. The absence of order unity fluctuations on large scales, as evidenced by the small anisotropy of the CMB, is further indication that inflation lasted for a long time.

Exactly how small do we expect this mean curvature to be? Roughly speaking, we expect the ensemble average of the curvature to be such that $\Omega_k h^2 \lesssim 10^{-60}$. However, no

*Electronic address: lknox@ucdavis.edu

observations are sensitive to this ensemble average. The best we can do is determine the mean curvature as averaged over our Hubble volume. Because of the nearly scale-invariant spectrum of fluctuations, we expect the curvature averaged over our Hubble volume to be such that $|\Omega_k h^2| \sim 10^{-5}$.

Detecting $|\Omega_k h^2| \gtrsim 10^{-5}$ would have important consequences for our understanding of the early Universe and the origin of all structure. Within the context of inflation, it would imply unusually large fluctuation power on super-Hubble scales. Other probes of super-Hubble scales [16,17] are sensitive to gradients across our Hubble patch and thus suppressed by factors of (k/H_0). The probe we consider here is sensitive to the average departure in our Hubble volume of the curvature from its mean value; it is not suppressed by factors of k/H_0.

Although one could design an inflaton field effective potential to produce extra super-Hubble fluctuation power, it is far from what is generically expected. Generically, the fluctuations are better described by a power law at earlier times, with departure from a power law occurring as one approaches the end of inflation. Within a broader context, such a detection might be evidence for some alternative to inflation.

Recent CMB observations have revealed some puzzling properties of the largest scales [18–25]. These peculiar features may have their origin in systematic error. Exploring the possibility of a cosmological origin, by acquiring more and relevant data, is difficult due to the small number of large scale modes in our Hubble volume. It is therefore highly desirable to probe beyond our Hubble volume. Measuring mean curvature provides us with such a probe. By exploiting the prediction of the ensemble average, the average measured over our local Hubble volume is then a measure of fluctuations on scales larger than the Hubble radius.

Anthropic arguments can alter one's intuition about the likelihood of detectably nonzero mean curvature. If the final epoch of inflation, prior to the hot big bang, begins from a tunneling event one no longer needs long inflation to explain homogeneity on large scales. The tunneling event itself creates a highly homogeneous open universe [26,27]. Inflation following tunneling is a natural consequence of the string theory landscape which has many metastable vacua [28,29]. Adopting a particular prior on the distribution of inflaton effective potential shapes, and including anthropically-motivated constraints on the amount of structure growth, [30] finds a limit on the allowable magnitude of the curvature comparable to the current observational limit which they take to be $\Omega_{\rm total} > 0.98$. Further, they find a significant probability that the curvature is near the upper-bound, with 10% of the probability lying between $1 - \Omega_{\rm tot} = 0.02$ and 4×10^{-4}. Universes with small but nonzero mean curvature also have been discussed recently in [31].

On the other hand, detection of a *positive* mean curvature would severely challenge this picture of inflation in the string theory landscape.

For the above reasons, precision measurement of the mean curvature is very well motivated. We therefore consider here the challenge of increasing the precision. Given our lack of understanding of the dark energy, the answer is straightforward: constrain the contribution from the low-redshift, dark energy-polluted universe by directly measuring it; i.e., measure distances from here into the matter-dominated era. In the following we expand upon this idea, work out the resulting uncertainties in the curvature for given CMB data and measurements into the matter-dominated era, and discuss how these distances might be measured.

II. THE PROBLEM

Defining r_s^* as the comoving extent of the sound horizon at the time of last scattering, we can write the angle it subtends as

$$\theta_s = r_s^*/D_A(z_*) \tag{1}$$

by definition of the angular-diameter distance to redshift z, $D_A(z)$ [32] and use of the small angle approximation. This angular size can be determined to very high accuracy from analysis of cosmic microwave background data. It sets the scale for the acoustic peaks, $l_A = \pi/\theta_s$ [33]. Thus if we can calculate r_s^*, and how D_A depends on curvature, we can determine the curvature.

To see how D_A depends on curvature we turn to the line-element for the Friedmann-Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right), \tag{2}$$

where $k = \Omega_k H_0^2$ and $\Omega_k \equiv 1 - \Omega_{\rm tot}$. The comoving length from the origin to a point with coordinate value r is

$$= 1/\sqrt{|k|}\sinh^{-1} \quad (\sqrt{|k|}r) \quad (k < 0), \tag{3}$$

$$l = \int \frac{dr}{\sqrt{1-kr^2}} = r \quad (k = 0), \tag{4}$$

$$= 1/\sqrt{|k|}\sin^{-1} \quad (\sqrt{|k|}r) \quad (k > 0). \tag{5}$$

We can now calculate the angular-diameter distance as a function of l by recognizing that an object at distance r subtending an angle $d\theta$ has length $rd\theta$. Therefore,

$$D_A = r = l + kl^3/6 \tag{6}$$

to lowest order in k.

If we knew the comoving distance to z, $l(z)$, and measured D_A, we could solve for the geometry, k. Of course, we do not know $l(z)$. We could calculate it though *if we knew the matter content*. Using the Friedmann equation we find that a photon suffers a redshift z in the course of traveling a comoving distance

$$l(z) = \sqrt{\frac{3}{8\pi G}} \int_0^z \frac{dz'}{\sqrt{\rho_{m,0}(1+z')^3 + \rho_{k,0}(1+z')^2 + \rho_x(z')}}, \tag{7}$$

where we have defined $\rho_{k,0} \equiv -3k/(8\pi G)$. Thus we see the sensitivity of $D_A(z)$ to the matter content in addition to geometry. Allowing arbitrary freedom in $\rho_x(z)$ destroys our ability to use $D_A(z)$ to determine geometry.

III. A SOLUTION

Despite contamination from dark energy, we can use the angular-diameter distance to the CMB last-scattering surface, which we will now call D_{OL}, to determine the curvature, as long as we can measure some angular-diameter distance, D_{OM}, to some redshift, in the matter-dominated era, z_M. The measurement of D_{OM} effectively controls the dark energy contribution to D_{OL} so that their difference only depends on the curvature and the matter density. Starting from $l_{OL} = l_{OM} + l_{ML}$ and solving for k we find

$$k = 6\left[\frac{D_{OL} - (D_{OM} + l_{ML})}{D_{OL}^3 - D_{OM}^3}\right] \tag{8}$$

to lowest order in k where

$$l_{ML} = \int_{z_M}^{z_*} dz/H(z) \tag{9}$$

$$\simeq \sqrt{\frac{3}{8\pi G \rho_{m,0}}}[(1+z_M)^{-1/2} - (1+z_*)^{-1/2}], \tag{10}$$

and in the final line we have assumed only the matter density contributes to l_{ML}.

Our goal now is to understand how well k can be determined assuming that we have some measurement of D_{OM} with error $\sigma(D_{OM})$ and D_{OL} and ω_m constrained by CMB measurements. The result is displayed in the left panel of Fig. 1. We first discuss the inference from CMB data and then speculate about how D_{OM} might be measured. Depending on how D_{OM} is measured one will get different values for $\sigma(\Omega_k h^2)$ as shown in the center and right panels, to be explained below.

Since θ_s can be determined with very high accuracy, the fractional error in D_{OL} is simply equal to the fractional error in the sound horizon. The sound horizon depends on the baryon-to-photon ratio, because of how this affects the sound speed, and the matter and radiation densities because of how these affect the expansion rate. Assuming the standard radiation content the only degrees of freedom can be taken to be the baryon density today and the matter density today. A fit of the sound horizon at last scattering (defined as the epoch at which the optical depth reaches unity) is given in [34] as

$$r_s^*/\mathrm{Mpc} = 144.4(\omega_m/0.14)^{-0.252}(\omega_b/0.024)^{-0.083}. \tag{11}$$

Here we have used the common notation for densities, $\omega_m = \rho_{m,0}/\rho_{\mathrm{scale}}$ and $\omega_b = \rho_{b,0}/\rho_{\mathrm{scale}}$ where $\rho_{\mathrm{scale}} \equiv 3[100\ \mathrm{km/sec/Mpc}]^2/(8\pi G) = 1.8791 \times 10^{-29}\ \mathrm{g/cm}^3$.

Thus to calculate k, we need to know θ_s (which we will assume we know perfectly), ω_m, ω_b, and a distance into the matter-dominated era, D_{OM}. With ω_m, ω_b, and θ_s we can calculate D_{OL}, and with ω_m we can calculate l_{ML}. Thus we have what we need to use Eq. (8) to get $k = \Omega_k H_0^2$. To express it in more convenient units, we calculate $\Omega_k h^2$ where $h \equiv H_0/[100\ \mathrm{km/sec/Mpc}]$.

The matter and baryon densities can be determined from the acoustic peak morphology [33]. We took the three independent elements of their error covariance matrix, forecasted for 4 years of WMAP and 1 year of Planck, from [35]. For D_{OM} we simply assume a measurement with some variance $\sigma^2(D_{OM})$. We discuss these measurements in the next section.

To calculate the error in $\Omega_k h^2$ we create 1000 realizations of the error in ω_m and ω_b from their assumed error covariance matrix, assuming a normal distribution, and add these errors to their fiducial value. We also create 1000 samples of the error in the distance to z_M assuming the distance error is a Gaussian with variance $\sigma^2(D_{OM})$ and add these errors to our fiducial value of D_{OM}. For each sample of D_{OM}, ω_b, ω_m we calculate $\Omega_k h^2$ using Eq. (8) and the other equations as described above. The statistical error in $\Omega_k h^2$ is then taken to be the square root of the variance of our derived $\Omega_k h^2$ values.

Of course the strategy depends on there being a redshift range $z_M < z < z_*$ during which the contribution of dark energy to comoving distances is negligible. To estimate the level of contamination one expects from dark energy at $z > z_M$, we have calculated it for the case of a cosmological constant and plotted the resulting systematic error in $\Omega_k h^2$ as the horizontal lines in Fig. 1. Since our fiducial model was a cosmological constant, the systematic error is simply $\Omega_k h^2$ averaged over all the samples.

For constant w models consistent with current data, taking $w > -1$ will increase the level of contamination slightly. More worrisome are models with more complicated time-dependence, such as the oscillating model of [36]. Unexpectedly large contamination by dark energy in the $z_M < z < z_*$ redshift range can be guarded against by measuring the growth of the matter power spectrum from last scattering to $z = z_M$. If dark energy is making a significant contribution to the expansion rate in this range, then it will suppress growth.

As seen in Fig. 1, at high $\sigma(D_{0M})$ the statistical error increases as z_M increases. We expect this since in the limit that $z_M = z_*$, the redshift of last scattering, our measurement of D_{OM} brings us no new information. At lower values of $\sigma(D_{0M})$ the trend with z_M is more complicated due to a cancellation between the D_{OL} error and the l_{ML} error. In the limit that the only uncertain parameter is ω_m

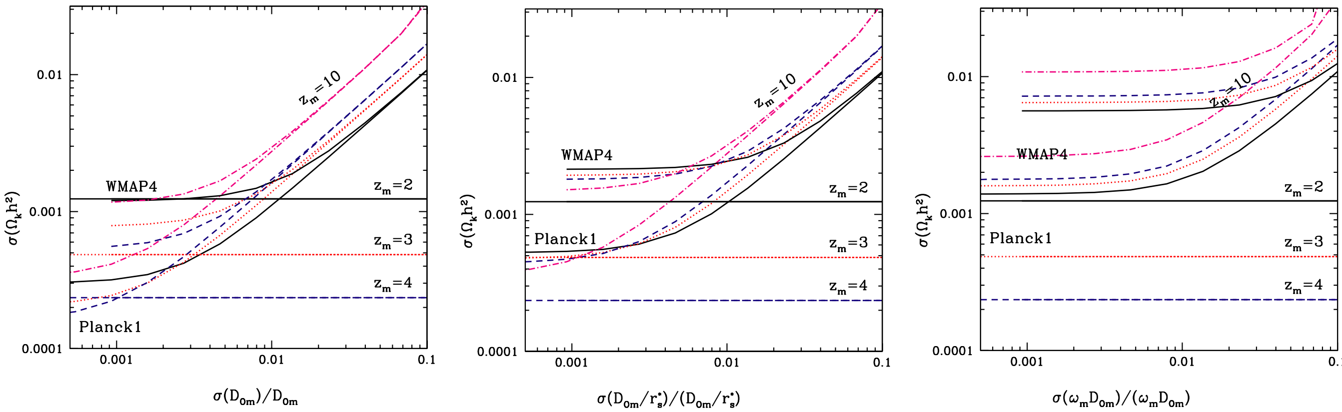

FIG. 1 (color online). Uncertainty in $\Omega_k h^2$ for WMAP (higher curves) and Planck (lower curves) as a function of the fractional error in D_{0M} (left panel), D_{0M}/r_s^* (middle panel), and $\sqrt{\omega_m}D_{0M}$ (right panel) for $z_M = 2$ (solid line), $z_M = 3$ (dotted line), $z_M = 4$ (dashed line), and $z_M = 10$ (dotted-dashed line). The straight lines show the amount of error resulting from neglecting the dark energy contribution at $z > z_M$ assuming our fiducial model with the dark energy as a cosmological constant.

we have

$$\sigma(\Omega_k h^2) = 6\frac{h^2/H_0^2}{D_{OL}^3 - D_{OM}^3}(a_1 D_{OL} + a_2 l_{ML})\frac{\sigma(\omega_m)}{\omega_m}, \tag{12}$$

with $a_1 = -0.252$ and $a_1 = 0.5$. In this limit, the error goes to zero at $D_{OL} = -a_2/a_1 l_{ML} \simeq 2l_{ML}$.

The chief benefit of increasing z_M is a reduced systematic error from dark energy at $z > z_M$. The statistical error is also smaller for 1 year of Planck than for 4 years of WMAP as long as D_{OM} is measured well enough that uncertainties in the matter and baryon densities are important.

As we will discuss in the next session, a variety of techniques can be used to get distances from standard rulers in the matter power spectrum. These standard rulers are r_s^* and the size of the comoving horizon at matter-radiation equality, $r_{\rm EQ} \propto \omega_m^{-1}$. The measurements thus determine the combinations D_{OM}/r_s^* and $\omega_m D_{0M}$. Thus we also plot in the middle and right panels of Fig. 1 results from two more calculations, just like the first one, except we assume independent errors in D_{OM}/r_s^* and then $\omega_m D_{0M}$. These assumptions lead to correlations between the D_{OM} error and the D_{OL} error, and therefore we get different results when the D_{OL} errors are significant.

Although the above analysis only assumes a measurement to one redshift, any particular methodology for distances measurements will result in distances to a series of redshifts, some of which may be heavily contaminated by dark energy. The actual analysis of the data will involve simultaneous fitting for dark energy and mean curvature. The calculations here are done in two illustrative limits: no dark energy at $z > z_m$ and completely neglected cosmological constant at $z > z_m$. Any conclusions about mean curvature from future precision measurements will be complicated with arguments about the possibility of residual amounts of dark energy. Going to high redshifts mitigates this confusion greatly but does not completely remove it. Thus distances to more than one redshift in the matter-dominated era will be useful.

Using our idealized calculation as a rough guide though, it appears that distance measurements in the $z \sim 3$ to $z \sim 4$ range will be most helpful. Lower redshifts are too susceptible to errors arising from modeling the dark energy (estimated by the horizontal lines in the figures) and the statistical errors for $z_M > 4$ become large since they must diverge as $z_M \rightarrow z_*$, the redshift of last scattering.

Another solution was recently proposed in [37] that exploits the fact that r_{ML} is not the angular-diameter distance from z_M to z_L. Thus, although $r_{OL} = r_{OM} + r_{ML}$, $D_{OL} - (D_{OM} + D_{ML}) \propto \Omega_k$. Cosmic shear observables are sensitive to all three of these distances.

IV. DISTANCE MEASUREMENTS

We have provided motivation for measurement of distances into the matter-dominated era. In this section we briefly discuss different methods for obtaining these distances.

A. Supernovae

The distance-redshift relation at $z \lesssim 1$ is currently determined best from observations of type Ia supernovae [38]. Although these are luminosity distances, in the absence of unknown scattering or absorption effects, $D_L = (1+z)^2 D_A$. However, the prospects for percent level determination of distances at $z \gtrsim 2$ are not good. Difficulties arise from the $(1+z)^4$ cosmological dimming and the challenge of determining the amount of reddening (in order to correct for dust extinction in the host galaxy) with light that was bluer in the rest frame than is the case for lower z supernovae. Further, gravitational lensing of the light from the supernovae will add additional dispersion to the observed fluxes, increasing the number needed to obtain a given level of precision [39–42]. Finally, current supernova constraints on Ω_m and Ω_Λ come from distance ratios, rather than absolute distance determinations, due to the (nearly) constant but unknown luminosity of the standard candle. The supernova absolute luminosity calibration would have to improve dramatically in order for percent level determination of absolute distances. A sufficient calibration would be possible with a 1% determination of H_0, which may be achievable with square kilometer array observations of water masers [43].

Linder [44] has recently considered the impact of allowing for nonzero curvature on the ability of a space-based supernova mission such as Supernova/Acceleration Probe (combined with CMB data) to constrain dark energy parameters. He assumes a distribution of supernovae in the interval $0 < z < 1.7$. He finds that dropping the assumption of zero mean curvature greatly weakens the constraints on w_0 and w_a when it is assumed that the equation of state parameter as a function of scale factor is given by $w(a) = w_0 + (1 - a)w_a$.

Experiments will not just determine the distance to a single redshift, as assumed above, but to a range of redshifts. Thus it is interesting to see how an actual experiment, making these multiple distance measurements, can constrain the curvature. Since the supernovae measurements do not go beyond $z = 1.7$, we expect the results to be dependent on assumptions made about the dark energy. Indeed, Linder [44] finds this to be the case. If one assumes $w_a = 0$ then $\sigma(\Omega_K) = 0.011$ but if one allows for nonzero w_a (still assuming the form for $w(a)$ above) then the curvature error weakens by more than a factor of 4 to $\sigma(\Omega_K) = 0.047$. We see here in this result a quantification of the degradation of curvature constraints due to uncertainty about the dark energy, as expected qualitatively from the discussion and idealized calculations above.

B. Cosmic shear

The statistical properties of cosmic shear are sensitive to both the distance-redshift relation and the growth of the matter power spectrum as a function of redshift. Of course they are also sensitive to the shape and amplitude of the primordial power spectrum, the matter density, and the baryon density. With these parameters constrained by CMB observations, and with sufficient knowledge of the redshift of the source galaxies (such as from photometrically-determined redshifts) one can use cosmic shear data to simultaneously reconstruct distance as a function of redshift and growth as a function of redshift [45]. The combination of Planck's measurement of the CMB and a deep multiband ground-based survey of half the sky, such as planned with the large-aperture synoptic survey telescope, can determine the distance to $z = 3$ with an error of about 1% [46]. The errors in the $z = 3$ measurements are highly correlated with the errors in the $z < 1$ measurements. The combination of the cosmic shear data with low-redshift distance measurements (for example, from supernovae) can therefore improve the $z = 3$ distance determination.

The standard ruler that allows for cosmic shear data to be sensitive to the distance-redshift relation is the turnover in the matter power spectrum at the comoving size of the horizon at matter-radiation equality [46] which is proportional to $1/\omega_m$. The distance determination is only possible to the extent that ω_m has been determined. The errors in the distance will thus be correlated with errors in ω_m and therefore with the errors in D_{OL} and l_{ML}. Errors in the product $D_{OM}\omega_m$ will, in contrast, be only very weakly correlated with those in D_{OL} and l_{ML}.

The error in $\Omega_k h^2$ in the limit of perfect knowledge of ω_b and a perfect measurement of $D_{OM}\omega_m$ (instead of D_{OM}) is again given by Eq. (12) but now with $a_1 = 0.748$ and $a_2 = -0.5$. The increased magnitude in a_1 means a larger contribution from the error in D_{OL} and the impossibility of any significant cancellation with the error in l_{ML}. As a result, one can see in the right panel of the figure at low values of $\sigma(D_{OM}\omega_m)/(D_{OM}\omega_m)$ the greatly increased errors in $\sigma(\Omega_k h^2)$ compared to the case of the left panel.

C. High-z galaxy power spectra

Acoustic oscillations prior to recombination create a feature in the matter correlation function with a length scale of r_s^* [47–50]. This feature can be used as a standard ruler to infer distances from measurement of D_A/r_s^* [7,51–55] and has recently been used to do so [12,56]. Seo and Eisenstein [54] find that a photometric redshift survey with redshift errors σ_z over a survey spanning $z = 2.5$ to $z = 3.5$ with solid angle Ω could achieve an angular-diameter distance determination to $z = 3$ of

$$\frac{\sigma(D_{OM})/r_s^*}{D_{OM}/r_s^*} = .01\sqrt{\frac{2000\text{ sq deg}}{\Omega}}\sqrt{\frac{\sigma_z/(1+z)}{0.04}}. \qquad (13)$$

With spectroscopic redshifts, clustering in the redshift direction also can be used to determine $H^{-1}(z)/r_s^*$ [54] and thereby provide a check on the prediction that $H(z) = 8\pi G\rho_{m,0}(1+z)^3$ where $\rho_{m,0}$ is determined by the CMB observations.

Achieving a 1% determination of D_{0M}/r_s^* to $z_M = 3$ would reduce the statistical error on the Ω_K determination, as reported in [12], by a factor of 10. It would also *greatly* reduce the level of systematic error due to their assumption of a cosmological constant to a level roughly about that of the dotted horizontal lines in the figures.

Using baryon oscillations to determine distances also mitigates a potential source of systematic error [57]. Rather than $\rho_{m,0}$, the quantity well determined from CMB observations is the redshift of matter-radiation equality, $z_{\rm EQ}$, because of how it affects the evolution of gravitational potentials (for a review, see [58]). If there were a nonstandard radiation content, then $\rho_{m,0}$ might be erroneously inferred from $z_{\rm EQ}$. This would then lead to an error in l_{ML} and therefore possibly a nonzero k, by Eq. (8), even for a flat cosmology.

However, as pointed out in [59], CMB observations robustly determine the combination $\sqrt{\rho_{m,0}}r_s^*$ *independent* of the value of $\rho_{m,0}$. Therefore, since baryon oscillations are sensitive to D_A/r_s^*, they robustly determine $\sqrt{\rho_{m,0}}D_A$. A check for nonzero $\sqrt{\rho_{m,0}}D_{\rm 0L} - (\sqrt{\rho_{m,0}}D_{\rm 0M} + \sqrt{\rho_{m,0}}l_{\rm ML})$ is thus a robust check for nonzero curvature. Note that $\sqrt{\rho_{m,0}}l_{\rm ML}$ has no cosmological parameter dependence (assuming complete matter domination); in particular, it does not depend on $\rho_{m,0}$.

D. 21 cm radiation from intergalactic neutral hydrogen

Perhaps the best prospect for measuring distances to redshifts very deep into the matter-dominated era comes from fluctuations in the brightness temperature of 21 cm radiation from neutral hydrogen prior to the complete reionization of the intergalactic medium [60,61]. By requiring statistical isotropy of the fluctuations, in particular, that correlation lengths along the line of sight are equal to correlation lengths perpendicular to the line of sight, one determines the ratio $D_A(z)/H^{-1}(z)$ [62,63]. With $H(z)$ determined from the CMB then this can be converted to a measurement of $D_A(z)$. The $D_A(z)$ might also be determined from observing baryon oscillations in the 21 cm power spectra [64]. The meter wavelength signals from redshifted 21 cm radiation are much smaller than contamination from a number of other astrophysical sources, e.g. [65,66]. Quantitative studies show a good prognosis for the ability to clean out these foregrounds based on their high coherence across frequency, e.g. [67–69].

Such high-z measurements would have the benefit of very low dark energy model dependence; e.g., the horizontal $z_M = 10$ curve is off-scale low. Unfortunately, these measurements are restricted to redshifts in the prereionization era ($z_M > 6$) which have the drawback of larger statistical errors as discussed above.

V. DISCUSSION

Given the prediction of zero mean curvature, we can use any detection of curvature averaged over our Hubble volume as evidence of curvature fluctuations on even larger length scales. Indeed, inflationary models predict the existence of these fluctuations, since inflationary models predict a spectrum of nearly scale-invariant fluctuations. These predictions are consistent with determinations of the subhorizon scale power spectrum from which we infer an rms amplitude of about 10^{-5}.

We have not calculated the dependence of the mean curvature in our Hubble volume, as measured by the means described above, on the power spectrum on super-Hubble scales. This would be interesting to do in order to more completely understand the implications of a nonzero determination of mean curvature. A useful starting point for such a calculation can be found in [70] and most recently in [71] who consider spatial fluctuations in the luminosity distance due to scalar perturbations.

VI. CONCLUSIONS

We have emphasized the importance of testing the robust prediction of inflation that the mean curvature is zero. We have pointed out that it is our uncertainty in the dark energy that limits our current determinations of the mean curvature and that precise measurements of the distance to redshifts in the matter-dominated era can circumvent this problem. Thus, measurements of the distance-redshift relation are not only probes of the dark energy, but also of inflation.

Important as some experiments are that expect null results, it is always attractive to have a nonzero signal to chase. It may actually be possible to reach the level of precision necessary to see a signal from super-Hubble curvature fluctuations. Unfortunately this prospect is a long shot since the signal will have to be 2 orders of magnitude larger than naïve expectations. Such a detection would be a unique datum on these largest scales, allowing us to reach back a bit further toward the onset of inflation.

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