Clarifying inflation models: Slow roll as an expansion in $1/N_{\text{efolds}}$

D. Boyanovsky,[1,3,*] H. J. de Vega,[2,3,†] and N. G. Sanchez[3,‡]

[1]*Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA*
[2]*LPTHE, Laboratoire Associé au CNRS UMR 7589, Université Pierre et Marie Curie (Paris VI) et Denis Diderot (Paris VII), Tour 24, 5 ème. étage, 4, Place Jussieu, 75252 Paris, Cedex 05, France*
[3]*Observatoire de Paris, LERMA, Laboratoire Associé au CNRS UMR 8112, 61, Avenue de l'Observatoire, 75014 Paris, France*

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Slow-roll inflation is studied as an effective field theory. We find that the form of the inflaton potential consistent with Wilkinson Microwave Anisotropy Probe (WMAP) data and slow roll is $V(\phi) = NM^4 w(\frac{\phi}{\sqrt{N}M_{\text{Pl}}})$, where ϕ is the inflaton field, M is the inflation energy scale, and $N \sim 50$ is the number of e-folds since the cosmologically relevant modes crossed the Hubble radius until the end of inflation. The inflaton field scales as $\phi = \sqrt{N}M_{\text{Pl}}\chi$. The dimensionless function $w(\chi)$ and field χ are generically $\mathcal{O}(1)$. The WMAP value for the amplitude of scalar adiabatic fluctuations $|\Delta_{kad}^{(S)}|^2$ fixes the inflation scale $M \sim 0.77 \times 10^{16}$. This form of the potential makes manifest that the slow-roll expansion is an expansion in $1/N$. A Ginzburg-Landau realization of the slow-roll inflaton potential reveals that the Hubble parameter, inflaton mass and nonlinear couplings are of the seesaw form in terms of the small ratio M/M_{Pl}. For example, the quartic coupling $\lambda \sim \frac{1}{N}(\frac{M}{M_{\text{Pl}}})^4$. The smallness of the nonlinear couplings is not a result of fine-tuning but a natural consequence of the validity of the effective field theory and slow-roll approximation. We clarify Lyth's bound relating the tensor/scalar ratio and the value of ϕ/M_{Pl}. The effective field theory is valid for $V(\phi) \ll M_{\text{Pl}}^4$ for general inflaton potentials allowing amplitudes of the inflaton field ϕ well beyond M_{Pl}. Hence bounds on r based on the value of ϕ/M_{Pl} are overly restrictive. Our observations lead us to suggest that slow-roll, single field inflation may well be described by an almost critical theory, near an infrared stable Gaussian fixed point.

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I. INTRODUCTION AND RESULTS

Inflation was originally proposed to solve several outstanding problems of the standard big bang model [1–5] thus becoming an important paradigm in cosmology. At the same time, it provides a natural mechanism for the generation of scalar density fluctuations that seed large scale structure, thus explaining the origin of the temperature anisotropies in the cosmic microwave background (CMB), as well as that of tensor perturbations (primordial gravitational waves). A distinct aspect of inflationary perturbations is that these are generated by quantum fluctuations of the scalar field(s) that drive inflation. After their wavelength becomes larger than the Hubble radius, these fluctuations are amplified and grow, becoming classical and decoupling from causal microphysical processes. Upon reentering the horizon, during the matter era, these classical perturbations seed the inhomogeneities which generate structure upon gravitational collapse. While there is a great diversity of inflationary models, most of them predict fairly generic features: a Gaussian, nearly scale invariant spectrum of (mostly) adiabatic scalar and tensor primordial fluctuations. These generic predictions of most inflationary models make the inflationary paradigm fairly robust.

Inflationary dynamics is typically studied by treating the inflaton as a homogeneous classical scalar field [2–5] whose evolution is determined by a classical equation of motion, while the quantum fluctuations of the inflaton provide the seeds for the scalar density perturbations of the metric and are treated in the Gaussian approximation. In quantum field theory, the classical inflaton corresponds to the expectation value of a quantum field operator in a translational invariant state.

Although there is a wide variety of inflationary models, the Wilkinson Microwave Anisotropy Probe (WMAP) [6] data can be fit outstandingly well by simple single field slow-roll models.

Inflation based on a scalar inflaton field should be considered as an effective theory, that is, not necessarily a fundamental theory but as a low energy limit of a microscopic fundamental theory. The inflaton may be a coarse-grained average of fundamental scalar fields, or a composite (bound state) of fields with higher spin, just as in superconductivity. Bosonic fields do not need to be fundamental fields, for example, they may emerge as condensates of fermion-antifermion pairs $\langle\bar{\Psi}\Psi\rangle$ in a grand unified theory (GUT) in the cosmological background. In order to describe the cosmological evolution is enough to consider the effective dynamics of such condensates. The relation between the low energy effective field theory of inflation and the microscopic fundamental theory is akin to the relation between the effective Ginzburg-Landau theory of superconductivity and the microscopic BCS theory, or like

*Electronic address: boyan@pitt.edu
†Electronic address: devega@lpthe.jussieu.fr
‡Electronic address: Norma.Sanchez@obspm.fr

the relation of the $O(4)$ sigma model, an effective low energy theory of pions, photons and chiral condensates with QCD [7]. The guiding principle to construct the effective theory is to include the appropriate symmetries [7]. Contrary to the sigma model where the chiral symmetry strongly constraints the model [7], only general covariance can be imposed to the inflaton model.

While inflationary cosmology is currently studied from the point of view of classical field theory with small quantum corrections, nonperturbative quantum aspects of the dynamics of inflation were studied in Refs. [8–11]. More recently particle decay in a de Sitter background as well as during slow-roll inflation has been studied in Ref. [12] together with its implication for the decay of the density fluctuations. Quantum corrections to slow-roll inflation including quantum corrections to the effective inflaton potential and its equation of motion are derived in Ref. [13].

Recent studies of quantum corrections during inflation [12,13] revealed the robustness of classical single field slow-roll inflationary models as a result of the validity of the effective field theory description. The reliability of an effective field theory of inflation hinges on a wide separation between the energy scale of inflation, determined by H and that of the underlying microscopic theory which is taken to be the Planck scale $M_{\rm Pl}$.

The data from WMAP provides an upper bound on the scale of the inflationary potential [6] $V^{1/4} < 3.3 \times 10^{16}$ GeV (95% CL), thereby establishing an upper bound on the scale of inflation $H < 2.6 \times 10^{14}$ GeV. Hence, the smallness of the ratio $H/M_{\rm Pl} \lesssim 10^{-4}$ warrants the reliability of the effective field theory approach. A simple Ginzburg-Landau realization of the inflationary potential as an effective field theory has been recently shown to fit the WMAP data remarkably well [14].

As mentioned in Refs. [12,13] there are *two independent* expansions: the effective field theory (EFT) one based on the small dimensionless ratio $H/M_{\rm Pl}$ and the slow-roll expansion. The latter one is an *adiabatic* expansion which relies on a fairly flat inflationary potential and invokes a hierarchy of dimensionless ratios that involve derivatives of the inflationary potential [4,6,15,16].

A. The goal of this article

In this article we combine the results from WMAP and the slow-roll expansion to suggest that the inflationary potential has a *universal* form which helps to clarify both the (EFT) and slow-roll expansions. The main point of the argument is the presence of two independent small parameters in single field inflationary cosmology: $H/M_{\rm Pl}$ and $1/N$, where N is the number of e-folds before the end of inflation during which wavelengths of cosmological relevance today first cross the horizon. Consistent inflationary models require that $N \sim 50$–60. We argue that the form of the potential suggested by the data and slow roll leads naturally to the identification of the slow-roll expansion as an expansion in $1/N$.

While the (EFT) ratio $H/M_{\rm Pl}$ and the slow-roll parameters are logically independent, slow roll implies a large number of e-folds, therefore the hierarchy of slow-roll parameters is related to the smallness of $1/N$. While this point is widely known and understood, our main observation is that the slow-roll expansion is a *systematic* expansion in the small parameter $1/N$.

B. Brief summary of results

(i) We observe that combining the WMAP data with the slow-roll expansion *suggests* a *consistent* description of single field inflation in terms of a classical potential of the form

$$V(\phi) = NM^4 w(\chi) \tag{1.1}$$

where $w(\chi) \sim \mathcal{O}(1)$, N ~ 50 and χ is a dimensionless, slowly varying field

$$\chi = \frac{\phi}{\sqrt{N} M_{\rm Pl}} \tag{1.2}$$

The WMAP data constrains M to be at the grand unification (GUT) scale $M \sim 0.77 \times 10^{16}$ GeV, which suggests a connection between inflation and the physics at the GUT scale in a cosmological space-time.

(ii) The dynamics of the rescaled field χ exhibits the slow time evolution in terms of the *stretched* (slow) dimensionless time variable,

$$\tau = \frac{tM^2}{M_{\rm Pl}\sqrt{N}} \tag{1.3}$$

The form of the potential and the rescaled dimensionless field and time variable lead consistently to slow-roll as an expansion in powers of $1/N$.

(iii) The inflaton mass around the minimum is given by a seesaw formula

$$m = \frac{M^2}{M_{\rm Pl}} \sim 2.45 \times 10^{13}\ \text{GeV}.$$

The Hubble parameter when the cosmologically relevant modes exit the horizon is given by

$$H = \sqrt{N} m h \sim 1.0 \times 10^{14}\ \text{GeV} = 4.1\, m,$$

using $h \sim 1$. As a result, $m \ll M$ and $H \ll M_{\rm Pl}$. A Ginzburg-Landau realization of the inflationary potential that fits the WMAP data remarkably well [14], reveals that the Hubble parameter, the inflaton mass and nonlinear couplings are seesaw-like, namely, powers of the ratio $M/M_{\rm Pl}$ multiplied by further powers of $1/N$. Therefore, their smallness is not a result of fine-tuning but a natural consequence of the form of the potential and the validity of the effective field theory description and slow roll. The quantum expansion in loops is therefore a double expansion on $(H/M_{\rm Pl})^2$ and $1/N$. Notice that graviton corrections are also at least of order $(H/M_{\rm Pl})^2$ because the amplitude of tensor modes is or order $H/M_{\rm Pl}$. While the hierarchy between the amplitude of the field, H, and $M_{\rm Pl}$ is known, we argue that the form of the potential that fits the WMAP data and is consistent with slow roll suggests the above result for the nonlinear couplings.

(iv) We discuss critically Lyth's bound on the ratio r of the amplitudes of tensor to curvature perturbations. This bound has been often used to assess the feasibility of detection of tensor modes in forthcoming searches [17–19]. Since r can be related to the change of the inflaton field while the cosmologically relevant modes exit the horizon [17], the restriction to models with $\frac{\phi}{M_{\rm Pl}} \lesssim 1$ [17,18] yield very small values of r. We argue that Lyth's bound is overly restrictive by making precise the regime of validity of the effective field theory approach: the use of the inflaton potential $V(\phi)$ is consistent for $V(\phi) \ll M_{\rm Pl}^4$. This inequality is fulfilled for values of the inflaton field ϕ well beyond the Planck mass allowing $r \gtrsim 1$. We provide elementary, yet illuminating examples of potentials that help precise these statements. Furthermore, the true value of r may be very well much smaller than the present upper bound $r \lesssim 0.16$ from WMAP [6]. If for whatever reason, a restriction to values for $\chi \lesssim 1$ is invoked, then an improved Lyth bound emerges from our analysis. This improved bound allows values for r larger than the previous bounds due to the presence of the new factor $\sqrt{N} \sim 7$ [17,18].

(v) For polynomial realizations of the inflationary potential, our analysis yields that the nonlinear couplings scale with inverse powers of $1/N$. In an expanding cosmology the logarithm of the scale factor is the number of e-folds, thus these couplings scale as powers of $1/\ln(a)$. In a quantum field theory, the behavior of the couplings under a change of scale is dictated by the renormalization group. For example, at one-loop level, the quartic coupling of the Φ^4 model in four dimensional euclidean (flat) space precisely scales as $1/\log a = 1/N$, approaching the trivial (zero coupling) infrared stable (critical) point when $a \to \infty$. Hence, it is *suggestive* that the underlying reason for the scaling of the couplings in powers of $1/N$ in the potential Eq. (1.1) [see Eq. (2.30) below] may be the infrared renormalization group running (RG) of long wavelength modes. This observation leads us to *conjecture* that slow-roll inflation strongly suggests that the effective field theory is near (but not exactly) a trivial Gaussian infrared fixed point during the stage in which scales of cosmological relevance today crossed the Hubble radius. In this interpretation the theory hovers near the Gaussian fixed point with an almost scale invariant spectrum of scalar fluctuations during the slow-roll stage, but eventually it must move away from the neighborhood of this fixed point by the end of inflation to reach the standard radiation dominated stage.

Equation (1.1) for the inflaton potential resembles (besides the factor N) the moduli potential arising from supersymmetry breaking [5],

$$V_{\rm susy}(\phi) = m_{\rm susy}^4 v\left(\frac{\phi}{M_{\rm Pl}}\right), \tag{1.4}$$

where $m_{\rm susy}$ stands for the supersymmetry breaking scale. In our context, Eq. (1.4) indicates that $m_{\rm susy} \simeq M \simeq M_{\rm GUT}$. That is, the susy breaking scale $m_{\rm susy}$ turns out to be at the GUT and inflation scales. This may be a *first* observational indication of the presence of supersymmetry.

In summary, we find that a form of the inflaton potential for single field models consistent with the WMAP data and slow-roll is given by Eq. (1.1). There is no requirement of fine-tuning small parameters, as these appear in $w(\chi)$ naturally in terms of the effective field theory ratio $(H/M_{\rm Pl})^2$ and $1/N$. The form of the potential Eq. (1.1) encodes the energy scale of inflation as well as slow-roll, and is valid for practically all slow-roll inflaton potentials. In particular, Eq. (1.1) applies for all polynomial potentials investigated in [14].

II. BASIC MASS SCALES IN THE INFLATON POTENTIAL AND THE NUMBER OF E-FOLDS

The description of cosmological inflation is based on an isotropic and homogeneous geometry, which, assuming flat spatial sections, is determined by the invariant distance

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2. \tag{2.1}$$

The scale factor obeys the Friedmann equation

$$\left[\frac{1}{a(t)}\frac{da}{dt}\right]^2 = \frac{\rho(t)}{3M_{\rm Pl}^2}, \tag{2.2}$$

where $M_{\rm Pl} = 1/\sqrt{8\pi G} = 2.4 \times 10^{18}$ GeV. In single field inflation the energy density is dominated by a homogeneous scalar *condensate*, the inflaton, whose dynamics can be described by an *effective* Lagrangian

$$\mathcal{L} = a^3(t)\left[\frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2a^2(t)} - V(\phi)\right]. \tag{2.3}$$

The inflaton potential $V(\phi)$ is a slowly varying function of ϕ in order to permit a slow-roll solution for the inflaton field $\phi(t)$.

Slow-roll inflation corresponds to a fairly flat potential and the slow-roll approximation invokes a hierarchy of dimensionless ratios in terms of the derivatives of the potential. Some [4,6,15] of these (potential) slow-roll parameters are given by[1]

$$\epsilon_V = \frac{M_{\rm Pl}^2}{2}\left[\frac{V'(\phi)}{V(\phi)}\right]^2, \qquad \eta_V = M_{\rm Pl}^2 \frac{V''(\phi)}{V(\phi)},$$
$$\xi_V = M_{\rm Pl}^4 \frac{V'(\phi)V'''(\phi)}{V^2(\phi)}, \qquad \sigma_V = M_{\rm Pl}^6 \frac{[V'(\phi)]^2 V^{(IV)}(\phi)}{V^3(\phi)}. \tag{2.4}$$

where $V(\phi)$ is the inflaton potential and primes stand for the derivative with respect to the inflaton field. The slow-roll approximation [4,6,15,16] corresponds to $\epsilon_V \sim \eta_V \ll 1$ with the hierarchy $\xi_V \sim \mathcal{O}(\epsilon_V^2)$; $\sigma_V \sim \mathcal{O}(\epsilon_V^3)$, namely ϵ_V and η_V are first order in slow roll, ξ_V second order in slow roll, etc.

As stressed in Ref. [14], in order to reproduce the CMB data, the inflationary potentials in the slow-roll scenarios must have the structure

$$V(\phi) = M^4 v\left(\frac{\phi}{M_{\rm Pl}}\right), \tag{2.5}$$

where v is dimensionless and M determines the energy scale of the potential during inflation. The minimum of the potential $\phi_{\rm min}$ is chosen such that $V(\phi_{\rm min}) = 0$ in order to ensure that inflation ends after a finite number of e-folds. This form of the potential is also suggested by the dimensionless slow-roll parameters, because the combination $\phi/M_{\rm Pl}$ in the argument of the potential cancels the ubiquitous factors $M_{\rm Pl}$ in the slow-roll variables.

During slow-roll inflation the number of e-folds before the end of inflation at which the value of the scalar field is $\phi_{\rm end}$, is given by

$$N[\phi(t)] = -\frac{1}{M_{\rm Pl}^2}\int_{\phi(\tau)}^{\phi_{\rm end}} V(\phi)\frac{d\phi}{dV}\, d\phi. \tag{2.6}$$

It is convenient to introduce $N \sim 50$ as the typical scale of e-folds during which wavelengths of cosmological relevance today first cross the horizon during inflation, namely

$$50 = -\frac{1}{M_{\rm Pl}^2}\int_{\phi_{50}}^{\phi_{\rm end}} V(\phi)\frac{d\phi}{dV}\, d\phi. \tag{2.7}$$

where ϕ_{50} is the value of the scalar field 50 e-folds before the end of inflation.

The form of the potential Eq. (2.5) combined with the above definition, strongly suggests the following rescaling of the field and the potential

$$\phi = \sqrt{N} M_{\rm Pl}\chi, \qquad V(\phi) = N M^4 w(\chi), \tag{2.8}$$

where χ is dimensionless. With this definition, the expression (2.7) simply becomes

$$1 = -\int_{\chi_{50}}^{\chi_{\rm end}} \frac{w(\chi)}{w'(\chi)}\, d\chi, \tag{2.9}$$

where the prime stands for the derivative with respect to the argument. The dimensionless field χ is *slowly* varying during the stage of slow-roll inflation: a large amplitude change in the field ϕ results in a small amplitude change in χ,

$$\Delta\chi = \frac{1}{\sqrt{N}}\frac{\Delta\phi}{M_{\rm Pl}}, \tag{2.10}$$

a change in ϕ with $\Delta\phi \sim M_{\rm Pl}$ results in a change $\Delta\chi \sim 1/\sqrt{N}$. The number of e-folds during the cosmologically relevant stage of inflation can now be written as

$$\frac{N[\chi]}{N} = -\int_{\chi}^{\chi_{\rm end}} \frac{w(\chi)}{w'(\chi)}\, d\chi \leqslant 1. \tag{2.11}$$

In terms of the rescaled field and potential, the hierarchy of slow-roll parameters now becomes from Eqs. (2.4) and (2.8),

$$\epsilon_V = \frac{1}{2N}\left[\frac{w'(\chi)}{w(\chi)}\right]^2, \qquad \eta_V = \frac{1}{N}\frac{w''(\chi)}{w(\chi)}, \tag{2.12}$$

$$\xi_V = \frac{1}{N^2}\frac{w'(\chi)w'''(\chi)}{w^2(\chi)}, \qquad \sigma_V = \frac{1}{N^3}\frac{[w'(\chi)]^2 w^{(IV)}(\chi)}{w^3(\chi)}. \tag{2.13}$$

It is clear from Eqs. (2.9), (2.12), and (2.13) that during the inflationary stage when wavelengths of cosmological relevance cross the horizon $w(\chi)$, $w'(\chi) \sim \mathcal{O}(1)$ and that this statement leads to a consistent slow-roll expansion as an expansion in $1/N$, namely, the inverse of the number of e-folds.

This equivalence between the slow roll and the $1/N$ expansion can be made even more explicit by analyzing the Friedmann equation and the equation of motion for the inflaton in terms of the rescaled field and potential. For this purpose, let us also introduce a *stretched* (slow) dimensionless time variable τ and a rescaled dimensionless Hubble parameter h as follows

$$t = \sqrt{N}\frac{M_{\rm Pl}}{M^2}\tau, \qquad H = \sqrt{N}\frac{M^2}{M_{\rm Pl}} h \tag{2.14}$$

in terms of which the Friedmann equation reads

$$h^2(\tau) = \frac{1}{3}\left[\frac{1}{2N}\left(\frac{d\chi}{d\tau}\right)^2 + w(\chi)\right] \tag{2.15}$$

and the evolution equation for the field χ is given by

$$\frac{1}{N}\frac{d^2\chi}{d\tau^2} + 3h\frac{d\chi}{d\tau} + w'(\chi) = 0 \tag{2.16}$$

The slow-roll approximation follows by neglecting the $\frac{1}{N}$ terms in Eqs. (2.15) and (2.16). Both $w(\chi)$ and $h(\tau)$ are of order N^0 for large N. Both equations make manifest the

[1] We follow the definitions of ξ_V; σ_V in Ref. [6]. (ξ_V; σ_V are called ξ_V^2; σ_V^3, respectively, in [15]).

slow-roll expansion as an expansion in $1/N$. The possibility of using $1/N$ as an expansion to study inflationary dynamics was advocated previously in Ref. [20]. The analysis above confirms this early suggestion and establishes the slow-roll expansion as a systematic expansion in $1/N$.

In addition, at the absolute minimum of the potential $w(\chi)$, $\chi_{\min}$, one has to require $w(\chi_{\min}) = w'(\chi_{\min}) = 0$ to guarantee that inflation ends after a finite number of e-folds. Moreover, we can choose $|w''(\chi_{\min})| = 1$ without loss of generality. Then, the inflaton mass around the minimum is given by a seesaw formula

$$m = \frac{M^2}{M_{\rm Pl}}. \tag{2.17}$$

This seesaw form of the inflaton mass is again a hallmark of an effective field theory and its smallness compared both to $M_{\rm Pl}$ as well as $M \sim 10^{16}$ GeV is a consequence of the wide separation of scales.

In particular the equation of motion (2.16) can be solved in an *adiabatic* expansion in terms of $1/N$, with the following result to zeroth order

$$\frac{dw(\chi)}{d\tau} = -\frac{[w'(\chi)]^2}{3h(\tau)}\left[1 + \mathcal{O}\left(\frac{1}{N}\right)\right], \tag{2.18}$$

which again requires for consistency that $w'(\chi) \sim w(\chi) \sim \mathcal{O}(1)$ during slow roll. From Eqs. (2.16), (2.17), and (2.18) it is clear that the slow field χ is a function of the stretched (slow) time scale τ.

We can now input the results from WMAP [6] to constrain the scale M. The amplitude of adiabatic scalar perturbations in slow-roll is expressed as

$$|\Delta_{kad}^{(S)}|^2 = \frac{1}{12\pi^2 M_{\rm Pl}^6}\frac{V^3}{V'^2} = \frac{N^2}{12\pi^2}\left(\frac{M}{M_{\rm Pl}}\right)^4 \frac{w^3(\chi)}{w'^2(\chi)}, \tag{2.19}$$

Since, $w(\chi)$ and $w'(\chi)$ are of order one, we find

$$\left(\frac{M}{M_{\rm Pl}}\right)^2 \sim \frac{2\sqrt{3}\pi}{N}|\Delta_{kad}^{(S)}| \simeq 1.02 \times 10^{-5}. \tag{2.20}$$

where we used $N \simeq 50$ and the WMAP value for $|\Delta_{kad}^{(S)}| = (4.67 \pm 0.27) \times 10^{-5}$ [6]. This fixes the scale of inflation to be

$$M \simeq 3.19 \times 10^{-3} M_{\rm PL} \simeq 0.77 \times 10^{16} \text{ GeV}.$$

This value pinpoints the scale of the potential during inflation to be at the GUT scale suggesting a deep connection between inflation and the physics at the GUT scale in cosmological space-time.

That is, the WMAP data fix the scale of inflation M for single field potentials with the form given by Eq. (2.10). This value for M is below the WMAP upper bound on the inflation scale 3.3×10^{16} GeV [6].

Furthermore, the Hubble scale during (slow-roll) inflation and the inflaton mass near the minimum of the potential are thereby determined from Eqs. (2.14) and (2.17) to be

$$m = \frac{M^2}{M_{\rm Pl}} = 2.45 \times 10^{13} \text{ GeV}, \qquad H = \sqrt{N} m h \sim 1.0 \times 10^{14} \text{ GeV} = 4.1 m. \tag{2.21}$$

since $h = \mathcal{O}(1)$.

In the absence of an underlying microscopic model from which the effective field theory description can be reliably extracted, we can only surmise the above form of the potential. Our main observation is that the current phenomenological success of single field slow-roll inflaton models, validated by the wealth of observational data from WMAP strongly suggests the universal form (2.10). Furthermore, such form yields a slow-roll expansion consistently organized in powers of the small parameter $1/N$.

A. A polynomial realization

The results obtained above are very general and they only depend on a recognition of the fast and slow fields and time scales during the stage of slow-roll inflation of cosmological relevance. For specific models this general form severely constrains the value of the couplings. In Ref. [14] a thorough study of a general quartic potential revealed that this simple Ginzburg-Landau type effective field theory fits the WMAP data remarkably well. Therefore, following Ref. [14] we consider the potential

$$V(\phi) = V_0 \pm \frac{m^2}{2}\phi^2 + \frac{mg}{3}\phi^3 + \frac{\lambda}{4}\phi^4. \tag{2.22}$$

where g, λ are dimensionless couplings. The sign + in the quadratic term corresponds to unbroken symmetry while the − sign describes the broken symmetry case. We choose $\lambda > 0$ as a stability condition in order to have a potential bounded from below while g may have any sign (but we always have $m^2 > 0$).

The WMAP results rule out the purely quartic potential ($m = 0$, $g = 0$). From the point of view of an effective field theory it is rather *unnatural* to set $m = 0$, since this is a particular point at which the correlation length is infinite and the theory is critical. Indeed the systematic study in Ref. [14] shows that the best fit to the WMAP data requires $m > 0$.

The general quartic Lagrangian Eq. (1.1) describes a renormalizable theory. However, one can choose in the present context arbitrary high order polynomials for $V(\phi)$. These nonrenormalizable models are also effective theories where $M_{\rm Pl}$ plays the rôle of UV cutoff. However, already a quartic potential is rich enough to describe the full physics and to reproduce accurately the data [14].

Introducing the slow field χ as in Eq. (2.8) the potential (2.22) can be written in the simple form

$$V(\phi) = Nm^2M_{\rm Pl}^2\left[w_0 \pm \frac{\chi^2}{2} + \frac{G_3}{3}\chi^3 + \frac{G_4}{4}\chi^4\right], \quad (2.23)$$

where

$$w_0 = \frac{V_0}{Nm^2M_{\rm Pl}^2}, \qquad G_3 = g\sqrt{N}\frac{M_{\rm Pl}}{m}, \qquad G_4 = \lambda N\left(\frac{M_{\rm Pl}}{m}\right)^2 \quad (2.24)$$

That is from Eq. (2.8),

$$w(\chi) = w_0 \pm \frac{1}{2}\chi^2 + \frac{G_3}{3}\chi^3 + \frac{G_4}{4}\chi^4.$$

Comparing Eqs. (2.8) and (2.23) we read off the relation (2.21) between the inflaton mass m, the scale of the potential M and the Hubble parameter H during slow-roll inflation

Since slow-roll inflation is consistently described with $w(\chi) \sim \mathcal{O}(1)$, this in turn implies that $G_3, G_4 \sim \mathcal{O}(1)$. This statement translates into the following seesaw-like relations,

$$g = \frac{G_3}{\sqrt{N}}\left(\frac{M}{M_{\rm Pl}}\right)^2, \qquad \lambda = \frac{G_4}{N}\left(\frac{M}{M_{\rm Pl}}\right)^4. \quad (2.25)$$

Therefore, we naturally find the order of magnitude of the couplings to be:

$$g \sim 10^{-6}, \qquad \lambda \sim 10^{-12}. \quad (2.26)$$

Since $M/M_{\rm Pl} \sim 3 \times 10^{-3}$, these relations are a natural consequence of the validity of the effective field theory and of slow roll and relieve the fine-tuning of the smallness of the couplings. We emphasize that the "seesaw-like" form of the couplings is a natural consequence of the form of the potential (2.8) and of the inequality (2.11). While the hierarchy between the Hubble parameter, the inflaton mass and the Planck scale during slow-roll inflation is well known, our analysis reveals that small couplings are naturally explained in terms of powers of the ratio between the inflationary and Planck scales *and* integer powers of $1/\sqrt{N}$.

Therefore, one of our main results in this article, is that the effective field theory and slow-roll descriptions of inflation, both validated by WMAP, lead us to conclude that there is no fine-tuning for the numerical values of the couplings. The smallness of the inflaton mass and the coupling constants in this polynomial realization of the inflationary potential is a *direct* consequence of the validity of both the effective field theory and the slow-roll approximations through a seesaw-like mechanism.

For a general potential $V(\phi)$,

$$V(\phi) = \sum_{n=0}^{\infty} \lambda_n \phi^n, \quad (2.27)$$

we find from Eq. (2.8)

$$\lambda_n = \frac{G_n m^2}{(NM_{\rm Pl}^2)^{(n/2)-1}} \quad \text{where } w(\chi) = \sum_{n=0}^{\infty} G_n \chi^n, \quad (2.28)$$

and the dimensionless coefficients G_n are of order one. We find the scaling behavior $\lambda_n \sim 1/N^{(n/2)-1}$. Equation (2.25) displays particular cases of Eq. (2.28) for $n = 3$ and 4.

There are several remarkable features and consistency checks of the relations (2.25):

(i) Note the relation $\lambda \sim g^2$. This is the correct consistency relation in a renormalizable theory because at one-loop level there is a renormalization of the quartic coupling (or alternatively a contribution to the four points correlation function) of orders λ^2, g^4 and λg^2 which are of the same order for $\lambda \sim g^2$. Similarly, at one-loop level there is a renormalization of the cubic coupling (alternatively, a contribution to the three point function) of orders g^3 and λg which again require $g^2 \sim \lambda$ for consistency.

(ii) In terms of the effective field theory ratio H/M_{Pl} and slow-roll parameters, the dimensionless couplings are[2]

$$\frac{mg}{H} \sim \frac{1}{N}\frac{H}{M_{\rm Pl}}, \qquad \lambda \sim \frac{1}{N^2}\left(\frac{H}{M_{\rm Pl}}\right)^2. \quad (2.29)$$

These relations agree with those found for the dimensionless couplings in Refs. [12,13] once the slow-roll parameters are identified with the expressions (2.12) and (2.13) in terms of $1/N$. The results of Refs. [12,13] revealed that the loop expansion is indeed an expansion in the effective field theory ratio $H/M_{\rm Pl}$ and the slow-roll parameters. Our study here allows us to go further and state that the loop expansion is a consistent double series in the effective field theory ratio $H/M_{\rm Pl}$ *and* $1/N$. Loops are either powers of g^2 or of λ which implies that for each loop there is a factor $H^2/M_{\rm Pl}^2$. The counting of powers of $1/N$ is more subtle: the nearly scale invariant spectrum of fluctuations leads to infrared enhancements of quantum corrections in which the small factor $1/N$ enters as an infrared regulator. Therefore, large denominators that feature the infrared regulator of order $1/N$ cancel out factors $1/N$ in the numerator. The final power of $1/N$ must be computed in detail in each loop contribution.

(iii) We find the relation (2.25) to be very suggestive. Since the scale of inflation M is fixed, presumably by the underlying microscopic (GUT) theory, the scaling of λ with the inverse of the number of e-

[2] In Refs. [12,13] the cubic coupling g corresponds to mg here with g dimensionless in the Lagrangian Eq. (2.22). In Ref. [12,13] it was established that loop corrections involve the ratio g/H in the notation of that reference.

folds strongly suggests a *renormalization group explanation of the effective field theory* because the number of e-folds is associated with the logarithm of the scale $N = \ln a$. A renormalization group improved scale dependent quartic coupling behaves [21] as $\lambda(K) \propto 1/\ln K$ with K the scale at which the theory is studied. Since in an expanding cosmology the physical scale grows with the scale factor it is natural to expect that a renormalization group resummation program would yield that the renormalized coupling scales as

$$\lambda \sim 1/\ln a \sim 1/N.$$

This of course requires further study.

B. Gauge invariant scalar perturbations

Let us now see that the slow-roll approximation appears as a consistent expansion in $1/N$ in the mode equations for the gauge invariant scalar perturbations $u_k(\eta)$. These can be written as [16]

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{1}{z}\frac{d^2 z}{d\eta^2}\right]u_k(\eta) = 0, \tag{2.30}$$

where η is the conformal time related, as usual, to cosmic time t by $dt = a(t)d\eta$ and $z(\eta)$ stands for

$$z(\eta) \equiv a^2 \frac{d\phi}{da}.$$

In terms of the slow and dimensionless variables

$$\bar{\eta} \equiv \eta\sqrt{N}m, \qquad \chi = \frac{\phi}{M_{\rm Pl}\sqrt{N}},$$
$$h = \frac{H}{m\sqrt{N}}, \qquad \bar{k} = \frac{k}{m\sqrt{N}},$$

Equation (2.30) takes the form

$$\left[\frac{d^2}{d\bar{\eta}^2} + \bar{k}^2 - \frac{1}{z}\frac{d^2 z}{d\bar{\eta}^2}\right]u_k(\bar{\eta}) = 0 \quad . \tag{2.31}$$

We can compute the "potential" in Eq. (2.31) in terms of $w(\chi)$ and its derivatives using the evolution Eqs. (2.16). We find to first order in $1/N$,

$$\frac{1}{z}\frac{d^2 z}{d\bar{\eta}^2} = \frac{2}{\bar{\eta}^2}\left\{1 + \frac{3}{2N}\left[-\frac{w''(\chi)}{w(\chi)} + \frac{3w'^2(\chi)}{2w^2(\chi)}\right] + \mathcal{O}\left(\frac{1}{N^2}\right)\right\}$$

where we used the following expression for the dimensionless conformal time

$$\bar{\eta} = -\frac{1}{ha}\left[1 + \frac{1}{2N}\frac{w'^2(\chi)}{w^2(\chi)} + \mathcal{O}\left(\frac{1}{N^2}\right)\right].$$

Equation (2.31) takes the familiar form [16] in terms of the above dimensionless variables,

$$\left\{\frac{d^2}{d\bar{\eta}^2} + \bar{k}^2 - \frac{2}{\bar{\eta}^2}\left[1 + \frac{1}{2}(9\epsilon_V - 3\eta_V) + \mathcal{O}\left(\frac{1}{N^2}\right)\right]\right\}u_k(\bar{\eta}) = 0, \tag{2.32}$$

and explicitly exhibiting [see Eq. (2.12)], once again, that the slow-roll approximation is an expansion in $1/N$.

Relevant modes for the large scale structure and the CMB are today in the range from 0.1 Mpc to 103 Mpc. These scales at the beginning of inflation correspond to physical wave numbers in the range

$$e^{N_T - 60}10^{16}\ \text{GeV} < k < e^{N_T - 60}10^{20}\ \text{GeV}$$

where N_T stands for the total number of e-folds (see, for example, Ref. [22]). Therefore, $\bar{k} \gg 1$ and Eq. (2.32) is deep in the semiclassical regime.

III. THE TENSOR/SCALAR RATIO *r* AND LYTH'S BOUND CLARIFIED

The next step in CMB observations is the search for B-modes which, if observed, can place a direct bound on the scale of inflation. The measurement of B-modes or tensor perturbations with CMB experiments depends on the magnitude of the ratio r between tensor and scalar perturbations.

As noticed by Lyth [17] the ratio of tensor/scalar fluctuations r can be related to the change of the inflaton field while the cosmologically relevant modes exit the horizon by

$$\frac{\Delta\phi}{M_{\rm Pl}} \sim \sqrt{\frac{r}{8}}\Delta N. \tag{3.1}$$

For $\Delta N \simeq 4$ this gives

$$\frac{\Delta\phi}{M_{\rm Pl}} \sim \sqrt{2r}. \tag{3.2}$$

A more stringent bound has been found in Ref. [18] by a statistical analysis of over 2×10^6 slow-roll inflationary models

$$\frac{\Delta\phi}{M_{\rm Pl}} \sim 6\sqrt{8\pi}r^{1/4}. \tag{3.3}$$

Inflationary model building proposes an effective field theory description of the inflationary potential of the form [17,19]

$$V(\phi) = V_0 + \frac{m^2}{2}\phi^2 + \phi^4\sum_{p=0}^{\infty}\lambda_p\left(\frac{\phi}{M_P}\right)^p. \tag{3.4}$$

Within this framework, it is often stated that the validity of the effective field theory description entails that

$$\frac{\phi}{M_{\rm Pl}} \ll 1. \tag{3.5}$$

A tension between this stringent constraint on the validity of an effective field theory and the bounds (3.2) and (3.3) is evident. The validity of the constraint (3.5) entails that $\Delta\phi/M_{\rm Pl} \ll 1$ suggesting that effective field theory predicts values of $r \ll 1$ probably too small to be observed in the next generation of CMB experiments [19]. Alternatively if larger values of r are measured then this would entail a breakdown of the effective field theory approach to inflation.

We find this line of reasoning incorrect for three different but complementary reasons:

(i) The validity of an effective field theory expansion does *not* rely on $\phi/M_{\rm Pl} \ll 1$, instead it relies on a wide separation between the scale of inflation and the higher energy scale of the underlying microscopic theory. If the effective field theory emerges from integrating out degrees of freedom at the GUT scale, then $H/M_{\rm GUT} \sim 10^{-2}$, if such scale is instead the Planck scale then $H/M_{\rm Pl} \sim 4 \times 10^{-5}$, and in either case an effective field theory description is valid. Indeed, detailed calculations in Refs. [12,13] reveal that the quantum corrections to slow-roll inflation are of the order $(H/M_{\rm Pl})^2$. Any breakdown of an effective field theory is typically manifest in large quantum corrections but the results of [12,13] unambiguously point out that quantum corrections are well under control for $H/M_{\rm Pl} \ll 1$. This provides a reassuring confirmation of the validity of the effective field theory for a scale of inflation consistent with the WMAP data.

(ii) Three simple examples highlight that the criterion $\phi/M_{\rm Pl} \ll 1$ *cannot* be the deciding factor for the reliability of the effective field theory: consider the following series

$$\sum_{p=0}^{\infty}(-1)^p\left(\frac{\phi}{M_{\rm Pl}}\right)^{2p} = \frac{1}{1+(\frac{\phi}{M_{\rm Pl}})^2} \tag{3.6}$$

$$\sum_{p=0}^{\infty}\frac{(-1)^p}{(2p)!}\left(\frac{\phi}{M_{\rm Pl}}\right)^{2p} = \cos\left(\frac{\phi}{M_{\rm Pl}}\right) \tag{3.7}$$

$$\sum_{p=0}^{\infty}\left(\frac{\phi}{M_{\rm Pl}}\right)^{2p} = \frac{1}{1-(\frac{\phi}{M_{\rm Pl}})^2}. \tag{3.8}$$

The sum of both series (3.6) and (3.7) is perfectly well defined for $\phi > M_{\rm Pl}$. In particular, the series (3.7) is a prototype for an axion-type potential [23], while certainly the series (3.8) breaks down for $\phi \sim M_{\rm Pl}$. The series (3.6) and (3.7) do not feature any real singularity in the variable $\phi/M_{\rm Pl}$ whereas Eq. (3.8) has a singularity at $\phi = M_{\rm Pl}$. These elementary examples highlight that what constraints the reliability of the effective field theory description of the inflationary potential is not the value of the ratio $\phi/M_{\rm Pl}$ but the position of the singularities as a function of this variable. These singularities are determined by the large order behavior of the coefficients in the series. If the series has a nonzero radius of convergence or if it is just Borel summable, it defines the function $V(\phi)$ uniquely. Clearly, a thorough knowledge of the radius of convergence of the series in the effective field theory requires a detailed knowledge of the underlying microscopic theory. However, it should also be clear that the requirement $\phi/M_{\rm Pl} \ll 1$ is overly restrictive in general.

(iii) One of the main results of this article is that the combination of WMAP data and slow-roll expansion suggest a universal form of the inflation potential,

$$V(\phi) = NM^4 w(\chi), \qquad \chi = \frac{\phi}{\sqrt{N}M_{\rm Pl}} \quad . \tag{3.9}$$

Even in the case when the coefficients in a χ-series expansion of $w(\chi)$ lead to a breakdown of the series at $\chi \sim 1$, namely, at $\phi \sim \sqrt{N}M_{\rm Pl}$, there is still room for values of $M_{\rm Pl} < \phi < \sqrt{N}M_{\rm Pl}$ for which the series would be reliable. However, no *a priori* physical reason for such a breakdown can be inferred without a reliable calculation of the effective field theory from a microscopic fundamental theory. Therefore, we expect that the effective field theory potential $V(\phi) = NM^4 w(\chi)$ would be reliable at least up to $\phi \sim \sqrt{N}M_{\rm Pl}$ and most generally for values $\chi \gg 1$ and hence $\phi \gg M_{\rm Pl}$.

As mentioned above, the studies in Refs. [12,13] reveal that quantum corrections in the effective field theory yields an expansion in $(\frac{H}{M_{\rm Pl}})^2$ for general inflaton potentials. This indicates that the use of the inflaton potential $V(\phi)$ from effective field theory is consistent for

$$\left(\frac{H}{M_{\rm Pl}}\right)^2 \ll 1 \quad \text{and hence} \quad V(\phi) \ll M_{\rm Pl}^4.$$

We find using Eq. (2.8):

$$w(\chi) \ll \frac{1}{N}\left(\frac{M_{\rm Pl}}{M}\right)^4 \sim \frac{1}{N}10^{12}. \tag{3.10}$$

This condition yields an upper bound in the inflaton field ϕ depending on the large field behavior of $w(\chi)$. We find for relevant behaviors of the inflaton potential the following upper bounds on χ and ϕ:

$$
\begin{aligned}
&w(\chi) \simeq^{\chi \gg 1} \chi^2 : \chi \ll \frac{10^6}{\sqrt{N}}, \quad \phi \ll 10^6 M_{\rm Pl}, \\
&w(\chi) \simeq^{\chi \gg 1} \chi^4 : \chi \ll \frac{10^3}{N^{1/4}}, \quad \phi \ll 2659 M_{\rm Pl} \quad \text{for} \quad N \simeq 50, \\
&w(\chi) \simeq^{\chi \gg 1} \chi^n : \chi \ll \left(\frac{10^{12}}{N}\right)^{1/n}, \quad \phi \ll 10^{12/n} N^{1/2-1/n} M_{\rm Pl}, \\
&w(\chi) \simeq^{\chi \gg 1} e^{\chi} : \chi \ll 12 \ln 10 - \ln N = 23.72, \quad \phi \ll 167 M_{\rm Pl} \quad \text{for} \quad N \simeq 50.
\end{aligned} \tag{3.11}
$$

We see that the effective field theory is consistent for values of the inflaton field well beyond the Planck mass even for very steep potentials, such as the exponential function e^{χ}.

We summarize this discussion with the following two remarks, which when taken together, relieve the tension between the accessible experimental values of r and the validity of the effective field theory description:

(i) The inflaton potential $V(\phi)$ may be applicable even for large values of $\frac{\phi}{M_{\rm Pl}}$ invalidating the arguments leading to the bounds Eqs. (3.2) and (3.3) for r. The WMAP data constrains $r \lesssim 0.16$ [6], the true value for r may be very well much smaller than the present upper bound.

(ii) If for some reason (or prejudice) one wishes to restrict the analysis to values of the field where $\chi \lesssim 1$, then we provide from our study the following improved bounds:

$$
\begin{aligned}
&\Delta\chi = \frac{\Delta\phi}{\sqrt{N} M_{\rm Pl}} \sim \sqrt{\frac{2r}{N}} \quad \text{for the bound in Ref. [17]} \\
&\Delta\chi = \frac{\Delta\phi}{\sqrt{N} M_{\rm Pl}} \simeq \frac{6}{\sqrt{N}} \sqrt{8\pi} r^{1/4} \\
&\qquad \text{for the bound in Ref. [18]}
\end{aligned} \tag{3.12}
$$

This gives for $\Delta\chi \sim 1$, the **improved** bounds on r:

$$
\begin{aligned}
&r \lesssim \frac{N}{2} \simeq 25 \qquad \text{for the bound in Ref. [17]} \\
&r \lesssim \frac{N^2}{6^4 (8\pi)^2} \simeq 0.003 \quad \text{for the bound in Ref. [18]} \\
&\qquad \text{and} \quad N \simeq 50.
\end{aligned} \tag{3.13}
$$

It must be noticed that these bounds depend on whether one uses as Planck's mass $M_{\rm Pl} = 1/\sqrt{8\pi G}$ or $m_{\rm Pl} = 1/\sqrt{G}$, G being Newton's constant. Here we have used the first definition $M_{\rm Pl}$ as in ref.[17]. Reference [18] uses the second definition $m_{\rm Pl}$. We find using $m_{\rm Pl} = 1/\sqrt{G}$

$$
\begin{aligned}
&\frac{\Delta\chi}{\sqrt{8\pi}} = \frac{\Delta\phi}{\sqrt{N} m_{\rm Pl}} \sim \frac{1}{2}\sqrt{\frac{r}{\pi N}} \quad \text{for the bound in Ref. [17]} \\
&\frac{\Delta\chi}{\sqrt{8\pi}} = \frac{\Delta\phi}{\sqrt{N} m_{\rm Pl}} \simeq \frac{6}{\sqrt{N}} r^{1/4} \quad \text{for the bound in Ref. [18].}
\end{aligned} \tag{3.14}
$$

And we then find for $\frac{\Delta\chi}{\sqrt{8\pi}} \sim 1$:

$$
\begin{aligned}
&r \lesssim 4\pi N \simeq 628 \quad \text{for the bound in Ref. [17]} \\
&r \lesssim \frac{N^2}{6^4} \simeq 1.93 \quad \text{for the bound in Ref. [18]} \\
&\qquad \text{and} \quad N \simeq 50.
\end{aligned} \tag{3.15}
$$

In conclusion, even under the more conservative assumption $\chi \lesssim 1$, we provide the improved bounds Eqs. (3.12), (3.13), (3.14), and (3.15) which allow values of r substantially larger than the original ones [17,18]. This is a consequence of the factor $\sqrt{N} \sim 7$ in our slow variable χ, which in turn is the result of a consistency between the WMAP data and slow roll in an effective field theory description. The structure Eq. (2.8) of the inflaton potential therefore relieves the tension between the values of $\frac{\phi}{M_{\rm Pl}}$ and r.

We remark that the arguments presented above suggest that the reluctance to use the inflaton potential $V(\phi)$ for $\phi \gtrsim M_{\rm Pl}$ arises from a prejudice which is unwarranted under the most general circumstances, unless of course, the inflaton potential features singularities. The true upper bound for the validity of the effective field theory description of inflation is $V(\phi) \ll M_{\rm Pl}^4$ which in fairly general cases allows large values of $\frac{\phi}{M_{\rm Pl}}$ as emphasized by Eqs. (3.11).

IV. CONCLUSIONS

In this article we show that the consistency of an effective field theory description of inflation with the WMAP data and the slow-roll approximation provide a universal form of the inflationary potential. This form leads to a clear understanding of the validity of the effective field theory and makes manifest that the slow-roll expansion is an expansion in $1/N$ where $N \sim 50$ is the number of e-folds before the end of inflation when cosmologically relevant scales exit the Hubble radius. The inflaton potential is of the form,

$$V(\phi) = NM^4 w(\chi),$$

where the WMAP data pinpoints M at the GUT scale $M \sim 0.77 \times 10^{16}$ GeV, χ is a slowly varying dimensionless field,

$$\chi = \frac{\phi}{M_{\rm Pl}\sqrt{N}},$$

and $w(\chi) \sim \mathcal{O}(1)$.

The dynamics of the field χ depends solely on the *stretched* (slow) time variable

$$\tau = \frac{tM^2}{M_{\rm Pl}\sqrt{N}} = \frac{mt}{\sqrt{N}},$$

and is determined by the equation of motion (2.16) which can be solved consistently in an expansion in $1/N$.

This form of the potential makes explicit that the slow-roll expansion is a consistent expansion in $1/N$, see Eqs. (2.12) and (2.13). This also shows up in the equations of motion for the mode functions of gauge invariant scalar perturbations.

A polynomial realization of the inflaton potential as an effective field theory of the Ginzburg-Landau form, which has recently been shown to fit the WMAP data remarkably well [14], indicates that the Hubble parameter, the inflaton mass and the nonlinear couplings emerge as powers of the seesaw-like ratio $M/M_{\rm Pl} \sim 3 \times 10^{-3}$. The smallness of which warrants the validity of the effective field theory. Thus, it is clear that the smallness of the nonlinear self-couplings is not a result of fine-tuning, but a natural consequence of an effective field theory in which self-couplings emerge from seesaw like mechanisms with two widely different scales: the inflation (or GUT) and Planck scales. Furthermore, the consistency between the slow-roll and $1/N$ expansions implies that the quartic self-coupling λ scales as $\lambda \sim 1/N \sim 1/\ln a$ and that the cubic self-coupling g and the quartic self-coupling are such that $g^2 \sim \lambda$.

Our observations and results here relieve the tension between the validity of the effective field theory approach and the values of the ratio r between tensor and scalar perturbations. This tension is actually an artificial result of a prejudice on the validity of the effective field theory. This validity is not determined by the maximum value of ϕ/M_{Pl} but rather on the smallness of the ratio $H/M_{\rm Pl}$. These arguments pave the way for an unprejudiced observational exploration of B-modes in the next generation of CMB experiments within the theoretical description of slow-roll inflation based on an effective field theory.

The effective field theory describing slow-roll inflation during the stage relevant for cosmology features remarkable properties which indicate that inflation is hovering near a trivial Gaussian infrared fixed point in the renormalization group sense. Three important aspects are behind this conjecture:

(i) the nearly scale invariant power spectrum of scalar perturbations,

(ii) the fact that the coupling constant associated with a dimension four operator, λ (the quartic coupling) scales with the scale factor as $\lambda \sim 1/\ln a$ and

(iii) the fact that the quantum corrections [12,13] are in terms of the effective field theory ratio $H/M_{\rm Pl}$ and powers of $1/\ln a$.

This behavior is similar to that of a renormalizable field theory near its trivial fixed point. We will continue to explore this remarkable aspect of slow-roll inflation and expect to report on these studies in the future.

We can summarize our main present results as follows: we trade the small parameters in the inflationary models for appropriate slow variables by introducing two crucial and independent ingredients:

(i) First, by introducing the inflationary or GUT energy scale $M \sim 10^{16}$ GeV and the Planck scale $M_{\rm Pl}$ in the inflaton field and in the inflaton potential.

(ii) Second, by rescaling the inflaton field with the square root of the number of e-folds $\sqrt{N}$. This turn to introduce a dependence of the couplings on N similar to a renormalization group running of the couplings.

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