Simultaneous flavor transformation of neutrinos and antineutrinos with dominant potentials from neutrino-neutrino forward scattering

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In astrophysical environments with intense neutrino fluxes, neutrino-neutrino forward scattering contributes both diagonal and off-diagonal potentials to the flavor-basis Hamiltonian that governs neutrino flavor evolution. We examine a special case where adiabatic flavor evolution can produce an off-diagonal potential from neutrino-neutrino forward scattering that dominates over both the corresponding diagonal term and the potential from neutrino-matter forward scattering. In this case, we find a solution that, unlike the ordinary Mikeyhev-Smirnov-Wolfenstein scenario, has both neutrinos and antineutrinos maximally mixed in medium over appreciable ranges of neutrino and antineutrino energy. Employing the measured solar and atmospheric neutrino mass-squared differences, we identify the conditions on neutrino fluxes that are required for this solution to exist deep in the supernova environment, where it could affect the neutrino signal, heavy-element nucleosynthesis, and even the revival of the supernova shock. We speculate on how this solution might or might not be attained in realistic supernova evolution. Though this solution is ephemeral in time and/or space in supernovae, it may signal the onset of subsequent appreciable flavor mixing for both neutrinos and antineutrinos.

I. INTRODUCTION

In this paper we study the problem of coherent nonlinear flavor evolution of active neutrinos in environments where neutrino-neutrino forward scattering makes a significant contribution to the effective neutrino mass in medium. In particular, we examine a special case where the off-diagonal potential from neutrino-neutrino forward scattering becomes the dominant term in the flavor-basis Hamiltonian that governs neutrino flavor evolution.

In both post-core-bounce supernovae and an early universe with net lepton numbers, local net neutrino number densities can exceed electron and baryon number densities. These large neutrino number densities or fluxes, sometimes referred to as a ''neutrino background,'' require that the usual Mikheyev-Smirnov-Wolfenstein (MSW) formalism [1] for calculating the evolution of neutrino flavors be modified to include the effects of neutrino-neutrino forward scattering. Though the resulting problem of neutrino flavor evolution can be complicated, we can identify a key parameter governing the relevant physics: the ratio of the product of neutrino energy and the off-diagonal potential from neutrino-neutrino forward scattering relative to the difference of the squares of the vacuum neutrino mass eigenvalues. If this ratio becomes very large as a result of adiabatic neutrino flavor evolution, we can even find an interesting solution, which has neutrinos and antineutrinos simultaneously maximally mixed in medium. This is different from the usual MSW case where, at a given location or time, neutrino flavor mixing in medium is maximal for a specific range of neutrino energies (this range is narrow for small vacuum mixing angles), while antineutrino mixing is suppressed; or vice versa. As we discuss below, the off-diagonal potential from neutrino-neutrino forward scattering plays a unique role: it can alter neutrino flavor evolution into a form that is utterly unlike the MSW case.

In the presence of a significant neutrino background, neutrino flavor histories can be followed by solving a mean-field Schrödinger-like equation in the modified-MSW format. This is a long-standing and vexing problem. In fact, it has defied general and complete solutions for the supernova environment, even with sophisticated numerical treatments. The existence and importance of the flavor-diagonal potential from neutrino-neutrino forward scattering and how it might modify MSW-like neutrino flavor evolution in supernovae were pointed out early on [2] (see Ref. [3] for a subsequent formal treatment). However, the existence of the corresponding off-diagonal potential was established only later [4]. This latter discovery may prove to be a watershed event in supernova neutrino physics.

There have been several attempts to elucidate how flavor-diagonal and/or off-diagonal potentials from neutrino-neutrino forward scattering can affect active-active neutrino flavor transformation in the post-core-bounce supernova regime, especially as regards shock reheating [5–7] and *r*-process nucleosynthesis [6,8–10]. The rationale for these studies was that the energy spectra and/or the fluxes of the various neutrino flavors could differ on emergence from the neutron star surface or neutrino sphere, and therefore, flavor interconversion above this surface could alter these spectra and/or fluxes to change supernova dynamics and nucleosynthesis or the neutrino

signal in a detector. If the energy distribution functions and the associated net energy luminosities are the same for all flavors of neutrinos and antineutrinos, then, obviously, flavor transformation will have no effect. However, on account of core deleptonization and concomitant changes in composition, size, and equation of state, this is unlikely to be the case over the entire post-core-bounce period of ~ 20 s during which neutrino fluxes are appreciable.

If at any epoch in the supernova environment there develops a hierarchy of average neutrino energies or luminosities among the different neutrino flavors, then flavor conversion could alter the rates of electron neutrino and antineutrino capture on free nucleons:

$$\nu_e + n \rightarrow p + e^-, \tag{1}$$

$$\bar{\nu}_e + p \rightarrow n + e^+. \tag{2}$$

These are the processes principally responsible for depositing energy in the material behind the shock after core bounce. Therefore, altering their rates by, for example, swapping flavor labels between possibly less energetic electron neutrinos and more energetic mu and/or tau neutrinos could significantly affect the prospects for a supernova explosion [5]. Also, the competition between these processes and their reverse reactions sets the neutron-to-proton ratio in neutrino-heated material [8]. In turn, this ratio is sometimes a crucial parameter for r-process [6,8] and other heavy-element nucleosynthesis [11] associated with slow neutrino-heated outflows.

Most of the studies cited above posited the existence of neutrino mass-squared differences ≥ 0.2 eV^2. This was required for normal MSW resonances to occur in the high-density regions most relevant for supernova shock reheating and r-process nucleosynthesis. These regions lie above but relatively close to the neutron star, generally within a few hundred kilometers. Without the hypothesized high neutrino mass-squared differences, conventional MSW evolution in these regions would not result in any significant neutrino flavor conversion.

Although we do not know the absolute vacuum neutrino mass eigenvalues, m_1, m_2, and m_3, the two independent differences of their squares are now measured to be $\delta m^2 \approx 7 \times 10^{-5}$ and 3×10^{-3} eV^2 by observations of solar and atmospheric neutrinos, respectively. The lower δm^2 has also been measured directly by the KamLAND reactor experiment. (See Ref. [12] for a review of neutrino properties.) As these δm^2 values are certainly small compared with the scale previously believed to be most relevant for supernovae, one may tend to conclude that they have no consequence for supernova dynamics and nucleosynthesis. However, as we will discuss below, neutrino background effects could alter this conclusion dramatically.

Similar to the supernova case, when there are net lepton numbers residing in the neutrino sector in the early universe, neutrino flavor conversion can be important in, for example, setting the neutron-to-proton ratio and, hence, the ^{4}He abundance yield in primordial nucleosynthesis. It was recognized in Ref. [13] that the flavor-diagonal potential from neutrino-neutrino forward scattering affects neutrino propagation in the coherent limit of the problem. However, a complete treatment of active-active neutrino flavor conversion in the early universe requires a coupled calculation including both flavor-diagonal and off-diagonal potentials from neutrino-neutrino forward scattering as well as general inelastic neutrino scattering [14–16]. In fact, the seminal numerical work in Refs. [15,16] shows that the measured neutrino mass-squared differences and mixing angles result in an "evening up" of the initially disparate lepton numbers residing in neutrinos of different flavors. The flavor oscillations in these calculations exhibit "synchronization" in time and space and can correspond to near-maximal flavor mixing for neutrinos and antineutrinos. This numerical result may be closely related to our solution (discussed below) for the special case where the off-diagonal potential from neutrino-neutrino forward scattering dominates.

In Sec. II we will outline how neutrino flavor transformation proceeds in the coherent limit when neutrino backgrounds are non-negligible. In Sec. III we discuss the particular limit of domination by the off-diagonal potential from neutrino-neutrino forward scattering and the corresponding solution. We also speculate under what conditions and to what extent this solution could be attained in supernovae. Conclusions are given in Sec. IV.

II. COHERENT FLAVOR EVOLUTION WITH NEUTRINO BACKGROUNDS

Here we give a brief synopsis of coherent neutrino flavor amplitude development in the supernova and early universe environments. In the supernova core and the dense environment immediately above it, and in the early universe prior to weak decoupling, nonforward neutrino scattering can result in neutrino flavor conversion through decoherence. We will ignore this in what follows and instead concentrate on the purely coherent evolution of the neutrino fields. It should always be kept in mind, however, that our considerations may need to be modified at high density or in high neutrino flux regimes.

Even in the purely coherent limit, following the effects of neutrino-neutrino forward scattering in the most general case is daunting in scope. Part of the difficulty in following neutrino flavor evolution in the supernova environment is geometric: flavor evolution histories on different neutrino trajectories are coupled. This is because two neutrino states will experience quantum entanglement to the future of a forward-scattering event occurring at the intersection of their world lines. In light of this entanglement there has been considerable speculation about whether neutrino flavor evolution can be modeled adequately by a mean-field treatment with Schrödinger equations [17,18]. The mean

field in this case is the potential seen by a neutrino by virtue of forward scattering on particles in the environment that carry weak charge. Here we will follow the conclusions of Ref. [17] and take the mean-field treatment as sufficient in a statistical sense. This seems reasonable for the early universe and supernova environments because a statistically large number of neutrino scattering events and entanglements occur in these places.

Further complicating the supernova problem is the non-isotropic nature of the neutrino fields above the neutrino sphere. Neutrinos traveling along trajectories nearly tangential to the neutrino sphere may have quite different flavor amplitude histories from those moving radially or near radially. For now we will ignore this feature of flavor development and instead approximate all neutrinos as evolving the way radially propagating neutrinos evolve. This approximation is *not* a good one (it has been made in all previous numerical work [6,10]), but it will suffice in our analytic arguments here.

A. Overview

In vacuum, the flavor (weak interaction) eigenstates of neutrinos are related to the mass (energy) eigenstates by a unitary transformation U_m:

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U_m \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}, \quad (3)$$

where the mass eigenstates $|\nu_1\rangle$, $|\nu_2\rangle$, and $|\nu_2\rangle$ correspond to the vacuum-mass eigenvalues m_1, m_2, and m_3, respectively. The unitary transformation U_m can be written in terms of a sequence of rotations,

$$U_m = U_{23}U_{13}U_{12}. \quad (4)$$

A convenient representation for these rotations is

$$U_{23} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix},$$

$$U_{13} \equiv \begin{pmatrix} \cos\theta_{13} & 0 & e^{i\delta}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{-i\delta}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}, \quad (5)$$

$$U_{12} \equiv \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

In the above representation, the mixing angles, θ_{12} and θ_{23}, have been measured by observations of solar and atmospheric neutrinos and related experiments. In particular, the best fit for the vacuum mixing of the mu and tau neutrinos is very near maximal, which gives $\theta_{23} \approx \pi/4$. However, the mixing angle, θ_{13}, and the *CP*-violating phase, δ, have not been measured yet.

Here we consider a neutrino mixing scenario where $m_3 > m_2 > m_1$ with $\delta m_{12}^2 \equiv m_2^2 - m_1^2 \approx 7 \times 10^{-5}$ eV^2

and $\delta m_{13}^2 \equiv m_3^2 - m_1^2 \approx 3 \times 10^{-3}$ eV^2, $\theta_{23} = \pi/4$, and $\delta = 0$. With the definitions

$$|\nu_\mu^*\rangle \equiv \frac{|\nu_\mu\rangle - |\nu_\tau\rangle}{\sqrt{2}}, \quad (6)$$

$$|\nu_\tau^*\rangle \equiv \frac{|\nu_\mu\rangle + |\nu_\tau\rangle}{\sqrt{2}}, \quad (7)$$

it is straightforward to show that

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu^*\rangle \\ |\nu_\tau^*\rangle \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12} & c_{12} & 0 \\ -c_{12}s_{13} & -s_{12}s_{13} & c_{13} \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}, \quad (8)$$

where, for example, $c_{12} \equiv \cos\theta_{12}$ and $s_{12} \equiv \sin\theta_{12}$. The states $|\nu_\mu^*\rangle$ and $|\nu_\tau^*\rangle$ are still useful in medium. This is because in the supernova medium the mu and tau neutrinos have very nearly the same interactions, so that matter effects on mixing and effective mass for these species are nearly identical (likewise for the mu and tau antineutrinos). This will also be true for the early universe if the net muon and tau lepton numbers are identical. For the sake of our arguments here, we will take the symmetry between mu and tau neutrinos and that between their antiparticles to be rigorously true so that $|\nu_\mu^*\rangle$ and $|\nu_\tau^*\rangle$ are effective flavor eigenstates in medium. Since $\delta m_{13}^2 \gg \delta m_{12}^2$, the regions of neutrino flavor mixing governed by these parameters should be well separated. As neutrinos propagate outward from the neutrino sphere at high density in supernovae, $\delta m_{13}^2 \approx 3 \times 10^{-3}$ eV^2 becomes relevant first. We will focus on neutrino flavor mixing with this parameter, for which ν_μ^* is effectively decoupled [see Eq. (8)] and we only need to consider mixing of ν_e and ν_τ^* [19,20]. Thus, the general problem of 3ν mixing in medium is reduced to one of 2ν mixing in our scenario.

With the above simplification we can hereafter follow the notation of Ref. [6]. In particular, we now simply refer to $|\nu_\tau^*\rangle$ as $|\nu_\tau\rangle$ and write the effective 2ν unitary transformation in vacuum as

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle, \quad (9)$$

$$|\nu_\tau\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle, \quad (10)$$

where $|\nu_1\rangle$ and $|\nu_2\rangle$ refer generically to the light and heavy mass eigenstates, respectively, and θ is the effective 2ν vacuum mixing angle. The relevant vacuum-mass-squared difference is $\delta m^2 \approx 3 \times 10^{-3}$ eV^2. The corresponding effective vacuum mixing angle is $\theta \sim \theta_{13}$, with the current reactor experiment limit being $\sin^2 2\theta_{13} < 0.1$ (see, e.g., Ref. [12]).

Consider a neutrino of initial flavor $\alpha = e$ or τ. As it propagates outward from the neutrino sphere in supernovae, the evolution of its state, the ket $|\Psi_{\nu_\alpha}(t)\rangle$, can be described as

$$|\Psi_{\nu_\alpha}(t)\rangle = a_{e\alpha}(t)|\nu_e\rangle + a_{\tau\alpha}(t)|\nu_\tau\rangle, \qquad (11)$$

where $a_{e\alpha}(t)$ or $a_{\tau\alpha}(t)$ is the amplitude for the neutrino to be a ν_e or ν_τ, respectively, at time t. (Note that t could be any affine parameter such as radius along the neutrino's world line.) Alternatively, the evolution of $|\Psi_{\nu_\alpha}(t)\rangle$ can be described as

$$|\Psi_{\nu_\alpha}(t)\rangle = a_{1\alpha}(t)|\nu_1(t)\rangle + a_{2\alpha}(t)|\nu_2(t)\rangle, \qquad (12)$$

where $|\nu_1(t)\rangle$ and $|\nu_2(t)\rangle$ are the instantaneous mass (energy) eigenstates in medium, and $a_{1\alpha}(t)$ and $a_{2\alpha}(t)$ are the corresponding amplitudes. The flavor eigenstates are related to $|\nu_1(t)\rangle$ and $|\nu_2(t)\rangle$ as

$$|\nu_e\rangle = \cos\theta_M(t)|\nu_1(t)\rangle + \sin\theta_M(t)|\nu_2(t)\rangle, \qquad (13)$$

$$|\nu_\tau\rangle = -\sin\theta_M(t)|\nu_1(t)\rangle + \cos\theta_M(t)|\nu_2(t)\rangle, \qquad (14)$$

where $\theta_M(t)$ is the effective 2ν mixing angle in medium at time t. In matrix form, the ket $|\Psi_{\nu_\alpha}(t)\rangle$ can be represented by

$$\Psi_f \equiv \begin{bmatrix} a_{e\alpha}(t) \\ a_{\tau\alpha}(t) \end{bmatrix} \qquad (15)$$

in the flavor basis and by

$$\Psi_M \equiv \begin{bmatrix} a_{1\alpha}(t) \\ a_{2\alpha}(t) \end{bmatrix} \qquad (16)$$

in the energy basis. In analogous fashion we will employ a $2\bar{\nu}$ scheme to follow separately the flavor evolution of the antineutrino sector.

B. Characterizing neutrino densities

The single neutrino density operator at time t projected into the energy basis is

$$\begin{aligned} |\Psi_{\nu_\alpha}(t)\rangle\langle\Psi_{\nu_\alpha}(t)| = &|a_{1\alpha}(t)|^2|\nu_1(t)\rangle\langle\nu_1(t)| \\ &+ |a_{2\alpha}(t)|^2|\nu_2(t)\rangle\langle\nu_2(t)| \\ &+ a_{1\alpha}(t)a_{2\alpha}^*(t)|\nu_1(t)\rangle\langle\nu_2(t)| \\ &+ a_{1\alpha}^*(t)a_{2\alpha}(t)|\nu_2(t)\rangle\langle\nu_1(t)|. \end{aligned} \qquad (17)$$

The third and fourth lines of Eq. (17) contain cross terms which, in general, have complex coefficients. However, these cross terms vanish in the limit where neutrino flavor evolution is adiabatic. This is because for neutrinos produced in regions of high weak charge densities (e.g., near the neutrino sphere in supernovae) the flavor eigenstates essentially coincide with energy eigenstates and a neutrino evolving adiabatically is always in a single energy state. For example, in this limit we might have $|a_{1\alpha}(t)| = 1$, which would imply that $|a_{2\alpha}(t)| = 0$ due to the normalization condition

$$\langle\Psi_\alpha(t)|\Psi_\alpha(t)\rangle = |a_{1\alpha}(t)|^2 + |a_{2\alpha}(t)|^2 = 1. \qquad (18)$$

The density operator for the neutrinos or antineutrinos with momentum centered around **p** in a pencil of neutrino or antineutrino momenta and directions $d^3\mathbf{p}$ can be defined as in Ref. [6]:

$$\hat{\rho}_{\mathbf{p}}(t)d^3\mathbf{p} \equiv \sum_\alpha dn_{\nu_\alpha}|\Psi_{\nu_\alpha}(t)\rangle\langle\Psi_{\nu_\alpha}(t)|, \qquad (19)$$

$$\hat{\bar{\rho}}_{\mathbf{p}}(t)d^3\mathbf{p} \equiv \sum_\alpha dn_{\bar{\nu}_\alpha}|\Psi_{\bar{\nu}_\alpha}(t)\rangle\langle\Psi_{\bar{\nu}_\alpha}(t)|. \qquad (20)$$

Note that the traces of these operators over neutrino flavor do not give unity but rather the total number density of neutrinos or antineutrinos of all kinds in the pencil.

We assume that neutrinos and antineutrinos of all flavors are emitted from the same sharp neutrino sphere of radius R_ν in supernovae. (This is not a particularly good approximation for neutrinos very near the neutron star surface, but it will suffice for our arguments.) At a radius $r > R_\nu$, the neutrino sphere subtends a solid angle of

$$\Delta\Omega_\nu(r) = 2\pi\left(1 - \sqrt{1 - R_\nu^2/r^2}\right). \qquad (21)$$

Within this solid angle, the number density of ν_α in a pencil of directions and momenta is

$$dn_{\nu_\alpha} = \frac{L_{\nu_\alpha}}{\pi R_\nu^2}\frac{1}{\langle E_{\nu_\alpha}\rangle}\left(\frac{d\Omega_\nu}{4\pi}\right)f_{\nu_\alpha}(E_\nu)dE_\nu, \qquad (22)$$

where L_{ν_α} is the energy luminosity of ν_α, $d\Omega_\nu$ is the pencil of directions, E_ν is the neutrino energy, $f_{\nu_\alpha}(E_\nu)$ is the normalized energy distribution function for ν_α, and $\langle E_{\nu_\alpha}\rangle$ is the corresponding average ν_α energy. Here and in the rest of this paper we assume that neutrinos have relativistic kinematics and employ natural units where $\hbar = c = 1$. The function $f_{\nu_\alpha}(E_\nu)$ can be fitted to the results from supernova neutrino transport calculations and is commonly taken to be of the form

$$f_{\nu_\alpha}(E_\nu) = \frac{1}{T_{\nu_\alpha}^3 F_2(\eta_{\nu_\alpha})}\frac{E_\nu^2}{e^{E_\nu/T_{\nu_\alpha} - \eta_{\nu_\alpha}} + 1}, \qquad (23)$$

where T_{ν_α} and η_{ν_α} are two fitting parameters and $F_2(\eta_{\nu_\alpha})$ is the Fermi integral of order 2 and argument η_{ν_α}. The Fermi integral of order k and argument η is defined as

$$F_k(\eta) \equiv \int_0^\infty \frac{x^k dx}{e^{x-\eta} + 1}. \qquad (24)$$

In terms of these integrals, the average ν_α energy is

$$\langle E_{\nu_\alpha}\rangle \equiv \int_0^\infty E_\nu f_{\nu_\alpha}(E_\nu)dE_\nu = T_{\nu_\alpha}\frac{F_3(\eta_{\nu_\alpha})}{F_2(\eta_{\nu_\alpha})}. \qquad (25)$$

C. Neutrino propagation in medium

For a neutrino originating as a ν_α at $t = 0$, its subsequent flavor evolution along a radially directed trajectory with affine parameter t is described by

$$i\frac{\partial}{\partial t}|\Psi_{\nu_\alpha}\rangle = (\hat{H}_{\rm vac} + \hat{H}_{e\nu} + \hat{H}_{\nu\nu})|\Psi_{\nu_\alpha}\rangle, \qquad (26)$$

where we have decomposed the overall evolution Hamiltonian into contributions from vacuum neutrino masses and from mean-field ensemble averages for neutrino-electron and neutrino-neutrino forward scattering. These contributions are discussed individually below.

For a neutrino with energy E_ν and vacuum mass $m \ll E_\nu$, we have $E_\nu = \sqrt{p^2 + m^2} \approx p + m^2/(2p)$, where p is the magnitude of the neutrino momentum $\mathbf{p}$. In this limit, the vacuum-mass contribution to the flavor evolution Hamiltonian is

$$\hat{H}_{\rm vac} \approx p\hat{I} + \frac{1}{2p}(m_1^2|\nu_1\rangle\langle\nu_1| + m_2^2|\nu_2\rangle\langle\nu_2|), \qquad (27)$$

where $\hat{I}$ is the identity operator.

Electron neutrinos and antineutrinos can forward scatter on electrons and positrons through exchange of $W^\pm$. In contrast, there is no such charged-current forward scattering for ν_μ, ν_τ, and their antiparticles due to the absence of $\mu^\pm$ and $\tau^\pm$ in the environments of interest here. Consequently, the effective contribution from charged-current neutrino-electron forward scattering to the flavor evolution Hamiltonian is

$$\hat{H}_{e\nu}(t) = A(t)|\nu_e\rangle\langle\nu_e|, \qquad (28)$$

where

$$A(t) \equiv \sqrt{2}G_F(n_{e^-} - n_{e^+}) = \sqrt{2}G_F n_b Y_e. \qquad (29)$$

In the above equation, n_{e^-}, n_{e^+}, and n_b are the proper number densities of electrons, positrons, and baryons, respectively, at the position corresponding to time t, and $Y_e = (n_{e^-} - n_{e^+})/n_b$ is the net electron number per baryon, or electron fraction.

For a specific neutrino with momentum $\mathbf{p}$, the effective neutral-current neutrino-neutrino forward-scattering contribution [2] to the flavor evolution Hamiltonian is

$$\hat{H}_{\nu\nu}(t) = \sqrt{2}G_F \int (1 - \cos\theta_{\mathbf{pq}})[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)]d^3\mathbf{q}, \qquad (30)$$

where $\mathbf{q}$ is the momentum of the background neutrinos and $\cos\theta_{\mathbf{pq}} = \mathbf{p} \cdot \mathbf{q}/pq$. The term $(1 - \cos\theta_{\mathbf{pq}})$ stems from the structure of the weak current [2]. This can be seen from the limit where completely relativistic neutrinos are traveling in the same direction along the same spacetime path. In this limit $1 - \cos\theta_{\mathbf{pq}} = 0$ and neutrinos never forward scatter on one another. Obviously, for the homogeneous and isotropic neutrino distribution functions characteristic of the early universe, $\cos\theta_{\mathbf{pq}}$ averages to zero and the ensemble average of $(1 - \cos\theta_{\mathbf{pq}})$ is unity. In the supernova environment the term $(1 - \cos\theta_{\mathbf{pq}})$ will be largest close to the neutron star, where the neutrino trajectories can intersect at

high angles. At sufficiently large radii above the neutron star, the neutrino-neutrino forward-scattering contribution to the flavor evolution Hamiltonian will scale as r^{-4}. As the neutrino-electron forward-scattering contribution will scale roughly as r^{-3}, it may be dominated by the neutrino-neutrino forward-scattering contribution at small to moderate distances from the neutron star.

In matrix form, the neutrino flavor evolution equation in the flavor basis is

$$i\frac{\partial\Psi_f}{\partial t} = \left[\left(p + \frac{m_1^2 + m_2^2}{4p} + \frac{A}{2} + \alpha_\nu\right)\hat{I} + \frac{1}{2}\right.$$
$$\left.\times\begin{pmatrix} A + B - \Delta\cos2\theta & \Delta\sin2\theta + B_{e\tau} \\ \Delta\sin2\theta + B_{\tau e} & \Delta\cos2\theta - A - B \end{pmatrix}\right]\Psi_f,$$
$$(31)$$

where we have separated the Hamiltonian into a traceless term and a term proportional to the identity matrix. The latter term gives only an overall phase to the neutrino states, and is therefore unimportant in neutrino flavor conversion. In the above equation, $\Delta \equiv \delta m^2/2E_\nu$, and α_ν, B, and $B_{e\tau}$ ($B_{\tau e} = B_{e\tau}^\dagger$) are the potentials from neutrino-neutrino forward scattering. Specifically,

$$\alpha_\nu = \frac{\sqrt{2}}{2}G_F \int (1 - \cos\theta_{\mathbf{pq}})([\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)]_{ee}$$
$$+ [\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)]_{\tau\tau})d^3\mathbf{q}, \qquad (32)$$

$$B = \sqrt{2}G_F \int (1 - \cos\theta_{\mathbf{pq}})([\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)]_{ee}$$
$$- [\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)]_{\tau\tau})d^3\mathbf{q}, \qquad (33)$$

$$B_{e\tau} = 2\sqrt{2}G_F \int (1 - \cos\theta_{\mathbf{pq}})[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)]_{e\tau}d^3\mathbf{q}, \quad (34)$$

where the matrix elements of the density operators are defined as

$$[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)]_{ee}d^3\mathbf{q} \equiv \langle\nu_e|\hat{\rho}_{\mathbf{q}}(t)d^3\mathbf{q}|\nu_e\rangle$$
$$- \langle\bar{\nu}_e|\hat{\bar{\rho}}_{\mathbf{q}}(t)d^3\mathbf{q}|\bar{\nu}_e\rangle, \qquad (35)$$

$$[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)]_{\tau\tau}d^3\mathbf{q} \equiv \langle\nu_\tau|\hat{\rho}_{\mathbf{q}}(t)d^3\mathbf{q}|\nu_\tau\rangle$$
$$- \langle\bar{\nu}_\tau|\hat{\bar{\rho}}_{\mathbf{q}}(t)d^3\mathbf{q}|\bar{\nu}_\tau\rangle, \qquad (36)$$

$$[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)]_{e\tau}d^3\mathbf{q} \equiv \langle\nu_e|\hat{\rho}_{\mathbf{q}}(t)d^3\mathbf{q}|\nu_\tau\rangle$$
$$- \langle\bar{\nu}_e|\hat{\bar{\rho}}_{\mathbf{q}}(t)d^3\mathbf{q}|\bar{\nu}_\tau\rangle. \qquad (37)$$

The physical interpretation of these matrix elements is straightforward, even if the notation is cumbersome. For example, $[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)]_{ee}d^3\mathbf{q}$ gives the expectation value for the net ν_e number density in the pencil of momenta and directions $d^3\mathbf{q}$ centered on $\mathbf{q}$. Note that the off-diagonal matrix element vanishes and makes no contribution to $B_{e\tau}$ if neutrinos remain in their initial flavor states. This is

evident if we expand out the first term in Eq. (37):

$$\langle \nu_e | \hat{\rho}_{\mathbf{q}}(t) d^3\mathbf{q} | \nu_\tau \rangle = \sum_\alpha dn_{\nu_\alpha} \langle \nu_e | \Psi_{\nu_\alpha} \rangle \langle \Psi_{\nu_\alpha} | \nu_\tau \rangle. \quad (38)$$

In the above equation, one or the other amplitude in the sum on the right-hand side will be zero unless some neutrino flavor transformation has occurred at the time t when this matrix element is evaluated.

For real $B_{e\tau} = B_{\tau e}$, it is convenient to define the effective mixing angle θ_M in medium by

$$\cos 2\theta_M(t) \equiv (\Delta \cos 2\theta - A - B)/\Delta_{\mathrm{eff}}, \quad (39)$$

$$\sin 2\theta_M(t) \equiv (\Delta \sin 2\theta + B_{e\tau})/\Delta_{\mathrm{eff}}, \quad (40)$$

where

$$\Delta_{\mathrm{eff}} = \sqrt{(\Delta \cos 2\theta - A - B)^2 + (\Delta \sin 2\theta + B_{e\tau})^2}. \quad (41)$$

With the term proportional to the identity matrix dropped, Eq. (31) can be transformed to the instantaneous energy basis to give

$$i\frac{\partial \Psi_M}{\partial t} = \begin{bmatrix} -\Delta_{\mathrm{eff}}/2 & -i\dot{\theta}_M(t) \\ i\dot{\theta}_M(t) & \Delta_{\mathrm{eff}}/2 \end{bmatrix} \Psi_M, \quad (42)$$

where $\dot{\theta}_M(t) = d\theta_M/dt$. In the limit where $|\dot{\theta}_M(t)| \ll \Delta_{\mathrm{eff}}/2$, Eq. (42) becomes two decoupled equations and flavor amplitude evolution is adiabatic. The flavor evolution equations and the corresponding effective mixing angle $\bar{\theta}_M$ in medium for antineutrinos can be obtained from those for neutrinos by replacing A, B, and $B_{e\tau}$ in the latter with $-A$, $-B$, and $-B_{e\tau}$, respectively.

The condition $|\dot{\theta}_M(t)| \ll \Delta_{\mathrm{eff}}/2$ for adiabatic neutrino flavor evolution is most stringent when Δ_{eff} reaches the minimum value $|\Delta \sin 2\theta + B_{e\tau}|$ at a MSW resonance corresponding to

$$\Delta \cos 2\theta = A + B. \quad (43)$$

At resonance, the effective in-medium mixing angle is $\theta_M(t_{\mathrm{res}}) = \pi/4$ and mixing is maximal with $\sin^2 2\theta_M = 1$. We can define an adiabaticity parameter

$$\gamma \equiv \frac{\Delta_{\mathrm{eff}}(t_{\mathrm{res}})}{2|\dot{\theta}_M(t_{\mathrm{res}})|} = \frac{(\Delta \sin 2\theta + B_{e\tau})^2}{\Delta \cos 2\theta} \mathcal{H}, \quad (44)$$

where

$$\mathcal{H} \equiv \left| \frac{V}{\dot{V}} \right|_{\mathrm{res}} = \left| \frac{A + B}{\dot{A} + \dot{B}} \right|_{\mathrm{res}} \quad (45)$$

is the scale height for the total potential $V \equiv A + B$ at resonance with $\dot{V} = dV/dt$. We can gain more insight into the adiabaticity parameter by further defining a resonance region corresponding to $1/2 \leq \sin^2 2\theta_M \leq 1$. In this region the change in V around the resonance value $\Delta \cos 2\theta$ is $\delta V = |\Delta \sin 2\theta + B_{e\tau}|$, so the width of this region is

$$(\delta t)_{\mathrm{res}} = \frac{\delta V}{|\dot{V}|_{\mathrm{res}}} = \frac{|\Delta \sin 2\theta + B_{e\tau}|}{\Delta \cos 2\theta} \mathcal{H}. \quad (46)$$

As the oscillation length at resonance is

$$L_{\mathrm{res}} = \frac{2\pi}{\Delta_{\mathrm{eff}}(t_{\mathrm{res}})} = \frac{2\pi}{|\Delta \sin 2\theta + B_{e\tau}|}, \quad (47)$$

we have

$$\gamma = 2\pi \frac{(\delta t)_{\mathrm{res}}}{L_{\mathrm{res}}}. \quad (48)$$

In summary, large $B_{e\tau}$ increases γ in two ways: (1) by increasing the resonance width $(\delta t)_{\mathrm{res}}$ and; (2) by decreasing the oscillation length L_{res}.

Clearly, neutrino flavor evolution will be adiabatic for $\gamma \gg 1$. For the small effective vacuum mixing angle $\theta \ll 1$ of interest here, neutrino flavor conversion will be complete in this limit. For arbitrary γ, the probability of neutrino flavor conversion after propagation through resonance is well approximated by $1 - P_{\mathrm{LZ}}$, where $P_{\mathrm{LZ}} = \exp(-\pi\gamma/2)$ is the Landau-Zener probability for a neutrino to jump from one energy eigenstate to the other in traversing the resonance region.

D. Neutrino potentials in the adiabatic limit

Because of the cross terms in the single neutrino density operator in Eq. (17), $B_{e\tau}$ is generally complex. If these cross terms are unimportant, then both B and $B_{e\tau}$ are real and their expressions in Eqs. (33) and (34) can be simplified as

$$B = -\sqrt{2}G_{\mathrm{F}} \sum_\alpha \int (1 - \cos\theta_{\mathbf{pq}})[\cos 2\theta_M(1 - 2|a_{1\alpha}|^2)dn_{\nu_\alpha} - \cos 2\bar{\theta}_M(1 - 2|\bar{a}_{1\alpha}|^2)dn_{\bar{\nu}_\alpha}], \quad (49)$$

$$B_{e\tau} = \sqrt{2}G_{\mathrm{F}} \sum_\alpha \int (1 - \cos\theta_{\mathbf{pq}})[\sin 2\theta_M(1 - 2|a_{1\alpha}|^2)dn_{\nu_\alpha} - \sin 2\bar{\theta}_M(1 - 2|\bar{a}_{1\alpha}|^2)dn_{\bar{\nu}_\alpha}]. \quad (50)$$

As mentioned in Sec. II B, the cross terms in Eq. (17) vanish if neutrino states evolve adiabatically. In this limit, the above expressions of B and $B_{e\tau}$ can be simplified further. As the electron and neutrino number densities at the neutrino sphere are far above those satisfying the resonance condition, ν_e and ν_τ are born essentially as the energy eigenstates $|\nu_2\rangle$ and $|\nu_1\rangle$, respectively. For adiabatic evolution, $|a_{1e}|^2 \approx 0$ and $|a_{1\tau}|^2 \approx 1$ for all subsequent time t. For antineutrinos, adiabatic evolution gives $|\bar{a}_{1e}|^2 \approx 1$ and $|\bar{a}_{1\tau}|^2 \approx 0$. Thus, we have

$$B \approx \sqrt{2}G_{\mathrm{F}} \int (1 - \cos\theta_{\mathbf{pq}})[(dn_{\nu_\tau} - dn_{\nu_e})\cos2\theta_M$$

$$+ (dn_{\bar{\nu}_\tau} - dn_{\bar{\nu}_e})\cos2\bar{\theta}_M], \qquad (51)$$

$$B_{e\tau} \approx \sqrt{2}G_{\mathrm{F}} \int (1 - \cos\theta_{\mathbf{pq}})[(dn_{\nu_e} - dn_{\nu_\tau})\sin2\theta_M$$

$$+ (dn_{\bar{\nu}_e} - dn_{\bar{\nu}_\tau})\sin2\bar{\theta}_M]. \qquad (52)$$

III. FLAVOR MIXING WITH LARGE OFF-DIAGONAL POTENTIAL

We now focus on adiabatic neutrino flavor evolution, for which the potentials from neutrino-neutrino forward scat-

tering are given by Eqs. (51) and (52). Our main concern is the effects of these potentials on neutrino flavor evolution in supernovae. In Sec. II C we have outlined this evolution for a specific neutrino as it propagates outward from the neutrino sphere. At a given radius r with potentials A and B, the resonance condition in Eq. (43) will be met for a particular neutrino energy E_{res}, i.e.,

$$\frac{\delta m^2}{2E_{\mathrm{res}}}\cos2\theta = A + B. \qquad (53)$$

As B and $B_{e\tau}$ at a given radius involve integration of $\cos2\theta_M$, $\cos2\bar{\theta}_M$, $\sin2\theta_M$, and $\sin2\bar{\theta}_M$ over the neutrino energy distribution functions, it is convenient to write

$$\cos2\theta_M = \frac{1 - E_\nu/E_{\mathrm{res}}}{\sqrt{(1 - E_\nu/E_{\mathrm{res}})^2 + [\tan2\theta + (E_\nu/E_{\mathrm{res}})(2E_{\mathrm{res}}B_{e\tau})/(\delta m^2\cos2\theta)]^2}}, \qquad (54)$$

$$\sin2\theta_M = \frac{\tan2\theta + (E_\nu/E_{\mathrm{res}})(2E_{\mathrm{res}}B_{e\tau})/(\delta m^2\cos2\theta)}{\sqrt{(1 - E_\nu/E_{\mathrm{res}})^2 + [\tan2\theta + (E_\nu/E_{\mathrm{res}})(2E_{\mathrm{res}}B_{e\tau})/(\delta m^2\cos2\theta)]^2}}, \qquad (55)$$

$$\cos2\bar{\theta}_M = \frac{1 + E_\nu/E_{\mathrm{res}}}{\sqrt{(1 + E_\nu/E_{\mathrm{res}})^2 + [\tan2\theta - (E_\nu/E_{\mathrm{res}})(2E_{\mathrm{res}}B_{e\tau})/(\delta m^2\cos2\theta)]^2}}, \qquad (56)$$

$$\sin2\bar{\theta}_M = \frac{\tan2\theta - (E_\nu/E_{\mathrm{res}})(2E_{\mathrm{res}}B_{e\tau})/(\delta m^2\cos2\theta)}{\sqrt{(1 + E_\nu/E_{\mathrm{res}})^2 + [\tan2\theta - (E_\nu/E_{\mathrm{res}})(2E_{\mathrm{res}}B_{e\tau})/(\delta m^2\cos2\theta)]^2}}. \qquad (57)$$

Let us now consider the limits of the above expressions for the in-medium mixing angles when $\theta \ll 1$ and $B_{e\tau}$ is positive and so large that the second terms in the square root of these expressions dominate the first terms. In this limit we have

$$\cos2\theta_M \to 0, \qquad (58)$$

$$\sin2\theta_M \to 1, \qquad (59)$$

$$\cos2\bar{\theta}_M \to 0, \qquad (60)$$

$$\sin2\bar{\theta}_M \to -1, \qquad (61)$$

for which both neutrinos and antineutrinos have maximal in-medium mixing with

$$\theta_M \to \frac{\pi}{4}, \qquad (62)$$

$$\bar{\theta}_M \to \frac{3\pi}{4}. \qquad (63)$$

A large negative $B_{e\tau}$ clearly changes the signs of the limits in Eqs. (58)–(61), and the in-medium mixing angles in this case are $\theta_M \to 3\pi/4$ and $\bar{\theta}_M \to \pi/4$.

In general, we see that large in-medium mixing will occur simultaneously for neutrinos and antineutrinos over

a broad range of energies if adiabatic flavor evolution results in

$$|B_{e\tau}| \gg \frac{\delta m^2}{2E_{\mathrm{res}}}\cos2\theta \qquad (64)$$

at some radius above the neutrino sphere. Using Eq. (53), we can rewrite the above equation as $|B_{e\tau}| \gg A + B$. As $\cos2\theta_M \to 0$ and $\cos2\bar{\theta}_M \to 0$ when this is achieved, Eq. (51) gives $B \to 0$. Therefore, Eq. (64) reduces to

$$|B_{e\tau}| \gg A. \qquad (65)$$

Note that even a $|B_{e\tau}|$ only as large as $(\delta m^2/2E_{\mathrm{res}}) \times \cos2\theta$ already has important effects on the in-medium mixing of neutrinos and antineutrinos. While a neutrino with resonance energy $E_\nu = E_{\mathrm{res}}$ has maximal in-medium mixing independent of $B_{e\tau}$, the energy range over which neutrinos have large in-medium mixing with $1/2 \leq \sin^2 2\theta_M \leq 1$ is strongly affected by $B_{e\tau}$. For $B_{e\tau} = 0$, this energy range corresponds to $E_{\mathrm{res}}(1 - \tan2\theta) \leq E_\nu \leq E_{\mathrm{res}}(1 + \tan2\theta)$, which is very narrow for $\theta \ll 1$. In contrast, for example, with $B_{e\tau} = (\delta m^2/2E_{\mathrm{res}})\cos2\theta$, all neutrinos with $E_\nu \geq E_{\mathrm{res}}/2$ have $1/2 \leq \sin^2 2\theta_M \leq 1$ even for $\theta \ll 1$. Furthermore, $B_{e\tau}$ also affects the in-medium mixing for antineutrinos, which is strongly suppressed ($\sin^2 2\bar{\theta}_M \ll 1$) in the absence of neutrino-neutrino for-

ward scattering. For $B_{e\tau} = (\delta m^2/2E_{res})\cos 2\theta$ and $\theta \ll 1$, antineutrinos with $E_\nu \gtrsim E_{res}$ have substantial in-medium mixing with $1/5 \leq \sin^2 2\bar{\theta}_M \leq 1/2$.

A. Towards a self-consistent solution with a large $B_{e\tau}$

Here we outline a possible self-consistent $B_{e\tau}$-dominant solution (BDS) which meets two criteria: (1) $|B_{e\tau}| \gg A$; and (2) adiabaticity, $\gamma \gg 1$. An immediate question is as follows: can adiabatic neutrino flavor evolution ever produce a large $B_{e\tau}$ as in Eq. (65)? Obviously, the answer is yes if one can demonstrate that this result is obtained under some conditions. Such demonstration requires following the flavor evolution of neutrinos with a wide range of energies covered by their energy distributions. As mentioned in Sec. II, this process may sound straightforward but turns out to be computationally difficult. On the other hand, if adiabatic flavor evolution can indeed give rise to a large $B_{e\tau}$, then the conditions required for this to occur must depend on the following: the neutrino mixing parameters δm^2 and $\sin^2 2\theta$, the profile of electron number density that gives the potential A, and the neutrino luminosities and energy distribution functions that are related to the potentials B and $B_{e\tau}$. Our goal here is to examine these dependences. In so doing, we will not be able to answer the question posed at the beginning of this paragraph, but we will be able to provide a range of conditions that can guide future numerical calculations in search of a complete solution for neutrino flavor mixing.

We start with the basic input for our discussion. As explained in Sec. II A, the mixing parameters of interest here are $\delta m^2 \approx 3 \times 10^{-3}$ eV^2 and $\sin^2 2\theta < 0.1$. To characterize the potential A, we need the electron number density $n_e = Y_e n_b$. We note that the envelope above the post-core-bounce neutron star can be approximated as a quasistatic configuration with a constant entropy per baryon S in the gravitational field of the neutron star. In this case, the enthalpy per baryon, TS, is roughly the gravitational binding energy of a baryon, so that the temperature T scales with the radius as

$$T \approx \frac{M_{NS} m_p}{m_{Pl}^2} S^{-1} r^{-1}, \qquad (66)$$

where $m_{Pl} \approx 1.221 \times 10^{22}$ MeV is the Planck mass, m_p is the proton mass, and M_{NS} is the neutron star mass. At late times relevant for r-process nucleosynthesis, the environment above the neutron star is radiation dominated, so

$$S \approx \frac{2\pi^2}{45} g_s \frac{T^3}{n_b} \qquad (67)$$

in units of Boltzmann constant k_B per baryon. In the above equation, g_s is the statistical weight in relativistic particles: $g_s \approx 11/2$ when $e^\pm$ pairs are abundant and $g_s \approx 2$ otherwise. Combining Eqs. (66) and (67), we obtain the run of baryon number density for the r-process epoch as

$$n_b \approx \frac{2\pi^2}{45} g_s \left(\frac{M_{NS} m_p}{m_{Pl}^2}\right)^3 S^{-4} r^{-3}. \qquad (68)$$

The potential A is given by

$$A = \sqrt{2} G_F Y_e n_b \approx \frac{2\sqrt{2}\pi^2}{45} g_s Y_e G_F \left(\frac{M_{NS} m_p}{m_{Pl}^2}\right)^3 S^{-4} r^{-3}$$

$$\approx (5.2 \times 10^{-13} \text{ MeV}) g_s Y_e \left(\frac{M_{NS}}{1.4 M_\odot}\right)^3 S_{100}^{-4} r_6^{-3}, \qquad (69)$$

where S_{100} is S in units of $100 k_B$ per baryon and r_6 is r in units of 10^6 cm. The r-process epoch corresponds to a time post-core-bounce $t_{pb} > 3$s. This is a relatively long time after core bounce, at least compared with the time scale of the shock reheating epoch and the time scale for evolution of neutrino emission characteristics such as luminosities and average energies. The potential A in Eq. (69) can also be used to describe crudely the *shocked* regions of the envelope above the core in the shock reheating epoch, $t_{pb} < 1$s, if we take $g_s \sim 1$ and employ a low entropy [6].

To evaluate B and $B_{e\tau}$, we need the differential number density of each neutrino species at radius $r > R_\nu$ above the neutrino sphere. The differential ν_α number density in the absence of flavor evolution is given by Eq. (22), which depends on the luminosity L_{ν_α} and the average energy $\langle E_{\nu_\alpha} \rangle$. For some illustrative numerical estimates we will assume that all neutrino species have the same luminosity,

$$L_\nu \equiv L_{\nu_e} = L_{\bar{\nu}_e} = L_{\nu_\tau} = L_{\bar{\nu}_\tau}, \qquad (70)$$

and take the average neutrino energies to be

$$\langle E_{\nu_e} \rangle = 10 \text{ MeV}, \qquad \langle E_{\bar{\nu}_e} \rangle = 15 \text{ MeV},$$
$$\langle E_{\nu_\tau} \rangle = \langle E_{\bar{\nu}_\tau} \rangle = 27 \text{ MeV}. \qquad (71)$$

Assuming that adiabatic flavor evolution up to some radius $r > R_\nu$ results in a BDS described by the criterion in Eq. (65), we now examine the implications of this criterion for supernova conditions. For definiteness, we discuss the case of a large positive $B_{e\tau}^{BDS}$, which gives $\sin 2\theta_M \to 1$ and $\sin 2\bar{\theta}_M \to -1$. In this case, Eq. (52) gives

$$B_{e\tau}^{BDS} \approx \sqrt{2} G_F \int (1 - \cos\theta_{pq})(dn_{\nu_e} - dn_{\bar{\nu}_e}), \qquad (72)$$

where we have assumed that the luminosities and energy distribution functions for ν_τ and $\bar{\nu}_\tau$ are very nearly the same in the supernova environment. This is a good approximation because these species experience nearly identical interactions both in the dense environment of the core and in the more tenuous outer regions. For a radially propagating test neutrino, the intersecting angles of the background neutrinos, θ_{pq}, are coincident with the polar angle in the integration over $d\Omega_\nu$ for the test neutrino. Assuming that neutrinos of all flavors originate on the same neutrino sphere and using dn_{ν_e} and $dn_{\bar{\nu}_e}$ of the form in Eq. (22), $B_{e\tau}^{BDS}$ in Eq. (72) can be evaluated as

$$B_{e\tau}^{\text{BDS}} \approx \frac{\sqrt{2}G_{\text{F}}}{4\pi R_\nu^2}\left[1 - \sqrt{1 - R_\nu^2/r^2}\right]^2\left(\frac{L_{\nu_e}}{\langle E_{\nu_e}\rangle} - \frac{L_{\bar{\nu}_e}}{\langle E_{\bar{\nu}_e}\rangle}\right)$$

$$\approx (2.1 \times 10^{-10} \text{ MeV})R_{\nu6}^{-2}\left(1 - \sqrt{1 - R_{\nu6}^2/r_6^2}\right)^2$$

$$\times \left[\frac{L_{\nu_e52}}{\langle E_{\nu_e}\rangle/(10 \text{ MeV})} - \frac{L_{\bar{\nu}_e52}}{\langle E_{\bar{\nu}_e}\rangle/(10 \text{ MeV})}\right], \quad (73)$$

where $R_{\nu6} \equiv R_\nu/(10^6 \text{ cm})$, $L_{\nu_e52} \equiv L_{\nu_e}/(10^{52} \text{ ergs s}^{-1})$,

and $L_{\bar{\nu}_e52} \equiv L_{\bar{\nu}_e}/(10^{52} \text{ ergs s}^{-1})$. For $r \gg R_\nu \sim 10^6$ cm and the assumptions in Eqs. (70) and (71), we obtain

$$B_{e\tau}^{\text{BDS}} \approx (1.8 \times 10^{-11} \text{ MeV})\frac{L_{\nu52}R_{\nu6}^2}{r_6^4}. \quad (74)$$

Using Eqs. (69) and (73), we can rewrite the criterion $B_{e\tau}^{\text{BDS}} \gg A$ as

$$\frac{R_\nu^3/r^3}{(1 - \sqrt{1 - R_\nu^2/r^2})^2} \ll \frac{45}{8\pi^3}\left(\frac{m_{\text{Pl}}^2}{M_{\text{NS}}m_p}\right)^3\left(\frac{R_\nu S^4}{g_s Y_e}\right)\left(\frac{L_{\nu_e}}{\langle E_{\nu_e}\rangle} - \frac{L_{\bar{\nu}_e}}{\langle E_{\bar{\nu}_e}\rangle}\right)$$

$$\approx (398)\left(\frac{1.4 \text{ M}_\odot}{M_{\text{NS}}}\right)^3\left(\frac{R_{\nu6}S_{100}^4}{g_s Y_e}\right)\left[\frac{L_{\nu_e52}}{\langle E_{\nu_e}\rangle/(10 \text{ MeV})} - \frac{L_{\bar{\nu}_e52}}{\langle E_{\bar{\nu}_e}\rangle/(10 \text{ MeV})}\right]. \quad (75)$$

For $r \gg R_\nu$ and the assumptions in Eqs. (70) and (71), we obtain

$$r_6 \ll 33\left(\frac{1.4 \text{ M}_\odot}{M_{\text{NS}}}\right)^3\left(\frac{S_{100}^4 L_{\nu52}R_{\nu6}^2}{g_s Y_e}\right). \quad (76)$$

We have assumed adiabatic neutrino flavor evolution in the above discussion. Though the adiabaticity of the general flavor evolution can be ascertained only with a sophisticated numerical treatment, the BDS clearly will not be self-consistent if flavor evolution is not adiabatic at the radius where $B_{e\tau}^{\text{BDS}} \gg A$ is achieved. On the other hand, if we can show that at this radius the adiabaticity parameter for the neutrino with energy E_{res} satisfies $\gamma^{\text{BDS}} \gg 1$, then the BDS is more likely to be obtained. With $\mathcal{H} \approx |A/\dot{A}| \approx r/3$, this criterion [see Eq. (44)] can be rewritten as

$$\gamma^{\text{BDS}} \approx \frac{B_{e\tau}^2}{A}\mathcal{H}$$

$$\approx \frac{15}{16\sqrt{2}\pi^4}\left(\frac{m_{\text{Pl}}^2}{M_{\text{NS}}m_p}\right)^3\left(\frac{1 - \sqrt{1 - R_\nu^2/r^2}}{R_\nu/r}\right)^4\left(\frac{G_{\text{F}}S^4}{g_s Y_e}\right)$$

$$\times \left(\frac{L_{\nu_e}}{\langle E_{\nu_e}\rangle} - \frac{L_{\bar{\nu}_e}}{\langle E_{\bar{\nu}_e}\rangle}\right)^2 \gg 1, \quad (77)$$

which reduces to

$$\gamma^{\text{BDS}} \approx 10^7\left(\frac{1.4 \text{ M}_\odot}{M_{\text{NS}}}\right)^3\left(\frac{S_{100}^4 L_{\nu52}^2}{g_s Y_e}\right)\frac{R_{\nu6}^4}{r_6^4} \gg 1 \quad (78)$$

for $r \gg R_\nu$ and the assumptions in Eqs. (70) and (71).

The criteria in Eqs. (75) and (77) can be met in some regions with significant scale and duration above the neutron star during both the r-process and shock reheating epochs. For example, an r-process environment with modest entropy might have $Y_e \approx 0.4$, $R_{\nu6} \approx 1$, $g_s \approx 11/2$, $S_{100} \approx 1.5$, and $L_{\nu52} \approx 0.1$. For these parameters Eqs. (75) and (77) would give $r_6 \ll 8$ and 22, respectively, so the BDS may be obtained over an extended region above the neutrino sphere. For a higher entropy, $S_{100} = 2.5$, but

with all the other parameters remaining the same, Eqs. (75) and (77) would give $r_6 \ll 60$ and 40, respectively. We can put these limits in perspective by noting the temperature at which salient events or processes occur above the neutron star. The radius corresponding to a temperature T_9 (measured in units of 10^9 K) is very roughly $r_6 \approx 22.5/(T_9 S_{100})$ [see Eq. (66)]. Weak freeze-out, where the neutron-to-proton ratio is set, occurs at $T_9 \sim 10$. The neutron capture regime in the r process is typically further out, occurring between $T_9 \approx 3$ and $T_9 \approx 1$. Therefore, the limits on the radius discussed above are so generous that maximal in-medium mixing for both neutrinos and antineutrinos associated with the BDS could affect important weak interaction processes in the envelope, the r process, and the neutrino signal.

Taking $Y_e = 0.4$, $g_s = 11/2$, $M_{\text{NS}} = 1.4 \text{ M}_\odot$, $R_\nu = 10$ km, $L_{\nu_e} = L_{\bar{\nu}_e} = L_\nu$, $\langle E_{\nu_e}\rangle = 10$ MeV, and $\langle E_{\bar{\nu}_e}\rangle = 15$ MeV, we use Eq. (75) to calculate the combinations of L_ν and S for which the criterion $B_{e\tau}^{\text{BDS}} \gg A$ can be met below a fixed radius. The results are shown in Fig. 1 as contours labeled by the limiting radius. Except for the region corresponding to the larger values of the limiting radius, these results are generally more stringent than those from the criterion $\gamma^{\text{BDS}} \gg 1$. Compared with the r-process regime at later times, the shock reheating epoch is characterized by much higher neutrino luminosities. In general, both L_{ν_e} and $L_{\bar{\nu}_e}$ are $\sim 10^{52}$ ergs s^{-1}. Taking $Y_e = 0.35$, $g_s = 1.5$, and $R_\nu = 40$ km (other parameters remaining the same as for Fig. 1), we present in Fig. 2 the constraints on L_ν and S for which $B_{e\tau}^{\text{BDS}} \gg A$ can be met below various radii during the shock reheating epoch. Based on these results, both shock reheating and the neutrino signal could be affected by maximal neutrino flavor mixing [5,21] if there were a hierarchy of neutrino energies at this epoch.

Note that average ν_e and $\bar{\nu}_e$ energies may be quite similar during much of the shock reheating epoch, but the luminosities for ν_e can be significantly larger than those for $\bar{\nu}_e$. This is especially true for shock breakout through the neutrino sphere, the so-called neutronization
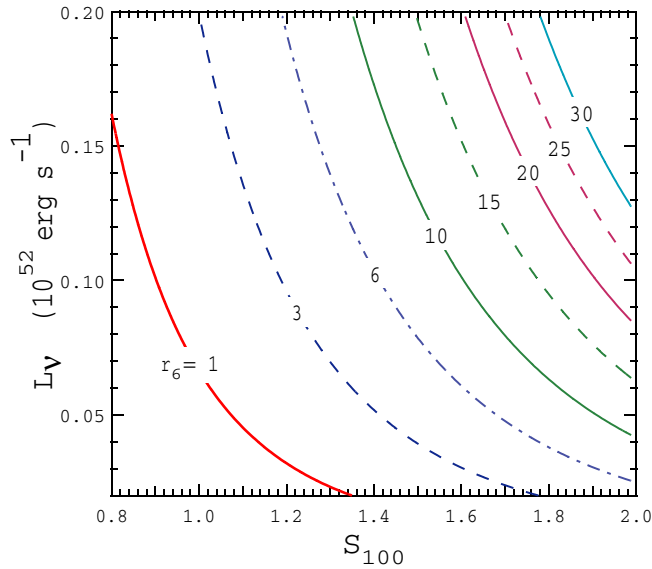
FIG. 1 (color online). Contours of limiting radius (in units of 10^6 cm) beneath which $B_{e\tau}^{\mathrm{BDS}} \gg A$ may be obtained. Except for the region corresponding to the larger values of the limiting radius, these results are generally more stringent than those from $\gamma^{\mathrm{BDS}} \gg 1$. The chosen range of parameters is meant to be characteristic of the r-process epoch. The horizontal axis is entropy in units of $100 k_B$ per baryon, while the vertical axis is neutrino luminosity L_ν in units of 10^{52} ergs s^{-1}. Here we take $Y_e = 0.4$, $g_s = 11/2$, $M_{\mathrm{NS}} = 1.4\,M_\odot$, and $R_\nu = 10$ km. We also assume that $L_{\nu_e} = L_{\bar{\nu}_e} = L_\nu$, $\langle E_{\nu_e} \rangle = 10$ MeV, $\langle E_{\bar{\nu}_e} \rangle = 15$ MeV, and that all mu and tau neutrinos and antineutrinos have identical luminosities and energy spectra.
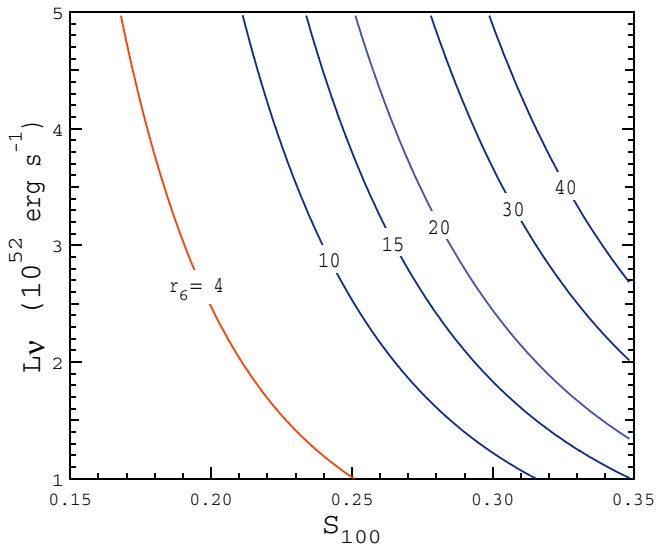

FIG. 2 (color online). Same as Fig. 1, except that now the conditions are meant to be representative of the shock reheating epoch. In this case we take $Y_e = 0.35$, $g_s = 1.5$, and $R_\nu = 40$ km.

burst. For a time span of ~ 10 ms we could have $L_{\nu_e,52} \sim 10$, while $L_{\bar{\nu}_e,52}$ is an order of magnitude smaller. Since neutrino flavor mixing in the coherent limit is a phase effect, the 10 ms duration of this high-luminosity burst may be long enough to establish the BDS. Neglecting $L_{\bar{\nu}_e}$ and taking $L_{\nu_e,52} = 10$, $\langle E_{\nu_e} \rangle = 10$ MeV, $R_{\nu 6} = 4$, $Y_e = 0.35$, $g_s = 1.5$, and $S_{100} = 0.15$, we find that $B_{e\tau}^{\mathrm{BDS}} \gg A$ may be obtained for $r_6 \ll 15$ [Eq. (75)]. This limit becomes $r_6 \ll 50$ if $S_{100} = 0.2$. For both cases the limit from Eq. (77) is much weaker. Therefore, the neutrino signal from the neutronization burst and the early shock reheating process could be affected by maximal neutrino flavor mixing associated with the BDS.

B. Is the BDS ever attained?

Achieving the BDS is dependent on a number of conditions, many of which are unlikely to strictly and generally obtain in astrophysical environments with high neutrino fluxes. However, the essence of the BDS is the dominance of the flavor off-diagonal potential and, in particular, $2E_\nu B_{e\tau} \gg \delta m^2 \cos 2\theta$. Since the measured neutrino mass-squared differences are small, it will not take a large flavor off-diagonal potential to force the system into something like the BDS.

As Eq. (38) shows, a necessary condition for $B_{e\tau}$ to be nonzero at some time/position is that *some* neutrinos must have transformed their flavors prior to arriving there. This can be problematic because in both the early universe and the post-shock supernova environment the region of interest has very high density. This will tend to suppress in-medium neutrino flavor mixing. However, consider a hydrodynamic flow carrying a fluid element away from the neutron star surface through the envelope described in Sec. II A. Both the density and the net neutrino fluxes experienced by this fluid element will decrease with the radius. At large enough radius the neutrino-electron potential will scale like $A \sim r^{-3}$, while the flavor-diagonal and off-diagonal neutrino-neutrino potentials will scale as r^{-4}. As a result, there may be some region where the neutrino-neutrino potentials dominate.

A neutrino propagating through a fluid element will experience resonance there if its energy is $E_{\mathrm{res}} = \delta m^2 \cos 2\theta / 2(A + B)$. Near the neutron star surface E_{res} will be extremely small. Further out, as the fluid element moves through the envelope (notation as in the last section), the resonance energy at radius r will be

$$E_{\mathrm{res}} \approx \frac{45}{4\sqrt{2}\pi^2} \left(\frac{m_{\mathrm{Pl}}^2}{M_{\mathrm{NS}} m_p} \right)^3 \frac{\delta m^2 \cos 2\theta S^4 r^3}{g_s G_{\mathrm{F}}(Y_e + Y_\nu^{\mathrm{eff}})} \tag{79}$$

$$\approx (2.85 \times 10^{-3}\ \mathrm{MeV}) \left(\frac{1.4\ M_\odot}{M_{\mathrm{NS}}} \right)^3 \left(\frac{\delta m^2 \cos 2\theta}{3 \times 10^{-3}\ \mathrm{eV}^2} \right)$$

$$\times \frac{S_{100}^4 r_6^3}{g_s(Y_e + Y_\nu^{\mathrm{eff}})}, \tag{80}$$

where we define the effective net number of neutrinos per baryon through $B = \sqrt{2}G_{\text{F}}n_{\text{b}}Y_{\nu}^{\text{eff}}$. Because δm^2 is small, the resonance energy also tends to be small at distances where neutrino fluxes are appreciable.

However, if neutrinos transform their flavors via a strict MSW evolution then B (and Y_{ν}^{eff}) will drop with the radius of the fluid element and eventually will be driven *negative* [6]. To see this, consider first example energy spectra for ν_e and ν_τ with average energies $\langle E_{\nu_\tau} \rangle > \langle E_{\nu_e} \rangle$ as they leave the neutrino sphere. In Fig. 3 we show Fermi-Dirac-type energy spectra for these species, taking the neutrino degeneracy parameter for both to be $\eta_{\nu_\alpha} = 3$, and taking average energies $\langle E_{\nu_e} \rangle = 10$ MeV and $\langle E_{\nu_\tau} \rangle = 27$ MeV. The actual supernova neutrino energy spectra may differ significantly from these, but they serve to illustrate general trends. Note that for our chosen spectral parameters, the ν_e population at lower energies is *larger* than the ν_τ population for comparable luminosity in the two neutrino species.

As our reference fluid element moves out to larger r, the resonance energy will also increase. It could increase significantly if $|A + B| \rightarrow 0$. If neutrino flavor conversion in the channel $\nu_e \rightleftharpoons \nu_\tau$ is efficient and complete, then at some point we will have the situation depicted in Fig. 4. Here B could be negative because we have swapped flavors at low neutrino energy and, for our chosen spectral parameters, the ν_e population now may be *smaller* than the ν_τ population. Furthermore, in this situation the material may be driven more neutron rich (lower Y_e) on account of the now altered competition between the processes in Eqs. (1) and (2). Eventually, of course, the resonance will sweep through the higher energy regions of the distribution
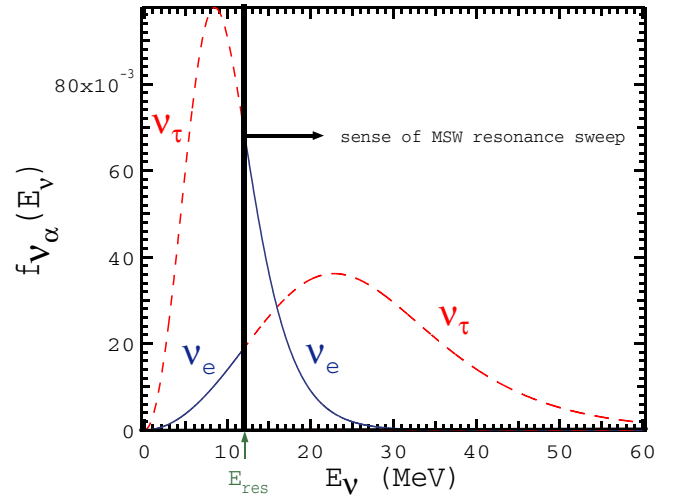
FIG. 4 (color online).   Neutrino energy distribution functions in a fluid element at some time t corresponding to position r. These have the same neutrino spectral parameters as in Fig. 3. Here, however, complete flavor conversion between ν_e and ν_τ has taken place from $E_\nu = 0$ to the MSW resonance energy at this time/position $E_{\text{res}} = 12$ MeV. As the fluid element moves out the resonance energy will increase and sweep from left to right through the neutrino energy distributions.

functions and the fluid element will move further out to where neutrino fluxes are lower.

The higher the resonance energy, the greater the neutrino population which has been appreciably mixed and, hence, the larger will be $B_{e\tau}$. The importance of this can be ascertained by comparing $B_{e\tau}^{\text{BDS}}$ to the vacuum term

$$\frac{\delta m^2 \cos 2\theta}{2E_\nu} \approx (1.5 \times 10^{-16} \text{ MeV})\left(\frac{\delta m^2 \cos 2\theta}{3 \times 10^{-3} \text{ eV}^2}\right)$$
$$\times \left(\frac{10 \text{ MeV}}{E_\nu}\right). \tag{81}$$

From Eq. (73) it is clear that this term could be substantially smaller than $B_{e\tau}^{\text{BDS}}$ if E_ν is a typical neutrino energy. Even if this is not true for $E_\nu = E_{\text{res}}$ at very high density where E_{res} is small, higher energy neutrinos and antineutrinos may experience significant in-medium mixing angles over a broad range of energy. Though not strictly our BDS, this may nevertheless approximate it.

Previous numerical simulation work on neutrino flavor evolution in the supernova environment may offer only limited guidance here. The simulation in Ref. [6] made the same 2×2 and one-dimensional approximations as we make here. (By "one-dimensional" we mean that flavor histories on neutrino trajectories of any polar angle are taken to be the same as a radially directed path for the same lapse of affine parameter along these trajectories.) Additionally, the work in Ref. [6] employed the density profiles and neutrino fluxes of the Mayle & Wilson late time supernova models and it adopted a range for δm^2
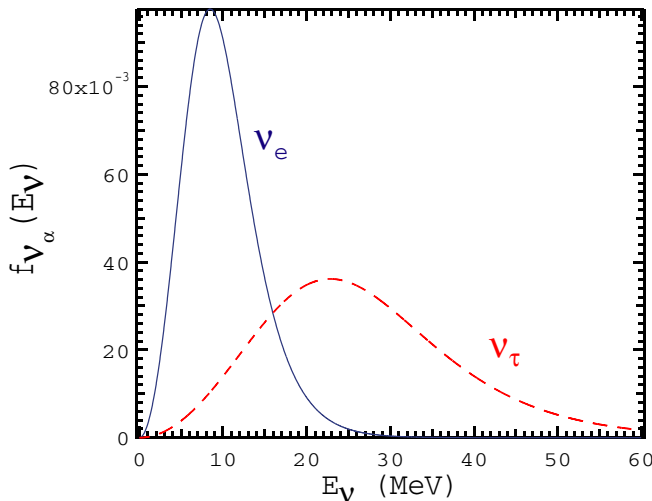

FIG. 3 (color online).   Example of normalized energy distribution functions for $\alpha = e, \tau$ neutrinos at the neutrino sphere in the supernova environment. Here we take $\langle E_{\nu_e} \rangle = 10$ MeV and $\langle E_{\nu_\tau} \rangle = 27$ MeV and the neutrino degeneracy parameter for both flavors to be $\eta_{\nu_\alpha} = 3$.

which is now known to be unphysically large for active-active neutrino evolution. Both of these features combined to produce only minimal effects from rather small values of $B_{e\tau}$.

Likewise, the numerical simulation of Ref. [10] considered one-dimensional, 2×2 neutrino flavor evolution with an unphysically large mass-squared difference. The conclusions in this work regarding real supernovae are suspect because (1) the large δm^2 used would in reality demand the incorporation of sterile neutrinos which mix significantly with active neutrinos and this was left out; and (2) the feedback of neutrino flavor conversion on Y_e was not correctly modeled since effects such as the threshold (neutron-proton mass difference) and weak magnetism corrections [9] were neglected in calculating the rates for the processes in Eqs. (1) and (2) [8]. However, this numerical simulation was the first to follow neutrino phases in detail in this environment. Synchronization of large amplitude neutrino flavor oscillations was seen. This behavior is at least qualitatively like some aspects of the BDS, especially as regards significant in-medium mixing.

Though the conditions for establishment of the BDS are manifest in many regions of the post-shock supernova environment, it has not been seen unambiguously in simulations to date. However, there is considerable room for improvement in the sophistication of these simulations. For example, flavor evolution histories on different neutrino trajectories need to be followed in detail, including all the effects of quantum entanglement. The role of density fluctuations [22] in getting some neutrino conversion going at high density also needs to be investigated. Likewise, legitimate three-neutrino mixing of neutrinos and antineutrinos must be followed. Finally, the effects of neutrino mixing on neutrino transport in the neutron star core may be important and recent formulations [23,24] of this problem represent significant progress.

C. The ephemeral nature of the BDS

Changing neutrino luminosities and fluxes and changing matter density will quickly lead to the development of complex amplitudes in the unitary transformation between the neutrino mass/energy and flavor bases which, in turn, will lead to complex potentials. This will signal the end of the strict validity of our particular BDS discussed above. However, it may not signal the immediate end of appreciable in-medium mixing among the flavors of neutrinos and antineutrinos.

If we ride along with a fluid element being driven from the neutron star's surface by heating we will see a local falloff in matter density and neutrino fluxes and, hence, a decrease in neutrino-electron and neutrino-neutrino forward-scattering-induced potentials in this Lagrangian frame. What is the effect of this time dependence on in-medium flavor mixing? Using the flavor-basis evolution equation [Eq. (31)] and ignoring the term proportional to

the identity we can find a second order equation for, e.g., $a_{e\alpha}$, the amplitude for a neutrino of initial flavor α to be a ν_e:

$$\ddot{a}_{e\alpha} + \omega^2 a_{e\alpha} = \left(\frac{\dot{B}_{e\tau}}{B_{e\tau} + \Delta \sin 2\theta}\right)\dot{a}_{e\alpha}. \qquad (82)$$

Here the dots over quantities denote time derivatives and

$$\omega^2 = \frac{1}{4}\Bigg[|B_{e\tau} + \Delta \sin 2\theta|^2 + \beta^2 + 2i\dot{\beta} \\ - 2i\frac{\beta \dot{B}_{e\tau}}{B_{e\tau} + \Delta \sin 2\theta}\Bigg], \qquad (83)$$

with $\beta \equiv A + B - \Delta \cos 2\theta$ and $\dot{\beta} = \dot{A} + \dot{B}$.

Our BDS corresponds to $\omega \approx |B_{e\tau}|/2$ and $\ddot{a}_{e\alpha} + \omega^2 a_{e\alpha} = 0$. In this case we have $a_{ee} = a_{\tau e} = \exp(-i\omega t)/\sqrt{2}$ and $a_{e\tau} = -a_{\tau\tau} = -\exp(i\omega t)/\sqrt{2}$. Likewise, we have $\bar{a}_{ee} = \bar{a}_{\tau e} = -\exp(i\omega t)/\sqrt{2}$ and $\bar{a}_{e\tau} = -\bar{a}_{\tau\tau} = \exp(-i\omega t)/\sqrt{2}$. If we employ these solutions in the general flavor-basis form for the off-diagonal potential,

$$B_{e\tau} = 2\sqrt{2}G_F \sum_\alpha \int (1 - \cos\theta_{\mathbf{pq}}) \\ \times [dn_{\nu_\alpha} a_{e\alpha} a_{\tau\alpha}^* - dn_{\bar{\nu}_\alpha} \bar{a}_{e\alpha} \bar{a}_{\tau\alpha}^*], \qquad (84)$$

we will recover the BDS form for this [cf. Eq. (72)] discussed above:

$$B_{e\tau}^{\text{BDS}} \approx \sqrt{2}G_F \int (1 - \cos\theta_{\mathbf{pq}})[(dn_{\nu_e} - dn_{\bar{\nu}_e}) \\ - (dn_{\nu_\tau} - dn_{\bar{\nu}_\tau})]. \qquad (85)$$

However, once we take into account the change of the potentials with time, amplitudes will quickly acquire a nonsinusoidal time dependence which will lead to the development of potentials with imaginary components. With complex potentials we will lose a key assumption used in obtaining the BDS of Eq. (85). Flavor evolution from that point on will be complicated, but there is nothing in the evolution equations that demands an immediate return to medium-suppressed flavor mixing for most neutrino energies.

IV. CONCLUSION

We have investigated the complicated problem of 2×2 coherent neutrino flavor evolution in the limit of large flavor off-diagonal neutrino-neutrino forward-scattering potential in the post-shock supernova environment. We have identified a simple solution/limit in this problem. This solution (BDS) is governed by a dominant off-diagonal potential. This constitutes a viable solution only under a number of restrictive assumptions, but it is evident that even a rough facsimile to this solution will retain key principal features of the BDS. These include maximal or near-maximal in-medium mixing angles for both neutrinos and antineutrinos over broad ranges of energies. These

features are very different from conventional neutrino flavor amplitude evolution with the MSW effect.

Indeed, it has been generally thought that the small values of the neutrino mass-squared difference among the active neutrinos preclude significant effects from medium-enhanced neutrino flavor conversion in the dense environment above the neutron star in supernova models. This *is* largely the case for conventional MSW neutrino evolution. It need not be the case when neutrino-neutrino forward-scattering potentials are large. Indeed, we have outlined above how the neutrino potentials may conspire to engineer significant neutrino/antineutrino transformation even when δm^2 values are small.

We have identified regions and plausible conditions in the post-bounce and post-shock supernova environment where the BDS, or something approximating it, could be obtained. We speculate that early universe scenarios with significant lepton numbers may also provide conditions favorable for the BDS solution to reign. It is not yet clear that either of these venues provides a clear and compelling evolutionary path into the BDS regime. However, numerical simulations have provided hints that something like the BDS may occur in these environments. A central question that we leave for the computational community is whether, and/or to what extent, the BDS is attained.

The stakes may be high. If neutrino energy spectra or fluxes for different neutrino flavors are appreciably different at any point in the ~ 20 s time frame following core bounce, then neutrino and antineutrino mixing could affect shock reheating physics, conditions in the neutrino-driven wind, and r-process nucleosynthesis scenarios associated with slow outflows. Of course, the neutrino signal could be affected by any kind of neutrino/antineutrino flavor mixing. The effect we point out here, if it is ever realized deep in the supernova envelope, could appreciably alter the emergent neutrino energy spectra and fluxes over those calculated via conventional MSW evolution alone.

Finally, our considerations extend to any environment where neutrino fluxes are appreciable and where neutrino flavor mixing may have important consequences for the neutron-to-proton ratio and/or energetics and dynamics. Fireball models for gamma-ray bursts sited in the vicinity of a hot or collapsed compact object are a case in point. In this environment, as in supernovae and the early universe, we are hard pressed to follow numerically the flavor evolution of neutrinos/antineutrinos. It is unsatisfactory that this remains true even in the face of the tremendous strides in experimental neutrino physics which have given us the neutrino mass-squared differences and most of the vacuum mixing angles.

[1] S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42,** 1441 (1985) [Sov. J. Nucl. Phys. **42,** 913 (1985)]; L. Wolfenstein, Phys. Rev. D **17,** 2369 (1978).

[2] G. M. Fuller, R. W. Mayle, J. R. Wilson, and D. N. Schramm, Astrophys. J. **322,** 795 (1987).

[3] G. Sigl and G. Raffelt, Nucl. Phys. **B406,** 423 (1993).

[4] J. Pantaleone, Phys. Lett. B **342,** 250 (1995).

[5] G. M. Fuller, B. S. Meyer, R. W. Mayle, and J. R. Wilson, Astrophys. J. **389,** 517 (1992).

[6] Y.-Z. Qian and G. M. Fuller, Phys. Rev. D **51,** 1479 (1995).

[7] A. Mezzacappa and S. Bruenn, in *Proceeding of the Second International Workshop on the Identification of Dark Matter*, edited by N. J. C. Spooner and V. Kudryavtsev (World Scientific, Singapore, 1999), p. 665.

[8] Y.-Z. Qian, G. M. Fuller, G. J. Mathews, R. Mayle, J. R. Wilson, and S. E. Woosley, Phys. Rev. Lett. **71,** 1965 (1993).

[9] C. J. Horowitz, Phys. Rev. D **65,** 043001 (2002); C. J. Horowitz and G. Li, Phys. Rev. Lett. **82,** 5198 (1999).

[10] S. Pastor and G. Raffelt, Phys. Rev. Lett. **89,** 191101 (2002); S. Pastor, G. Raffelt, and D. V. Semikoz, Phys. Rev. D **65,** 053011 (2002).

[11] R. D. Hoffman, S. E. Woosley, G. M. Fuller, and B. S. Meyer, Astrophys. J. **460,** 478 (1996).

[12] H. Back *et al.*, hep-ex/0412016.

[13] M. Savage, R. A. Malaney, and G. M. Fuller, Astrophys. J. **368,** 1 (1991).

[14] R. Foot and R. R. Volkas, Phys. Rev. Lett. **75,** 4350 (1995); M. Prakash, J. M. Lattimer, R. F. Sawyer, and R. R. Volkas, Annu. Rev. Nucl. Part. Sci. **51,** 295 (2001).

[15] A. D. Dolgov, S. H. Hansen, S. Pastor, S. T. Petcov, G. G. Raffelt, and D. V. Semikoz, Nucl. Phys. **B632,** 363 (2002); K. N. Abazajian, J. F. Beacom, and N. F. Bell, Phys. Rev. D **66,** 013008 (2002); Y. Y. Y. Wong, Phys. Rev. D **66,** 025015 (2002).

[16] K. N. Abazajian, Astropart. Phys. **19,** 303 (2003); P. Di Bari, Phys. Rev. D **65,** 043509 (2002); **67,** 127301 (2003).

[17] A. Friedland and C. Lunardini, Phys. Rev. D **68,** 013007 (2003); J. High Energy Phys. 10 (2003) 43.

[18] N. F. Bell, A. A. Rawlinson, and R. F. Sawyer, Phys. Lett. B **573,** 86 (2003).

[19] A. B. Balantekin and G. M. Fuller, Phys. Lett. B **471,** 195

(1999).

[20] D. O. Caldwell, G. M. Fuller, and Y.-Z. Qian, Phys. Rev. D **61**, 123005 (2000).

[21] R. Schirato and G. M. Fuller, astro-ph/0205390.

[22] R. F. Sawyer, Phys. Rev. D **42**, 3908 (1990); F. Loreti,

Y.-Z. Qian, G. M. Fuller, and A. B. Balantekin, Phys. Rev. D **52**, 6664 (1995).

[23] R. F. Sawyer, Phys. Rev. D **72**, 045003 (2005).

[24] P. Strack and A. Burrows, Phys. Rev. D **71**, 093004 (2005).