

Detecting a small perturbation through its non-Gaussianity

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A highly non-Gaussian cosmological perturbation with a flat spectrum has unusual stochastic properties. We show that they depend on the size of the box within which the perturbation is defined, but that for a typical observer the parameters defining the perturbation “run” to compensate for any change in the box size. Focusing on the primordial curvature perturbation, we show that an uncorrelated Gaussian-squared component is bounded at around the 10% level by the Wilkinson Microwave Anisotropy Probe (WMAP) bound on the bispectrum, and we show that a competitive bound may follow from the trispectrum when it too is bounded by WMAP. Similar considerations apply to a highly non-Gaussian isocurvature perturbation.

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Introduction.—The origin of structure in the Universe seems to be the primordial curvature perturbation ζ , present already a few Hubble times before cosmological scales enter the horizon and come into causal contact [1]. Within the observational uncertainty, ζ is Gaussian with a practically scale-independent spectrum. Future observation though may find non-Gaussianity or scale dependence and in this paper we focus on the former. We shall discount the possible contribution of topological defects to observables, but we shall consider briefly the possible contribution of a primordial isocurvature perturbation.

We assume that ζ originates from the vacuum fluctuations during slow-roll inflation of one or more light scalar fields. These fluctuations are promoted to practically Gaussian [2] classical perturbations around the time of horizon exit. Expanding ζ in powers of the field perturbations, the linear terms are Gaussian and quadratic terms are expected to account adequately for non-Gaussianity [3].

To obtain detectable non-Gaussianity some field other than the inflaton must contribute to ζ [2,4]. Possible examples of such a field in the literature are the extra field in double inflation [5], the curvaton [6] (see also [7]), and a field causing inhomogeneous reheating [8] or preheating [9].

When discussing non-Gaussianity, it is usually supposed that the contribution of the extra field will actually dominate ζ . Then ζ is of the form

$$\zeta(\mathbf{x}) = \sigma(\mathbf{x}) - \frac{3}{5}f_{\text{NL}}(\sigma^2(\mathbf{x}) - \langle\sigma^2(\mathbf{x})\rangle), \quad (1)$$

where $\sigma(\mathbf{x})$ is the perturbation of the extra field, normalized so that the linear term has unit coefficient.¹

We will suppose instead that the additional contribution is subdominant so that

$$\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + \zeta_\sigma(\mathbf{x}), \quad (2)$$

¹We are defining the curvature perturbation as in [10], and the factor $-3/5$ appears because the original authors [11] worked (using first-order perturbation theory) with $\Phi \equiv -\frac{3}{5}\zeta$.

$$\zeta_\sigma(\mathbf{x}) = a\sigma(\mathbf{x}) + \sigma^2(\mathbf{x}) - \langle\sigma^2\rangle, \quad (3)$$

where ζ_g is the practically Gaussian contribution of the inflaton, and we now normalize σ so that the quadratic term has unit coefficient. For the moment we take the linear term to be negligible so that²

$$\zeta_\sigma(\mathbf{x}) = \sigma^2(\mathbf{x}) - \langle\sigma^2\rangle. \quad (4)$$

Basic Definitions.—To describe the stochastic properties of cosmological perturbations one formally invokes an ensemble of universes, of which the observable Universe is supposed to be a typical member. A sampling of the ensemble may be regarded as a sampling of different locations for the region under consideration. The stochastic properties are conveniently described using a Fourier expansion, which we make inside a finite box of coordinate size L much bigger than the region of interest. In terms of physically significant wave numbers, this means $k \gg L^{-1}$.

We will denote a generic cosmological perturbation, evaluated at some instant, by $g(\mathbf{x})$, and assume $\langle g \rangle = 0$. The fields responsible for ζ are smoothed on some scale k_{max}^{-1} and we consider the era when this scale is outside the horizon. Taking the box size to be formally infinite, the spectrum $P_g(k)$ is defined by

$$\langle g_{\mathbf{k}} g_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P_g(k), \quad (5)$$

where $g_{\mathbf{k}} = \int d^3x e^{i\mathbf{k}\cdot\mathbf{x}} g(\mathbf{x})$ are the Fourier modes of $g(\mathbf{x})$. The δ -function and the dependence of the spectrum only on $k \equiv |\mathbf{k}|$ express the fact that stochastic properties are invariant under rotations and translations. The variance is

$$\langle g^2(\mathbf{x}) \rangle = \frac{1}{(2\pi)^3} \int d^3k P_g(k) \equiv \int_{L^{-1}}^{k_{\text{max}}} \frac{dk}{k} \mathcal{P}_g(k), \quad (6)$$

and $\mathcal{P}_g \equiv (k^3/2\pi^2) P_g$ is the typical value of g^2 . On cos-

²The literature includes examples where the linear term dominates [12] and where the quadratic term dominates [3,13]. The most closely related previous discussion is [14], whose authors consider Eq. (1) but with a sharply peaked spectrum for σ instead of the usual flat spectrum.

mological scales $\mathcal{P}_\zeta$ is almost scale-invariant with $\mathcal{P}_\zeta^{1/2} = 5 \times 10^{-5}$.

Non-Gaussianity is signaled by additional connected correlators. The bispectrum B_g is defined by

$$\langle g_{\mathbf{k}_1} g_{\mathbf{k}_2} g_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_g(k_1, k_2, k_3). \quad (7)$$

If the curvature perturbation has the form (1) its bispectrum is given to leading order by [11]

$$B_\zeta(k_1, k_2, k_3) = -\frac{6}{5} f_{\text{NL}} [P_\zeta(k_1) P_\zeta(k_2) + \text{cyclic}]. \quad (8)$$

(Only the term linear in f_{NL} is kept, which is justified because the second term of Eq. (1) is much less than the first term.) Current observation [15,16] gives $|f_{\text{NL}}| \lesssim 100$, which makes the non-Gaussian fraction of ζ less than $100 \mathcal{P}_\zeta^{1/2} \sim 10^{-3}$. Absent a detection, PLANCK [17] will bring this down to roughly $|f_{\text{NL}}| \lesssim 1$.

Following Maldacena [4], we define f_{NL} by Eq. (8) irrespectively of its origin, making it in general a function of the wave numbers.

The trispectrum T_g is defined in terms of the connected four-point correlator by as

$$\langle g_{\mathbf{k}_1} g_{\mathbf{k}_2} g_{\mathbf{k}_3} g_{\mathbf{k}_4} \rangle_c = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4) T_g. \quad (9)$$

It is a function of six scalars, defining the quadrilateral formed by $\{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4\}$. If the curvature perturbation is given by (1), its trispectrum to leading order is of the form [18]

$$T_\zeta = \frac{1}{2} \tau_{\text{NL}} [P_\zeta(k_1) P_\zeta(k_2) P_\zeta(k_4) + 23 \text{ perms.}], \quad (10)$$

with $\tau_{\text{NL}} = 36 f_{\text{NL}}^2 / 25$. In this expression, $k_{ij} \equiv |\mathbf{k}_i + \mathbf{k}_j|$, and the permutations are of $\{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4\}$ giving actually 12 distinct terms.

The authors of [19,20] have measured some connected terms of the angular trispectrum on the COBE DMR data, with no significant detection of signals. Nondetection of the trispectrum by the COBE DMR data may be interpreted as $|f_{\text{NL}}| < 10^4$ (95% c.l.) based upon theoretical expectations (Figure 3 of [18] with $\ell_{\text{max}} = 10$). Expressed as a bound on τ_{NL} , the current bound becomes $|\tau_{\text{NL}}| \lesssim 10^8$. When a bound on the trispectrum becomes available from WMAP data, it might constrain non-Gaussianity of the type (8) more strongly than the bispectrum. There is currently no estimate of the bound on the trispectrum which will eventually be possible.

Following Maldacena's strategy for f_{NL} , we will define τ_{NL} by Eq. (10) irrespectively of its origin, so that in general it depends on the wave vectors. In this paper we will calculate f_{NL} and τ_{NL} in the case that the curvature perturbation has the form (4).

The bispectrum.—We take the spectrum $\mathcal{P}_\sigma$ to be scale-invariant, which will be a good approximation if σ is sufficiently light. Since ζ_g and σ are uncorrelated and the curvature perturbation is dominated by ζ_g , the spectrum and the bispectrum of the curvature perturbation are

$$P_\zeta(k) = P_{\zeta_g}(k) + P_{\sigma^2}(k) \simeq P_{\zeta_g}(k), \quad (11)$$

$$B_\zeta(k_1, k_2, k_3) = B_{\sigma^2}(k_1, k_2, k_3). \quad (12)$$

The Fourier components of σ^2 are given by

$$(\sigma^2)_{\mathbf{k}} = \frac{1}{(2\pi)^3} \int d^3 q \sigma_{\mathbf{q}} \sigma_{\mathbf{k}-\mathbf{q}}. \quad (13)$$

For nonvanishing $\mathbf{k}$ and $\mathbf{k}'$

$$\begin{aligned} \langle (\sigma^2)_{\mathbf{k}} (\sigma^2)_{\mathbf{k}'} \rangle &= \frac{1}{(2\pi)^6} \int d^3 p d^3 p' \langle \sigma_{\mathbf{p}} \sigma_{\mathbf{k}-\mathbf{p}} \sigma_{\mathbf{p}'} \sigma_{\mathbf{k}'-\mathbf{p}'} \rangle \\ &= 2 \delta^{(3)}(\mathbf{k} + \mathbf{k}') \int d^3 p P_\sigma(p) P_\sigma(|\mathbf{k} - \mathbf{p}|). \end{aligned}$$

Taking $\mathcal{P}_\sigma$ to be scale-independent [21],

$$\mathcal{P}_{\sigma^2}(k) = \frac{k^3}{2\pi} \mathcal{P}_\sigma^2 \int_{L^{-1}} \frac{d^3 p}{p^3 |\mathbf{p} - \mathbf{k}|^3}. \quad (14)$$

The subscript L^{-1} indicates that the integrand is set equal to zero in a sphere of radius L^{-1} around each singularity, and the discussion makes sense only for $L^{-1} \ll k \ll k_{\text{max}}$. In this regime one finds [21]

$$\mathcal{P}_{\sigma^2}(k) = 4 \mathcal{P}_\sigma^2 \ln(kL). \quad (15)$$

This expression gives the correct variance $\langle (\sigma^2 - \langle \sigma^2 \rangle)^2 \rangle = 2 \langle \sigma^2 \rangle^2$, confirming the consistency of the finite box approach.

Now consider the bispectrum $B_\zeta = B_{\sigma^2}$. Repeating the calculation leading to Eq. (14), we find eight terms which can be brought into a common form by a redefinition of the integration variables, giving

$$B_\zeta = (2\pi)^3 \mathcal{P}_\sigma^3 \int_{L^{-1}} \frac{d^3 p}{p^3 |\mathbf{p} - \mathbf{k}_1|^3 |\mathbf{p} + \mathbf{k}_2|^3}, \quad (16)$$

where again L^{-1} means that a sphere of radius L^{-1} is cut out around each singularity. The integral may be estimated by adding the contributions of the three singularities, and comparing the result with Eq. (8) one finds

$$f_{\text{NL}} = -A \frac{20}{3} \frac{\mathcal{P}_\sigma^3}{\mathcal{P}_\zeta^2} \ln(kL), \quad (17)$$

with $A = O(1)$ and $k = \min k_i$. Expressed in terms of the non-Gaussian fraction $r_{\zeta_\sigma} \equiv (\mathcal{P}_{\sigma^2} / \mathcal{P}_\zeta)^{1/2}$, this becomes $r_{\zeta_\sigma} \simeq 0.04 f_{\text{NL}}^{1/3}$

To compare with observations, we choose a minimal box size, so that $\ln(kL)$ is roughly of order 1 on cosmological scales. Since f_{NL} has only mild scale dependence, we can

use observational bounds on that are obtained by taking f_{NL} to be scale-independent. The current bound $|f_{\text{NL}}| \lesssim 100$ corresponds to $r_{\sigma^2} \lesssim 0.18$. Planck [17] will be able to detect an uncorrelated Gaussian-squared component of the primordial curvature perturbation, provided that its relative amplitude r_{σ^2} is bigger than 0.04.

The trispectrum.—Computing the connected four-point correlator of σ^2 we find

$$T_\zeta = 4\pi^5 \mathcal{P}_\sigma^4 \int_{L^{-1}} \frac{d^3 p}{p^3 |\mathbf{p} - \mathbf{k}_1|^3 |\mathbf{p} + \mathbf{k}_2|^3 |\mathbf{p} + \mathbf{k}_{24}|} + 23 \text{ perms}, \quad (18)$$

where the permutations are of $\{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4\}$ giving actually 3 distinct terms. A sphere of radius L^{-1} is cut out around each singularity, and imposing $k \gg L^{-1}$ on the wave numbers in Eq. (13) gives $|\mathbf{k}_{ij}| \gg L^{-1}$. The correspondence between Eqs. (18) and (10) is seen by drawing Feynman-like diagrams as for instance in [22]. Estimating the integral by adding the contributions of the four singularities, and comparing with Eq. (10), one finds

$$\tau_{\text{NL}} = 16B \frac{\mathcal{P}_\sigma^4}{\mathcal{P}_\zeta^3} \ln(kL), \quad (19)$$

with $B = O(1)$ and $k = \min\{k_i, |\mathbf{k}_{jm}|\}$. For the fraction of non-Gaussianity this corresponds to $r_{\sigma^2} \sim 10^{-1} \tau_{\text{NL}}^{1/4}$. This result might be competitive with the one coming from f_{NL} , when τ_{NL} is bounded by WMAP.

A Gaussian-squared isocurvature perturbation.—In addition to the primordial curvature perturbation, there might be a primordial isocurvature perturbation. Like the former, the latter is to be evaluated a few Hubble times before cosmological scales start to enter the horizon, and is specified by

$$S_i = \frac{\delta n_i}{n_i} - \frac{\delta n_\gamma}{n_\gamma}, \quad (20)$$

where the n 's are number densities and i runs over cold dark matter (CDM) particles (C), baryonic matter (B), and neutrinos (ν). Observation requires roughly $|S_i| \lesssim 0.1\zeta$. Depending on its origin, $S_i(\mathbf{x})$ might be a constant multiple of $\zeta(\mathbf{x})$, but it might instead be uncorrelated with it [23]. The classic example of the latter case is a CDM isocurvature perturbation caused by a perturbation in the axion field. In that situation S_C will be the sum of terms linear and quadratic in the axion field [21]. Let us suppose that the latter dominates and that ζ is Gaussian. On large angular scales, ignoring baryons, the temperature anisotropies are given in terms of the primordial curvature and isocurvature perturbation through

$$\left(\frac{\Delta T}{T}\right)_{\text{sw}} = \left(\frac{\Delta T}{T}\right)_A + \left(\frac{\Delta T}{T}\right)_S = -\frac{\zeta}{5} - \frac{2}{5} S_C. \quad (21)$$

The angle-averaged bispectrum defined e.g. in [11] will

consist only of the isocurvature one i.e. $B_{\ell_1, \ell_2, \ell_3} = B_{\ell_1, \ell_2, \ell_3}^{(S_C)}$. Since the coefficients of ζ and S_C are similar, the observational bound on $\mathcal{P}_{S_C}/\mathcal{P}_\zeta$ from non-Gaussianity will be similar to the one on $\mathcal{P}_{\sigma^2}/\mathcal{P}_\zeta$, and hence competitive with the existing bound.

The logarithmic scale dependence.—We have found that the spectrum, bispectrum, and trispectrum all increase like $\ln(kL)$ with k a representative wave number. This factor arises from the infrared divergence of Eq. (13), which in turn is due to the interference of very long wavelength components. We adopted a minimal box size making the factor of order one on cosmological scales, though it might still be significant on very small scales relevant for instance for primordial black hole formation. But irrespective of practicalities, the appearance of the factor seems to contradict a basic tenet of physics concerning the use of Fourier series; that the box size should be irrelevant if it is much bigger than the scale of interest.

To see what is going on, we need to consider the generic perturbation (3) which contains a linear term $a\sigma$. The crucial point is that the mean $\langle\sigma\rangle$ is supposed to vanish *within the chosen box* of size L . Suppose now that we go to box of size $M \ll L$, still comfortably surrounding the region of observational interest. (It might be the whole observable Universe, or a much smaller region around us where say the distribution of primordial black holes has been detected.) Inside the small box the appropriate variable is $\hat{\sigma} \equiv \sigma - \langle\sigma\rangle_M$, and we will denote expectation values inside the small box by $\langle\rangle_M$. Then

$$\zeta_\sigma(\mathbf{x}) = a(M)\hat{\sigma}(\mathbf{x}) + (\hat{\sigma}^2(\mathbf{x}) - \langle\hat{\sigma}^2\rangle), \quad (22)$$

with $a(M) = a + 2\langle\sigma\rangle_M$. Finally, instead of locating the small box around a particular observed region, let its location vary so that $\langle\sigma\rangle_M$ becomes the original perturbation σ smoothed on the scale M . It is clear that the stochastic properties of the perturbations within a randomly located small box will be the same, whether we use such a box directly or whether we calculate them using instead the big box.

Let us see how this works for the spectrum, bispectrum, and trispectrum of ζ_σ . The spectrum is $\mathcal{P}_{\zeta_\sigma} = a^2 \mathcal{P}_\sigma + \mathcal{P}_{\sigma^2}$. Defined within a particular small box it becomes

$$\mathcal{P}_{\zeta_\sigma}(M) = a^2(M) \mathcal{P}_\sigma + \mathcal{P}_{\sigma^2}. \quad (23)$$

Using Eq. (15), the expectation value is

$$\langle\mathcal{P}_{\zeta_\sigma}(M)\rangle = 4\mathcal{P}_\sigma^2(\ln(L/M) + \ln(kL)) = \mathcal{P}_\zeta. \quad (24)$$

These equations translate to the following words: going to the small box increases the size of the cutoff around the singularities, from L^{-1} to M^{-1} , but this is compensated by the change in the coefficient $a(M)$.

Exactly the same thing happens for the bispectrum and trispectrum. Using our previous results, the bispectrum within the large box is

$$B_{\zeta} = 8\pi^4 a^2 \mathcal{P}_{\sigma}^2 \left(\frac{1}{k_1^3 k_2^3} + \text{cyclic} \right) + (2\pi)^3 \mathcal{P}_{\sigma}^3 \int_{L^{-1}} \frac{d^3 p}{p^3 |\mathbf{k}_1 - \mathbf{p}|^3 |\mathbf{k}_2 + \mathbf{p}|^3}. \quad (25)$$

Replacing a by $a(M)$ and L by M gives the bispectrum $B_{\zeta}(M)$ within a small box, leading to

$$\langle B_{\zeta}(M) \rangle = (2\pi)^3 \mathcal{P}_{\zeta}^3 \left[4\pi \ln(L/M) \left(\frac{1}{k_1^3 k_2^3} + \text{cyclic} \right) + \int_{M^{-1}} \frac{d^3 p}{p^3 |\mathbf{k}_1 - \mathbf{p}|^3 |\mathbf{k}_2 + \mathbf{p}|^3} \right]. \quad (26)$$

The first term changes $\int_{M^{-1}}$ to $\int_{L^{-1}}$, and a similar calculation works for the trispectrum.

What we are finding here is a logarithmic “running” of the (typical value of) the parameter a with the box size, similar to the running of field theory parameters with the renormalization scale. The logarithm would become a power if $\mathcal{P}_{\sigma}(k)$ went like a positive power. If more fields and/or higher terms in the expansion of ζ were allowed, we would arrive at “renormalization group” equations relating the running of the coefficients in the expansion. That is hardly of practical interest for the primordial curvature perturbation, but it will be applicable when the formalism developed in [22] is extended to include non-Gaussianity.

Returning to the primordial perturbation, consider again our assumption that ζ (or S_i) is quadratic in σ , corresponding to $a \simeq 0$. If this assumption is valid within a box exponentially larger than the observable Universe, then it

can be valid also for the minimal box that we adopted only if our location is to some extent untypical. This situation was noticed for the axion [21,24], providing an early example of a theoretically motivated anthropic consideration.

Conclusion.—We have considered the effect of a subdominant primordial perturbation, which is uncorrelated with the main component. Unlike the main component, this one may be highly non-Gaussian, with the result that its stochastic properties are quite different from those of the main component. They have to be defined with respect to a comoving box of finite size, but a change in the box size is compensated by a change in the coefficient of the Gaussian component as measured by a typical observer, so that physical results are unchanged for such an observer.

Coming to the observational side, and assuming that the subdominant is the square of a Gaussian, it will eventually be detectable if it contributes more than a few percent of the total. A smoking gun for this setup would be the detection of both primordial gravitational waves (indicating that the inflaton gives the dominant contribution to the primordial curvature perturbation) and non-Gaussianity (indicating that this contribution cannot be the only one). Another smoking gun would be provided by the observation of both the bispectrum and the trispectrum, the latter being at least equally important.

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