Influence of finite chemical potential on the critical number of fermion flavors in QED_3

Hong-tao Feng,[1] Feng-yao Hou,[1] Xiang He,[1] Wei-min Sun,[2,1] and Hong-shi Zong[3,1,2,*]

[1]*Department of Physics, Nanjing University, Nanjing 210093, China*

[2]*Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing 210093, China*

[3]*CCAST(World Laboratory), P.O. Box 8730, Beijing 100080, China*

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Based on the rainbow approximation of Dyson-Schwinger equation and the assumption that the inverse dressed fermion propagator at finite chemical potential is analytic in the neighborhood of $\mu = 0$, A new method for calculating the dressed fermion propagator at finite chemical potential in QED_3 is developed. From this the effects of the chemical potential on the critical number of the fermion flavors is evaluated.

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I. INTRODUCTION

Quantum electrodynamics in $(2 + 1)$ dimensions (QED_3) is a well studied field-theoretical model. It exhibits several interesting features, similar to quantum chromodynamical (QCD), for instance dynamical chiral symmetry breaking (DCSB) [[1–14]] and confinement [[15–17]]. Moreover, it is superrenormalizable so that it is not plagued with ultraviolet divergences. Thus, QED_3 is an ideal model to study nonperturbative phenomena. In addition, QED_3 can be applied in condense matter physics to unpuzzle some realistic microscopic mechanisms [[18,19]]. Especially, since the discovery of the high-T_c superconductivity, QED_3 have attracted more attention. It is general believed that QED_3 with N flavors can be regarded as a possible effective theory for high-T_c superconductivity in underdoped cuprates [[20–26]].

As is well known, the Lagrangian of QED_3 with N fermion flavors in Euclidean space is

$$\mathcal{L} = \sum_{i=0}^{N} \bar{\psi}_i (i\not{\partial} + e\not{A})\psi_i + \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2\xi}(\partial_\nu A_\nu)^2. \quad (1)$$

This Lagrangian has chiral symmetry in the absence of mass term $m_0\bar{\psi}\psi$. However, chiral symmetry is broken spontaneously due to the generation of a fermion mass, i.e. dynamical chiral symmetry breaking. Several research works show that DCSB occurs when the number of the fermion flavor N is less than a critical number N_c [[3,9,10]].

The above result holds at zero density but is expected to change when the density is nonzero. Since high-T_c superconductivity is discovered in condense matter systems where the density is not zero, one should consider the influence of the density or chemical potential. Therefore, it is very interesting to study the effects of the chemical potential on the critical number of the fermion flavors.

In this paper, the relation between the critical number of the fermion flavors N_c and the chemical potential μ is investigated by analyzing the Dyson-Schwinger equation (DSE) for the self-energy in the rainbow approximation of the fermion propagator. We shall see that N_c decreases with the increasing chemical potential.

II. THE FERMION PROPAGATOR AT FINITE CHEMICAL POTENTIAL

In Euclidean space, the Lagrangian of massless QED_3 with N fermion flavors at finite chemical potential μ reads, ignoring the issues discuss in [16],

$$\mathcal{L} = \sum_{i=0}^{N} \bar{\psi}_i (i\not{\partial} + e\not{A} - \mu\gamma_3)\psi_i + \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2\xi}(\partial_\nu A_\nu)^2, \quad (2)$$

where the 4×1 spinor ψ_i represent the fermion field and $i = 1, \ldots,$ N are flavor indices. The term $-\mu\bar{\psi}\gamma_3\psi$ shows the influence of chemical potential μ.

In order to motivate our new method for calculating the dressed fermion propagator at finite chemical potential, let us first recall the methods adopted in the previous literatures [27–29]. As just described in Eq. (2), the introduction of chemical potential to Euclidean QED_3 breaks the original $O(3)$ symmetry to $O(2)$. In this case, the most general form for the dressed fermion propagator at finite chemical potential reads (due to the presence of medium)

$$\begin{aligned}\hat{S}^{-1}(\mu, p) = {} & i\vec{\gamma}\cdot\vec{p}\,\mathcal{A}(p^2, u\cdot p) - \mu\gamma_3 C(p^2, u\cdot p) \\ & + \mathcal{B}(p^2, u\cdot p) - \mu\gamma_3\vec{\gamma}\cdot\vec{p}\,\mathcal{D}(p^2, u\cdot p),\end{aligned} \quad (3)$$

where $u_i (i = 1, 2, 3)$ denotes the relative velocity of the medium which in the rest frame of the medium can be written as $u_i = (\vec{0}, 1)$. Substituting Eq. (3) into the DSE satisfied by the dressed fermion propagator at finite chemical potential and by means of suitable projection procedure (multiplying by appropriate gamma matrices and then taking the trace), the four independent scalar function $\mathcal{A}(p^2, u\cdot p)$, $\mathcal{B}(p^2, u\cdot p)$, $C(p^2, u\cdot p)$, and $\mathcal{D}(p^2, u\cdot p)$ are found to satisfy a coupled set of Dyson-Schwinger equations in QED_3. In principle, for a given model fermion-boson vertex, one can numerically solve these

*Electronic address: zonghs@chenwang.nju.edu.cn

coupled DSE. However, the numerical solution turns out to be rather difficult in practice (the last term on the right hand of Eq. (3) was dropped in Refs. [28,29]). In order to avoid the difficulty of numerically solving the four independent coupled integral equations in QED_3, here we shall adopt a new method for calculating the dressed fermion propagator at finite chemical potential.

In the lattice treatment of finite-density QCD it is generally believed that physical quantities are analytic in the neighborhood of $\mu = 0$ and two kinds of methods, i.e the Taylor expansion in powers of μ and analytic continuation from simulations at imaginary μ are adopted [30–32]. We expect that this is also the case in QED_3. Thus it is interesting to study the full fermion propagator at finite chemical potential by assuming its analyticity property. According to this assumption, we can expand the full inverse fermion propagator at finite chemical potential, $\hat{S}^{-1}(\mu, p)$ in powers of μ as follows:

$$\begin{aligned}\hat{S}^{-1}(\mu, p) &= \hat{S}^{-1}(\mu, p)|_{\mu=0} + \frac{\partial \hat{S}^{-1}(\mu, p)}{\partial \mu}\bigg|_{\mu=0}\mu \\ &\quad + \frac{1}{2!}\frac{\partial^2 \hat{S}^{-1}(\mu, p)}{\partial \mu^2}\bigg|_{\mu=0}\mu^2 + \cdots \\ &\quad + \frac{1}{n!}\frac{\partial^n \hat{S}^{-1}(\mu, p)}{\partial \mu^n}\bigg|_{\mu=0}\mu^n + \cdots \\ &= \Gamma^{(0)} + \Gamma^{(1)}(p)\mu + \frac{1}{2!}\Gamma^{(2)}(p)\mu^2 + \cdots \\ &\quad + \frac{1}{n!}\Gamma^{(n)}(p) + \cdots, \end{aligned} \tag{4}$$

where $\Gamma^{(0)}$, $\Gamma^{(1)}$ and $\Gamma^{(n)}$ are defined as

$$\begin{aligned}\Gamma^{(0)}(p) &= \hat{S}^{-1}(\mu, p)|_{\mu=0} \equiv S^{-1}(p) \\ &\equiv i\gamma \cdot pA(p^2) + B(p^2), \end{aligned} \tag{5}$$

$$\Gamma^{(1)}(p) = \frac{\partial \hat{S}^{-1}(\mu, p)}{\partial \mu}\bigg|_{\mu=0}, \tag{6}$$

$$\Gamma^{(n)}(p) = \frac{\partial^n \hat{S}^{-1}(\mu, p)}{\partial \mu^n}\bigg|_{\mu=0}. \tag{7}$$

Obviously, Eq. (4) is only valid within the radius of convergence of μ expansion and may be inadequate at large values of μ, particularly near any critical one.

Using the Lagrangian in Eq. (2) and adopting the rainbow approximation, we can get the DSE for the inverse fermion propagator at nonzero μ

$$\begin{aligned}\hat{S}^{-1}(\mu, p) &= i\gamma \cdot p - \mu\gamma_3 \\ &\quad - \int \frac{d^3k}{(2\pi)^3}\gamma_\mu \hat{S}(\mu, k)\gamma_\nu D_{\mu\nu}(q). \end{aligned} \tag{8}$$

with $q = p - k$ and $e^2 = 1$. It should be noted here that the photon propagator $D_{\mu\nu}(q)$ does not evolve with μ (where we have assumed that the effect of chemical potential on the photon propagator arising through fermion loops is small in comparison with that on the fermion propagator. This is a commonly used approximation in calculating the dressed fermion propagator at finite chemical potential [27–29,33]).

Applying the differential operation $\frac{\partial}{\partial \mu}$ on both sides of Eq. (8), we get

$$\begin{aligned}\frac{\partial \hat{S}^{-1}(\mu, p)}{\partial \mu} &= -\gamma_3 - \int \frac{d^3k}{(2\pi)^3}\gamma_\mu \frac{\partial \hat{S}(\mu, k)}{\partial \mu}\gamma_\nu D_{\mu\nu}(q) \\ &= -\gamma_3 + \int \frac{d^3k}{(2\pi)^3}\gamma_\mu \hat{S}(\mu, k)\frac{\partial \hat{S}^{-1}(\mu, k)}{\partial \mu} \\ &\quad \times \hat{S}(\mu, k)\gamma_\nu D_{\mu\nu}(q), \end{aligned} \tag{9}$$

where we have employed the following identity:

$$\frac{\partial \hat{S}(\mu, k)}{\partial \mu} = -\hat{S}(\mu, k)\frac{\partial \hat{S}^{-1}(\mu, k)}{\partial \mu}\hat{S}(\mu, k). \tag{10}$$

Setting $\mu = 0$, Eq. (9) is reduced to an integral equation satisfied by $\Gamma^{(1)}$,

$$\Gamma^{(1)}(p) = -\gamma_3 + \int \frac{d^3k}{(2\pi)^3}\gamma_\mu S(k)\Gamma^{(1)}(k)S(k)\gamma_\nu D_{\mu\nu}(q). \tag{11}$$

Applying the differential operation $\frac{\partial}{\partial \mu}$ on both sides of Eq. (9) successively $n(n \geq 2)$ times and subsequently setting $\mu = 0$, we obtain the following:

$$\Gamma^{(n)}(p) = \frac{\partial^n \hat{S}^{-1}(\mu, p)}{\partial \mu^n}|_{\mu=0} = \int \frac{d^3k}{(2\pi)^3}\gamma_\mu \left\{\frac{\partial^{n-1}}{\partial \mu^{n-1}}\left[\hat{S}(\mu, k)\frac{\partial \hat{S}^{-1}(\mu, k)}{\partial \mu}\hat{S}(\mu, k)\right]\right\}\bigg|_{\mu=0}\gamma_\nu D_{\mu\nu}(q). \tag{12}$$

For instance, when $n = 2$, we have the following:

$$\Gamma^{(2)}(p) = \int \frac{d^3k}{(2\pi)^3}\gamma_\mu\left\{\frac{\partial}{\partial\mu}\left[\hat{S}(\mu,k)\frac{\partial\hat{S}^{-1}(\mu,k)}{\partial\mu}\hat{S}(\mu,k)\right]\right\}\Big|_{\mu=0}\gamma_\nu D_{\mu\nu}(q)$$
$$= \int \frac{d^3k}{(2\pi)^3}\gamma_\mu\hat{S}(0,k)\Gamma^{(2)}(k)\hat{S}(0,k)\gamma_\nu D_{\mu\nu}(q) - 2\int \frac{d^3k}{(2\pi)^3}\gamma_\mu\hat{S}(0,k)\Gamma^{(1)}(k)\hat{S}(0,k)\Gamma^{(1)}(k)\hat{S}(0,k)\gamma_\nu D_{\mu\nu}(q), \quad (13)$$

where we have used Eq. (10).

In the following, we will discuss the relation between $\frac{\partial^n\hat{S}^{-1}(p)}{\partial(-ip_3)^n}$ and $\Gamma^{(n)}(p)$ and show that they are equal to each other under the rainbow approximation and the assumption that the photon propagator is μ-independent.

Now we will prove that the equation is tenable at $n = 1$, i.e.,

$$\Gamma^{(1)}(p) = \frac{\partial S^{-1}(p)}{\partial(-ip_3)}. \quad (14)$$

Setting $\mu = 0$ and then applying the differential operation $\frac{\partial}{\partial(-ip_3)}$ on both sides of Eq. (8), we obtain

$$\frac{\partial S^{-1}(p)}{\partial(-ip_3)} = -\gamma_3 - \int \frac{d^3k}{(2\pi)^3}\gamma_\mu S(k)\gamma_\nu\frac{\partial D_{\mu\nu}(q)}{\partial(-ip_3)}$$
$$= -\gamma_3 + \int \frac{d^3k}{(2\pi)^3}\gamma_\mu S(k)\gamma_\nu\frac{\partial D_{\mu\nu}(q)}{\partial(-ik_3)}$$
$$= -\gamma_3 - \int \frac{d^3k}{(2\pi)^3}\gamma_\mu\frac{\partial S(k)}{\partial(-ik_3)}\gamma_\nu D_{\mu\nu}(q), \quad (15)$$

where we have used integration by parts. Making use of an identity to analogous Eq. (10), we can rewrite Eq. (15) as

$$\frac{\partial S^{-1}(p)}{\partial(-ip_3)} = -\gamma_3 + \int \frac{d^3k}{(2\pi)^3}\gamma_\mu S(k)\frac{\partial S^{-1}(k)}{\partial(-ik_3)}S(k)\gamma_\nu D_{\mu\nu}(q). \quad (16)$$

From Eqs. (11) and (16), we see that $\Gamma^{(1)}(p)$ and $\frac{\partial S^{-1}(p)}{\partial(-ip_3)}$ satisfy identical integral equation and are therefore equal to each other, so the so-called vector Ward identity hold.

Applying the differential operation $\frac{\partial}{\partial(-ip_3)}$ on both sides of Eq. (16), we obtain

$$\frac{\partial^2 S^{-1}(p)}{\partial(-ip_3)^2} = \int \frac{d^3k}{(2\pi)^3}\gamma_\mu\left[S(k)\frac{\partial S^{-1}(k)}{\partial(-ik_3)}S(k)\right]\gamma_\nu\frac{\partial D_{\mu\nu}(q)}{\partial(-ip_3)} = -\int \frac{d^3k}{(2\pi)^3}\gamma_\mu S(k)\frac{S^{-1}(k)}{\partial(-ik_3)}S(k)\gamma_\nu\frac{\partial D_{\mu\nu}(q)}{\partial(-ik_3)}$$
$$= \int \frac{d^3k}{(2\pi)^3}\gamma_\mu\frac{\partial}{\partial(-ik_3)}\left[S(k)\frac{\partial S^{-1}(k)}{\partial(-ik_3)}S(k)\right]\gamma_\nu D_{\mu\nu}(q)$$
$$= \int \frac{d^3k}{(2\pi)^3}\gamma_\mu S(k)\frac{\partial^2 S^{-1}(k)}{\partial(-ik_3)^2}S(k)\gamma_\nu D_{\mu\nu}(q) - 2\int \frac{d^3k}{(2\pi)^3}\gamma_\mu S(k)\frac{\partial S^{-1}(k)}{\partial(-ik_3)}S(k)\frac{\partial S^{-1}(k)}{\partial(-ik_3)}S(k)\gamma_\nu D_{\mu\nu}(q). \quad (17)$$

Comparing Eqs. (13) and (17), we find that $\Gamma^{(2)}(p) \equiv \frac{\partial S^{-1}(p)}{\partial(-ip_3)^2}$ comes into existence, too.

The general case can be proved by induction. Assuming

$$\Gamma^{(n)}(p) \equiv \frac{\partial^n S^{-1}(p)}{\partial(-ip_3)^n}, \quad (18)$$

holds for $n \le m (m \ge 2)$. Then we have

$$\Gamma^{(m+1)}(p) = \frac{\partial^{m+1}\hat{S}^{-1}(\mu,p)}{\partial\mu^{m+1}}\Big|_{\mu=0} = \int \frac{d^3k}{(2\pi)^3}\gamma_\mu\left\{\frac{\partial^m}{\partial\mu^m}\left[\hat{S}(\mu,k)\frac{\partial\hat{S}^{-1}(\mu,k)}{\partial\mu}\hat{S}(\mu,k)\right]\right\}\Big|_{\mu=0}\gamma_\nu D_{\mu\nu}(q)$$
$$= \int \frac{d^3k}{(2\pi)^3}\gamma_\mu\hat{S}(\mu,k)\frac{\partial^{m+1}\hat{S}^{-1}(\mu,k)}{\partial\mu^{m+1}}\hat{S}(\mu,k)\Big|_{\mu=0}\gamma_\nu D_{\mu\nu}(q)$$
$$- \int \frac{d^3k}{(2\pi)^3}\gamma_\mu f\left(\frac{\partial^m\hat{S}^{-1}(\mu,k)}{\partial\mu^m},\cdots,\frac{\partial\hat{S}^{-1}(\mu,k)}{\partial\mu},\hat{S}(\mu,k)\right)\Big|_{\mu=0}\gamma_\nu D_{\mu\nu}(q)$$
$$= \int \frac{d^3k}{(2\pi)^3}\gamma_\mu S(k)\Gamma^{(m+1)}(k)S(k)\gamma_\nu D_{\mu\nu}(q) - \int \frac{d^3k}{(2\pi)^3}\gamma_\mu f(\Gamma^{(m)}(k),\cdots,\Gamma^{(1)}(k),S(k))\gamma_\nu D_{\mu\nu}(q), \quad (19)$$

and

$$\begin{aligned}\frac{\partial^{m+1}S^{-1}(p)}{\partial(-ip_3)^{m+1}} &= \int\frac{d^3k}{(2\pi)^3}\gamma_\mu\left[S(k)\frac{\partial S^{-1}(k)}{\partial(-ik_3)}S(k)\right]\gamma_\nu\frac{\partial^m D_{\mu\nu}(q)}{\partial(-ip_3)^m}\\ &= \int\frac{d^3k}{(2\pi)^3}\gamma_\mu\frac{\partial^m}{\partial(-ik_3)^m}\left[S(k)\frac{\partial S^{-1}(k)}{\partial(-ik_3)}S(k)\right]\gamma_\nu D_{\mu\nu}(q)\\ &= \int\frac{d^3k}{(2\pi)^3}\gamma_\mu S(k)\frac{\partial^{m+1}S^{-1}(k)}{\partial(-ik_3)^{m+1}}S(k)\gamma_\nu D_{\mu\nu}(q)\\ &\quad-\int\frac{d^3k}{(2\pi)^3}\gamma_\mu h\left(\frac{\partial^m S^{-1}(k)}{\partial(-ik_3)^m},\frac{\partial^{m-1}S^{-1}(k)}{\partial(-ik_3)^{m-1}},\cdots,\frac{\partial S^{-1}(k)}{\partial(-ik_3)},S(k)\right)\gamma_\nu D_{\mu\nu}(q),\end{aligned}\tag{20}$$

It is apparent that $f(\Gamma^{(m)}(k),\cdots,\Gamma^{(1)}(k),S(k))$ and $h(\frac{\partial^m S^{-1}(k)}{\partial(-ik_3)^m},\cdots,\frac{\partial S^{-1}(k)}{\partial(-ik_3)},S(k))$ have identical structures. According to the induction hypothesis, we see that $\Gamma^{m+1}(p)$ and $\frac{\partial^{m+1}S^{-1}(p)}{\partial(-ip_3)^{m+1}}$ satisfy the same integral equation and are therefore equal.

Form Eq. (4) and the above results, we can write

$$\begin{aligned}\hat{S}^{-1}(\mu,p) &= \Gamma^{(0)}+\Gamma^{(1)}\mu+\frac{1}{2!}\Gamma^{(2)}\mu^2+\cdots+\Gamma^{(n)}\mu^n+\cdots\\ &= S^{-1}(p)+\frac{\partial S^{-1}(p)}{\partial(-ip_3)}\mu+\frac{1}{2!}\frac{\partial^2S^{-1}(p)}{\partial(-ip_3)^2}\mu^2+\cdots+\frac{1}{n!}\frac{\partial^n S^{-1}(p)}{\partial(-ip_3)^n}+\cdots\\ &= S^{-1}(p)+\frac{\partial S^{-1}(p)}{\partial p_3}(i\mu)+\frac{1}{2!}\frac{\partial^2S^{-1}(p)}{\partial(p_3)^2}(i\mu)^2+\cdots+\frac{1}{n!}\frac{\partial^n S^{-1}(p)}{\partial(p_3)^n}(i\mu)^n+\cdots\\ &= S^{-1}(\vec{p},p_3+i\mu)\equiv S^{-1}(\tilde{p})=i\gamma\cdot\tilde{p}A(\tilde{p}^2)+B(\tilde{p}^2),\end{aligned}\tag{21}$$

where $\tilde{p}=(\vec{p},p_3+i\mu)$. Comparing Eq. (21) with Eq. (3) obtained from general Lorentz structure analysis, we see that the function $\mathcal{D}(p^2,u\cdot p)$ vanishes and the other two functions $\mathcal{A}(p^2,u\cdot p)$ and $\mathcal{C}(p^2,u\cdot p)$ are not independent under the rainbow approximation and the assumption that the photon propagator is μ-independent. Therefore, the number of independent functions contained in the dressed fermion propagator at finite μ is reduced from 4 to 2.

Equation (21) clearly shows that under the rainbow approximation and the assumption that the photon propagator is μ-independent there are only two independent Lorentz structures in the dressed fermion propagator at finite chemical potential and furthermore the dressed fermion propagator at finite μ can be obtained by making the substitution $p_3\rightarrow p_3+i\mu$ in the dressed fermion propagator at $\mu=0$ (In QCD a similar result for the dressed quark propagator at finite chemical potential was also obtained in Ref. [33]). This feature obviously facilitates the numerical calculations considerably.

III. NUMERICAL CALCULATIONS OF FERMION PROPAGATOR AT FINITE CHEMICAL POTENTIAL

The photon propagator and its Dyson-Schwinger equation reads

$$D_{\mu\nu}(q)=\frac{\delta_{\mu\nu}-(1-\xi)q_\mu q_\nu/q^2}{q^2+\Pi(q^2)},\tag{22}$$

$$\Pi_{\mu\nu}(q^2)=-N\int\frac{d^3k}{(2\pi)^3}Tr[S(k)\gamma_\mu S(q+k)\gamma_\nu],\tag{23}$$

with the relation between the vacuum polarization $\Pi(q^2)$ and $\Pi_{\mu\nu}(q^2)$

$$\Pi_{\mu\nu}(q^2)=\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\Pi(q^2),\tag{24}$$

where ξ is the gauge parameter. For convenience, we will employ Landau gauge in the following.

Using Eqs. (8) and (21)–(24), we obtain the coupled DS equations in QED_3 at finite chemical potential.

$$\begin{aligned}A(\tilde{p}^2) &= 1-\frac{1}{4\tilde{p}^2}\int\frac{d^3k}{(2\pi)^3}Tr[i(\gamma\cdot\tilde{p})\gamma_\mu\hat{S}(\mu,k)\\ &\quad\times\gamma_\nu D_{\mu\nu}(p-k)]\\ &= 1+\frac{2}{\tilde{p}^2}\int\frac{d^3k}{(2\pi)^3}\\ &\quad\times\frac{A^2(\tilde{k}^2)(\tilde{p}\cdot q)(\tilde{k}\cdot q)/q^2}{[A^2(\tilde{k}^2)\tilde{k}^2+B^2(\tilde{k}^2)][q^2+\Pi(q^2)]}\end{aligned}\tag{25}$$

$$\begin{aligned}B(\tilde{p}^2) &= \frac{1}{4}\int\frac{d^3k}{(2\pi)^3}Tr[\gamma_\mu\hat{S}(\mu,k)\gamma_\nu D_{\mu\nu}(q)]\\ &= \int\frac{d^3k}{(2\pi)^3}\frac{2B(\tilde{k})}{[A^2(\tilde{k}^2)\tilde{k}^2+B(\tilde{k}^2)][q^2+\Pi(q^2)]}.\end{aligned}\tag{26}$$

$$\Pi(q^2) = 2N \int \frac{d^3k}{(2\pi)^3} \times \frac{A(k^2)A(p^2)(2k^2 - 4(k\cdot q) - 6(k\cdot q)^2/q^2)}{[A^2(k^2)k^2 + B^2(k^2)][A^2(p^2)p^2 + B^2(p^2)]}, \tag{27}$$

where $q = p - k$. Here we note that the derivation of Eq. (27) involves some mathematical tricks. For more details we refer the readers to Ref. [9]. If μ is set to be zero, Eqs. (25) and (26) reduce to the corresponding equations at zero chemical potential. This is just what one would expect in advance.

It is well known that under certain conditions there are two solutions for the above equations, the Nambu solution and the Wigner solution . If Eqs. (25)–(27) has only trivial solution, i.e the Wigner solution, then the fermions remain massless and DCSB will not occur. T. Appelquist, *et al.* found that Eq. (26) has only Wigner solution when N is larger than $N_c = \frac{32}{\pi^2} \simeq 3.24$ [3].

In principle, $A(\tilde{p}^2)$ and $B(\tilde{p}^2)$ can be obtained by numerically solving Eqs. (25)–(27) by iteration method just as one solves the corresponding set of coupled equations at $\mu = 0$. However, the numerical solution for Eqs. (25)–(27) by iteration turns out to be rather difficult in practice. Because of the presence of the μ in the third component of the momentum variable, the number of arguments of the dressed fermion propagator and the independent integration dimension in Eqs. (25) and (26) is larger than those at zero chemical potential and this fact makes it difficult to find a stationary solutions by means of iteration method. This situation forces us to seek another effective method instead of the iteration method to solve Eqs. (25)–(27).

Applying the differential operation $\frac{\partial}{\partial(p_3)}$ on both sides of Eq. (8) at $\mu = 0$, we get

$$\frac{\partial S^{-1}(p)}{\partial p_3} = i\gamma_3 - \int \frac{d^3k}{(2\pi)^3} \gamma_\mu S(k) \gamma_\nu \frac{\partial[D_{\mu\nu}(q)]}{\partial p_3}. \tag{28}$$

Similarly, applying the differential operation $\frac{\partial}{\partial(p_3)}$ on both sides of Eq. (28) $(n-1)(n \geq 2)$ times, we obtain the following:

$$\frac{\partial^n S^{-1}(p)}{\partial(p_3)^n} = - \int \frac{d^3k}{(2\pi)^3} \gamma_\mu S(k) \gamma_\nu \frac{\partial^n[D_{\mu\nu}(q)]}{\partial(p_3)^n} \tag{29}$$

Using Eqs. (8), (22), (28), and (29), the full inverse fermion propagator at finite chemical potential can be written as

$$\begin{aligned}
\hat{S}^{-1}(\mu, p) &= i\gamma\cdot\tilde{p}A(\tilde{p}^2) + B(\tilde{p}^2) \\
&= S^{-1}(p) + \frac{\partial S^{-1}(p)}{\partial p_3}(i\mu) + \frac{1}{2!}\frac{\partial^2 S^{-1}(p)}{\partial(p_3)^2}(i\mu)^2 + \cdots + \frac{1}{n!}\frac{\partial^n S^{-1}(p)}{\partial(p_3)^n}(i\mu)^n + \cdots \\
&= S^{-1}(p) - \mu\gamma_3 - \int \frac{d^3k}{(2\pi)^3} \gamma_\mu S(k) \gamma_\nu \left[\frac{\partial D_{\mu\nu}(q)}{\partial(p_3)^3} i\mu + \cdots + \frac{1}{n!}\frac{\partial^n D_{\mu\nu}(q)}{\partial(p_3)^n}(i\mu)^n + \cdots \right] \\
&= S^{-1}(p) - \mu\gamma_3 - \int \frac{d^3k}{(2\pi)^3} \gamma_\mu S(k) \gamma_\nu [D_{\mu\nu}(\tilde{p} - k) - D_{\mu\nu}(p - k)] \\
&= i\gamma\cdot\tilde{p} - \int \frac{d^3k}{(2\pi)^3} \gamma_\mu S(k) \gamma_\nu D_{\mu\nu}(\tilde{p} - k).
\end{aligned} \tag{30}$$

At this point some remarks concerning the derivation of Eq. (30) are in order. Equation (30) is formally obtained from the Dyson-Schwinger equation in rainbow truncation at zero chemical potential (Eq. (8) with $\mu = 0$) by replacing p by $\tilde{p} = (\vec{p}, p_3 + i\mu)$ (analytical continuation to complex values of p_3) and the whole procedure seems trivial. Here we would like to point out that this is not so. The reason is twofold. First, as a first step, one should prove (under certain assumptions) that the dressed fermion propagator at finite μ can be obtained from the one at $\mu = 0$ by the substitution $p_3 \rightarrow p_3 + i\mu$ as shown in Eq. (21) and then derive Eq. (30) as we did above, otherwise the expression $\hat{S}^{-1}(\tilde{p}) = i\gamma\cdot\tilde{p}A(\tilde{p}^2) + B(\tilde{p}^2)$ does not represent the dressed fermion propagator at finite μ and Eq. (30) is physically irrelevant. Second, if one directly obtains Eq. (30) from the quark DSE at $\mu = 0$ by the substitution $p \rightarrow \tilde{p} = (\vec{p}, p_3 + i\mu)$, one needs to assume that $S^{-1}(p)$ and $\int \frac{d^3k}{(2\pi)^3} \gamma_\mu S(k) \gamma_\nu D_{\mu\nu}(p - k)$ are analytic functions of p_3(at least in some domain of the complex p_3 plain). The analyticity of $S^{-1}(p)$ is expected to hold but the analyticity of $\int \frac{d^3k}{(2\pi)^3} \gamma_\mu S(k) \gamma_\nu D_{\mu\nu}(p - k)$ needs justification. The usual argument resorting to the differentiation lemma in complex analysis [34] depends on the properties of $S(k)$ and $D_{\mu\nu}(p - k)$ and is difficult to implement directly. Our derivations (Eq. (28) to Eq. (30)) have avoided this and provides a direct proof.

From the above equation it is clearly seen that the whole nontrivial μ-dependence of the fermion propagator is determined by the μ-dependence of $D_{\mu\nu}(\tilde{p} - k)$, while the $i\gamma\cdot\tilde{p}$ term only gives the trivial μ-dependence. Here we warn the reader that the introduction of μ-dependence in $D_{\mu\nu}(\tilde{p} - k)$ is only a mathematical trick and the function

$D_{\mu\nu}(\tilde{p}-k)$ should not be interpreted as the actual μ dependent photon propagator.

Using Eq. (30), Eqs. (25) and (26) can be rewritten as

$$A(\tilde{p}^2) = 1 + \frac{2}{\tilde{p}^2}\int\frac{d^3k}{(2\pi)^3} \times \frac{A^2(k^2)(\tilde{p}\cdot\hat{q})(k\cdot\hat{q})/\hat{q}^2}{[A^2(k^2)k^2 + B^2(k^2)][(\hat{q}^2 + \Pi(\hat{q}^2)]}, \quad (31)$$

$$B(\tilde{p}^2) = \int\frac{d^3k}{(2\pi)^3}\frac{2B(k^2)}{[A^2(k^2)k^2 + B^2(k^2)][\hat{q}^2 + \Pi(\hat{q}^2)]}, \quad (32)$$

with $\hat{q} = \tilde{p} - k$. By means of the mathematical trick illustrated in Eq. (30), we have shown that Eqs. (25) and (26) is equivalent to Eqs. (31) and (32) and thus reduced the hard problem of numerically solving the coupled set of DSEs satisfied by $A(\tilde{p}^2)$ and $B(\tilde{p}^2)$ (Eqs. (25) and (26)) using iteration methods into the much easier one of simple integration (Eqs. (31) and (32)).

Just as pointed out above, it is not difficult to numerically solve Eqs. (25)–(27) at $\mu = 0$ by iteration method to obtain $A(p^2)$, $B(p^2)$ and $\Pi(q)$. Subsequently we use the $\mu = 0$ solution for $A(p^2)$, $B(p^2)$ and $\Pi(q)$ to obtain the wave-function renormalization factor $A(\tilde{p}^2)$ and self-energy $B(\tilde{p}^2)$ at $\mu \neq 0$ using Eqs. (31) and (32). Here one encounters the problem of how to determine $\Pi(\hat{q}^2)$ in the numerical calculation of Eqs. (31) and (32). Our method is to use the method of curve fitting to get an analytical formula for $\Pi(q^2)$ from its numerical solution obtained by solving Eqs. (25)–(27) at $\mu = 0$. Once the analytical formula for $\Pi(q^2)$ is obtained one directly obtains $\Pi(\hat{q}^2)$ by analytic continuation $q \rightarrow \hat{q} = \tilde{p} - k$. In order to check the consistency of our procedure, we have substituted the obtained form of $A(\tilde{p}^2)$, $B(\tilde{p}^2)$ and $\Pi(\hat{q}^2)$ into Eqs. (25)–(27) to see whether they indeed satisfy this set of equations. We have checked some discrete momentum values $(\vec{p}, p_3)$ for fixed N and μ and found that they indeed satisfy Eqs. (25)–(27).

In principle one could alternatively use the method of curve fitting to determine the analytical forms of $A(p^2)$ and $B(p^2)$ from their numerical solutions and then get $A(\tilde{p}^2)$ and $B(\tilde{p}^2)$ by analytic continuation. However, such a fitting procedure for $A(p^2)$ and $B(p^2)$ turns out to be much more complicated than that for $\Pi(q^2)$, so in order to improve the accuracy of numerical computation, we adopt the former method in this paper.

Let us now consider the DCSB phase. Here $B(p^2) > 0$ at $\mu = 0$, which is a solution of Eq. (32) at $N < N_c$ and $\mu = 0$. The originally massless bare fermion can acquire a dynamical mass through nonperturbative effects. An order parameter for DCSB is the chiral condensate $\langle\bar{\psi}\psi\rangle$ which is trivially obtained from the fermion propagator $S(x)$ via:

$$\langle\bar{\psi}\psi\rangle = -\mathrm{Re}\{\mathrm{Tr}[\hat{S}(x=0)]\} = 4\int\frac{d^3k}{(2\pi)^3}\mathrm{Re}\left\{\frac{B(\tilde{k}^2)}{A^2(\tilde{k}^2)\tilde{k}^2 + B^2(\tilde{k}^2)}\right\}. \quad (33)$$

The typical behaviors of the fermion condensate for several values of small chemical potential μ are plotted in Fig. 1. We observe that for all values of chemical potential $\langle\bar{\psi}\psi\rangle_\mu$ decreases monotonically as N increases and goes to zero when N approaches a critical number N_c. For different ranges of N, $\langle\bar{\psi}\psi\rangle_\mu$ shows different behaviors with μ. When $N < 0.5$, $\langle\bar{\psi}\psi\rangle_\mu$ increases with μ increasing. When $0.5 < N < 2.25$, $\langle\bar{\psi}\psi\rangle_\mu$ almost does not change with μ. Finally, when $N > 2.25$, $\langle\bar{\psi}\psi\rangle_\mu$ decreases with μ increasing. For N near N_c, it can be clearly seen that the larger μ is, the more rapidly $\langle\bar{\psi}\psi\rangle_\mu$ decreases with N increasing. This shows clearly that the critical number of fermion flavor N_c decreases as μ increases.

In order to further illustrate the influence of the chemical potential μ on the fermion condensate, we take the ratio of $\langle\bar{\psi}\psi\rangle_\mu$ for $\mu > 0$ to $\langle\bar{\psi}\psi\rangle_0$ and plot the dependence of $\frac{\langle\bar{\psi}\psi\rangle_\mu}{\langle\bar{\psi}\psi_0\rangle}$ on N for several values of μ in Fig. 2.

From Fig. 2, we can see that when $0.5 < N < 2.25$ $\frac{\langle\bar{\psi}\psi\rangle_\mu}{\langle\bar{\psi}\psi\rangle_0}$ almost keeps constant, whereas in the two ends ($N < 0.5$ and $N > 2.25$) the chemical potential μ has very large influence (about a factor of 2.5 in the left end and a factor of 10 in the right end) on $\frac{\langle\bar{\psi}\psi\rangle_\mu}{\langle\bar{\psi}\psi\rangle_0}$. In physical applications of QED_3 (such as high-T_c superconductivity), the fermion number N is 1 or 2 and one therefore concludes that the small chemical potential has little influence on the fermion condensate. It should be noted that our numerical results is only valid for small values of chemical potential. This is

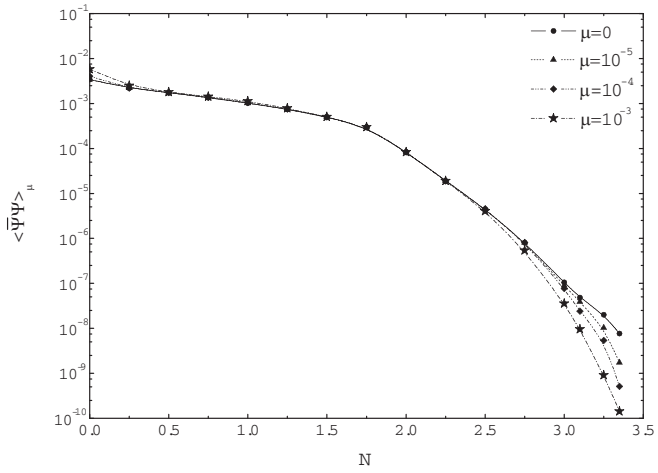

FIG. 1. The dependence of the fermion condensate $\langle\bar{\psi}\psi\rangle_\mu$ on N for several values of μ.

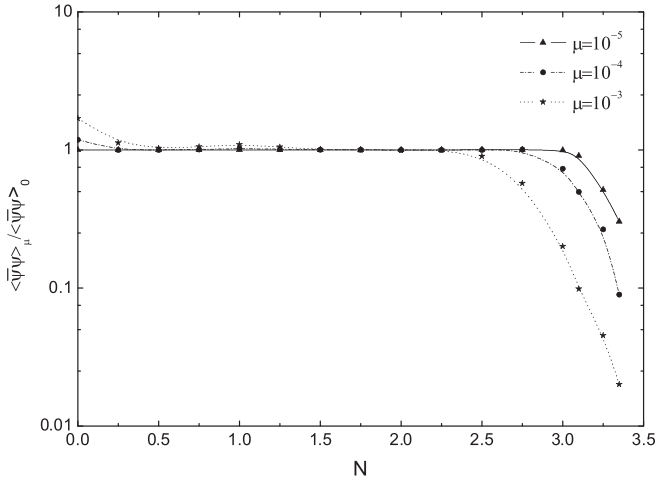

FIG. 2. The dependence of $\frac{\langle\bar{\psi}\psi\rangle_\mu}{\langle\bar{\psi}\psi\rangle_0}$ on N for several values of μ.

because we ignore the influence of the chemical potential μ on the photon propagator while the actual photon propagator should be μ dependent due to fermion vacuum polarizations. As such it may be inadequate at large value of μ.

IV. CONCLUSION

Ignoring the influence of the chemical potential μ on the photon propagator and working in the rainbow approximation of DS equations, we first prove, using the method of Taylor-expansion in terms of chemical potential μ, that the fermion propagator at nonzero chemical potential can be written as $\hat{S}^{-1}(p) = i\gamma\cdot\tilde{p}A(\tilde{p}^2) + B(\tilde{p}^2)$. Using this form, we provide a general recipe to calculate the chemical potential dependence of the dressed fermion propagator under the rainbow approximation to the DS equation. This approach has the advantage that the dressed fermion propagator at finite chemical potential can be obtained directly from the one at zero chemical potential without the necessity of numerically solving the corresponding coupled integral equations by iteration methods. This feature facilitates numerical calculations considerably. From this it is found that $\frac{\langle\bar{\psi}\psi\rangle_\mu}{\langle\bar{\psi}\psi\rangle_0}$ decreases monotonically as N increases. With the chemical potential μ increasing, the ratio of the fermion condensate decreases near N_c. This shows that N_c decreases with μ increasing.

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