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Light neutralino dark matter in the next-to-minimal supersymmetric standard model

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Neutralino dark matter is generally assumed to be relatively heavy, with a mass near the electroweak scale. This does not necessarily need to be the case, however. In the next-to-minimal supersymmetric standard model (NMSSM) and other supersymmetric models with an extended Higgs sector, a very light *CP*-odd Higgs boson can naturally arise making it possible for a very light neutralino to annihilate efficiently enough to avoid being overproduced in the early Universe. In this article, we explore the characteristics of a supersymmetric model needed to include a very light neutralino, 100 MeV $< m_{\tilde{\chi}^0_1} <$ 20 GeV, using the NMSSM as a prototype. We discuss the most important constraints from Upsilon decays, $b \to s\gamma$, $B_s \to \mu^+\mu^-$ and the magnetic moment of the muon, and find that a light bino or singlino neutralino is allowed, and can be generated with the appropriate relic density. It has previously been shown that the positive detection of dark matter claimed by the DAMA collaboration can be reconciled with other direct dark matter experiments such as CDMS II if the dark matter particle is rather light, between about 6 and 9 GeV. A singlino or binolike neutralino could easily fall within this range of masses within the NMSSM. Additionally, models with sub-GeV neutralinos may be capable of generating the 511 keV gamma-ray emission observed from the galactic bulge by the INTEGRAL/SPI experiment. We also point out measurements which can be performed immediately at CLEO, *BABAR*, and Belle using existing data to discover or significantly constrain this scenario.

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I. INTRODUCTION

Despite the substantial effort which has gone into its detection, the nature of dark matter remains unknown [1]. The dark matter candidates which have received the most attention fall into the category of weakly interacting massive particles (WIMPs), which can emerge from a variety of theoretical frameworks, including supersymmetry. Of the supersymmetric candidates for dark matter, the lightest neutralino is often considered to be the most attractive.

Neutralinos produced in the early Universe must annihilate into standard model particles at a sufficient rate to avoid overproducing the density of dark matter. Within the framework of the minimal supersymmetric standard model (MSSM), the lightest neutralino can annihilate through a variety of channels, exchanging other sparticles, Z bosons, or Higgs bosons. The masses of sparticles such as sleptons or squarks, as well as the masses of Higgs bosons, are limited by collider constraints, with typical lower limits of around $\sim$100 GeV. For lighter neutralinos, it becomes increasingly difficult for these heavy propagators to generate neutralino annihilation cross sections that are large enough. The most efficient annihilation channel for very light neutralinos in the MSSM is the s-channel exchange of a pseudoscalar Higgs boson. It has been shown that this channel can, in principle, be sufficiently efficient to allow for neutralinos as light as 6 GeV [2]. Such models require a careful matching of a number of independent parameters, however, making viable models with neutralinos lighter than $\sim$20 GeV rather unlikely [3]. Measurements of rare B-decays are also particularly constraining in this regime. If we do not require that the lightest supersymmetric particle (LSP) be the dominant component of dark matter, its mass can be zero [4].

More generally speaking, Lee and Weinberg have demonstrated that a fermionic dark matter candidate which annihilates through its couplings to the weak gauge bosons must be heavier than a few GeV to avoid over-closing the Universe [5]. Therefore, if a neutralino is to be very light, it requires another annihilation channel which enables it to sufficiently annihilate in the early Universe. This can be provided within the context of the next-to-minimal supersymmetric standard model (NMSSM) by the lightest of the two *CP*-odd Higgs bosons, which can be considerably lighter than the single *CP*-odd Higgs boson of the MSSM without violating collider constraints. Furthermore, it has been shown that models within the NMSSM which require the smallest degree of fine tuning often contain a light *CP*-odd Higgs boson [6].

In addition to these theoretical arguments, there are experimental motivations to consider light dark matter particles. The observation of 511 keV gamma rays from the galactic bulge [7] indicates the presence of a Gaussian profile of low-velocity positrons throughout our galaxy's inner kiloparsec. It is challenging to explain this observation with traditional astrophysics [8]. Annihilating [9] or decaying [10] dark matter particles have been suggested as a possible source of these positrons. If such a dark matter particle were in the mass range usually considered, however, their annihilation would produce positrons with far too much energy to annihilate at rest. Furthermore, they would almost certainly generate far too many gamma rays and violate the constraints placed by EGRET [11]. Thus a dark matter candidate capable of generating the observed 511 keV line must be exceptionally light.

Additionally, it has been shown that the claims of dark matter detection made by the DAMA collaboration [12] can be reconciled with null results of CDMS II [13] and other experiments if one considers a WIMP lighter than approximately 10 GeV [14,15].

In this article, we explore the phenomenology of supersymmetric models with a neutralino in the mass range of 100 MeV to 20 GeV within the context of the NMSSM. We find that many such models can be found which are not highly fine tuned and are consistent with all constraints including direct collider searches, rare decays, and relic abundance considerations. We find examples of consistent models in which a light neutralino can potentially produce the 511 keV emission observed by INTEGRAL as well as models that can potentially reconcile DAMA with CDMS II. However, we have not found models in which all these observations can be simultaneously explained.

II. NEUTRALINO DARK MATTER WITH A SINGLET HIGGS

The simplest possible extension of the particle content of the MSSM is the addition of a new gauge singlet chiral supermultiplet. There are several ways to do this including the NMSSM [16,17], the minimal nonminimal supersymmetric standard model (MNSSM) [18], and larger models [19] with interesting implications for dark matter [20]. For concreteness and the availability of dominant 1-loop and 2-loop corrections to the Higgs sector via the code NMHDECAY [21], we choose to study the NMSSM.

Adding a Higgs singlet is attractive for several reasons. Most interesting, perhaps, is that it provides an elegant solution to the μ-problem present in the MSSM [22]. Additionally, the "little fine tuning problem," which results in the MSSM from the lack of a detection of a CP-even Higgs at LEP II, is less severe within the NMSSM [6,23], and is completely absent if the lightest CP-odd Higgs is light enough to allow $H \to A_1 A_1$ decays [6]. Thirdly, baryogenesis considerations leave the MSSM in disfavor, requiring the right-handed stop squark to be lighter than the top quark and the Higgs lighter than about 117 GeV [24]. Recent studies of baryogenesis within the NMSSM indicate that parameter points with a light singlet Higgs and a corresponding light neutralino are favored [25]. Finally, the domain wall problem [26] in the NMSSM can be avoided by the introduction of appropriate nonrenormalizable Planck-suppressed operators, and imposing a discrete R-symmetry on them [27].

In the NMSSM, the physical CP-even and CP-odd Higgs states are mixtures of MSSM-like Higgses and singlets. The lightest neutralino therefore has, in addition to the four MSSM components, a singlino component which is the superpartner of the singlet Higgs. The eigenvector of the lightest neutralino, $\tilde{\chi}_1^0$, in terms of gauge eigenstates is:

$$\tilde{\chi}_1^0 = \epsilon_u \tilde{H}_u^0 + \epsilon_d \tilde{H}_d^0 + \epsilon_W \tilde{W}^0 + \epsilon_B \tilde{B} + \epsilon_s \tilde{S}, \quad (1)$$

where ϵ_u, ϵ_d are the up-type and down-type Higgsino components, ϵ_W, ϵ_B are the wino and bino components and ϵ_s is the singlet component of the lightest neutralino.

Likewise, for the lightest CP-even Higgs state we can define:

$$H_1 = \left[\xi_u \Re\left(\frac{H_u^0}{\sqrt{2}} - v_u\right) + \xi_d \Re\left(\frac{H_d^0}{\sqrt{2}} - v_d\right) + \xi_s \Re\left(\frac{S}{\sqrt{2}} - x\right) \right]. \quad (2)$$

Here, $\Re$ denotes the real component of the respective state, and we take vacuum expectation values to be those of the complex states (e.g. $v = \sqrt{v_u^2 + v_d^2} \simeq 174$ GeV).

Lastly, we can write the lightest CP-odd Higgs as:

$$A_1 = \cos\theta_A A_{\rm MSSM} + \sin\theta_A A_s, \quad (3)$$

where A_s is the CP-odd piece of the singlet and $A_{\rm MSSM}$ is the state that would be the MSSM pseudoscalar Higgs if the singlet were not present. θ_A is the mixing angle between these two states. There is also a third imaginary linear combination of H_u^0, H_d^0, and S that we have removed by a rotation in β. This field becomes the longitudinal component of the Z after electroweak symmetry is broken.

The NMSSM can contain either an approximate global $U(1)$ R-symmetry in the limit that the Higgs-sector trilinear soft SUSY-breaking terms are small, or a $U(1)$ Peccei-Quinn symmetry in the limit that the cubic singlet term in the superpotential vanishes [28]. In either case, one ends up with the lightest CP-odd Higgs boson, A_1, as the pseudogoldstone boson of this broken symmetry, which can be very light. In some regions of the NMSSM parameter space, one can also get the lightest CP-even state, H_1, to be very light as well. This is discussed in more detail in Sec. IV. As shown in Sec. III, it is easy to get a light largely singlino LSP in the $U(1)_{PQ}$ symmetry limit.

While we confine our analysis to the NMSSM, it should be noted that such symmetries are generically present in other singlet models such as the minimal nonminimal supersymmetric standard model (MNSSM) [18]. The combination of a light A_1 and a light neutralino is not uncommon in a wide class of models with extra singlets and/or extra gauge groups [19]. Implications of such models for the relic neutralino density have been considered in [29].

The NMSSM is defined by the superpotential

$$\lambda \hat{H}_u \hat{H}_d \hat{S} + \frac{\kappa}{3} \hat{S}^3 \quad (4)$$

and associated soft-supersymmetry-breaking terms

$$\lambda A_\lambda H_u H_d S + \frac{\kappa}{3} A_\kappa S^3, \quad (5)$$

where the hatted objects are chiral superfields and unhatted objects are their scalar components. An effective μ parameter (as defined by the superpotential form $\mu \hat{H}_u \hat{H}_d$ of the MSSM) is generated from the first term of Eq. (4) when $\langle S \rangle \equiv x$ is nonzero: $\mu = \lambda x$. We follow the sign conventions for NMSSM parameters of Refs. [30,31] in which λ and $\tan\beta \equiv v_u/v_d$ are positive while κ, A_λ, and A_κ can have either sign.

III. LIGHT NEUTRALINOS IN THE NMSSM

In the basis $\tilde{\chi}^0 = (-i\tilde{\lambda}_1, -i\tilde{\lambda}_2, \psi_u^0, \psi_d^0, \psi_s)$, the tree-level neutralino mass matrix takes the form

$$\mathcal{M}_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & \frac{g_1 v_u}{\sqrt{2}} & -\frac{g_1 v_d}{\sqrt{2}} & 0 \\ 0 & M_2 & -\frac{g_2 v_u}{\sqrt{2}} & \frac{g_2 v_d}{\sqrt{2}} & 0 \\ \frac{g_1 v_u}{\sqrt{2}} & -\frac{g_2 v_u}{\sqrt{2}} & 0 & -\mu & -\lambda v_d \\ -\frac{g_1 v_d}{\sqrt{2}} & \frac{g_2 v_d}{\sqrt{2}} & -\mu & 0 & -\lambda v_u \\ 0 & 0 & -\lambda v_d & -\lambda v_u & 2\kappa x \end{pmatrix}. \tag{6}$$

In the above, the upper 4×4 matrix corresponds to $\mathcal{M}_{\tilde{\chi}^0}^{\rm MSSM}$. From the lower 3×3 matrix, we find that if $\lambda v_{u,d} = (\mu/x) v_{u,d}$ are small compared to $|\mu|$ and/or $2|\kappa x|$ then the singlino decouples from the MSSM and has mass

$$m_{\rm singlino} \simeq \sqrt{\lambda^2 v^2 + 4\kappa^2 x^2} = \sqrt{\mu^2 v^2/x^2 + 4\kappa^2 x^2}, \tag{7}$$

as found from $[M_{\tilde{\chi}^0}]^2_{55}$. Thus, if $2|\kappa x|$ and λv are both $< M_1, M_2, |\mu|$, then the lightest neutralino will tend to be singlinolike [32]. Since $|x|$ is typically substantial (given that $\lambda < 1$ and $|\mu| = \lambda |x|$ must be substantial to satisfy chargino mass limits), a singlinolike $\tilde{\chi}_1^0$ (formally defined by $\epsilon_s^2 < 0.5$) emerges mainly for small κ. In fact, for very small λ, $|x|$ must be quite large and thus the singlino will be the LSP only if $|\kappa|$ is also very small; otherwise $2|\kappa x|$ would exceed one or more of the typically moderate M_1, M_2, $|\mu|$ values considered here and the singlino would not be the LSP. For larger λ (e.g. $\gtrsim 0.3$), $|x|$ need not be extremely large and the singlino LSP condition $2|\kappa x| < M_1, M_2, |\mu|$ can hold for slightly larger $|\kappa|$. These behaviors can be seen in Fig. 1 obtained by scanning using NMHDECAY 1.1 [21]. NMHDECAY tests for theoretical consistency of the model and for consistency with LEP constraints on the Higgs sector, neutralinos, and the chargino). It also includes radiative corrections to the tree-level mass matrix that are often quite important for small $|\kappa|$. As expected, the neutralino is singlinolike ($\epsilon_s^2 > 0.5$) when $|\kappa|$ is small. Consistent solutions are found primarily in two regions of parameter space—one at small λ with very small $|\kappa|$, and another at large λ with slightly larger $|\kappa|$ allowed, see Fig. 1. Since $\tan\beta$ also induces singlino mixing, the singlino points at large λ also have small $\tan\beta \lesssim 4$ while the points at small λ can have any value of $\tan\beta$.

For $|\kappa|$ not close to zero, binolike $\tilde{\chi}_1^0$'s can easily emerge for small values of M_1. In this case, the bino does not have a large degree of mixing with the other neutralinos and the LSP mass is nearly fixed to M_1.

We will find that a light neutralino which is mostly bino or a combination of bino and singlino, with a small admixture of Higgsino, can generate the observed dark matter density and evade all relevant collider constraints.

IV. LIGHT *CP*-ODD HIGGS BOSONS IN THE NMSSM

After removing the *CP*-odd degree of freedom that is absorbed in giving the Z its mass, the remaining *CP*-odd states have the squared-mass matrix

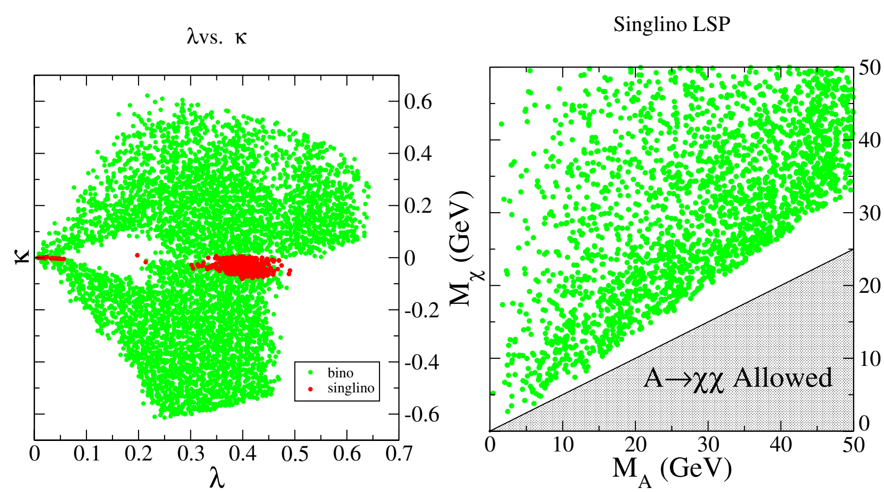

FIG. 1 (color online). On the left, we show regions of $\lambda - \kappa$ parameter space for which the $\tilde{\chi}_1^0$ is singlinolike (defined by $\epsilon_s^2 > 0.5$) and binolike (defined by $\epsilon_s^2 \leq 0.5$). On the right, we plot m_A vs $m_{\tilde{\chi}_1^0}$ for singlinolike neutralinos with $\epsilon_s^2 > 0.9$. Each point shown is consistent with all LEP constraints.

$$\mathcal{M}_A^2 = \begin{pmatrix} \frac{2\lambda x}{\sin 2\beta}(A_\lambda + \kappa x) & \lambda v(A_\lambda - 2\kappa x) \\ \lambda v(A_\lambda - 2\kappa x) & \left(2\lambda\kappa + \frac{\lambda A_\lambda}{2x}\right)v^2 \sin 2\beta - 3x\kappa A_\kappa \end{pmatrix}, \tag{8}$$

where $v^2 = v_u^2 + v_d^2$. For physically acceptable solutions, the lightest state must have $m_{A_1}^2 > 0$. In addition, the lightest CP-even Higgs boson and the charged Higgs boson must have positive mass-squared. To avoid spontaneous CP-violation several other conditions must be satisfied [30]. In our conventions these are as follows:

(a) For $\kappa > 0$, we must have one of three situations:
  (1) $\text{sign}(\mu) = \text{sign}(A_\lambda) = -\text{sign}(A_\kappa)$;
  (2) $\text{sign}(\mu) = -\text{sign}(A_\lambda) = -\text{sign}(A_\kappa)$ with $|A_\kappa| > 3\lambda v_u v_d |A_\lambda|/(-|xA_\lambda| + \kappa x^2)$, where the denominator has to be positive;
  (3) $\text{sign}(\mu) = \text{sign}(A_\lambda) = \text{sign}(A_\kappa)$ with $|A_\kappa| < 3\lambda v_u v_d |A_\lambda|/(|xA_\lambda| + \kappa x^2)$.

(b) For $\kappa < 0$, a CP-conserving minimum requires
  (1) $\text{sign}(\mu) = \text{sign}(A_\lambda) = \text{sign}(A_\kappa)$ with $|A_\kappa| > 3\lambda v_u v_d |A_\lambda|/(|xA_\lambda| - \kappa x^2)$.

To find a model which has a light CP-odd Higgs boson, we can require that one of the $U(1)_R$ or $U(1)_{PQ}$ symmetries approximately holds. The $U(1)_R$ symmetry appears in the limit that the trilinear terms A_κ and A_λ vanish. This is well motivated from models of gaugino-mediated SUSY breaking [33] in which trilinear terms are generated radiatively and, therefore, are suppressed relative to the gaugino masses by a loop factor of 4π. One would expect A_λ to be smaller than the gaugino masses by a factor of 4π and A_κ to be smaller by a factor of $16\pi^2$ because S is not charged under gauge symmetries, and only receives a trilinear term at two loops. The small trilinear terms are also radiatively protected and remain small when evolved via renormalization group equations (RGEs) from the SUSY breaking scale to the weak scale [28]. In this limit, the lightest CP-odd Higgs is a pseudogoldstone boson of the broken $U(1)_R$ symmetry and has a mass of $m_{A_1}^2 \simeq -3\kappa A_\kappa x$ in the large $\tan\beta$ or large $|x|$ limits. Alternatively, we can make the substitution, $x = \mu/\lambda$, and write this as $m_{A_1}^2 \simeq -3\frac{\kappa}{\lambda}A_\kappa \mu$.

More generally, in the limit of small A_λ and A_κ one finds

$$\tan\theta_A \sim \frac{x}{v\sin 2\beta}, \qquad \cos^2\theta_A \sim \frac{v^2\sin^2 2\beta}{v^2\sin^2 2\beta + x^2}, \tag{9}$$

and

$$m_{A_1}^2 \sim \frac{\frac{9}{2}\lambda A_\lambda v^2 x \sin 2\beta - 3\kappa A_\kappa x^3}{x^2 + v^2\sin^2 2\beta}. \tag{10}$$

Since $|x| > v$ is preferred, $|\cos\theta_A|$ is typically small at small to moderate $\tan\beta$, with $\cos^2\theta_A \to 0$ at large $\tan\beta$. If we only take $A_\kappa \to 0$, one finds the results

$$\tan\theta_A \sim \frac{x}{v\sin 2\beta}\left[\frac{1 + A_\lambda/(\kappa x)}{1 - A_\lambda/(2\kappa x)}\right], \tag{11}$$

and

$$m_{A_1}^2 \sim \frac{\frac{9}{2}\lambda A_\lambda v^2 x \sin 2\beta}{x^2 + v^2\sin^2 2\beta + A_\lambda x/\kappa}, \tag{12}$$

valid whenever the numerator of the preceding equation is much smaller than the square of the denominator, as, for example, if $\tan\beta \to \infty$ or x is large. Again, $\cos\theta_A$ will be quite small typically and the A_1 relatively singletlike. In practice, this limit is very frequently applicable.

The $U(1)_{PQ}$ symmetry appears in the limit that κ vanishes (and therefore the soft SUSY-breaking term κA_κ also vanishes) and also results in a light A_1. To leading order, one finds

$$\tan\theta_A \sim -\frac{2x}{v\sin 2\beta}, \qquad \cos^2\theta_A \sim \frac{v^2\sin^2 2\beta}{v^2\sin^2 2\beta + 4x^2} \tag{13}$$

and

$$m_{A_1}^2 \sim \frac{6\kappa x^2(3\lambda v^2 \sin 2\beta - 2A_\kappa x)}{4x^2 + v^2\sin^2 2\beta}. \tag{14}$$

For $|x| > v$, $|\cos\theta_A|$ is small for moderate $\tan\beta$ and approaches 0 at large $\tan\beta$.

It is useful to note that if λ is small (implying large $|x|$), then singlet mixing in both $M_{\tilde{\chi}_0}$ and $\mathcal{M}_A^2$ is small. If $2|\kappa x| < M_1, M_2, |\mu|$, and $2\mu(A_\lambda + \kappa x)/\sin 2\beta > -3\kappa A_\kappa x$ then the $\tilde{\chi}_1^0$ and A_1 will both be singlet in nature. In particular, for small $|\kappa|$ both the A_1 and the $\tilde{\chi}_1^0$ can easily be singletlike. At large λ, the A_1 can have a more mixed nature ($\cos\theta_A$ tends to be larger) but as we have seen a moderately-singlino $\tilde{\chi}_1^0$ is still allowed despite the somewhat larger mixing in the neutralino mass matrix.

In either of the $A_\lambda, A_\kappa \to 0$ or $\kappa \to 0$ cases, a light A_1 is technically natural since it is protected by an approximate symmetry. From an effective field theory perspective, the small terms that break the symmetry will not receive large radiative corrections. It is technically natural for the A_1, H_1, or $\tilde{\chi}_1^0$ to be very light as a result of $U(1)_R$ and/or $U(1)_{PQ}$ symmetries.

The fermion Yukawas break the $U(1)_R$ symmetry, leading to contributions arising at one loop for the H_u and H_d components of the Higgs sector, and at two loops for the S component. However the radiative corrections to the singlet component are proportional to either λ or κ, and thus are suppressed for small values of λ, κ. These symmetries therefore result in the hierarchy $m_h \gg A_\lambda \gg A_\kappa$.

It will be helpful in understanding dark matter relic density issues to examine whether or not a light (singlet) A_1 can decay to a pair of nearly pure $\tilde{\chi}_1^0$ singlinos. From the right-hand plot of Fig. 1, we observe that it is impossible to obtain $m_{A_1} > 2m_{\tilde{\chi}_1^0}$ when the LSP is nearly purely singlino ($\epsilon_s^2 > 0.9$). Using the fact that a highly singlino $\tilde{\chi}_1^0$ is achieved by taking λ to be very small (so as to remove mixing in $M_{\tilde{\chi}^0}$), implying large $|x|$, we are able to analytically understand this in two cases: (i) $|\kappa x|$ moderate in size and (ii) $|\kappa x|$ small. For moderate $|\kappa x|$, the inability to satisfy the mass requirements for the decay $A_1 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$ stems from the inability to simultaneously satisfy $m_{S_3}^2 > 0$ and $m_{A_1}^2/4m_{\tilde{\chi}_1^0}^2 > 1$. Here, S_3 is the third CP-even (largely singlet) Higgs mass eigenstate as defined in [16] and is the lightest CP-even Higgs state in the limit of interest. For small λ and finite $|\kappa x|$, we have

$$m_{\tilde{\chi}_1^0} \sim 2\kappa x, \qquad m_{A_1}^2 \sim -3\kappa A_\kappa x, \tag{15}$$

the latter requiring $\kappa A_\kappa < 0$. Further, if one expands the CP-even mass matrix in the large $|x|$ limit, holding μ and κx fixed, one finds [16] (after correcting for differences in sign conventions)

$$m_{S_3}^2 = 4\kappa^2 x^2 + \kappa A_\kappa x + \frac{\mu^2 v^2}{\kappa^2 x^2}\left[\frac{\mu}{x} - \frac{1}{2}\left(-2\kappa + \frac{A_\lambda}{x}\right)\sin 2\beta\right]^2 + \frac{\mu^2 v^2 (-2\kappa + \frac{A_\lambda}{x})^2 \cos^2 2\beta}{4\kappa^2 x^2 - \frac{2\mu x A_\Sigma}{\sin 2\beta}}, \tag{16}$$

where $A_\Sigma \equiv A_\lambda - \kappa x$. For fixed $|\kappa x|$ with $|x|$ very large, the last two terms approach zero and we have $m_{S_3}^2 \sim 4\kappa^2 x^2 + \kappa A_\kappa x$ which is positive only if $-\kappa A_\kappa < 4\kappa^2 x$. However, in the same limit, the mass condition for the $A_1 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$, written as $m_{A_1}^2 > 4m_{\tilde{\chi}_1^0}^2$, becomes $-\kappa A_\kappa > (16/3)\kappa^2 x$. These two conditions cannot be simultaneously satisfied, and thus the decay $A_1 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$ is not allowed for a pure singlino in the large $|x|$, fixed κx limit. Some admixture of bino, and therefore moderate λ and $|x|$ must be required for this decay to be open.

The $|\kappa| \to 0$ case (the Peccei-Quinn symmetry limit) at small λ is defined by

$$|\kappa| \ll \mathcal{O}\left(\lambda, \frac{|A_\lambda|}{v}, \frac{|A_\kappa|}{v}, \frac{|\mu|}{|x|}, \frac{v}{|x|}\right), \qquad v \ll |x|. \tag{17}$$

In this limit, Eq. (7) implies $m_{\tilde{\chi}_1^0}^2 \simeq \lambda^2 v^2 = \mu^2 v^2/x^2$. Meanwhile, for $|\kappa x| \to 0$ and $|x|$ large (λ small), it is easily seen that $m_{A_1}^2 \propto 1/|x|^3$. Thus, once again, the $A_1 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$ decay is disallowed.

The fact that $m_{A_1} < 2m_{\tilde{\chi}_1^0}$ in these singlino limits implies that a singlino $\tilde{\chi}_1^0$ is disfavored cosmologically. This is because $m_{A_1} \simeq 2m_{\tilde{\chi}_1^0}$ is required to enhance the annihilation cross section to the level needed to obtain the correct relic density. Therefore some bino mixing is required to get an appropriate relic density. This, in turn, requires M_1 to be small as well.

Finally, we should note that the constraints on a light A_1 are rather weak. This is because direct searches at LEP [34] require a light A_1 to be radiated off of a quark or a tau lepton and, due to the small fermion Yukawa couplings, bounds are only obtainable when the A_1 coupling to fermions is enhanced by $\tan\beta$. However, in the $U(1)_{PQ}$ and $U(1)_R$ symmetry limits discussed earlier, $\cos\theta_A$ (the nonsinglet part of A_1) is proportional to $\sin 2\beta$ so that the product $\tan\beta \cos\theta_A$ remains modest in size. If a light A_1 exists and is near in mass to the η (547 MeV), it may be discovered via invisible decays of the η at low energy lepton colliders [35]. This mass range is extremely interesting if a light A_1 exchange is the explanation for the recent galactic 511 keV line from the INTEGRAL/SPI experiment [9].

We now give some additional remarks concerning the singlet and nonsinglet A_1 possibilities.

A. Models with a singletlike A_1

As we have seen, a singletlike A_1 ($\cos\theta_A \ll 1$) is extremely easy to obtain by making some combination of $|\kappa|$, $|A_\kappa|$, and $|A_\lambda|$ small. Indeed, the mass of the A_1 can be driven to zero at tree level. Radiative corrections increase this mass, however. The dominant source of these radiative corrections to the singlet mass is from standard model couplings since the number of degrees of freedom is much larger in the MSSM than in the singlet supermultiplet. These radiative corrections are therefore proportional to λ, since the λ superpotential term is the *only* coupling connecting the singlet with the rest of the MSSM. Therefore, if the light singlet mass is to be radiatively stable, λ must be small. λ being small also has the effect of reducing the mixing with the singlet component in both the CP-even mass matrix and the neutralino mass matrix. *All* terms which mix the singlino with the MSSM neutralinos and the singlet S with H_u and H_d are proportional to λ. We find $\lambda \lesssim 0.1$ to be natural, with larger values of λ requiring an increasing amount of cancellation between the various radiative contributions to its mass. λ being this small necessarily implies that the singlet vacuum expectation value, $|x|$, is large since $\mu = \lambda x$. Chargino searches generally imply $|\mu| \gtrsim 100$ GeV, leading to $|x| \gtrsim 1$ TeV for $\lambda \lesssim 0.1$. Furthermore, with all four of λ, κ, A_κ, and A_λ small in magnitude, the entire supermultiplet is light and A_1, H_1 (largely the singletlike S_3) and $\tilde{\chi}_1^0$ tend to be nearly degenerate.

B. Models with an MSSM-like A_1

An MSSM-like (nonsinglet) A_1 ($\cos\theta_A \simeq 1$) can also be obtained, but is subject to more stringent constraints. If $\cos\theta_A \simeq 1$, couplings of the A_1 to down-type fermions go like $\cos\theta_A \tan\beta$, therefore phenomenological constraints become significant at large $\tan\beta$. If such an A_1 is very light, it will be further constrained by rare decays such as $K \to \pi\nu\bar{\nu}$ and $\Upsilon \to \gamma X$ as discussed in Sec. V E.

V. CONSTRAINTS

In this section, we will consider a series of constraints which may be relevant to light neutralinos and/or a light

CP-odd Higgs bosons in the NMSSM. Except for the LEP and Υ decay limits, most of the constraints discussed below are easily avoided by appropriate choices of SUSY parameters to which our dark matter calculations are not sensitive.

A. LEP limits

If the lightest neutralino is lighter than $m_Z/2$, Z decays to neutralino pairs may violate the bounds obtained at LEP for the Z's invisible decay width. In particular, we require $\Gamma_{Z\to\tilde{\chi}_1^0\tilde{\chi}_1^0} < 4.2$ MeV, which corresponds to 1 standard deviation from the measured neutrino contribution. Since binos, winos, and singlinos do not couple to the Z, this constraint can only limit the Higgsino components of the lightest neutralino. In the mass range we are most interested in here ($m_{\tilde{\chi}_1^0} \lesssim 20$ GeV), this constraint is satisfied for all models with $|\epsilon_u^2 - \epsilon_d^2| \lesssim 6\%$.

Direct chargino searches also limit the wino component of the lightest neutralino. This is due to the fact that if the lightest neutralino has a significant wino component, then it will have a mass that is a significant fraction of the chargino mass (with near degeneracy if the $\tilde{\chi}_1^0$ is mainly wino).

In combination, these constraints imply that a very light $\tilde{\chi}_1^0$ must be dominantly a linear combination of bino and singlino.

B. The magnetic moment of the muon

The one-loop contribution to the magnetic moment of the muon from a light neutralino comes from a triangle diagram with a smuon along two sides and the neutralino around the third. This contribution is given by [36]:

$$\delta a_\mu^{\tilde{\chi}_1^0} = \frac{m_\mu}{16\pi^2}\sum_{m=1,2}\left[\frac{-m_\mu}{12m_{\tilde{\mu}_m}^2}(|n_m^L|^2 + |n_m^R|^2)F_1^N(x_m) + \frac{m_{\tilde{\chi}_1^0}}{3m_{\tilde{\mu}_m}^2}\mathrm{Re}[n_m^L n_m^R]F_2^N(x_m)\right], \tag{18}$$

where $n_m^R = \sqrt{2}g_1\epsilon_B X_{m2} + y_\mu\epsilon_u X_{m1}$, $n_m^L = (g_2\epsilon_W X_{m2} + g_1\epsilon_B)X_{m1}^*/\sqrt{2} - y_\mu\epsilon_u X_{m2}^*$, $y_\mu = g_2 m_\mu/(\sqrt{2}m_W\cos\beta)$, and $X_{m,n}$ are elements of the unitary matrix which diagonalizes the smuon mass matrix. The functions, F^N, are defined by:

$$F_1^N(x_m) = \frac{2}{(1-x_m)^4}(1 - 6x_m + 3x_m^2 + 2x_m^3 - 6x_m^2\ln x_m), \tag{19}$$

$$F_2^N(x_m) = \frac{3}{(1-x_m)^3}(1 - x_m^2 + 2x_m\ln x_m), \tag{20}$$

where $x_m = m_{\tilde{\chi}_1^0}^2/m_{\tilde{\mu}_m}^2$. For a binolike neutralino, this reduces to:

$$\delta a_\mu^{\tilde{\chi}_1^0} \simeq \frac{g_1^2}{48\pi^2}\frac{m_\mu m_{\tilde{\chi}_1^0}}{(m_{\tilde{\mu}_2}^2 - m_{\tilde{\mu}_1}^2)}\Delta_{\tilde{\mu}}\left[\frac{F_2^N(x_1)}{m_{\tilde{\mu}_1}^2} - \frac{F_2^N(x_2)}{m_{\tilde{\mu}_2}^2}\right], \tag{21}$$

where $\Delta_{\tilde{\mu}}$ is the real part of the off diagonal elements of the smuon mass matrix, $\Delta_{\tilde{\mu}} = \Re[m_\mu(A_{\tilde{\mu}} - \mu^*\tan\beta)]$. This further simplifies to:

$$\delta a_\mu^{\tilde{\chi}_1^0} \sim 2.3\times 10^{-11}\left(\frac{m_{\tilde{\chi}_1^0}}{10\text{ GeV}}\right)\left(\frac{200\text{ GeV}}{m_{\tilde{\mu}}}\right)^4 \times\left(\frac{\mu\tan\beta - A_{\tilde{\mu}}}{1000\text{ GeV}}\right). \tag{22}$$

In addition to this contribution from a light neutralino, a light CP-odd Higgs can contribute non-negligibly to δa_μ through both one-loop and two-loop processes [37]. The one-loop contribution (corresponding to a triangle diagram with a muon along two sides and the CP-odd Higgs along the third side) is given by:

$$\delta a_\mu^{A1\text{ loop}} = \frac{g_2^2 m_\mu^2\cos^2\theta_A\tan^2\beta L_A}{32m_W^2\pi^2}, \tag{23}$$

where the function, L_A, is given by:

$$L_A = \frac{m_\mu^2}{m_A^2}\int_0^1\frac{-x^3dx}{x^2(m_\mu^2/m_A^2) + (1-x)}. \tag{24}$$

Numerically, this function yields $L_A = -0.032$, -0.00082, and -0.000014 for 1, 10, and 100 GeV CP-odd Higgs bosons, respectively. Thus we arrive at:

$$\delta a_\mu^{A1\text{ loop}} \approx L_A\cos^2\theta_A\tan^2\beta\times 2.7\times 10^{-9}, \tag{25}$$

which is a negative contribution due to the sign of L_A.

The contribution from two-loop diagrams involving a heavy fermion loop is given by:

$$\delta a_\mu^{A2\text{ loop}} = \frac{g_2^2 m_\mu^2\alpha c_f Q_f^2\chi_f^2 L_f}{32m_W^2\pi^2}, \tag{26}$$

where c_f is the color factor of the fermion in the loop (3 for quarks, 1 for leptons), Q_f is the electric charge of the fermion, $\alpha\approx 1/137$, $\chi_f = \cos\theta_A\tan\beta$ for up-type fermions and $\cos\theta_A\cot\beta$ for down-type fermions, and L_f is given by:

$$L_f = \frac{m_f^2}{2m_A^2}\int_0^1\frac{dx}{x(1-x) - (m_f^2/m_A^2)}\ln\left(\frac{x(1-x)}{m_f^2/m_A^2}\right). \tag{27}$$

For a top quark loop, this function yields the values $L_t = 6.2$, 3.9, and 1.7 for 1, 10, and 100 GeV CP-odd Higgs bosons, respectively. For a bottom quark loop, these values are $L_b = 2.5$, 0.59, and 0.038. Numerically, these contributions are:

$$\delta a_\mu^{A2\text{ loop}} \approx L_t\cos^2\theta_A\cot^2\beta\times 2.6\times 10^{-11} + L_b\cos^2\theta_A\tan^2\beta\times 6.6\times 10^{-12}. \tag{28}$$

Combining the results of the one and two-loop contributions from the light A_1, we arrive at the following:

$$\delta a_\mu^{\rm A1+2\ loop} \approx -7 \times 10^{-11} \times \cos^2\theta_A \tan^2\beta \quad \text{for } m_{A_1} = 1 \text{ GeV},$$
$$\delta a_\mu^{\rm A1+2\ loop} \approx 1.7 \times 10^{-12} \times \cos^2\theta_{\rm A} \tan^2\beta \quad \text{for } {\rm m}_{\rm A_1} = 10 \text{ GeV}. \tag{29}$$

It is somewhat difficult to know how best to interpret the current status of the measurement of the muon's magnetic moment. Using e^+e^- data, the measured value exceeds the theoretical prediction by $\delta a_\mu(e^+e^-) = [23.9 \pm 7.2_{\text{had-lo}} \pm 3.5_{\text{lbl}} \pm 6_{\text{exp}}] \times 10^{-10}$, where the error bars correspond to theoretical uncertainties in the leading order hadronic and the hadronic light-by-light contributions as well as from experimental contributions. Combined, this result is 2.4σ above the standard model prediction. Experiments using $\tau^+\tau^-$ data, on the other hand, find $\delta a_\mu(\tau^+\tau^-) = [7.6 \pm 5.8_{\text{had-lo}} \pm 3.5_{\text{lbl}} \pm 6_{\text{exp}}] \times 10^{-10}$, which is only 0.9σ above the standard model prediction [38].

Comparing the expression shown in Eq. (22) for a light neutralino to these experimental results illustrates that only in extreme models, with a combination of small $m_{\tilde{\mu}}$, large $\tan\beta$, and large μ is there any danger of exceeding these bounds with a light neutralino. For fairly moderate choices of parameters, i.e. $m_{\tilde{\mu}} \sim 200$ GeV, $\tan\beta \sim 20$, and $\mu \sim 500$ GeV, the experimental values can be matched. Since $m_{\tilde{\mu}}$ is not currently known and the dark matter scenarios we consider are not sensitive to $m_{\tilde{\mu}}$, we will not consider this constraint in our dark matter calculations.

The contribution from a light CP-odd Higgs also should not violate the δa_μ constraint. If one considered the A_1 contribution alone, one might conclude that $\cos\theta_A \tan\beta$ is strongly limited in the case of small m_{A_1}. However, contributions to δa_μ from other sources such as the charged Higgs, charginos, and sfermions can easily overwhelm or cancel any contribution from a light A_1. Furthermore, LEP and other indirect limits such as Υ decays (discussed in Sec. V E) constrain $\cos\theta_A \tan\beta$ to be small, so it is generally not possible to see a large enhancement in δa_μ. Finally we note that if the A_1 and $\tilde{\chi}_1^0$ are both light, as considered here, their contributions to δa_μ are of opposite sign, and can cancel.

Thus, we do not explicitly include the δa_μ constraints in our computations. Their inclusion would only become appropriate if a specific model for soft SUSY-breaking is being considered.

C. Rare kaon decays

The $K^+ \to \pi^+\nu\bar{\nu}$ branching ratio was recently measured by the E787 and E949 experiments to be $BR(K^+ \to \pi^+\nu\bar{\nu}) = (1.47^{+1.30}_{-0.89}) \times 10^{-10}$, which is nearly twice the value predicted in the standard model, $(0.67^{+0.28}_{-0.27}) \times 10^{-10}$ [39]. A CP-even Higgs boson lighter than a few hundred MeV can contribute to this branching ratio via a triangle diagram involving $W^\pm$ bosons on two sides, and an up or charm quark on the third. This contribution is suppressed by ξ_u and ξ_d. CP-odd Higgs bosons, on the other hand, cannot contribute to this process at the one-loop level since the vertex involving W's and the A_1 is $W^\mu W_\mu A_1 A_1$ and, therefore, the leading contribution to $K^+ \to \pi^+ +$ invisible has *four* $\tilde{\chi}_1^0$'s in the final state. This requires a $\tilde{\chi}_1^0$ lighter than 88.5 MeV, which is lighter than the range we consider in this study.

Other rare kaon decays such as $K^0 \to e^+e^-$ and $K^+ \to \pi^+e^+e^-$ are similarly unconstraining for a light A_1, but potentially important for a light H_1 for the same reasons.

A recent study [40] analyzed this in detail and concluded that extremely light $m_{A_1} < 2m_\mu$ can be ruled out. However, this can be evaded if $|\kappa|$ is small enough.

D. Rare B-meson decays

The transitions $b \to s\gamma$ and $B_s \to \mu^+\mu^-$ are usually considered sensitive probes of supersymmetry, however both are flavor changing, while a light A_1 and χ_1^0 are not flavor changing by themselves. These and other flavor-changing processes involving a light χ_1^0 propagator can always be suppressed by making the appropriate squark or slepton mass heavy since the relevant diagrams must involve a $f\tilde{f}\chi_1^0$ vertex. Processes involving a light A_1 may be suppressed at one loop by assuming the minimal flavor violation mechanism [40,41]. A recent study of the B meson decays $b \to s\gamma$, $b \to sA_1$, and $b \to sl^+l^-$ in the NMSSM concluded that A_1 masses down to $2m_e$ cannot be excluded from these constraints [40].

Another rare B-decay is $B^+ \to K^+\nu\bar{\nu}$. This process also necessarily involves a quark flavor-changing $W^\pm$ vertex. A diagram in which the light A_1 couples to the $W^\pm$ must involve *two* A_1's and *two* $W^\pm$'s unless CP is violated, severely limiting the set of processes to which it can contribute. Diagrams where the light A_1 couples to the fermion also must have a $W^\pm$ to change the quark flavor and also receive a factor of $\cos\theta_A$ at each $f\bar{f}A_1$ vertex, strongly suppressing the A_1 contribution for the scenarios we focus on, all of which have small $\cos\theta_A$.

E. Upsilon and J/Ψ decays

The vector resonances J/Ψ and Υ may decay radiatively into an A_1 and a photon if A_1 is sufficiently light. There are two experimental limits on this process: firstly when the A_1 decays invisibly or is long-lived enough to leave the detector volume [42], and secondly when the A_1 decays to standard model particles [43]. This width, relative to the width to muons at leading order is given by [44]:

$$\frac{\Gamma(V \to \gamma A_1)}{\Gamma(V \to \mu\mu)} = \frac{G_F m_b^2}{\sqrt{2}\alpha\pi}\left(1 - \frac{M_{A_1}^2}{M_V^2}\right)X^2, \tag{30}$$

where V is either J/Ψ or Υ and $X = \cos\theta_A \tan\beta$ for Υ and $X = \cos\theta_A \cot\beta$ for J/Ψ. The A_1 is often referred to as the axion in this literature. Equation (30) is also applicable for a light CP-even H_1, with $X = \xi_d/\cos\beta$ for $V = \Upsilon$ and $X = \xi_u/\sin\beta$ for $V = J/\Psi$.

It is usually claimed that a light MSSM A is ruled out if it is light enough so that both the Υ and J/Ψ can decay to it ($m_A \lesssim 3.1$ GeV), due to the observation that $\Gamma(J/\Psi \to \gamma A) \times \Gamma(\Upsilon \to \gamma A)$ is independent of $\tan\beta$. However, within the NMSSM, this product is proportional to $\cos^4\theta_A$, which may be small.

The best existing measurement of $\Upsilon \to$ invisible $+ \gamma$ is from CLEO [42] in 1995. Significantly more data has been collected on the $\Upsilon(1S)$ resonance that could be used to improve this measurement, however. Modern B-factories such as *BABAR* and Belle can also produce the $\Upsilon(1S)$ in initial state radiation to improve this measurement.

In Fig. 2, we illustrate the correlations between $BR(\Upsilon \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0)$ via 3-body decay (i.e. *not* $\Upsilon \to \gamma A_1$ with $A_1 \to$ invisible or visible), $m_{\tilde{\chi}_1^0}$ and the relic density Ωh^2 (the calculation of which is discussed in the following section). Only Higgs exchange is included in these relic density values. Subleading Z and sfermion exchanges would further decrease the relic density of points with very large Ωh^2. The left-hand plot shows that a significant fraction of the parameter choices such that $\Upsilon \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0$ is allowed are eliminated by the experimental constraint on this mode, with additional ones being eliminated by the constraints on the 2-body $\Upsilon \to \gamma A_1$ decay mode. But, many are not excluded, especially those with a binolike $\tilde{\chi}_1^0$. Improvement in the experimental sensitivities to $BR(\Upsilon \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0)$ and $BR(\Upsilon \to \gamma A_1)$ will further constrain the light $\tilde{\chi}_1^0$ scenarios considered here, or could yield a signal. The right-hand plot of Fig. 2 shows that there are many parameter choices that yield $BR(\Upsilon \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0)$ and $BR(\Upsilon \to \gamma A_1)$ below the experimental limits while simultaneously predicting a relic density roughly consistent with observation. We observe that this dual consistency can be achieved for either a binolike or a singlinolike lightest neutralino.

The points on the left side of the right frame of Fig. 2 undergo $\Upsilon \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0$ dominantly via a CP-even, mostly-singlet scalar, H_1, which mediates this interaction. When the A_1 becomes light and mostly singlet, it often brings the CP-even scalar and singlino down in mass as well. For these points, the two body decay, $\Upsilon \to \gamma A_1$, followed by the decay, $A_1 \to \tilde{\chi}_1^0\tilde{\chi}_1^0$, is also just below the experimental limit.

VI. ANNIHILATION CROSS SECTION AND RELIC ABUNDANCE

The calculation of the neutralino annihilation cross section and relic abundance in the NMSSM is only slightly modified from the case of the MSSM. First, the diagonalization of the 5×5 neutralino mass matrix of the NMSSM yields different LSP compositions for given choices of input parameters (M_1, μ, etc.). Secondly, annihilations can occur through the exchange of a Higgs boson with a significant singlet component. On one hand, this weakens the respective couplings. On the other hand, much lighter Higgses can be considered, as collider constraints are weakened.

For the range of masses we are considering, the only final states available for the annihilations of light neutralinos are fermion pairs. This process can occur through s-channel Higgs exchange (both CP-even and CP-odd),

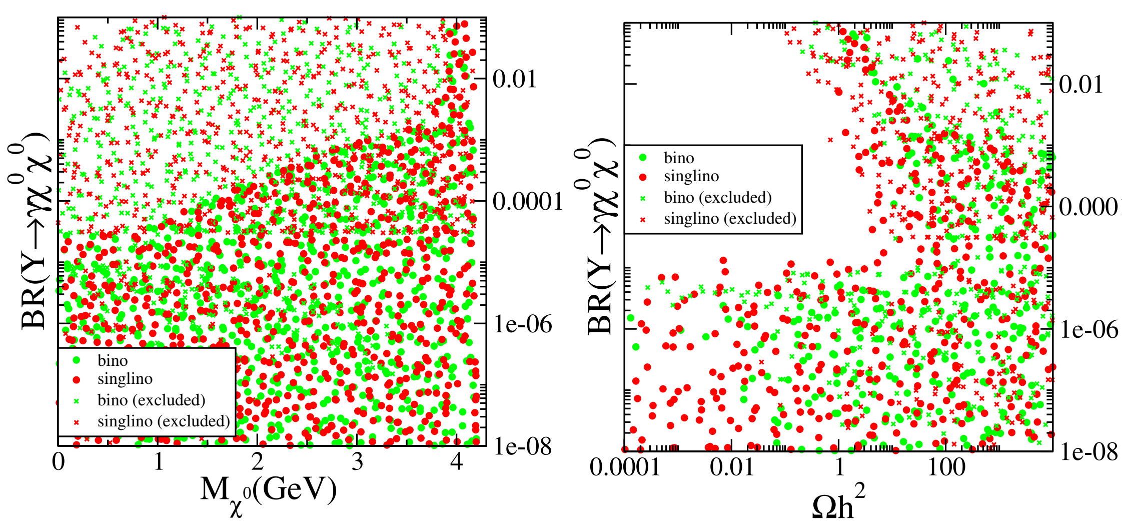

FIG. 2 (color online). The branching ratio for $\Upsilon(1S) \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0$ via 3-body decay (i.e. either $m_{A_1} < 2m_{\tilde{\chi}_1^0}$ or $m_{A_1} > m_\Upsilon$) is plotted vs the LSP mass (left) and relic density Ωh^2 (right). All points shown are consistent with all LEP constraints. Points marked by an x are excluded by one of: $\Upsilon \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0$ (3-body decay) (that which is plotted); $\Upsilon \to \gamma A_1$ (2-body decay) with $A_1 \to \tilde{\chi}_1^0\tilde{\chi}_1^0$ (2-body decay); or $\Upsilon \to \gamma A_1$ (2-body decay) where the A_1 decays visibly.

s-channel Z exchange, or t and u-channel sfermion exchange. With LEP constraints limiting sfermion masses to $m_{\tilde{f}} \gtrsim 100$ GeV, neutralinos lighter than approximately 25 GeV cannot annihilate efficiently enough through sfermions to yield the measured relic density. Similarly, Z exchange cannot dominate the annihilation cross section for light neutralinos. Therefore, we focus on the process of Higgs exchange.

The squared amplitudes for the processes, $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to A \to f\bar{f}$ and $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to H \to f\bar{f}$, averaged over the final state angle are given by [45]:

$$\omega^A_{f\bar{f}} = \frac{C^2_{f\bar{f}A} C^2_{\tilde{\chi}_1^0\tilde{\chi}_1^0 A}}{(s - m_A^2)^2 + m_A^2\Gamma_A^2} \frac{s^2}{16\pi}\sqrt{1 - \frac{4m_f^2}{s}}, \qquad (31)$$

$$\omega^H_{f\bar{f}} = \frac{C^2_{f\bar{f}H} C^2_{\tilde{\chi}_1^0\tilde{\chi}_1^0 H}}{(s - m_H^2)^2 + m_H^2\Gamma_H^2} \frac{(s - 4m^2_{\tilde{\chi}_1^0})(s - 4m_f^2)}{16\pi} \times \sqrt{1 - \frac{4m_f^2}{s}}, \qquad (32)$$

where the labels A and H denote a CP-odd and CP-even Higgs, respectively. Here, $C^2_{f\bar{f}A}$, $C^2_{f\bar{f}H}$, $C^2_{\tilde{\chi}_1^0\tilde{\chi}_1^0 A}$ and $C^2_{\tilde{\chi}_1^0\tilde{\chi}_1^0 H}$ are the fermion-fermion-Higgs couplings and the neutralino-neutralino-Higgs couplings, and $m_{A,H}$ and $\Gamma_{A,H}$ are the Higgs masses and widths. In the NMSSM case, we will be considering only $A = A_1$ and $H = H_1$, the lightest of the CP-odd and CP-even states, respectively. The relevant couplings are then given by:

$$\begin{aligned} C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 A} &= \cos\theta_A[(g_2\epsilon_W - g_1\epsilon_B)(\epsilon_d\cos\beta - \epsilon_u\sin\beta) \\ &\quad + \sqrt{2}\lambda\epsilon_s(\epsilon_u\sin\beta + \epsilon_d\cos\beta)] \\ &\quad + \sin\theta_A\sqrt{2}[\lambda\epsilon_u\epsilon_d - \kappa\epsilon_s^2], \end{aligned} \qquad (33)$$

$$\begin{aligned} C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 H} &= (g_1\epsilon_B - g_2\epsilon_W)(\epsilon_d\xi_u - \epsilon_u\xi_d) \\ &\quad + \sqrt{2}\lambda\epsilon_s(\epsilon_d\xi_d + \epsilon_u\xi_u) \\ &\quad + \sqrt{2}\xi_s(\lambda\epsilon_u\epsilon_d - \kappa\epsilon_s^2), \end{aligned} \qquad (34)$$

$$C_{f\bar{f}A} = \begin{cases} \frac{m_f}{\sqrt{2}v}\cos\theta_A\tan\beta, & f = d, s, b, l \\ \frac{m_f}{\sqrt{2}v}\cos\theta_A\cot\beta, & f = u, c \end{cases} \qquad (35)$$

$$C_{f\bar{f}H} = \begin{cases} \frac{m_f}{\sqrt{2}v}\frac{\xi_d}{\cos\beta}, & f = d, s, b, l \\ \frac{m_f}{\sqrt{2}v}\frac{\xi_u}{\sin\beta}, & f = u, c. \end{cases} \qquad (36)$$

We expect $\Gamma_A \approx$ eV-MeV if $A = A_1$ is mostly singlet and $\Gamma_A \approx$ 1-10 MeV otherwise. Similarly, we expect $\Gamma_H \approx$ 10 eV-100 keV if H_1 is mostly singlet and $\Gamma_H \approx$ keV-MeV if $H = H_1$ is mostly nonsinglet. These widths are strongly affected by the many kinematic thresholds due to hadronic resonances with masses less than 10 GeV. Therefore, any computation of the relic density is inherently limited by our ability to compute hadronic form factors and sum over hadronic decays which may be on-shell and may enhance the annihilation. We require only that the relic density is $\mathcal{O}(0.1)$. There is sufficient parameter space to make the relic density precisely the value measured by WMAP when all hadronic corrections are taken into account. In our computations, we neglect the widths since they are very small compared to the masses considered. Of course, one could always tune $2m_{\tilde{\chi}_1^0}$ to some hadronic resonance or threshold in order to drastically increase the cross section and thus reduce the thermal relic density, but we do not employ such precision tuning.

The squared amplitudes of Eqs. (31) and (32) can be used to obtain the thermally averaged annihilation cross section [46]. Using the notation $s_0 = 4m^2_{\tilde{\chi}_1^0}$, we have

$$\begin{aligned} \langle\sigma v\rangle &= \frac{\omega(s_0)}{m^2_{\tilde{\chi}_1^0}} - \frac{3}{m_{\tilde{\chi}_1^0}}\left[\frac{\omega(s_0)}{m^2_{\tilde{\chi}_1^0}} - 2\omega'(s_0)\right]T + \mathcal{O}(T^2) \\ &= \frac{1}{m^2_{\tilde{\chi}_1^0}}\left[1 - \frac{3T}{m_{\tilde{\chi}_1^0}}\right]\omega(s)|_{s\to 4m^2_{\tilde{\chi}_1^0} + 6m_{\tilde{\chi}_1^0}T} + \mathcal{O}(T^2), \end{aligned} \qquad (37)$$

where T is the temperature. Keeping terms to zeroth and first order in T should be sufficient for the relic abundance calculation. Writing this as an expansion in $x = T/m_{\tilde{\chi}_1^0}$, $\langle\sigma v\rangle = a + bx + \mathcal{O}(x^2)$, we arrive at:

$$\begin{aligned} a_{\chi\chi\to A\to f\bar{f}} &= \frac{g_2^4 c_f m_f^2 \cos^4\theta_A \tan^2\beta}{8\pi m_W^2} \frac{m^2_{\tilde{\chi}_1^0}\sqrt{1 - m_f^2/m^2_{\tilde{\chi}_1^0}}}{(4m^2_{\tilde{\chi}_1^0} - m_A^2)^2 + m_A^2\Gamma_A^2} \\ &\quad \times\Big[-\epsilon_u(\epsilon_W - \epsilon_B\tan\theta_W)\sin\beta \\ &\quad + \epsilon_d(\epsilon_W - \epsilon_B\tan\theta_W)\cos\beta \\ &\quad + \sqrt{2}\frac{\lambda}{g_2}\epsilon_s(\epsilon_u\sin\beta + \epsilon_d\cos\beta) \\ &\quad + \frac{\tan\theta_A}{g_2}\sqrt{2}(\lambda\epsilon_u\epsilon_d - \kappa\epsilon_s^2)\Big]^2, \end{aligned} \qquad (38)$$

$$b_{\chi\chi\to A\to f\bar{f}} \simeq 0, \qquad (39)$$

where c_f is a color factor, equal to 3 for quarks and 1 otherwise. For this result, we have assumed that the final state fermions are down-type. If they are instead up-type fermions, the couplings used must be modified as described above.

We have not written the result for CP-even Higgs exchange because the low-velocity term in the expansion is zero: $a_{\chi\chi\to H\to f\bar{f}} = 0$. Although the b-terms can, in principle, contribute to the freeze-out calculation, in the computations here such contributions do not have a significant impact.

The annihilation cross section can now be used to calculate the thermal relic abundance present today.

$$\Omega_{\tilde{\chi}_1^0} h^2 \approx \frac{10^9}{M_{\rm Pl}} \frac{x_{\rm FO}}{\sqrt{g_\star}} \frac{1}{(a + 3b/x_{\rm FO})}, \quad (40)$$

where $g_\star$ is the number of relativistic degrees of freedom available at freeze-out and $x_{\rm FO}$ is given by:

$$x_{\rm FO} \approx \ln\left(\sqrt{\frac{45}{8}} \frac{m_{\tilde{\chi}_1^0} M_{\rm Pl}(a + 6b/x_{\rm FO})}{\pi^3 \sqrt{g_\star x_{\rm FO}}}\right). \quad (41)$$

For the range of cross sections and masses we are interested in, $x_{\rm FO} \approx 20$.

As a benchmark for comparison, we consider a light bino which annihilates through the exchange of an MSSM-like CP-odd Higgs ($\cos\theta_A = 1$). The results for this case are shown in Fig. 3. In this figure, the thermal relic density of LSP neutralinos exceeds the measured value for CP-odd Higgses above the solid and dashed curves, for values of $\tan\beta$ of 50 and 10, respectively. Shown as a horizontal dashed line is the lower limit on the MSSM CP-odd Higgs mass from collider constraints. This figure demonstrates that even in the case of very large $\tan\beta$, the lightest neutralino must be heavier than about 7 GeV. For moderate values of $\tan\beta$, the neutralino must be heavier than about 20 GeV.

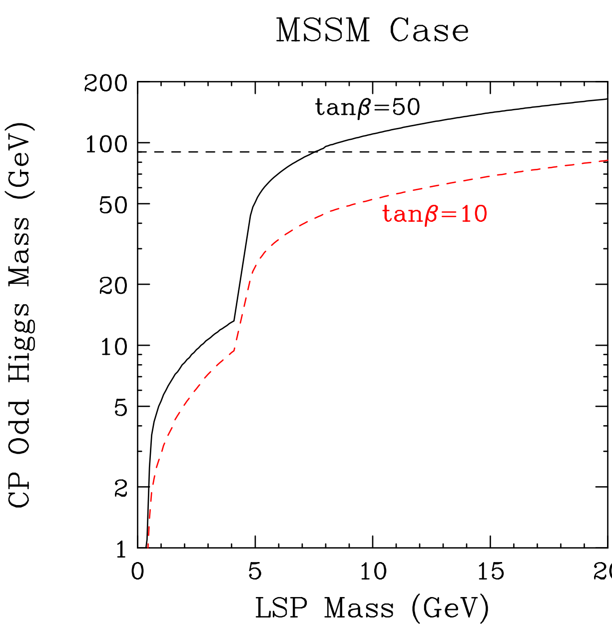

FIG. 3 (color online). The CP-odd Higgs mass required to obtain the measured relic density for a light neutralino in the MSSM. Models above the curves produce more dark matter than is observed. These results are for the case of a binolike neutralino with a small Higgsino admixture ($\epsilon_B^2 = 0.94$, $\epsilon_u^2 = 0.06$). Results for two values of $\tan\beta$ (10 and 50) are shown. The horizontal dashed line represents the lower limit on the CP-odd Higgs mass in the MSSM from collider constraints. To avoid overproducing dark matter, the neutralino must be heavier than about 8 (22) GeV for $\tan\beta = 50$ (10).

In Fig. 4, we show how this conclusion is modified within the framework of the NMSSM. Here, we have considered a CP-odd Higgs which is a mixture of MSSM-like and singlet components specified by $\cos^2\theta_A = 0.6$ and a neutralino with composition specified by $\epsilon_B^2 = 0.94$ and $\epsilon_u^2 = 0.06$. These specific values are representative of those that can be achieved for various NMSSM parameter choices satisfying all constraints. For each pair of contours (solid line, dashed line, and dot-dashed line), the region between the lines is the space in which the neutralino's relic density does not exceed the measured density. The solid, dashed, and dot-dashed lines correspond to $\tan\beta = 50$, 15, and 3, respectively. Also shown as a dotted line is the contour corresponding to the resonance condition, $2m_{\tilde{\chi}_1^0} = m_A$.

For the $\tan\beta = 50$ or 15 cases, neutralino dark matter can avoid being overproduced for any A_1 mass below ~20-60 GeV, as long as $m_{\tilde{\chi}_1^0} > m_b$. For smaller values of $\tan\beta$, a lower limit on m_{A_1} can apply as well.

For neutralinos lighter than the mass of the b-quark, annihilation is generally less efficient. This region is shown in detail in the right frame of Fig. 4. In this funnel region, annihilations to $c\bar{c}$, $\tau^+\tau^-$, and $s\bar{s}$ all contribute significantly. Despite the much smaller mass of the strange quark, its couplings are enhanced by a factor proportional to $\tan\beta$ (as with bottom quarks) and thus can play an important role in this mass range. In this mass range, constraints from Upsilon and J/ψ decays can be very important, often requiring fairly small values of $\cos\theta_A$.

For annihilations to light quarks, $c\bar{c}$, $s\bar{s}$, etc., the Higgs couplings to various meson final states should be considered, which include effective Higgs-gluon couplings induced through quark loops. In our calculations here, we have used the conservative approximation of the Higgs-quark-quark couplings alone, even for these light quarks, but with kinematic thresholds set by the mass of the lightest meson containing a given type of quark, rather than the quark mass itself. This corresponds to thresholds of 9.4 GeV, 1.87 GeV, 498 MeV, and 135 MeV for bottom, charm, strange, and down quarks, respectively. A more detailed treatment, which we will not undertake here, would include the proper meson form factors as well as allowing for the possibility of virtual meson states.

Thus far, we have focused on the case of a binolike LSP. If the LSP is mostly singlino, it is also possible to generate the observed relic abundance in the NMSSM. A number of features differ for the singlinolike case in contrast to a binolike LSP, however. First, the ratio $m_{\tilde{\chi}_1^0}/m_{A_1}$ cannot be arbitrarily small. The relationship between these two masses was shown for singlinolike LSPs in Fig. 1. As discussed earlier, and shown in this figure, an LSP mass that is chosen to be precisely at the Higgs resonance, $m_{A_1} \simeq 2m_{\tilde{\chi}_1^0}$, is not possible for this case: m_{A_1} is always less than $2m_{\tilde{\chi}_1^0}$ by a significant amount.

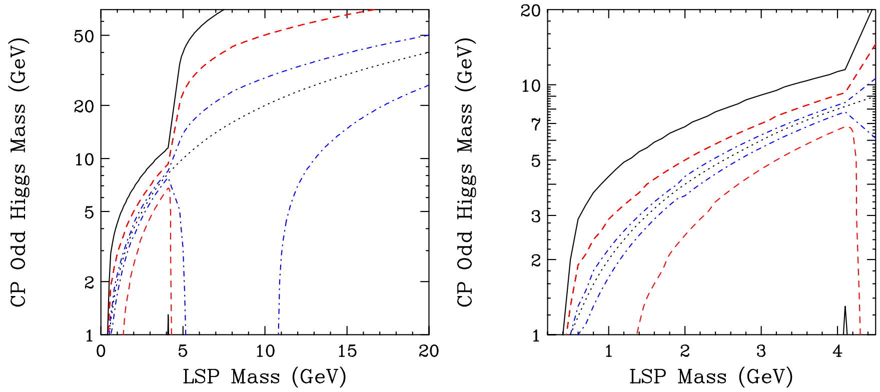

FIG. 4 (color online). We display contours in $m_{A_1} - m_{\tilde{\chi}_1^0}$ parameter space for which Eq. (40) yields $\Omega h^2 = 0.1$. Points above or below each pair of curves produce more dark matter than is observed; inside each set of curves less dark matter is produced than is observed. These results are for a binolike neutralino with a small Higgsino admixture ($\epsilon_B^2 = 0.94$, $\epsilon_u^2 = 0.06$). Three values of $\tan\beta$ (50, 15, and 3) have been used, shown as solid, dashed, and dot-dashed lines, respectively. The dotted line is the contour corresponding to $2m_{\tilde{\chi}_1^0} = m_A$. For each set of lines, we have set $\cos^2\theta_A = 0.6$. The $\tan\beta = 50$ case is highly constrained for very light neutralinos, and is primarily shown for comparison with the MSSM case.

Second, in models with a singlinolike LSP, the A_1 is generally also singletlike and the product of $\tan^2\beta$ and $\cos^4\theta_A$ is typically very small. This limits the ability of a singlinolike LSP to generate the observed relic abundance. The last term in Eq. (38) introduces an additional $\tan^2\theta_A$ dependence, however, which effectively reduces the impact of $\cos\theta_A$ on the annihilation cross section from four powers to two. But, this last term is suppressed when the singlet fraction ϵ_s is large and ϵ_u, ϵ_d are small by the factor of κ (which is small for a singlino) that multiplies ϵ_s^2. Alternatively, the second to last term in Eq. (38) can also be of importance. Overall, the inability to compensate the smallness of the coefficients in Eq. (38) by being nearly on-pole implies that annihilation is too inefficient for an LSP that is more than 80% singlino.

In the following section, we give sample cases for which m_{A_1} and $m_{\tilde{\chi}_1^0}$ are light and $\Omega h^2 \sim 0.11$. These are representative of the many different types of scenarios that are possible and include a case in which the $\tilde{\chi}_1^0$ is largely singlino.

VII. SAMPLE MODEL POINTS

In this section we present specific sample model points of the type we propose. These points are obtained using NMHDECAY 1.1 [21].

The first point (Table I) has a singletlike H_1, which would have escaped detection at LEP due to this singlet nature. In addition, the mass of the more SM-like H_2 is beyond the LEP reach. It also has a sizable $BR(\Upsilon \to \gamma + A_1)$ which could be discovered by a reanalysis of existing CLEO data.

The second point (Table II) has an MSSM-like H_1, but due to the presence of the light A_1 and the large λ coupling, this MSSM-like H_1 decays dominantly to a pair of A_1's [$BR(H_1 \to A_1 A_1) = 99.6\%$ for this point]. Such an H_1 would not be easily detected at the LHC.

The third point (Table III) has a singlinolike $\tilde{\chi}_1^0$ as well as a singletlike H_1. As for point #1, this point has a $BR(\Upsilon \to \gamma + A_1)$ that might be excluded by an appropriate reanalysis of existing data.

TABLE I. Sample model point #1.

λ	κ	$\tan\beta$	μ	A_λ	A_κ	M_1	M_2
0.436736	−0.049 955	1.79644	−187.931	−458.302	−40.4478	1.92375	390.053
M_{A_1}	$\cos\theta_A$						
7.173 07	−0.193 618						
M_{H_1}	ξ_u	ξ_d	ξ_s				
73.8217	0.1127	−0.0277	0.9932				
$M_{\tilde{\chi}_1^0}$	ϵ_B	ϵ_W	ϵ_u	ϵ_d	ϵ_s		
3.496 03	−0.781 466	−0.005 946 69	0.114 76	0.264 93	0.553 099		
$BR(\Upsilon \to \gamma + A_1)$	$\langle\sigma v\rangle$	Ωh^2					
8.123 31e-06	4.558 41e-26 cm^3/s	0.107 689					

TABLE II. Sample model point #2.

λ	κ	$\tan\beta$	μ	A_λ	A_κ	M_1	M_2
0.224 982	−0.479 12	7.587 31	−174.624	−421.908	−30.6106	21.0909	984.116
M_{A_1}	$\cos\theta_A$						
46.6325	−0.570 716						
M_{H_1}	ξ_u	ξ_d	ξ_s				
117.72	0.9823	0.1848	0.0316				
$M_{\tilde{\chi}_1^0}$	ϵ_B	ϵ_W	ϵ_u	ϵ_d	ϵ_s		
22.37	−0.9715	−0.0024	0.0020	0.2366	0.0128		
$BR(\Upsilon \to \gamma + A_1)$	$\langle\sigma v\rangle$	Ωh^2					
0	2.174 78e-25 cm^3/s	0.108 649					

VIII. ELASTIC SCATTERING OF LIGHT NEUTRALINOS

The spin-independent elastic scattering cross section of a light neutralino with nuclei is generally dominated by the t-channel exchange of a CP-even Higgs boson. The cross section for this process is approximately given by:

$$\sigma_{\text{elastic}} \approx \sum_H \frac{1}{\pi m_H^4}\left(\frac{m_p m_{\tilde{\chi}_1^0}}{m_p + m_{\tilde{\chi}_1^0}}\right)^2 \times C^2_{\tilde{\chi}_1^0\tilde{\chi}_1^0 H}\left(\sum_q C_{q\bar{q}H}\langle N|q\bar{q}|N\rangle\right)^2, \quad (42)$$

where the first sum is over the CP-even Higgs states of the NMSSM and m_H are their masses. The second sum is over the quark types and $\langle N|q\bar{q}|N\rangle$ are the matrix elements over the atomic nuclear state. Of course, one must be careful to use the correct form of $C_{q\bar{q}H}$ which differs for up-type quarks versus down-type quarks. In the sum over quark species, the strange quark contribution dominates with $m_s\langle N|s\bar{s}|N\rangle \approx 0.2$ GeV. For a binolike LSP and any one H, Eq. (42) reduces to

$$\sigma^{\text{bino}}_{\text{elastic}} \sim \frac{8G_F^2 m_Z^2}{\pi m_H^4}\left(\frac{m_p m_{\tilde{\chi}_1^0}}{m_p + m_{\tilde{\chi}_1^0}}\right)^2 \epsilon_B^2 \sin^2\theta_W(\epsilon_d\xi_u - \epsilon_u\xi_d)^2 \times\left(\sum_{q=d,s,b}\frac{m_q\xi_d}{\cos\beta}\langle N|q\bar{q}|N\rangle + \sum_{q=u,c}\frac{m_q\xi_u}{\sin\beta}\langle N|q\bar{q}|N\rangle\right)^2.$$

If the LSP is singlinolike, on the other hand, the appropriate approximation is

$$\sigma^{\text{singlino}}_{\text{elastic}} \sim \frac{8G_F^2 m_Z^2}{\pi m_H^4}\left(\frac{m_p m_{\tilde{\chi}_1^0}}{m_p + m_{\tilde{\chi}_1^0}}\right)^2 \frac{2\lambda^2\epsilon_s^2\cos^2\theta_W}{g_2^2} \times(\epsilon_d\xi_d + \epsilon_u\xi_u)^2\left(\sum_{q=d,s,b}\frac{m_q\xi_d}{\cos\beta}\langle N|q\bar{q}|N\rangle + \sum_{q=u,c}\frac{m_q\xi_u}{\sin\beta}\langle N|q\bar{q}|N\rangle\right)^2.$$

where, in $C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 H}$, we have dropped the term containing κ since it is expected to be very small and the term proportional to $\epsilon_u\epsilon_d$ which is also likely to be very small.

In assessing the implications of the above, it is useful to note that LEP limits on a Higgs boson with $m_H < 120$ GeV generally imply

$$\xi_{u,d} \lesssim \left(\frac{m_H}{120\ \text{GeV}}\right)^{3/2} + 0.1, \quad (43)$$

and for a light $\tilde{\chi}_1^0$ LEP limits on invisible Z decays roughly imply $\epsilon_{u,d} < 0.06$.

The claim of a positive WIMP detection made by the DAMA collaboration is not consistent with the limits placed by CDMS and others for a WIMP in the mass range normally considered (above a few tens of GeV). Very light WIMPs, however, scatter more efficiently with light target nuclei than with heavier nuclei, which can complicate this picture. For a WIMP with a mass between about 6 and 9 GeV, it has been shown that the DAMA results can be reconciled with the limits of CDMS and other experiments

TABLE III. Sample model point #3.

λ	κ	$\tan\beta$	μ	A_λ	A_κ	M_1	M_2
0.415 867	−0.029 989	1.788 74	−175.622	−455.387	−39.671	7.1098	289.115
M_{A_1}	$\cos\theta_A$						
8.35008	−0.187 349						
M_{H_1}	ξ_u	ξ_d	ξ_s				
63.3851	−0.1412	−0.1810	0.9733				
$M_{\tilde{\chi}_1^0}$	ϵ_B	ϵ_W	ϵ_u	ϵ_d	ϵ_s		
−3.98	−0.3697	−0.0262	0.2524	0.2560	0.8564		
$BR(\Upsilon \to \gamma + A_1)$	$\langle\sigma v\rangle$	Ωh^2					
3.96e-6	4.122 41e-26 cm^3/s	0.119 239					

[15].[1] This is made possible by the relatively light sodium ($A = 23.0$) component of the DAMA experiment compared to germanium ($A = 72.6$) and silicon ($A = 28.1$) of CDMS.

To produce the rate observed by DAMA, a light WIMP would need an elastic scattering cross section of 7×10^{-40} cm^2 to 2×10^{-39} cm^2 ($0.7 - 2$ fb). For the case of a binolike or singlinolike neutralino capable of resolving the DAMA discrepancy, the scale of this cross section is:

$$\sigma_{\rm elastic} \lesssim 1.4 \times 10^{-42}\ {\rm cm}^2 \left(\frac{120\ {\rm GeV}}{m_H}\right)^4 \times \left(\left(\frac{m_H}{120\ {\rm GeV}}\right)^{3/2} + 0.1\right)^2 \left(\frac{\tan\beta}{50}\right)^2 F_\lambda \quad (44)$$

assuming $m_{\tilde{\chi}_1^0} > m_p$ and $\tan\beta > 1$, using the $\xi_{u,d}$ limit of Eq. (43) and adopting $\epsilon_{u,d} \sim 0.06$. One has $F_\lambda = 1$ for the binolike case and $F_\lambda = 2\lambda^2/(g_2^2 \tan^2\theta_W) \approx 0.67 \times (\lambda/0.2)^2$ for the singlinolike case. For $\tan\beta = 50$, $\lambda = 0.2$, and a Higgs mass of 120 GeV, we estimate a neutralino-proton elastic scattering cross section on the order of 4×10^{-42} cm^2 (4×10^{-3} fb) for either a binolike or a singlinolike LSP. This value may be of interest to direct detection searches such as CDMS, DAMA, Edelweiss, ZEPLIN, and CRESST. To account for the DAMA data, the cross section would have to be enhanced by a local over-density of dark matter [15].

The cross section in Eq. (44) is small unless $\tan\beta$ is quite large, in which case the scenario will run into difficulty with LEP limits unless $\cos\theta_A$ is quite small. To explain the DAMA result, we can instead require m_H to be small. For instance, with $m_{\tilde{\chi}_1^0} = 6$ GeV, $m_H = 3$ GeV, and $\tan\beta = 10$, the DAMA result can be reproduced with $\sigma_{\rm elastic} \sim 4 \times 10^{-39}$ cm^2 (~ 4 fb), *without* requiring a dark matter wind through our solar system. It would not be unusual for a mostly-singlet H_1 to be this light if λ is small. In this case the singlet decouples from the MSSM and the whole singlet supermultiplet is light.

For a detailed study of direct detection prospects for heavier neutralinos in the NMSSM, see Refs. [30,31]. We find consistency with their results concerning annihilation through H and A resonances.

IX. EXTREMELY LIGHT NEUTRALINOS AND THE OBSERVATION OF 511 KEV EMISSION FROM THE GALACTIC BULGE

If the LSP's mass is even smaller, below $\sim$1 GeV, it may still be possible to generate the observed relic density. In this mass range, in addition to annihilations to strange quarks ($K^\pm$, K^0), final state fermions can include muons and even lighter quarks ($\pi^\pm$, π^0).

There is a $\tilde{\chi}_1^0$ mass range in which neutralinos will annihilate mostly to muon pairs. This range is $m_\mu < m_{\tilde{\chi}_1^0} < m_{\pi^+} + m_{\pi^0}/2$, or 106 MeV $< m_{\tilde{\chi}_1^0} < 207$ MeV. (The upper limit will be explained shortly.) This range of parameter space is of special interest within the context of the 511 keV emission observed from the galactic bulge by the INTEGRAL/SPI experiment. Muons produced in neutralino annihilations will quickly decay, generating electrons with energies of $\sim m_{\tilde{\chi}_1^0}/3$, which may be sufficiently small for them to come to rest in the galactic bulge before annihilating.

The upper limit above derives from the fact that the $\tilde{\chi}_1^0\tilde{\chi}_1^0$ annihilations should not create many π^0's. In this way, we avoid gamma-ray constraints from EGRET. If we assume that the annihilation mediator is the CP-odd A_1, $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to A_1 \to$ pions is only possible if $2m_{\tilde{\chi}_1^0} \gtrsim 2m_{\pi^+} + m_{\pi^0}$ since the lowest threshold channel is to three pions: $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to A_1 \to \pi^+\pi^-\pi^0$. Also note that by generating positrons through muon decays rather than directly allows gamma-ray constraints from final state radiation [11] to be easily evaded.

It has been shown that a $\sim$100 MeV dark matter particle annihilating through an a-term (low-velocity) cross section can simultaneously yield the measured relic density and generate the number of positrons needed to accommodate the INTEGRAL/SPI data [9]. These are precisely the features of a 106–207 MeV neutralino combined with the presence of a 100 MeV–1 GeV CP-odd Higgs.

The main difficulty with this scenario comes from the constraints on Upsilon decays, which we discussed in Sec. V E. To evade the CLEO limit [42] of $BR(\Upsilon \to \gamma A_1) < 2 \times 10^{-5}$ in this mass region, we must require $\cos^2\theta_A \tan^2\beta < 0.13$ [see Eq. (30)]. Given these constraints, and considering a binolike neutralino with a 6% Higgsino admixture and $m_{\tilde{\chi}_1^0} = 150$ MeV, the annihilation cross section needed to avoid overproducing dark matter can only be attained for a fairly narrow range of $m_{A_1} \approx 2m_{\tilde{\chi}_1^0} \pm 10$ MeV. This scenario, although not particularly attractive due to this requirement, does demonstrate that it is possible to generate the INTEGRAL signal with neutralinos in the NMSSM. This can be confirmed or ruled out by improving the limit on $BR(\Upsilon \to \gamma A_1)$ where the A_1 is not observed or where the A_1 decays to a muon pair. In the latter case, the A_1 may have a significant displaced vertex of a few cm, especially for small $\tan\beta$ and $m_{A_1} < 2m_{\tilde{\chi}_1^0}$ [47].

An A_1 this light (300 MeV) is too light to be technically natural, however. Radiative corrections pull up its mass and a cancellation between different orders in perturbation theory is required for A_1 to be this light. While we have found parameter points capable of yielding the INTEGRAL signal, we find that they are not stable in the sense that if any of the Higgs-sector parameters are adjusted by a very small amount, the A_1 is pulled up in mass

[1] If a tidal stream of dark matter is present in the local halo, WIMP masses over a somewhat wider range can reconcile DAMA with CDMS as well.

to $\mathcal{O}(10\ \text{GeV})$. From our numeric analysis, m_{A_1} as small as a few GeV is technically natural.

X. CONCLUSIONS

In this article, we have studied the possibility of light neutralinos (100 MeV to 20 GeV) being present within the next-to-minimal supersymmetric standard model (NMSSM) without conflicting with constraints on the dark matter of the Universe. We find that light *CP*-odd Higgs bosons with a substantial nonsinglet component, which appear naturally within this framework, can provide an efficient annihilation channel for light, bino, or singlino-like neutralinos. This channel makes it possible for very light neutralinos to generate the observed dark matter abundance, unlike in the case of neutralinos in the MSSM.

Within this model, we have discussed the implications of light neutralinos for direct detection and find that the NMSSM can naturally provide neutralinos in the mass range (6–9 GeV) as required to reconcile the DAMA claim of discovery with the limits placed by CDMS and other experiments. We have also explored the possibility that the 511 keV emission observed from the galactic bulge by INTEGRAL/SPI could be generated through neutralino annihilations into muon pairs. This scenario appears possible for a very light (106–207 MeV) neutralino and for a light *CP*-odd Higgs boson with mass close to twice the neutralino mass, provided $\tan\beta$ is not large.

This kind of scenario containing a light neutralino and/or light axionlike particles represents a challenge for the LHC and international linear collider (ILC), and is deserving of further analysis. We note that only the ILC will be able to study the properties of the $\tilde{\chi}_1^0$ and A_1 adequately to verify that they are consistent with the observed dark matter density.

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