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Corrections to the process $e^+e^- \to b\bar{b}$ in the topcolor assisted technicolor model

Gang Lv,[1] Jinshu Huang,[2] and Gongru Lu[1]

[1]*Department of Physics, Henan Normal University, Xinxiang 453007, China*
[2]*Department of Physics, Henan Nanyang Normal College, Nanyang 473061, China*
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In the framework of the topcolor assisted technicolor model we calculate the contributions from the pseudo Goldstone bosons and new gauge bosons to $e^+e^- \to b\bar{b}$. We find that, for reasonable ranges of the parameters, the top pions afford a dominate contribution, the corrections arising from technipions and new gauge bosons are negligibly small, the maximum of the total corrections is 43% with the c.m. energy $\sqrt{s} = 500$ GeV; whereas in the case of $\sqrt{s} = 1500$ GeV, the maximum of the relative corrections is only 3.1%. It might open a window to detect a topcolor assisted technicolor model in experiment of next generation linear colliders.

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I. INTRODUCTION

Despite the successful confirmation of the standard model (SM) of elementary particle physics by experimental precision tests during the past few years, the structure of the Higgs sector has been unconfirmed; the mechanism of electroweak symmetry breaking (EWSB) is still an open question. Technicolor theory [1,2] is one of the important candidates to probe new physics beyond the SM, especially the topcolor assisted technicolor models including the original topcolor assisted technicolor (TOPCTC) model [3] and the topcolor assisted multiscale technicolor (TOPCMTC) model [4]. They combine technicolor or multiscale walking technicolor with topcolor, with the former mainly responsible for EWSB and the latter for generating a major part of the top quark mass. Since they could give a rational interpretation to answer the questions, these models bring to people great interest. Several of the signals of these models have already been studied at the work environment of linear colliders and hadron-hadron colliders [5–7], but most of the attention has been focused on the neutral pseudo Goldstone bosons (PGBs). Here we wish to discuss the prospects of charged PGBs and new gauge bosons.

A bottom-quark pair can be produced at various high energy colliders. At the next generation linear colliders, a bottom-quark pair can be produced in sufficient abundance, and the event environment is very clean. Of special interest is to examine the ability of the suggested future TeV energy e^+e^- colliders in testing technicolor effects via $b\bar{b}$ production. This paper is organized as follows. In Sec. II, we present a brief review of the topcolor assisted technicolor models. In Sec. III, we give the corrections to the $e^+e^- \to b\bar{b}$ cross section at the center-of-mass (c.m.) energy $\sqrt{s} = 0.5$, 1.0, and 1.5 TeV colliders in the TOPCTC model. Both the effects of PGBs and new gauge bosons are evaluated. Discussions and conclusions are included in Sec. IV. The analytic formulas for the form factors in the production amplitudes are presented in Appendixes A and B.

II. TOPCOLOR ASSISTED TECHNICOLOR MODELS

First we discuss the original TOPCTC model by Hill [3]. The model assumes [3,8] that (i) electroweak symmetry is broken mainly by extended technicolor (ETC); (ii) the top quark mass is large because it is the combination of a dynamical condensate component, $(1-\varepsilon)m_t$, generated by a new strong dynamics, together with a small fundamental component, εm_t ($\varepsilon \approx 0.03$–0.1), generated by ETC; (iii) the new strong dynamics is assumed to be chiral critically strong but spontaneously broken by technicolor at the scale $\sim$1 TeV, and it generally couples preferentially to the third generation. This needs a new class of technicolor models incorporating "topcolor" (TOPC). The dynamics at the $\sim$1 TeV scale involves the following gauge structure:

$$\begin{aligned} &\mathrm{SU}(3)_1 \times \mathrm{SU}(3)_2 \times \mathrm{U}(1)_{Y_1} \times \mathrm{U}(1)_{Y_2} \\ &\quad \to \mathrm{SU}(3)_{\mathrm{QCD}} \times \mathrm{U}(1)_{\mathrm{EM}}, \end{aligned} \tag{1}$$

where $\mathrm{SU}(3)_1 \times \mathrm{U}(1)_{Y_1}[\mathrm{SU}(3)_2 \times \mathrm{U}(1)_{Y_2}]$ generally couples preferentially to the third (first and second) generation and is assumed to be strong enough to form chiral $\langle\bar{t}t\rangle$ but not $\langle\bar{b}b\rangle$ condensation by the $\mathrm{U}(1)_{Y_1}$ coupling. A residual global symmetry $\mathrm{SU}(3)' \times \mathrm{U}(1)'$ implies the existence of a massive color-singlet heavy Z' and an octet B^A_μ. A symmetry-breaking pattern outlined above will generically give rise to three top pions, neutral π_t^0 and charged $\pi_t^\pm$, near the top mass scale.

The new couplings $Z'e^+e^-$, $Z'b\bar{b}$ given by the topcolor interactions which for the process $e^+e^- \to b\bar{b}$ can be written as

$$\begin{aligned} &\tfrac{1}{2} g_1 \tan\theta' \gamma^\nu L + g_1 \tan\theta' \gamma^\nu R, \\ &\tfrac{1}{6} g_1 \cot\theta' \gamma^\mu L - \tfrac{1}{3} g_1 \cot\theta' \gamma^\mu R, \end{aligned} \tag{2}$$

where $L, R = (1 \mp \gamma_5)/2$ are the left- and right-handed projectors, and $g_1 = \alpha_{\mathrm{EM}}/\cos\theta_W$ is the $\mathrm{U}(1)_Y$ coupling constant at the scale $\sim$1 TeV. The SM $\mathrm{U}(1)_Y$ and the $\mathrm{U}(1)'$

field Z'_α are then defined by orthogonal rotation with mixing angle θ'.

The ETC gauge bosons Z^* exist, including the sideways and diagonal gauge bosons in this model. The coupling of Z^* to the fermions and technifermions can be found in Ref. [9]. For the sake of simplicity, we assume that the mass of the sideways gauge boson is equal to the mass of the diagonal gauge boson, namely, m_{Z^*}, so the $Z^* e^+ e^-$ and $Z^* b\bar{b}$ coupling by the ETC dynamics can be written as

$$\begin{aligned}
&-\frac{\varepsilon m_t}{16\pi f_\pi}\frac{e}{s_W c_W}\gamma_\mu\bigg\{\bigg[\frac{2N_C}{N_{\rm TC}+1}\xi_t^{-1}(1-\xi_e\xi_\nu)\xi_e \\
&\quad -(10\varepsilon)^{-2/3}\xi_e^2\bigg]L \\
&\quad -\bigg[\frac{2N_C}{N_{\rm TC}+1}(\xi_e\xi_\nu)\xi_t^{-1}(1-\xi_e\xi_\nu)\xi_e^{-1}\bigg]R\bigg\} \\
&\quad -\frac{\varepsilon m_t}{16\pi f_\pi}\frac{e}{s_W c_W}\gamma_\mu\bigg[\frac{N_C}{N_{\rm TC}+1}\xi_t(\xi_t^{-1}+\xi_b)-\xi_t^2)\bigg]L, \quad (3)
\end{aligned}$$

where $N_{\rm TC}$ and N_C are the numbers of technicolors and ordinary colors, respectively; $s_W = \sin\theta_W$, $c_W = \cos\theta_W$ with θ_W being the Weinberg angle; ξ_t, ξ_e, and ξ_ν are coupling coefficients and are ETC gauge-group-dependent. Following Ref. [9], we take $\xi_t = \xi_e = 1/\sqrt{2}$, $\xi_\nu = 0.1\xi_e^{-1}$, and $\xi_b = 0.028\xi_t^{-1}$.

According to the idea of TOPCTC, the masses of the first and second generation quarks are all generated by ETC interactions. Then, the difference between ξ_U and ξ_D reflects the mass difference between the charm and strange quarks [4,9]. So we have $m_{t1} = (m_c/m_s)m_{b1}$, where m_{t1} and m_{b1} are the top- and bottom-quark masses generated by ETC interactions, respectively. For $m_s = 0.18$ GeV, $m_c = 1.5$ GeV, we have $m_{t1} \simeq 10m_{b1}$. In this model, there are 60 technipions in the ETC sector with decay constant $f_\pi = 123$ GeV and three top pions π_t^0, $\pi_t^\pm$ in the TOPC sector with decay constant $f_{\pi_t} = 50$ GeV. The ETC sector is a one generation technicolor model [2]. In 60 technipions the color singlets π and color octets π_8 are extensively and universally studied. The color singlets π include the isosinglet π^0 and the isotriplets $(\pi^\pm, \pi^3)$, while the color octets π_8 involve the isosinglet π_8^0 and the iso-octets $(\pi_8^\pm, \pi_8^3)$. The coupling of these PGBs to the top (bottom) quark are given by [2,10]

$$\begin{aligned}
&\frac{ic_t}{\sqrt{2}f_\pi}[m_t\bar{t}\gamma_5 t\pi^0 + m_b\bar{b}\gamma_5 b\pi^0 + m_t\bar{t}\gamma_5 t\pi^3 + m_b\bar{b}\gamma_5 b\pi^3 \\
&\quad + \sqrt{2}V_{tb}\bar{t}(m_bR - m_tL)b\pi^+ + \sqrt{2}V_{tb}\bar{b}(m_tR - m_bL)t\pi^-], \quad (4)
\end{aligned}$$

$$\begin{aligned}
&\frac{i\lambda^a}{\sqrt{2}f_\pi}[m_t\bar{t}\gamma_5 t\pi_8^0 + m_b\bar{b}\gamma_5 b\pi_8^0 + m_t\bar{t}\gamma_5 t\pi_8^3 + m_b\bar{b}\gamma_5 b\pi_8^3 \\
&\quad + \sqrt{2}V_{tb}\bar{t}(m_bR - m_tL)b\pi_8^+ \\
&\quad + \sqrt{2}V_{tb}\bar{b}(m_tR - m_bL)t\pi_8^-], \quad (5)
\end{aligned}$$

where the coefficient $c_t = 1/\sqrt{6}$, λ^a is a Gell-Mann matrix acting on ordinary color indices. In this model $m_t \to m_{t1} = \varepsilon m_t$, $m_b \to m_{b1} = 0.1m_{t1}$.

The interaction of the top pions with the top and bottom quarks has the form

$$\begin{aligned}
&\frac{i}{f_{\pi_t}}\bigg[\frac{1}{\sqrt{2}}m_{t2}\bar{t}\gamma_5 t\pi_t^0 + \frac{1}{\sqrt{2}}m_{b2}\bar{b}\gamma_5 b\pi_t^0 + V_{tb}\bar{t}(m_{t2}L \\
&\quad + m_{b2}R)b\pi_t^+ + V_{tb}\bar{b}(m_{t2}R + m_{b2}L)t\pi_t^-\bigg], \quad (6)
\end{aligned}$$

where $m_{t2} = (1-\varepsilon)m_t$, and $m_{b2} = m_b - m_{b1}$ denote the masses of top and bottom quarks generated by TOPC interaction, respectively. More detailed Feynman rules needed in the calculations can be found in Refs. [3,10].

The topcolor assisted multiscale technicolor model [4] is different from the original TOPCTC model mainly by the ETC sector. In the original TOPCTC model, the ETC sector is the one generation technicolor model with $f_\pi = 123$ GeV, $N_{\rm TC} = 4$ and $c_t = 1/\sqrt{6}$ in Eq. (4), and in the TOPCMTC model the ETC sector is the multiscale walking technicolor model [11] with $f_\pi = 40$ GeV, $N_{\rm TC} = 6$, and $c_t = 2/\sqrt{6}$.

III. THE CORRECTIONS OF THE $e^+e^- \to b\bar{b}$ CROSS SECTION

A. The PGB contributions

The relevant Feynman diagrams for the corrections arising from PGBs to the $e^+e^- \to b\bar{b}$ production amplitudes are shown in Figs. 1(b)–1(e). In our calculation, we use the dimensional regularization to regulate all the ultraviolet divergences in the virtual loop corrections and adopt the Feynman gauge and on-mass-shell renormalization scheme [12]. The renormalized amplitude for $e^+e^- \to b\bar{b}$ contains

$$M = M^{\rm tree} + \Delta M, \quad (7)$$

where the tree-level amplitude

$$\begin{aligned}
M^{\rm tree} &= \bar{u}(b)ie\gamma^\mu[v_b(1+\gamma_5) + a_b(1-\gamma_5)]v(\bar{b}) \\
&\quad \times \frac{-ig_{\mu\nu}}{(P_b + P_{\bar{b}})^2 - m_Z^2}\bar{v}(e^+)ie\gamma^\nu[v_e(1+\gamma_5) \\
&\quad + a_e(1-\gamma_5)]u(e^-) + \bar{u}(b)\Big(-i\frac{1}{3}e\Big)\gamma^\mu v(\bar{b}) \\
&\quad \times \frac{-ig_{\mu\nu}}{(P_b + P_{\bar{b}})^2}\bar{v}(e^+)(-ie)\gamma^\nu u(e^-). \quad (8)
\end{aligned}$$

Here $P_{b,\bar{b}}$ denote the momentum of the outgoing bottom-quark pair. ΔM represents the PGBs one-loop corrections which contain

$$\delta M = \Delta M_Z + \Delta M_\gamma, \quad (9)$$

$$\begin{aligned}
\delta M_Z &= \bar{u}(b)\Gamma_Z v(\bar{b})\frac{-ig_{\mu\nu}}{(P_b + P_{\bar{b}})^2 - m_Z^2}\bar{v}(e^+)ie\gamma^\nu[v_e(1+\gamma_5) \\
&\quad + a_e(1-\gamma_5)]u(e^-), \quad (10)
\end{aligned}$$

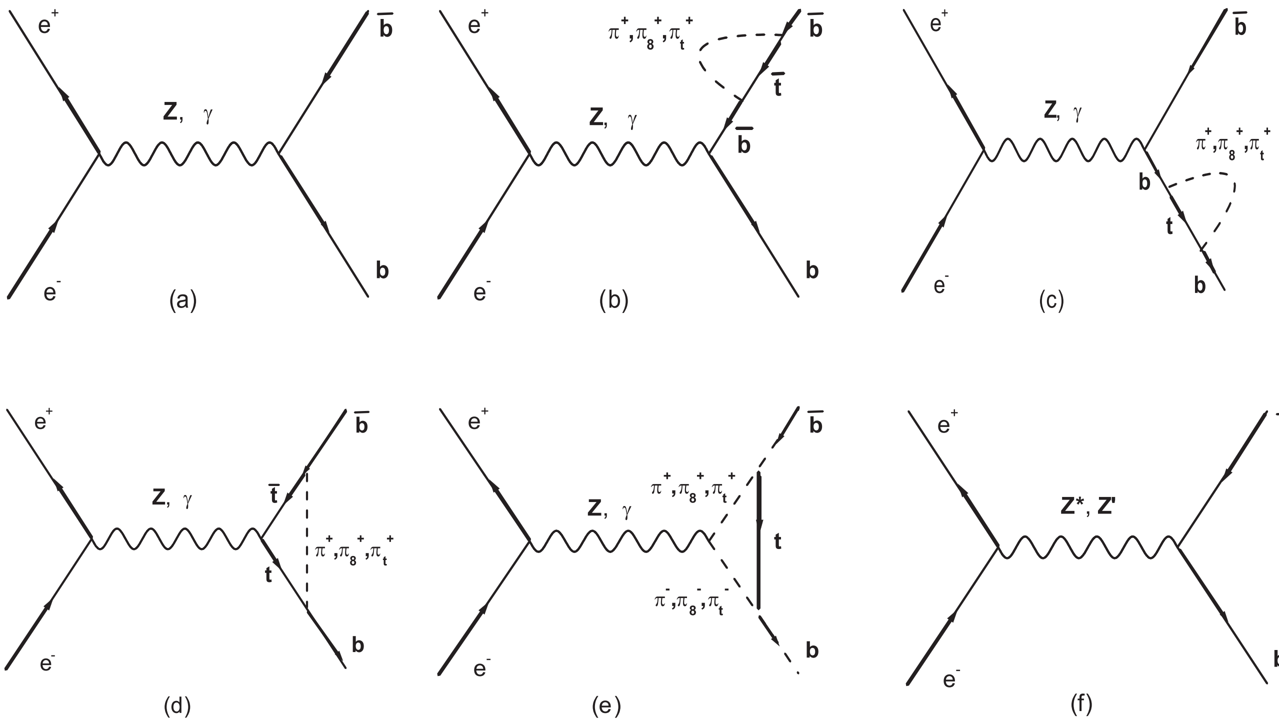

FIG. 1. Feynman diagrams for PGB and new gauge boson contributions to the $e^+e^- \rightarrow b\bar{b}$ process: (a) tree level; (b),(c) self-energy diagrams; (d),(e) vertex diagrams; (f) the corrective diagram of the gauge bosons Z^* and Z'.

$$\delta M_\gamma = \Delta M_Z|_{Z\rightarrow\gamma, m_Z=0, v_b=a_b=-1/6, v_e=a_e=-1/2}, \tag{11}$$

where

$$v_b = \frac{(2/3)s_W^2}{4s_Wc_W}, \qquad a_b = \frac{-1+(2/3)s_W^2}{4s_Wc_W},$$
$$v_e = \frac{2s_W^2}{4s_Wc_W}, \qquad a_e = \frac{1-2s_W^2}{4s_Wc_W}. \tag{12}$$

Γ_Z, Γ_γ denote the effect vertex $Zb\bar{b}$ and $\gamma b\bar{b}$ arising from PGB corrections, respectively, and

$$\Gamma_Z = ie[\gamma^\mu LF_{1Z} + \gamma^\mu RF_{2Z} + P_b^\mu LF_{3Z} + P_b^\mu RF_{4Z} + P_{\bar{b}}^\mu LF_{5Z} + P_{\bar{b}}^\mu RF_{6Z}], \tag{13}$$

$$\Gamma_\gamma = \Gamma_Z|_{F_{iz}\rightarrow F_{i\gamma}}. \tag{14}$$

The form factors Γ_{iZ} and $\Gamma_{i\gamma}$ are expressed in terms of two- and three-point scalar integrals. The expressions of Γ_{iZ} and $\Gamma_{i\gamma}$ arising from the color-octet technipions and top pions are presented in Appendixes A and B, respectively. It is easy to find that all the ultraviolet divergences cancel in the effective vertex.

For the contributions from color-singlet technipions to $e^+e^- \rightarrow b\bar{b}$, there is an enhanced factor 18 for the form factors arising from color-octet technipions compared with color singlets. Therefore, the magnitude of the processes involving the color-singlet technipions is at least 2 orders of magnitude smaller than those arising from color octets. So we shall neglect the color-singlet technipion contributions in our calculations.

The differential cross section of the process $e^+e^- \rightarrow b\bar{b}$ is

$$d\sigma = \frac{(2\pi)^4\delta^4(P_{e^+} + P_{e^-} - P_b - P_{\bar{b}})}{4\sqrt{(P_{e^+}P_{e^-})^2 - (m_{e^+}m_{e^-})^2}} \times \bar{\sum}|M|^2 \frac{d^3\vec{P}_b}{(2\pi)^3 2E_b}\frac{d^3\vec{P}_{\bar{b}}}{(2\pi)^3 2E_{\bar{b}}}. \tag{15}$$

Integrating out phase space, we get the total cross section

$$\sigma = \frac{\sqrt{s(s-4m_b^2)}}{32\pi s^2}\int_{-1}^{1}\bar{\sum}|M|^2 d\cos\theta = \sigma_0 + \Delta\sigma, \tag{16}$$

where σ_0 is the cross section at the tree level, and $\Delta\sigma$ denotes the Yukawa corrections arising from PGBs at the one-loop level.

Through the above equations, we calculate the relative corrections to the production rate σ arising from PGBs, since the ETC sector of this model is a one generation technicolor model. The masses of PGBs are model dependent. In Ref. [2], the masses of π and π_8 are taken to be in the range $100 < m_\pi < 300$ GeV, $200 < m_{\pi_8} < 500$ GeV. In the TOPC sector, the mass of the top pion, m_{π_t}, a reasonable value of the parameter is around 200 GeV. In the following calculation, we would rather take a slightly larger range, $150 < m_{\pi_t} < 500$ GeV, to see its effect, and shall take the mass of m_{π_8}, 246 GeV. The final numerical results are plotted in Figs. 2–5.

At first, we calculate the contributions of the color-octet technipions in the ETC sector to $e^+e^- \to b\bar{b}$. $\Delta\sigma/\sigma_0$ versus ε with $m_{\pi_8} = 246$ GeV are shown in Fig. 2. One can see that (i) the relative corrections $\Delta\sigma/\sigma_0$ increase with ε sensitively, and (ii) the maximum of the relative corrections is only 0.75% for $\varepsilon = 0.1$ when $\sqrt{s} = 1.0$ TeV.

Next we take a look at the corrections from the top pion in the TOPC sector. Figure 3 is the same as Fig. 2 but for the top pion with $m_{\pi_t} = 220$ GeV. From this figure, we can see the following: (i) As ε increases $\Delta\sigma/\sigma_0$ decrease slowly. (ii) The relative corrections $\Delta\sigma/\sigma_0$ decrease as $\sqrt{s}$ increases quickly. (iii) The maximum of the relative corrections can reach 43.0% for $\varepsilon = 0.03$ when $\sqrt{s} = 0.5$ TeV. The small decay constant $f\pi_t$ and main masses of the top and bottom quarks generated by TOP interaction give rise to a much large relative correction. Although there is small change in the pion mass, the contributions to the cross section arising from the TOPC sector are larger than the ETC sector because the couplings of the top pions to three family fermions are nonuniversal, and the top pions have large Yukawa couplings to the third generation.

Figure 4 presents the plots of $\Delta\sigma/\sigma_0$ vs m_{π_t} with $\varepsilon = 0.07$. When $\sqrt{s} = 0.5$ TeV, the size of the relative corrections decreases with m_{π_t} changes from 45.2% to 19.1%. However, the corrections increase with m_{π_t} at the outset stage and decrease at the back stage at $\sqrt{s} = 1.0$ TeV, and always increase quickly in the case of $\sqrt{s} = 1.5$ TeV.

We know, according to the idea of TOPCTC, technipions and top pions exist simultaneously and are also affected at the same time. The physics results should be the sum of the contributions of the ETC sector and TOPC sector. So we plot the total corrections arising from color-octet technipions and top pions to $e^+e^- \to b\bar{b}$ curves with ε for $m_{\pi_8} = 246$ GeV and $m_{\pi_t} = 220$ GeV in Fig. 5. Owing to the fact that the corrections arising from the technipion are negligibly small and the contributions of the top pion are rather large, the total corrections are similar to the corrections from the top pion (see Fig. 3).

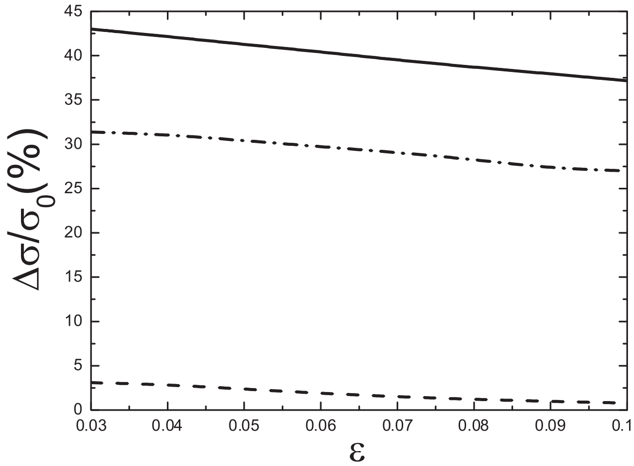

FIG. 3. The same as Fig. 2 but for a top pion with $m_{\pi_t} = 220$ GeV.

B. The gauge boson corrections

Now let us consider the contributions from the new gauge bosons to the $e^+e^- \to b\bar{b}$ cross section.

In the TOPCTC theory, there are two kinds of new gauge bosons: the ETC gauge bosons Z^* including the sideways and diagonal gauge bosons, and the TOPC gauge bosons including the color-octet colorons B_μ and color-singlet Z'. The relevant gauge bosons to the process $e^+e^- \to b\bar{b}$ are

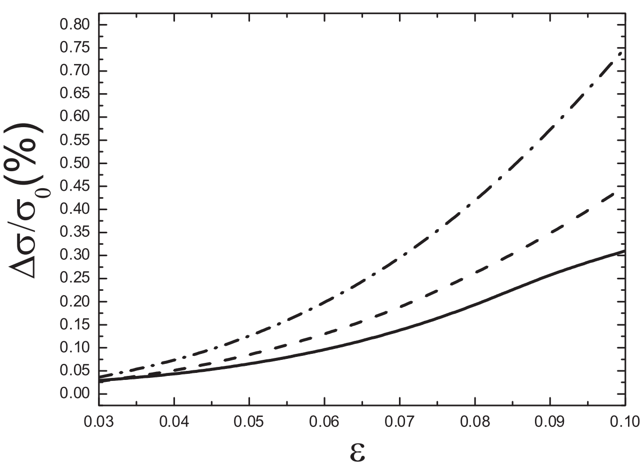

FIG. 2. The relative corrections from color-octet technipions to $e^+e^- \to b\bar{b}$ curves as function of ε with $m_{\pi_8} = 246$ GeV. The solid, dot-dashed, and dashed lines correspond to $\sqrt{s} = 0.5$, 1.0, and 1.5 TeV, respectively.

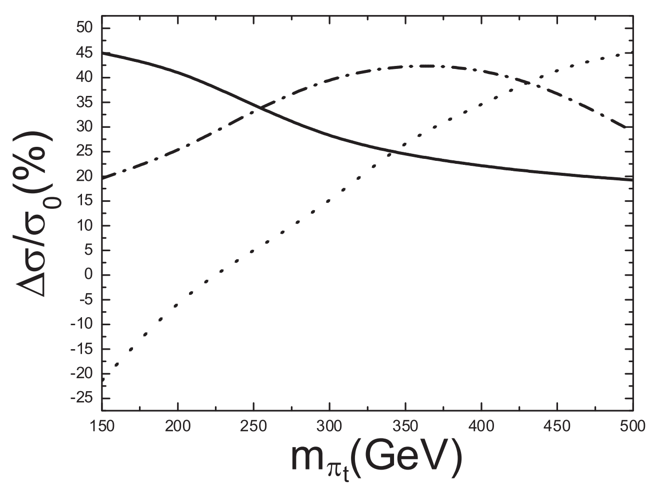

FIG. 4. The plots of $\Delta\sigma/\sigma_0$ vs m_{π_t} with $\varepsilon = 0.07$. The solid, dot-dashed, and dashed lines correspond to $\sqrt{s} = 0.5$, 1.0, and 1.5 TeV, respectively.

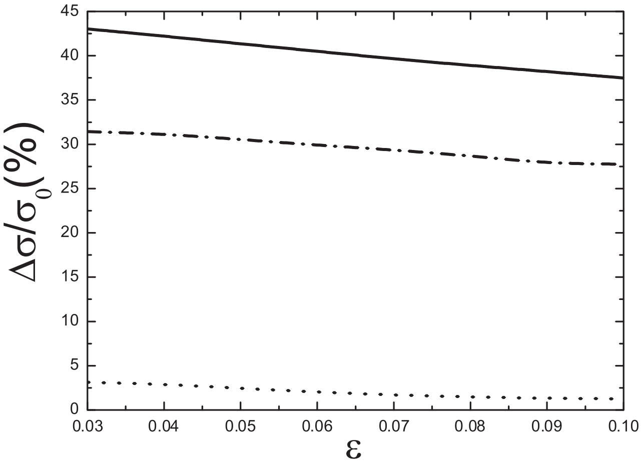

FIG. 5. The total corrections arising from color-octet technipions and top pions to $e^+e^- \to b\bar{b}$ curves with ε for $m_{\pi_8} = 246$ GeV and $m_{\pi_t} = 220$ GeV. The solid, dot-dashed, and dashed lines correspond to $\sqrt{s} = 0.5$, 1.0, and 1.5 TeV, respectively.

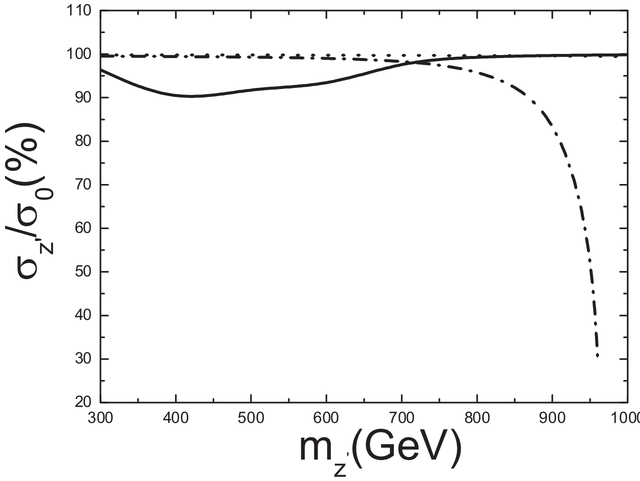

FIG. 6. The relative correction $\sigma_{z'}/\sigma_0$ of the gauge boson Z' as a function of $m_{Z'}$. The solid, dot-dashed, and dashed lines correspond to $\sqrt{s} = 0.5$, 1.0, and 1.5 TeV, respectively.

only Z^* in the ETC sector and Z' in the TOPC sector; the Feynman diagram is shown in Fig. 1(f).

Using Eqs. (2) and (3) we can obtain the effects of the gauge bosons. The contributions from the ETC gauge boson to the $e^+e^- \to b\bar{b}$ cross section increase quickly with ε and $\sqrt{s}$, and decrease slowly with m_{Z^*}. The maximum of the three bosons total relative corrections σ_{Z^*}/σ_0 is only the order of 10^{-14} whatever ε, $\sqrt{s}$ and m_{Z^*} take in the certain parameter ranges, and therefore, can be neglected safely.

As for the contributions from Z', it is obvious that the contributions to the $e^+e^- \to b\bar{b}$ cross section are not related to θ' [see Eq. (2)]. In our calculation, we assume the mass of the gauge boson Z' vary from 300 to 1000 GeV to study the effects of Z'. The numerical results are plotted in Fig. 6. From this figure we find that except for the Z' resonance region, the relative correction $\sigma_{Z'}/\sigma_0$ is negative and less than 1%. So it is also negligibly small.

For the topcolor assisted multiscale technicolor model, it is different from the original TOPCTC model mainly by the ETC sector. In TOPCMTC model, the corrections are similar to those of the original TOPCTC model. The contribution from PGBs in the TOPCMTC model is slightly larger than that of the other; the contribution from the gauge boson Z' is the same as that of the original TOPCTC model, and the correction from the ETC gauge boson Z^* is still negligibly small.

IV. DISCUSSION AND CONCLUSION

In this paper we have studied the contributions from the pseudo Goldstone bosons and new gauge bosons in the original topcolor assisted technicolor model to $b\bar{b}$ production at the $\sqrt{s} = 0.5$, 1.0, and 1.5 TeV e^+e^- colliders. We found for reasonable ranges of the parameters, the top pions afford a dominate contribution, the corrections arising from technipions and new gauge bosons are negligibly small, and the maximum of the total corrections is 43.0% with the c.m. energy $\sqrt{s} = 500$ GeV. We thus conclude that the $e^+e^- \to b\bar{b}$ experiments at the future colliders are really interesting in testing the standard model and searching for the signs of technicolor.

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APPENDIX A: THE FORM FACTORS ARISING FROM THE COLOR-OCTET TECHNIPION

The form factors F_{iV} from the contributions of the color-octet technipion can be written as

$$\begin{aligned} F_{1V} = \frac{3}{2\pi^2 f_\pi^2}\{ & k_1^V m_{t1}^2[-2C_{24} + m_{\pi_8}^2 C_0 \\ & + B_0(-\sqrt{s}, m_{t1}, m_{t1})] + k_2^V[m_{t1}^2 m_{b1}^2(2C_0 + C_{11} \\ & - m_{t1}^4 C_0 - m_{b1}^4(C_0 + C_{11})] \\ & + m_{t1}^2 k_4^V[(B_1(m_{b1}, m_{t1}, m_{\pi_8}) \\ & + m_{b1}^2 B_1'(m_{b1}, m_{t1}, m_{\pi 8})] - k_5^V m_{t1}^2 C_{24}^*\}, \end{aligned} \tag{A1}$$

$$\begin{aligned} F_{2V} = \frac{3}{2\pi^2 f_\pi^2}\{ & k_2^V[m_{t1}^2 m_{b1}^2(-C_{11}) + m_{b1}^4 C_{11} + m_{b1}^2 m_{\pi_8}^2 C_0 \\ & + m_{b1}^2 B_0(-\sqrt{s}, m_{t1}, m_{t1}) - 2m_{b1}^2 C_{24}] \\ & - k_5^V m_{b1}^2 C_{24}^* + m_{b1}^2 m_{t1}^2 k_3^V B_1'(m_{b1}, m_{t1}, m_{\pi_8})\}, \end{aligned} \tag{A2}$$

$$F_{3V} = \frac{3}{2\pi^2 f_\pi^2}\Big\{2m_{t1}^2 m_{b1} k_1^V (C_{23} - C_{22}) + 2k_2^V [m_{t1}^2 m_{b1}(C_{12} - C_{11}) + m_{b1}^3 (C_{21} + C_{22} - 2C_{23} + C_{11} - C_{12})] + \frac{1}{2} k_5^V [m_{t1}^2 m_{b1}(C_0^* + C_{12}^* + C_{11}^* - 2C_{22}^* + 2C_{23}^*) + m_{b1}^3 (C_{12}^* + 2C_{22}^*)]\Big\}, \quad \text{(A3)}$$

$$F_{4V} = \frac{3}{2\pi^2 f_\pi^2}\Big\{2m_{t1}^2 m_{b1} k_1^V (C_{21} + C_{22} - 2C_{23}) + 2m_{b1}^3 k_2^V (C_{23} - C_{22}) + \frac{1}{2} k_5^V [m_{t1}^2 m_{b1}(C_0^* + 3C_{12}^* + 2C_{22}^*) + m_{b1}^3 (C_{11}^* - C_{12}^* - 2C_{22}^* + 2C_{23}^*)]\Big\}, \quad \text{(A4)}$$

$$F_{5V} = \frac{3}{2\pi^2 f_\pi^2}\Big\{2k_1^V m_{t1}^2 m_{b1}(-C_{22}) + 2k_2^V m_{b1}^3 (C_{22} - C_{23}) - \frac{1}{2} k_5^V [m_{t1}^2 m_{b1}(C_0^* + 3C_{11}^* - 3C_{12}^* + 2C_{21}^* + 2C_{22}^* - 4C_{23}^*) + m_{b1}^3 (C_{12}^* - 2C_{22}^* + 2C_{23}^*)]\Big\}, \quad \text{(A5)}$$

$$F_{6V} = \frac{3}{2\pi^2 f_\pi^2}\Big\{2k_1^V [m_{t1}^2 m_{b1}(C_{22} - C_{23})] + 2k_2^V [m_{b1}^3 (-C_{22} - C_{12}) + m_{t1}^2 m_{b1} C_{12}] - \frac{1}{2} k_5^V [m_{t1}^2 m_{b1}(C_0^* + 2C_{11}^* - C_{12}^* - 2C_{22}^* + 2C_{23}^*) - m_{b1}^3 (C_{12}^* - C_{11}^* - 2C_{21}^* - 2C_{22}^* + 4C_{23}^*)]\Big\}, \quad \text{(A6)}$$

with

$$\begin{pmatrix} k_1^Z & k_1^\gamma \\ k_2^Z & k_2^\gamma \\ k_3^Z & k_3^\gamma \\ k_4^Z & k_4^\gamma \\ k_5^Z & k_5^\gamma \end{pmatrix} = \begin{pmatrix} \frac{-\frac{4}{3}s_w^2}{4s_w c_w} & 1/3 \\ \frac{1-\frac{4}{3}s_w^2}{4s_w c_w} & 1/3 \\ \frac{\frac{2}{3}s_w^2}{4s_w c_w} & -1/6 \\ \frac{-1+\frac{2}{3}s_w^2}{4s_w c_w} & -1/6 \\ \frac{1-2s_w^2}{2s_w c_w} & 1 \end{pmatrix}, \quad \text{(A7)}$$

where $C_{ij} = C_{ij}(m_{b1}, -\sqrt{s}, m_{\pi_8}, m_{t1}, m_{t1})$, $C_{ij}^* = C_{ij}(m_{b1}, \sqrt{s}, m_{t1}, m_{\pi_8}, m_{\pi_8})$; $B_i' = \partial B_i / \partial P_b^2 |_{P_b^2 = m_{b1}^2}$. The basic two- and three-scalar integrals are given in Ref. [13].

APPENDIX B: THE FORM FACTORS ARISING FROM THE TOP PION

The form factors F_{iV} from the contributions of the top pion can be written as

$$F_{1V} = \frac{1}{8\pi^2 f_{\pi_t}^2}\{m_{t2}^2 k_1^V [2C_{24} - 2m_{b2}^2 C_{11} - B_0(-\sqrt{s}, m_{t2}, m_{t2}) - m_{\pi_t}^2 C_0] + m_{b2}^4 k_2^V (C_0 + C_{11}) + m_{t2}^2 m_{b2}^2 k_2^V (2C_0 + C_{11}) + m_{t2}^4 k_2^V C_0 + m_{t2}^2 k_5^V C_{24}^* - m_{t2}^2 k_4^V [B_1(m_{b2}, m_{t2}, m_{\pi_t}) + m_{b2}^2 B_1'(m_{b2}, m_{t2}, m_{\pi_t})]\}, \quad \text{(B1)}$$

$$F_{2V} = \frac{1}{8\pi^2 f_{\pi_t}^2}\{k_2^V [2m_{b2}^2 C_{24} - m_{b2}^4 C_{11} - m_{b2}^2 B_0(-\sqrt{s}, m_{t2}, m_{t2}) - m_{b2}^2 m_{\pi_t}^2 C_0 - m_{t2}^2 m_{b2}^2 C_{11}] + k_1^V [2m_{b2}^2 m_{t2}^2 (2C_0 + C_{11}) + k_5^V m_{b2}^2 C_{24}^* - m_{b2}^2 m_{t2}^2 k_3^V B_1'(m_{b2}, m_{t2}, m_{\pi_t})\}, \quad \text{(B2)}$$

$$F_{3V} = \frac{1}{8\pi^2 f_{\pi_t}^2}\Big[-2m_{b2}^3 k_2^V (C_{21} + C_{22} - 2C_{23} + C_{11} - C_{12}) - 2m_{b2} m_{t2}^2 k_1^V (C_{23} - C_{22}) + 2m_{t2}^2 m_{b2} k_2^V (C_{12} - C_{11}) - \frac{1}{2} k_5^V m_{b2}^3 (C_{12}^* + 2C_{22}^*) + \frac{1}{2} k_5^V m_{t2}^2 m_{b2} (C_0^* + 3C_{12}^* - C_{11}^* + 2C_{22}^* - 2C_{23}^*)\Big], \quad \text{(B3)}$$

$$F_{4V} = \frac{1}{8\pi^2 f_{\pi_t}^2}\Big[-2m_{t2}^2 m_{b2} k_1^V (C_{21} + C_{22} - 2C_{23} + 2C_{11} - 2C_{12}) - 2m_{b2}^3 k_2^V (C_{23} - C_{22}) + \frac{1}{2} k_5^V m_{t2}^2 m_{b2} (C_0^* + C_{12}^* - 2C_{22}^*) - \frac{1}{2} k_5^V m_{b2}^3 (C_{11}^* - C_{12}^* - 2C_{22}^* + 2C_{23}^*)\Big], \quad \text{(B4)}$$

$$F_{5V} = \frac{1}{8\pi^2 f_{\pi_t}^2}\Big[2m_{b2}^3 k_2^V (-C_{22} + C_{23}) + 2m_{b2} m_{t2}^2 k_1^V (2C_{12} + C_{22}) - \frac{1}{2} k_5^V m_{b2}^3 (2C_{22} - 2C_{23} - C_{12}) - \frac{1}{2} k_5^V m_{t2}^2 m_{b2} (C_0^* + C_{11}^* - C_{12}^* - 2C_{21}^* - 2C_{22}^* + 4C_{23}^*)\Big], \quad \text{(B5)}$$

$$F_{6V} = \frac{1}{8\pi^2 f_{\pi_t}^2}\Big[2m_{t2}^2 m_{b2} k_1^V (-C_{22} + C_{23}) + 2m_{b2}^3 k_2^V (C_{22} + C_{12}) + 2m_{t2}^2 m_{b2} k_2^V C_{12} - \frac{1}{2} k_5^V m_{t2}^2 m_{b2} (C_0^* + 2C_{11}^* - 3C_{12}^* + 2C_{22}^* - 2C_{23}^*) - \frac{1}{2} k_5^V m_{b2}^3 (C_{12}^* - C_{11}^* - 2C_{21}^* - 2C_{22}^* + 4C_{23}^*)\Big], \quad \text{(B6)}$$

where k_i^V takes the same value as (A7) in Appendix A and $C_{ij} = C_{ij}(m_{b2}, -\sqrt{s}, m_{\pi_t}, m_{t2}, m_{t2})$, $C_{ij}^* = C_{ij}^*(m_{b2}, -\sqrt{s}, m_{t2}, m_{\pi_t}, m_{\pi_t})$.

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