

Electroweak constraints on effective theories with $U(2) \times U(1)$ flavor symmetry

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In a previous analysis presented in Phys. Rev. D **71**, 075009 (2005), electroweak constraints were given on arbitrary linear combinations of a set of dimension-6 operators. Flavor universality and thus $U(3)^5$ flavor symmetry were assumed for the operators. In this article, we expand the analysis to account for the flavor-dependent theories that distinguish the third generation of fermions from the light two generations. We still assume flavor universality for the light two generations to avoid large FCNCs. Consequently, the $U(3)^5$ flavor symmetry is relaxed to $[U(2) \times U(1)]^5$, and therefore more operators are added. We calculate the corrections to electroweak precision observables, assuming arbitrary coefficients for the operators. The corrections are combined with the standard model predictions and the experimental data to obtain the χ^2 distribution as a function of the operator coefficients. We apply our result to constrain two flavor-dependent extensions of the standard model: the simplest little Higgs model and a model with an $SU(2) \times SU(2) \times U(1)$ gauge group.

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I. INTRODUCTION

The standard model (SM) of electroweak physics is generally regarded as an effective theory with a cut-off far below the Planck scale. The cut-off is pinned down at the TeV scale if we require that the Higgs boson mass is not significantly fine-tuned. A variety of extensions of the SM at the TeV scale have been considered, including supersymmetry, technicolor, little Higgs and extra dimensions. All these models predict heavy states that are beyond the current experimental direct reach (with the Tevatron as a possible exception). While we are eagerly anticipating the operation of the LHC to produce and observe these heavy states, they could also leave their traces in the electroweak precision tests (EWPTs) [1], by means of quantum corrections. Because of the remarkable agreement between the SM predictions and the experimental results, the EWPTs do not tell us which direction of new physics is particularly promising. Rather, they often put stringent constraints on the model we are considering.

A model-independent approach to the electroweak constraints is desirable, in which one needs to calculate the constraints only once and then is able to apply them to different models. The oblique S , T and U parameter approach [2] has exhibited this merit, and has been effectively used to constrain various models. However, there exist models with corrections to the SM that cannot be fully described by the oblique parameters. A general method that can incorporate both the oblique and non-oblique corrections is the effective theory approach [3–5]. In this approach, all heavy particles are integrated out and their effects are manifested in the effective higher order operators suppressed by powers of the heavy mass scale. For a given order, there are only a limited number of such operators, to the contrast of the vast number of

possible theoretical extensions. Assuming arbitrary coefficients for the operators, one calculates the corrections to the electroweak precision observables (EWPOs) and compares them with the experimental data. The electroweak constraints are then obtained in terms of the operator coefficients.

In a previous publication [5], we focused on a set of dimension-6 operators that are consistent with the SM gauge symmetry, as well as CP , lepton and baryon number conservation. The electroweak symmetry breaking was assumed to be linearly realized so that one or more Higgs doublets were present in the SM particle spectrum. We also assumed $U(3)^5$ flavor symmetry, in other words, flavor universality for the operators. This is perhaps the simplest way to avoid large FCNCs and is well motivated in many theoretical frameworks. We chose those operators \mathcal{O}_i that could be tightly constrained and added them to the SM Lagrangian:

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i a_i \mathcal{O}_i = \mathcal{L}_{SM} + \sum_i \frac{1}{\Lambda_i^2} \mathcal{O}_i, \quad (1)$$

where Λ_i has mass dimension one, denoting the energy scale that suppresses the operator \mathcal{O}_i . We calculated the corrections to the EWPOs linearly in a_i and obtained the bounds on arbitrary linear combinations of these operators. The results have been used to constrain the mass of a generic Z' gauge boson [5], and the parameter spaces of a variety of little Higgs models [6].

Nevertheless, small FCNCs can be achieved without assuming flavor universality. Technically, each of the $U(3)$ symmetries can be relaxed to $U(1)^3$ with one $U(1)$ corresponding to one generation, as long as each generation is a mass eigenstate. With the $U(1)^3$ symmetry, operators involving different generations can have different coefficients, and still do not contribute to FCNCs. However, from the model-building point of view, it is

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unnatural to assume that the effective operators only contain mass eigenstates. In fact, in many models, operators are obtained by integrating out heavy gauge bosons, and involve gauge eigenstates instead. But natural flavor-dependent models at TeV scale are still possible, as long as universality is conserved for the first two generations and only the third generation is treated differently. This is mainly due to two reasons. First, the mass and gauge eigenstates of the third generation of quarks are quasialigned, namely, they are almost identical up to small rotations of order $\mathcal{O}(\lambda^2)$, where $\lambda \simeq 0.22$ is the Wolfenstein parameter in the CKM matrix. This is to the contrast of the first two generations of quarks, where the mixing is of order $\mathcal{O}(\lambda)$. Second, the most stringent experimental constraints on flavor violation come from the Kaon system, which can be avoided by assuming universality for the first two generations. The flavor changing experiments involving the third generation currently have limited statistics, and do not rule out TeV scale flavor physics [7].

Given the above observation, we extend the analysis in Ref. [5] on electroweak constraints to incorporate flavor-dependent operators. We relax each of the $U(3)$ symmetries to $U(2) \times U(1)$, with the $U(2)$ reflecting the universality of the first two generations. The $U(1)$ corresponds to flavor conservation in the third generation. We do not attempt to analyze the bounds from flavor changing measurements, which is a rich and interesting topic in itself. See Ref. [7] for a recent analysis. Rather, we will use the flavor-conserving EWPTs to constrain arbitrary linear combinations of all relevant flavor-dependent dimension-6 operators. We will use mass eigenstates as the basis of the operators. However, due to the universality in the first two generations and the quasialignment in the third generation discussed in the previous paragraph, the results apply as well to gauge eigenstates, and in general, to any basis that differs by only small rotations. Of course, since the symmetry is relaxed, there are more operators in this case than in the flavor-independent case. We enumerate and analyze the constraints for these operators in the next section. In Sec. III, the results are used to put bounds on two flavor-dependent models. Sec. IV contains the summary and discussion.

II. OPERATORS AND CONSTRAINTS

Assuming $U(3)^5$ flavor symmetry, we obtained 21 dimension-6 operators that are relevant to EWPTs in Ref. [5]. Using $W_{\mu\nu}^a$ and $B_{\mu\nu}$ to denote the $SU(2)_L$ and $U(1)_Y$ gauge boson field strength, l, q the left handed leptons and quarks, e, u, d the right handed leptons and quarks, and h the Higgs doublet, we list them below.

(1) Operators modifying gauge boson propagators:

$$O_{WB} = (h^\dagger \sigma^a h) W_{\mu\nu}^a B^{\mu\nu}, \quad O_h = |h^\dagger D_\mu h|^2; \quad (2)$$

(2) Four-fermion operators:

$$\begin{aligned} O_{ll}^s &= \frac{1}{2} (\bar{l} \gamma^\mu l) (\bar{l} \gamma_\mu l), & O_{ll}^t &= \frac{1}{2} (\bar{l} \gamma^\mu \sigma^a l) (\bar{l} \gamma_\mu \sigma^a l), \\ O_{lq}^s &= (\bar{l} \gamma^\mu l) (\bar{q} \gamma_\mu q), & O_{lq}^t &= (\bar{l} \gamma^\mu \sigma^a l) (\bar{q} \gamma_\mu \sigma^a q), \\ O_{le} &= (\bar{l} \gamma^\mu l) (\bar{e} \gamma_\mu e), & O_{qe} &= (\bar{q} \gamma^\mu q) (\bar{e} \gamma_\mu e), \\ O_{lu} &= (\bar{l} \gamma^\mu l) (\bar{u} \gamma_\mu u), & O_{ld} &= (\bar{l} \gamma^\mu l) (\bar{d} \gamma_\mu d), \\ O_{ee} &= \frac{1}{2} (\bar{e} \gamma^\mu e) (\bar{e} \gamma_\mu e), & O_{eu} &= (\bar{e} \gamma^\mu e) (\bar{u} \gamma_\mu u), \\ O_{ed} &= (\bar{e} \gamma^\mu e) (\bar{d} \gamma_\mu d); \end{aligned} \quad (3)$$

(3) Operators modifying gauge-fermion couplings:

$$\begin{aligned} O_{hl}^s &= i(h^\dagger D^\mu h) (\bar{l} \gamma_\mu l) + \text{h.c.}, \\ O_{hl}^t &= i(h^\dagger \sigma^a D^\mu h) (\bar{l} \gamma_\mu \sigma^a l) + \text{h.c.}, \\ O_{hq}^s &= i(h^\dagger D^\mu h) (\bar{q} \gamma_\mu q) + \text{h.c.}, \\ O_{hq}^t &= i(h^\dagger \sigma^a D^\mu h) (\bar{q} \gamma_\mu \sigma^a q) + \text{h.c.}, \\ O_{hu} &= i(h^\dagger D^\mu h) (\bar{u} \gamma_\mu u) + \text{h.c.}, \\ O_{hd} &= i(h^\dagger D^\mu h) (\bar{d} \gamma_\mu d) + \text{h.c.}, \\ O_{he} &= i(h^\dagger D^\mu h) (\bar{e} \gamma_\mu e) + \text{h.c.}; \end{aligned} \quad (4)$$

(4) Operator modifying the triple-gauge couplings:

$$O_W = \epsilon^{abc} W_\mu^{a\nu} W_\nu^{b\lambda} W_\lambda^{c\mu}. \quad (5)$$

Operators containing fermions are understood to be summed over flavor indices, consistent with the $U(3)^5$ symmetry. Note that the operators O_{WB} and O_h in Eqs. (2) correspond to the oblique S and T parameters. O_{WB} also contains a term that modifies the triple-gauge boson couplings. Four-fermion operators containing only quarks are not included in the list because they cannot be tightly constrained by available data.

With the relaxed $[U(2) \times U(1)]^5$ symmetry, the operators have similar structures. We only need to single out the third generation from the light two. By slightly abusing the notation, from now on, we will use l, q, e, u and d to represent only the first two generations of fermions. By this change of definition, the operators in Eqs. (2)–(5) are still present in the relaxed-symmetry case. Three of them, O_{WB} , O_h and O_W do not involve fermions. They are the same as in the $U(3)^5$ case. The other operators are now understood to be summed over only the first two generations, reflecting the $U(2)^5$ symmetry.

We then turn to the operators involving the third generation. We use L, Q, τ, t, b to denote the third generation counterparts of l, q, e, u, d , respectively. The relevant operators are listed below in Eqs. (6) and (7). We only consider those operators that can be tightly constrained by the current experimental data. Therefore, four-fermion

operators involving only the third generation have been omitted. We have also omitted the operator O_{ht} that modifies the coupling between the Z boson and the top quark, as well as the operators O_{lt} and O_{et} , since the top quark is not involved in any of the EWPTs.

(1) Four-fermion operators:

$$\begin{aligned} O_{lL}^s &= (\bar{l}\gamma^\mu l)(\bar{L}\gamma_\mu L), & O_{lL}^t &= (\bar{l}\gamma^\mu \sigma^a l)(\bar{L}\gamma_\mu \sigma^a L), \\ O_{lQ}^s &= (\bar{l}\gamma^\mu l)(\bar{Q}\gamma_\mu Q), \\ O_{lQ}^t &= (\bar{l}\gamma^\mu \sigma^a l)(\bar{Q}\gamma_\mu \sigma^a Q), \\ O_{Le} &= (\bar{L}\gamma^\mu L)(\bar{e}\gamma_\mu e), & O_{l\tau} &= (\bar{l}\gamma^\mu l)(\bar{\tau}\gamma_\mu \tau), \\ O_{Qe} &= (\bar{Q}\gamma^\mu Q)(\bar{e}\gamma_\mu e), & O_{lb} &= (\bar{l}\gamma^\mu l)(\bar{b}\gamma_\mu b), \\ O_{e\tau} &= (\bar{e}\gamma^\mu e)(\bar{\tau}\gamma_\mu \tau), & O_{eb} &= (\bar{e}\gamma^\mu e)(\bar{b}\gamma_\mu b); \end{aligned} \quad (6)$$

(2) Operators modifying gauge-fermion couplings:

$$\begin{aligned} O_{hL}^s &= i(h^\dagger D^\mu h)(\bar{L}\gamma_\mu L) + \text{h.c.}, \\ O_{hL}^t &= i(h^\dagger \sigma^a D^\mu h)(\bar{L}\gamma_\mu \sigma^a L) + \text{h.c.}, \\ O_{hQ}^s &= i(h^\dagger D^\mu h)(\bar{Q}\gamma_\mu Q) + \text{h.c.}, \\ O_{hQ}^t &= i(h^\dagger \sigma^a D^\mu h)(\bar{Q}\gamma_\mu \sigma^a Q) + \text{h.c.}, \\ O_{h\tau} &= i(h^\dagger D^\mu h)(\bar{\tau}\gamma_\mu \tau) + \text{h.c.}, \\ O_{hb} &= i(h^\dagger D^\mu h)(\bar{b}\gamma_\mu b) + \text{h.c.} \end{aligned} \quad (7)$$

The complete list of 37 operators relevant to our analysis are given by Eqs. (2)–(7). To obtain the constraints on these operators, we denote a_i the coefficients of the operators O_i and calculate to the linear order in a_i the corrections to the EWPOs. The validity of the linear approximation will be discussed later in this section.

The EWPOs contain all precisely measured observables relevant to our analysis. Observables not involving the third generation of fermions receive corrections only from operators (2)–(5), and the calculations are unaffected from those given in Ref. [5]. Observables involving the third generation also receive corrections from the new operators (6) and (7). Most of the observables were present in Ref. [5], although we have added a few that were previously omitted. These omitted observables are less precisely measured than some others that involve different generations but constrain the same operators due to the $U(3)^5$ flavor symmetry. With the flavor symmetry relaxed, they constrain different operators in the new analysis. The unaffected, affected and added observables are summarized in Table I. The corresponding new operators that contribute to the observables are also listed.

The corrections from the new operators (6) and (7) can be classified in two categories. First, the 4-fermion operators in (6) only affect the cross-sections of fermion pair productions from e^+e^- annihilation at LEP 2 energies. The linear order corrections come from the interference be-

tween the 4-fermion terms and the SM tree level processes, and are given in, for example, Ref. [8].

Second, the operators in (7) modify the couplings between the Z boson and the fermions τ and b :

$$\begin{aligned} \delta g_V^\tau &= -\frac{v^2}{2}(a_{hL}^s + a_{hL}^t + a_{h\tau}), \\ \delta g_A^\tau &= -\frac{v^2}{2}(a_{hL}^s + a_{hL}^t - a_{h\tau}), \\ \delta g_V^b &= -\frac{v^2}{2}(a_{hQ}^s + a_{hQ}^t + a_{hb}), \\ \delta g_A^b &= -\frac{v^2}{2}(a_{hQ}^s + a_{hQ}^t - a_{hb}), \end{aligned} \quad (8)$$

where $v = 246$ GeV is the Higgs vev and the convention is that in the SM the Z -fermion couplings are given by $g_V = T^3 - 2Q\sin^2\theta_W$ and $g_A = T^3$. We have calculated the SM tree level predictions for the observables in terms of arbitrary couplings g_V and g_A . Then it is straightforward to obtain the corrections from the operators by substituting the shifts in Eq. (8) to the SM results. These shifts affect the third generation fermion pair production at both LEP 1 and LEP 2 energies.

The three most precisely measured observables: α , M_Z and G_F , which we take as the input parameters, as well as the observables from the deep inelastic scattering and the atomic parity violation measurements, the W pair production cross-section and the W boson mass do not depend on the third generation at tree level. Therefore the only linear order corrections from the new operators are those discussed above.

After we have calculated the corrections to EWPOs in terms of arbitrary coefficients a_i , we compare them with the experimental data and obtain the total χ^2 distribution as a function of a_i . Since the corrections are assumed to be linear in a_i , the χ^2 is quadratic:

$$\chi^2(a_i) = \chi_{SM}^2 + a_i \hat{v}_i + a_i \mathcal{M}_{ij} a_j. \quad (9)$$

In the above equation, χ_{SM}^2 is the value of χ^2 when all a_i are set to zero. The vector \hat{v}_i and the matrix \mathcal{M} are our main results. Their numerical values are given in Ref. [9].

Unlike the $U(3)^5$ case, not all of the operators can be independently constrained by the current data. There exist “flat directions” in which some combinations of the operators have vanishing corrections to all of the EWPOs and thus can not be constrained. This is due to the increase in the number of operators and the lack of relevant measurements. To remove as many as possible flat directions, as mentioned above, we have added a few observables that were omitted in the previous analysis.

The added observables are mainly the total cross-sections and asymmetries in the processes $e^+e^- \rightarrow b\bar{b}$ and $e^+e^- \rightarrow c\bar{c}$ at LEP 2 [10]. They are useful for removing flat-directions for 4-fermion operators containing 2 leptons and 2 quarks. Without these data, there are 5 flat

TABLE I. Relevant measurements in standard notations. $Q_W(Cs)$, $Q_W(Tl)$: weak charges in Cs and Tl; g_L^2 , g_R^2 , R^ν , κ , $g_V^{\nu e}$, $g_A^{\nu e}$: deep inelastic scattering observables; Γ_Z , σ_h^0 , $A_{FB}^{0,f}$, A_f , Γ_{ff} ($f = e, \mu, \tau, s, c, b$), $\sin\theta_{\text{eff}}^{\text{lept}}(Q_{FB})$: Z-pole observables; σ_f , A_{FB}^f , ($f = \mu, \tau, c, b$), $d\sigma_e/d\cos\theta$, $d\sigma_W/d\cos\theta$: cross-sections for fermion pair and W pair production at LEP 2; M_W : the W boson mass. See Ref. [5] and references therein for detailed descriptions.

	Observables	Contributing New Operators
Unaffected ^a	$Q_W(Cs), Q_W(Tl),$ $g_L^2, g_R^2, R^\nu, \kappa, g_V^{\nu e}, g_A^{\nu e}$ $\Gamma_{ff}, A_{FB}^{0,f}, A_f$ ($f = e, \mu, c$), $\sin^2\theta_{\text{eff}}^{\text{lept}}(Q_{FB})$ $\sigma_\mu, A_{FB}^\mu, d\sigma_e/d\cos\theta, d\sigma_W/d\cos\theta$ M_W	
Affected	Γ_Z, σ_h^0 $\Gamma_{bb}, A_{FB}^{0,b}, A_b$ $\Gamma_{\tau\tau}, A_{FB}^{0,\tau}, A_\tau$ σ_q σ_τ, A_{FB}^τ	$O_{hL}^{s,t}, O_{hQ}^{s,t}, O_{h\tau}, O_{hb}$ $O_{hQ}^{s,t}, O_{hb}$ $O_{hL}^{s,t}, O_{h\tau}$ $O_{hQ}^{s,t}, O_{hb}, O_{lQ}^{s,t}, O_{lb}, O_{Qe}, O_{eb}$ $O_{hL}^{s,t}, O_{h\tau}, O_{lL}^{s,t}, O_{Le}, O_{l\tau}, O_{e\tau}$
Added	A_s σ_c, A_{FB}^c σ_b, A_{FB}^b	$O_{hQ}^{s,t}, O_{hb}, O_{lQ}^{s,t}, O_{lb}, O_{Qe}, O_{eb}$

^aFor the Z-pole observables, instead of the partial decay widths $\Gamma_{ee}, \Gamma_{\mu\mu}, \Gamma_{cc}$, often used are the ratios of the partial widths and the total hadronic width Γ_{had} : R_e, R_μ and R_c . If so, they are also affected by the new operators, but only through Γ_{had} .

directions involving such operators. This is what one would expect: because the third generation is separated from the first two, there are five new 4-fermion operators containing heavy quarks: $O_{lQ}^s, O_{lQ}^t, O_{Qe}, O_{lb}$ and O_{eb} . However the only data in the previous analysis that can constrain these operators is the total hadronic cross-section for e^+e^- scattering measured at LEP 2, and the constraints from these data are still correlated with the constraints on operators containing the light quarks. After adding the $e^+e^- \rightarrow b\bar{b}$ and $e^+e^- \rightarrow c\bar{c}$ data, 4 out of the 5 flat directions are removed. The remaining flat direction comes from the fact that the operators O_{lQ}^s and O_{lQ}^t contribute the same to the $e^+e^- \rightarrow b\bar{b}$ process:

$$a_{lQ}^s = -a_{lQ}^t, \quad \text{all other } a_i = 0. \quad (10)$$

Less important observables added to the analysis are the asymmetries for the strange quark: A_s and $A_{FB}^{0,s}$ [1] measured at the Z-pole. They are much less precisely measured than the asymmetries for the bottom quark: A_b and $A_{FB}^{0,b}$, which would constrain the same operators if $U(3)^5$ were assumed. It turns out that adding these observables does not remove any of the flat-directions, but helps to refine the constraints.

Besides the one in Eq. (11), there are three other flat directions:

$$\begin{aligned}
 a_{Le} &= -a_{l\tau}, & \text{all other } a_i &= 0; \\
 a_{lL}^s &= -a_{lL}^t, & \text{all other } a_i &= 0; \\
 a_{hQ}^s &= -a_{hQ}^t, & \text{all other } a_i &= 0.
 \end{aligned} \quad (11)$$

These flat directions reflect the lack of EWPOs involving top pair production and ν_τ -nucleon, ν_τ -lepton scattering.

Besides the exact flat directions, there also exist “-weakly-bounded” directions, which means that these directions cannot be tightly constrained by the available data. To illustrate this point, we consider the eigenvalues and the corresponding eigenvectors of the matrix \mathcal{M} in Eq. (9). By doing so, the correlations between different directions are removed. Each eigenvector represents a direction that can be independently constrained. The corresponding eigenvalue is the 1σ constraint on the scale Λ_i for this direction. Numerically, four of the eigenvalues are zeroes, corresponding to the four flat directions discussed above. The next few smallest eigenvalues are 190, 210, 270, 280 and 580 GeV. Bounds on Λ_i this small cannot be taken literally. This is because when calculating the corrections to EWPOs, we only work to the linear order in the coefficients a_i , corresponding to the linear order in v^2/Λ_i^2 or E^2/Λ_i^2 . Here $v = 246$ GeV, is the electroweak symmetry breaking scale, and E is the energy scale of a given measurement, which goes up to about 200 GeV for the LEP 2 measurements. Since the smallest nonzero eigenvalues mentioned above are close to v or E , the second order corrections in a_i are of the similar size and thus cannot be neglected. To obtain precise bounds one has to include these second order corrections, as well as corrections from higher order operators beyond dimension-6. In this case, a model-by-model approach would be more convenient.

The existence of the flat and weakly-bounded directions is not a deficiency of our method. Rather, it reflects the fact

that not enough measurements are available to tightly constrain all independent operators. In principle, these directions could also be constrained if new or more precise measurements were available. In practice, the chance is rare for the coefficients of the effective operators in a model to coincide with the flat or weakly-bounded directions. Moreover, one can easily check if the obtained bounds are large enough to ignore the higher order corrections. Usually a bound on Λ_i about 1 TeV is large enough, because higher order corrections are suppressed by extra powers of v^2/Λ_i^2 or E^2/Λ_i^2 , which are negligible for $\Lambda_i \gtrsim 1$ TeV. Even if we obtain bounds that are much smaller than 1 TeV and thus not precise, we will still be certain that the exact bounds will not be much tighter. Often, learning that a model is not tightly constrained by EWPTs is interesting enough.

In the following section, we apply our general result to obtain the bounds for two flavor-dependent models. We will first integrate out the heavy degrees of freedom and obtain the operator coefficients a_i as functions of the parameters in the theory. Then it is straightforward to obtain the constraints by substituting a_i in the χ^2 distribution, Eq. (9).

III. APPLICATIONS

A. The Simplest Little Higgs Model [11]

The first model we consider is the simplest little Higgs model discussed in Ref. [11]. Little Higgs models [11–13] are a set of models that aim at solving the fine-tuning problem associated with the Higgs mass, by introducing extra gauge and matter fields with masses at TeV scale. The simplest little Higgs model extends the electroweak gauge group to $SU(3) \times U(1)$. To each fermion doublet, a heavy fermion is added to form a triplet of the $SU(3)$ group. While the gauge group is fixed, there is still freedom to choose from different charge assignments for the fermions. In Ref. [11], two such charge assignments are discussed, which are called model 1 and model 2. In model 1, the three generations of fermions are assigned the same quantum numbers. The only flavor-dependent effect comes from the Yukawa couplings. Thus we obtain (approximately) flavor-universal operators and the constraints have been given in Ref. [6]. We focus on model 2 in this article. In model 2, the third generation of quark triplet is assigned different quantum numbers from the light two generations. The charge assignments for the three generations of lepton triplets, as well as lepton and quark singlets are the same. Thus we expect a $U(3)^4 \times U(2) \times U(1)$ group as the flavor symmetry. This symmetry is larger than $[U(2) \times U(1)]^5$ we assumed for the analysis, and is manifested by the relations between the operator coefficients, as described below. The notation follows Ref. [11].

We split the effective operators to two parts. The first part comes from integrating out the heavy gauge bosons and the coefficients a_i can be expressed in a condensed

form

$$\begin{aligned} a_h &= -\frac{9}{4F^2} \frac{(1 - \frac{2}{3}x^2)^2}{(3 + x^2)^2}, \\ a_{hf}^s &= \frac{9}{4F^2} \frac{1 - \frac{2}{3}x^2}{(3 + x^2)^2} (\sqrt{3}T^{8f} + x^2 Y_x^f), \\ a_{ff'}^s &= -\frac{9}{2F^2} \frac{1}{(3 + x^2)^2} (\sqrt{3}T^{8f} + x^2 Y_x^f) \\ &\quad \times (\sqrt{3}T^{8f'} + x^2 Y_x^{f'}), \end{aligned} \quad (12)$$

where $x = g_x/g$, $g_x^2 = 3g^2 g'^2/(3g^2 - g'^2)$, and $F^2 = f_1^2 + f_2^2$ depicting the heavy mass scale. The quantum numbers for the fermions $f = q, l, u, d, e, Q, L, b, \tau$ are given by

$$T^{8f} = -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}, 0, 0, 0, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}, 0, 0; \quad (13)$$

$$Y_x^f = 0, -\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}, -1, \frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, -1. \quad (14)$$

The second part of the operators comes from integrating out the heavy fermions. The Yukawa couplings come from the following Lagrangian.

$$\begin{aligned} &\lambda_1^u u_1^{c3} \Phi_1^\dagger \Psi_{Q^3} + \lambda_2^u u_2^{c3} \Phi_2^\dagger \Psi_{Q^3} + \lambda_1^d d_1^{c1,2} \Phi_1 \Psi_{Q^{1,2}} \\ &+ \lambda_2^d d_2^{c1,2} \Phi_2 \Psi_{Q^{1,2}} + \lambda^n n^c \Phi_1^\dagger \Psi_L + \dots \end{aligned} \quad (15)$$

We have omitted the terms that mix the third generation with the first two generations, such as $u^{c1,2} \Phi_1^\dagger \Psi_{Q^3}$, since the mixing is very small. In this case, the couplings $\lambda_{1,2}^d$ are 2×2 matrices. In order to avoid large FCNCs, similar to Ref. [14], we take one of them to be proportional to the identity matrix and of order one, and the other one approximately the down-type Yukawa matrix for the first two generations and thus much smaller in magnitude. When integrating out the heavy fermions, we can then neglect the smaller λ^d . In this way, we obtain operators that conserve the $U(2)$ symmetry for the light two generations:

$$\begin{aligned} a_{hl}^s &= -a_{hl}^t = \frac{1}{4} \frac{f_2^2}{F^2 f_1^2}, \\ a_{hq}^s &= a_{hq}^t = \begin{cases} \frac{1}{4} \frac{f_1^2}{F^2 f_2^2} & (\lambda_1^d \ll \lambda_2^d) \\ \frac{1}{4} \frac{f_2^2}{F^2 f_1^2} & (\lambda_2^d \ll \lambda_1^d) \end{cases}. \end{aligned} \quad (16)$$

The equality between a_{hq}^s and a_{hq}^t is a consequence of the fact that the heavy quarks mix with only the down-type quarks in the light two generations. For the third generation, the heavy quark mixes with the top quark and we get $a_{hQ}^s = -a_{hQ}^t$ similarly. According to Eq. (11), this is a flat direction, so we simply set them to zero.

Combining Eqs. (12) and (16) and substituting them to the χ^2 distribution, we can obtain the bounds on the scale F . For comparison with the bounds on model 1, given in

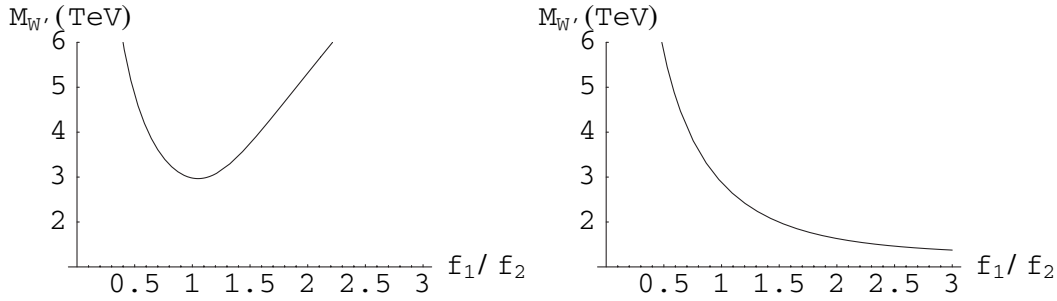


FIG. 1. Lower bounds at 95% CL on $M_{W'}$ as a function of f_1/f_2 in the simplest little Higgs model. Left: $\lambda_1^d \ll \lambda_2^d$; right: $\lambda_2^d \ll \lambda_1^d$.

Ref. [6], we translate the bounds on F to the bounds on the mass of the W' gauge boson by the relation

$$M_{W'}^2 = g^2 F^2 / 2. \quad (17)$$

The 95% confidence level (CL) bounds on $M_{W'}$ as a function of f_1/f_2 are shown in Fig. 1. One of the main motivations to consider the constraints is to estimate the associated fine-tuning. The heavy gauge bosons are introduced to cancel the quadratically divergent corrections to the Higgs boson mass-squared from the SM gauge boson loops. In order to avoid more than 10% fine-tuning, the W' boson mass should be smaller than about 5 TeV [13]. We see from Fig. 1 that $M_{W'} < 5$ TeV is allowed for a large portion of the parameter space. It is interesting that for the $\lambda_2^d \ll \lambda_1^d$ case, the bounds even go down to less than 1.5 TeV on the $f_1 > f_2$ side.

B. An $SU(2) \times SU(2) \times U(1)$ model [15]

The second model is an $SU(2) \times SU(2) \times U(1)$ model discussed in Ref. [15]. The electroweak gauge group is enlarged to $SU(2)_1 \times SU(2)_2 \times U(1)_Y$. The $U(1)_Y$ coincides with the SM $U(1)_Y$ group. The SM $SU(2)_L$ group is the diagonal subgroup of $SU(2)_1 \times SU(2)_2$. The authors of Ref. [15] mainly focus on the instanton effects associated with the larger gauge group, but electroweak constraints on the model are also given. Only the data of Z-pole measurements and the W boson mass are used in their analysis. In this subsection, we show that it is straightforward to obtain constraints from a much larger set of observables using our approach.

As in the previous model, we first obtain the effective operators. The third generation of fermion doublets are assigned different quantum numbers from the light two generations. In our notation, the quantum numbers under $SU(2)_1 \times SU(2)_2 \times U(1)_Y$ are

$$\begin{aligned} Q: (2, 1)_{1/6}, & \quad L: (2, 1)_{-1/2}, \\ q: (1, 2)_{1/6}, & \quad l: (1, 2)_{-1/2}. \end{aligned} \quad (18)$$

The $SU(2)_L$ singlet fields are singlets under both $SU(2)_1$ and $SU(2)_2$, with the $U(1)_Y$ charges the same as the SM hypercharges.

We have two choices for the Higgs doublet quantum numbers. In Ref. [15], they are called the “heavy” case $h = (2, 1)_{1/2}$ and the “light” case $h = (1, 2)_{1/2}$.

The $SU(2)_1 \times SU(2)_2$ gauge group is broken to $SU(2)_L$ by the vacuum expectation value of a bidoublet scalar $\langle \Sigma \rangle = \text{diag}\{u, u\}$. The gauge coupling g of the unbroken $SU(2)$ is related to the gauge couplings of $SU(2)_1$ and $SU(2)_2$ as

$$g = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}. \quad (19)$$

For convenience, we define

$$c = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \quad s = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}. \quad (20)$$

The symmetry breaking yields three heavy gauge bosons Z', W'^{\pm} with masses

$$M_{Z'}^2 = M_{W'^{\pm}}^2 = (g_1^2 + g_2^2)u^2. \quad (21)$$

We integrate out these gauge bosons in both the light and heavy cases and obtain the following operator coefficients

Light case:

$$\begin{aligned} a_{ll}^t &= a_{lq}^t = a_{hl}^t = a_{hq}^t = -\frac{1}{4u^2}s^4, \\ a_{lL}^t &= a_{lQ}^t = a_{hL}^t = a_{hQ}^t = \frac{1}{4u^2}s^2c^2. \end{aligned} \quad (22)$$

Heavy case:

$$\begin{aligned} a_{ll}^t &= a_{lq}^t = -\frac{1}{4u^2}s^4, \\ a_{lL}^t &= a_{lQ}^t = a_{hl}^t = a_{hq}^t = \frac{1}{4u^2}s^2c^2, \\ a_{hL}^t &= a_{hQ}^t = -\frac{1}{4u^2}c^4. \end{aligned} \quad (23)$$

The authors of Ref. [15] have calculated the corrections to the Z-pole observables and the W boson mass in terms of the parameters u and s . Given the simple form of operator coefficients in Eqs. (22) and (23), it is straightforward to reproduce their results from our general calculations with arbitrary operator coefficients. In fact, it does not take more effort to substitute the coefficients in Eq. (9) to obtain more

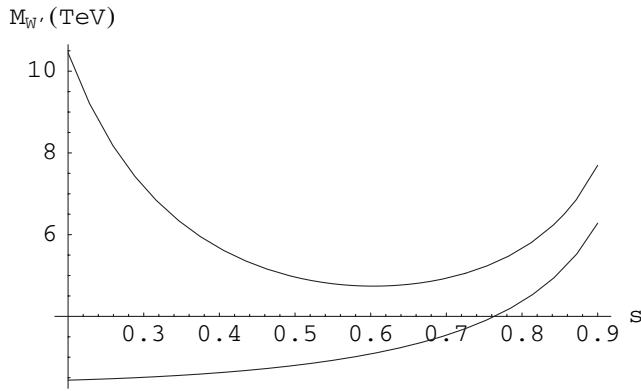


FIG. 2. Lower bounds at 95% CL on $M_{W'}$ as a function of s in the $SU(2) \times SU(2) \times U(1)$ model. The upper curve corresponds to the heavy case and the lower curve corresponds to the light case.

comprehensive constraints. We translate the bounds on u to the bounds on the physical mass $M_{W'}$ ($M_{Z'}$) using the relation (21). The 95% CL bounds using all data in our analysis are shown in Fig. 2, corresponding to $\Delta\chi^2 = 3.84$. If we restrict the data to those used in Ref. [15], we obtain curves of similar shapes for the bounds. But the bounds are generally lower, with the largest difference about 1 TeV for the heavy case and 2 TeV for the light case. Note that in Fig. 2, the bounds for the heavy case are quite stringent and always tighter than those for the light case. But in both cases, the heavy gauge bosons will have chances to be observed at the LHC.

IV. SUMMARY AND DISCUSSION

We have analyzed constraints from EWPTs on flavor-dependent extensions of the SM at TeV scale. Our results are model-independent in the sense that the constraints are given in terms of the bounds on arbitrary linear combinations of dimension-6 effective operators. The analysis is an extension to the previous work presented in Ref. [6]. In Ref. [6], $U(3)^5$ flavor symmetry is assumed for the matter fields, leaving no room for flavor-dependent effects. In this article, we have relaxed the flavor symmetry to $[U(2) \times U(1)]^5$, with the $U(1)$'s corresponding to the third generation. Constraints from FCNCs on the first two generations are much more stringent than the third generation, indicating that flavor-dependent physics for the first two gener-

ations have to arise at a much higher scale than 1 TeV. Therefore we still assume $U(2)$ symmetry for the first two generations.

There are 16 more operators that are relevant to EWPTs than in the previous analysis. There exists a potential problem with so many operators, that is, some operator combinations (which we call flat directions) can not be constrained by current experimental data because their net corrections to all EWPOs vanish. This is true even after we have added a few measurements that were omitted previously due to their low precision. However, in practice, the flat directions are almost never a problem, since there are usually much fewer parameters than effective operators in a given model. On the other hand, one might utilize the flat directions to avoid strong electroweak constraints when building a model. Since the flat directions given in Eqs. (10) and (11) all involve the third generation, it will be easier to avoid the constraints if the new physics couples exclusively to the third generation.

We have calculated the χ^2 distribution in terms of the operator coefficients. To constrain a given model, the only thing one still needs to do is integrating out the heavy degrees of freedom to obtain the operator coefficients in term of the model parameters. We have applied this procedure to two flavor-dependent models. The first one is the simplest little Higgs, model 2 [11]. We found that the bounds on the heavy W' gauge boson mass are even lower than that in model 1, indicating this model is a good solution to the Higgs mass fine-tuning problem. The second model we have considered is a model with the SM gauge group enlarged to $SU(2) \times SU(2) \times U(1)$, discussed in Ref. [15]. The authors of Ref. [15] obtained the electroweak constraints for this model from a subset of all available EWPOs. We have shown that it is straightforward to obtain more comprehensive constraints utilizing our result. Generally speaking, our result can be used to efficiently constrain any TeV scale flavor-dependent physics with $U(2) \times U(1)$ flavor symmetry.

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