

What do lattice baryonic susceptibilities tell us about quarks, diquarks, and baryons at $T > T_c$?

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Lattice data on QCD thermodynamics, especially recent study of high order susceptibilities by UK-Bielefeld Collaboration, have provided valuable information about matter properties around and above the critical temperature T_c . In this work we tried to understand what physical picture would explain these numerical data. We found two scenarios which will do it: (i) a quark quasiparticle gas, with the effective mass which is strongly *decreasing* near the phase boundary into the quark-gluon plasma (QGP) phase; or (ii) a picture including baryons at $T > T_c$, with the mass rapidly *increasing* across the phase boundary toward QGP. We further provide several arguments in favor of the latter scenario, one of which is a natural continuity with the baryon gas picture at $T < T_c$.

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I. INTRODUCTION

QGP is experimentally studied via heavy ion collisions, at CERN SPS and last years at Brookhaven National Laboratory and Relativistic Heavy Ion Collision (RHIC) collider, at temperatures reaching up to about $T \approx 2T_c$. Success of hydrodynamical description [1] of observed collective flows have indicated that all dissipative lengths are very short and thus the produced matter cannot be a weakly coupled gas but rather a near-perfect (small viscosity) liquid [2]. These features are further complemented by very high jet losses and robust heavy quark charm (equilibration) observed well beyond what pQCD predicted. As a result, a radically new picture of QGP at such temperatures is being developed, known as the *strongly coupled* quark-gluon plasma, or sQGP.

It has been pointed out by Shuryak and Zahed [3] that the interaction seems to be strong enough to preserve the mesonlike bound states above T_c although in a strongly modified form. In particular, the lowest charmonium states J/ψ , η_c are predicted to exist up to T as high as about $(2.5-3)T_c$. These charmonium states were observed on the lattice [4] as peaks in spectral densities of the correlation functions, and they indeed seem to survive till such high temperatures.

It was further pointed out in the next paper by Shuryak and Zahed [5] that in the deconfined phase also multiple binary *colored* bound states should exist, in about the same T domain, since the interaction is about the same. To put the discussion below into proper perspective, they argued that there should be three categories of bound states, in decreasing robustness: (i) glueballs, (ii) $(qg)_3$ and mesons $\bar{q}q$; and (iii) $(qg)_6$, diquarks and baryons. If the strength of the effective potential in $\bar{q}q$ states is counted as 1, the relative color Casimirs for categories (i), (ii), and (iii) are $9/4$, $9/8 \sim 1$, and $\approx 1/2$, respectively. In our recent work [6] we have extended the same approach to some many-body states. We found new 3-gluon configuration ggg belonging to category (i), the polymeric chains $\bar{q}.g.g\dots gq$

of the category (ii), and diquarks and baryons in category (iii).

The last two are the baryon number carrying states we will discuss in this work. Since these states belong to the *third*, most weakly bound category, they are naturally most vulnerable to uncertainties of the potential and their existence can be questioned. Besides, these states are relatively heavy: such states have not been included in [5] in pressure.

The reason we will discuss them now is because they are more important at increasing baryonic chemical potential μ . An alternative way to look at the same thing is to consider higher derivatives over μ at $\mu = 0$: this way the role of such states is enhanced due to powers of their baryon number. At some point the diquarks and baryons should become noticeable in these quantities even if their role in pressure is small—and this is precisely what we think happened in the lattice data of the UK-Bielefeld Collaboration (UKB) [7], especially in susceptibilities with four and six derivatives.

In this work we will concentrate on the so-called baryonic susceptibilities part of the free energy, which can be singled out via derivatives over quark chemical potentials $\mu_q = (\mu_u + \mu_d)/2$ and $\mu_l = (\mu_u - \mu_d)/2$ calculated recently by the UKB. They use it in a context of Taylor expansion of the thermodynamical quantities in powers of baryonic chemical potential μ/T^1 up to the order $O(\mu^6)$ of 2-flavor QCD, but we will not discuss this expansion *per se* and concentrate on (T -dependent) susceptibilities of the kind²:

¹We follow notations used in this work where μ is the chemical potential per quark, not per baryon. Thus the associated charge is $B = 1$ for a quark, $B = 2$ for a diquark, and $B = 3$ for a baryon.

²Since we would not discuss any Taylor series in this work, we would prefer to leave out the factorials and thus discuss susceptibilities d_n defined without them, not c_n .

$$d_n(T) = \left. \frac{\partial^n (p/T^4)}{\partial (\mu/T)^n} \right|_{\mu=0} = n! c_n(T) \quad (1)$$

for $n = 2, 4, 6$. (The odd ones vanish at $\mu = 0$ by symmetry.) These data are shown in Fig. 1 and also below. The UKB also studied what they called isospin susceptibilities defined as

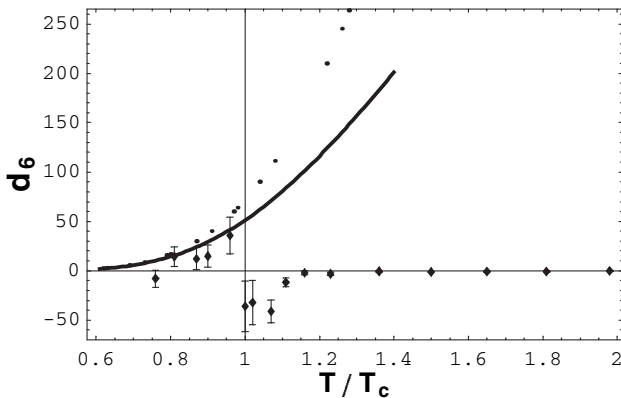
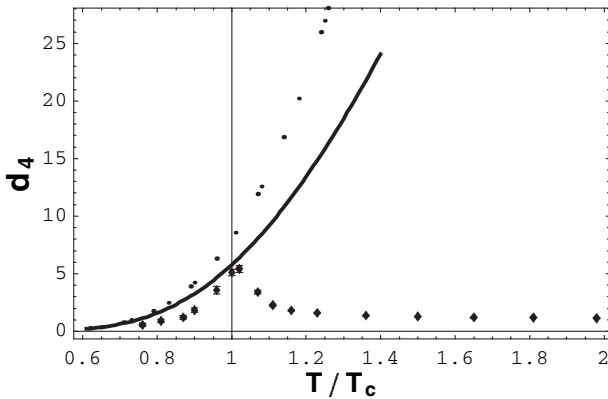
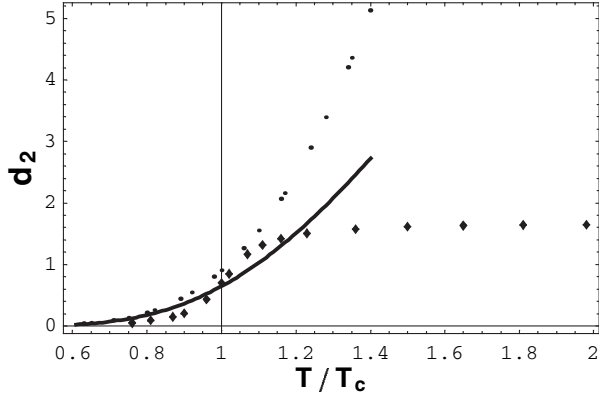


FIG. 1. The dotted lines correspond to a gas of baryonic resonances with both ground state and excited baryons, the solid lines correspond to that with only lowest (s -wave) nucleons and Δ , while the points with error bars are the susceptibilities $d_2(T)$, $d_4(T)$, $d_6(T)$ from lattice data (after removal of factorials in $c_{2,4,6}$).

$$d_n^I(T) = \left. \frac{\partial^n (p/T^4)}{\partial (\mu/T)^{n-2} \partial (\mu_I/T)^2} \right|_{\mu=\mu_I=0} = n! c_n^I(T), \quad (2)$$

and in a flavor-diagonal–nondiagonal language there are

$$d_n^{uu} = (d_n + d_n^I)/4, \quad d_n^{ud} = (d_n - d_n^I)/4. \quad (3)$$

Let us also mention another recent independent lattice studies on susceptibilities of 2-flavor QCD in [8], where they defined so-called nonlinear susceptibilities

$$\chi_{n_u, n_d} = \frac{\partial^{(n_u+n_d)} p}{\partial \mu_u^{n_u} \partial \mu_d^{n_d}} \quad (4)$$

and evaluated their values on lattice. To see the connection between these two approaches, we give the following relations

$$d_n^{uu} = T^{n-4} \sum_{l=0}^{n-2} C_{n-2}^l [\chi_{l+2, n-l-2} + \chi_{l, n-l}], \quad (5)$$

and

$$d_n^{ud} = T^{n-4} \sum_{l=0}^{n-2} C_{n-2}^l \chi_{l+1, n-l-1}. \quad (6)$$

While the two approaches are very closely related, their numerical results, however, are not quantitatively comparable, partly because they have used very different mass setup in the lattice calculation. We nevertheless emphasize that in a qualitative view both of them have found very similar and interesting patterns in those susceptibilities, especially for the fourth and the sixth, which are the central issues to be addressed in this paper.

To set the stage, we start with the hadronic phase below T_c . Here the relevant states are only the baryons with the baryon number (per quark) 3. Their spectrum is known at $T = 0$ experimentally, and thus an obvious question is can a simple resonance gas of known baryons explain the behavior of these susceptibilities below T_c ? Indeed it is the case, as shown by the dotted curves in Fig. 1 [obtained by including contributions of nucleon states from $N(940)$ to $N(1675)$ and Δ states from $\Delta(1232)$ to $\Delta(1700)$, for two-flavor theory one should not include strange baryons]. No T or μ dependence of these masses is assumed, nor do we take into account the fact that lattice is dealing with non-massive quarks³: tuning these will make the agreement even better. So the susceptibilities in the hadronic phase, $T < T_c$, can be well described by the usual resonance gas of baryons. As shown in Fig. 1, the contribution of all baryons starts overshooting the lattice data close to T_c . It probably indicates that excited baryons other than the s -wave states N , Δ may melt a bit below T_c .⁴ We will not include excited baryons in any discussion to follow.

³In fact the input quark mass in these calculations is $0.4T$.

⁴Note that the calculations like we have done in [6] (valid only above T_c , and not too close to it) do not support any states other than the s -wave ones, even for mesons.

The main issue to be discussed in this work is what these lattice data actually tell us about the nature of baryonic states *above* T_c and whether one can describe them with sQGP model or as well with some other model.

Before we proceed to the argumentation in literature, let us remind the reader of standard thermodynamical expressions for massive fermions, which can be put in the following well-known form:

$$\frac{p}{T^4} = N \frac{M^2}{2\pi^2 T^2} \sum_{l=1}^{\infty} \left[\frac{(-)^{l+1}}{l^2} (e^{lB\mu/T} + e^{-lB\mu/T}) K_2(lM/T) \right], \quad (7)$$

where B , M is the baryon number of the corresponding particle and its mass, N is the statistical weight, and K_2 is the Bessel function.⁵ This form is very convenient for taking derivatives over μ , for example, the first derivative, the baryon density is

$$\begin{aligned} \frac{n_B}{T^3} &= NB \frac{M^2}{2\pi^2 T^2} \sum_{l=1}^{\infty} \left[\frac{(-)^{l+1}}{l} (e^{lB\mu/T} - e^{-lB\mu/T}) K_2(lM/T) \right] \\ &= NB \mathcal{N}[B\mu/T, M/T], \end{aligned} \quad (8)$$

where the function $\mathcal{N}[x, y]$ is defined by these series. Note at this point we do not really consider mass as depending on μ so no extra derivatives against M appear.

In a number of talks Karsch (and also a recent preprint [9]) have presented what we would refer to as a “naïve” argument: the subsequent ratios

$$d_{n+2}/d_n \approx \langle B^2 \rangle \quad (9)$$

are directly related to the squared baryon number of the constituents. The argument goes as follows: (a) for massive particles with $M \gg T$ one can use the so-called Boltzmann approximation, keeping only the first term in the sum above; (b) after that the μ dependence factorizes, and thus each two derivatives over μ restore the same expression, modulo the factor B^2 . In the matter dominated by quark quasiparticles, or qg bound states, the right-hand side would be 1, but it would instead be 4 or 9 for matter dominated by diquarks or baryons, respectively. The measured ratio d_4/d_2 is ~ 10 at $T < T_c$ but at $T > T_c$ it rapidly drops and becomes close to 1. Comparing it to the formula above Karsch concluded that at $T > T_c$ matter is a gas of some $B = 1$ objects, while the contribution of the $B = 2$ diquarks is strongly restricted.

But if one looks closer at this argument, one finds it missing a lot of effects that should be there as well. For

⁵If there are more than one species of particle we then sum over different species. Yet there will be particular concern when dealing with quasiparticles instead of particles where some background term may arise in the pressure, as will be discussed in a later section.

example, the next similar ratio d_6/d_4 above T_c is nowhere close to 1 but is in fact a large negative number ~ -10 which cannot be interpreted as a B^2 of anything.

Furthermore, the idea that one can keep only the main term in the sum so that the μ and T dependence can factorize must be wrong by itself. The T dependence of $d_2(T)$, $d_4(T)$, and $d_6(T)$ is not at all similar: while $d_2(T)$ resembles the behavior of the pressure itself and can easily be interpreted as a transition from hadron to quark gas, the next one $d_4(T)$ has a sharp maximum near T_c , with even more complicated “wiggle” in the $d_6(T)$.

Another perspective on that issue can be made if one converts baryon number and isospin susceptibilities into flavor-diagonal (uu or dd) and flavor nondiagonal ud susceptibilities. The lattice data show that the second flavor-mixing derivatives are small⁶ $d_2^{ud}/d_2^{uu} \ll 1$, but similar ratios for higher derivatives $n = 4, 6$ are not small $d_n^{ud}/d_n^{uu} \sim 1/2$.

Does it imply that the quark gas model is also inadequate and should be excluded as well as the “bound state” gas? Or, if the argument is wrong, what exactly is missing?

- (i) Even if the Boltzmann approximation [keeping the first term in sum in (7)] may be good for pressure, it still fails for higher susceptibilities because the l th term has $l\mu$ in the exponent, and with subsequent differentiation their role grows as l^n . By the time one comes to the sixth derivative, these terms start canceling each other. In physics terms, this is a form of Fermi blocking effect not included in the simple Boltzmann approximation.
- (ii) The second item to recognize is the fact that quasiparticles are not particles and their effective masses depend on matter parameters, such as T and especially μ . Subsequent differentiation of this effective mass over μ would add powers of derivatives like

$$M'' = \frac{\partial^2 M}{\partial \mu^2}(T, \mu = 0) \quad (10)$$

to susceptibilities and to their ratios such as (9). Provided those are large enough, they may completely invalidate the naïve interpretation of those ratios as baryon number squared. This was already pointed out by Bluhm, Kempfer, and Soff [11], and we will refer to it below as the “BKS effect.” The same is true for bound states such as baryons, and similar derivatives of their masses $M_B''(T)$ would play an important role below.

- (iii) The contribution of diquarks has been grossly overestimated, while the contribution of baryons was not discussed at all. We will show below that it may naturally explain the features seen in higher derivatives.

⁶This is also the main point of the paper [10].

The outline of the paper is as follows: In Sec. II we will start with an “unconstrained” quark gas model and will use the lattice data to extract the quasiparticle mass together with its dependence on matter, $M(T, \mu)$. We would not need to rely on perturbative arguments used by BKS [11] (since even their own fit leads to rather strong coupling at $T \approx T_c$). Furthermore, we will conjecture a possible relation between the T and μ dependences due to known shape of the phase boundary on the phase diagram. In Sec. III we will further impose a number of constraints on quark mass, from other lattice data and also from confinement, a condition that there should not be any colored degrees of freedom at $T < T_c$. We will conclude that these constraints basically make it impossible to ascribe the observed features of the data to the BKS effect. After that we will proceed to Sec. IV in which we will discuss the contribution of diquark and baryons: here we will find good fits to the data satisfying all the needed constraints and nicely joining the baryon gas picture below T_c .

II. MODEL I: A QUARK GAS WITH AN UNCONSTRAINED MASS $M(T, \mu)$

The idea to use thermodynamical quantities calculated on the lattice to fit the mass parameters of quasiparticles is by itself quite old. For example, Levai and Heinz [12] have used the data on $p(T)$ for determination of quark and gluon effective masses $M(T)$.⁷

One well-known problem with quasiparticle gas models is that the derivatives over T and μ upset thermodynamical consistency between gaslike expressions for different thermodynamical quantities. Only one of them can be assumed to have a simple additive form over quasiparticles—then there is no freedom left and all other quantities can be calculated from it by thermodynamics. Thus only one “primary” expression can be additive, while others will have extra “derivative” terms complementing simple gas formulas.

Following conventions of the BKS paper, we will use as such primary expression that for the baryon number density (8). The expressions for pressure and energy density would then be corrected by some T, μ -dependent “bag terms.” Higher derivative terms d_n will be calculated by differentiating (8) $n - 1$ times. To be more specific, we explicitly give the baryon number density for this quark gas model

$$\begin{aligned} \frac{n_B}{T^3} &= \frac{\partial(p/T^4)}{\partial(\mu/T)} \\ &= \frac{g}{2\pi^2} \int dx x^2 n [F(\epsilon - n\tilde{\mu}) - F(\epsilon + n\tilde{\mu})]. \end{aligned} \quad (11)$$

⁷It was not as direct as our approach below, because one cannot get two functions out of one without assumptions.

Here $g = N_s * N_c * N_f = 12$ is the degeneracy factor for quarks in the two-flavor case, n is the baryon quantum number of quark which is defined here to be $n = 1$ by setting μ to be the quark chemical potential. $\tilde{\mu} = \mu/T$ is made to be dimensionless, and $\epsilon = \sqrt{x^2 + \tilde{m}}$ with $\tilde{m} = M/T$. And finally we have introduced Fermi distribution function $F(y) = \frac{1}{e^y + 1}$. Starting from (11) the explicit formulas for d_2, d_4, d_6 are given to be

$$d_2 = \left. \frac{\partial(n_B/T^3)}{\partial\tilde{\mu}} \right|_{\mu=0} = -\frac{2g}{2\pi^2} \int dx x^2 n^2 F^{(1)}(\epsilon_0), \quad (12)$$

$$\begin{aligned} d_4 &= \left. \frac{\partial^3(n_B/T^3)}{\partial\tilde{\mu}^3} \right|_{\mu=0} \\ &= -\frac{2g}{2\pi^2} \int dx x^2 \left[n^4 F^{(3)}(\epsilon_0) + 3n^2 F^{(2)}(\epsilon_0) \frac{\tilde{m}_0}{\epsilon_0} \right. \\ &\quad \left. \times \left(\frac{\partial^2\tilde{m}}{\partial\tilde{\mu}^2} \right) \right]_{\mu=0}, \end{aligned} \quad (13)$$

$$\begin{aligned} d_6 &= \left. \frac{\partial^5(n_B/T^3)}{\partial\tilde{\mu}^5} \right|_{\mu=0} \\ &= -\frac{2g}{2\pi^2} \int dx x^2 \left[n^6 F^{(5)}(\epsilon_0) + 10n^4 F^{(4)}(\epsilon_0) \frac{\tilde{m}_0}{\epsilon_0} \right. \\ &\quad \times \left(\frac{\partial^2\tilde{m}}{\partial\tilde{\mu}^2} \right)_{\mu=0} + 15n^2 F^{(3)}(\epsilon_0) \frac{\tilde{m}_0^2}{\epsilon_0^2} \left(\frac{\partial^2\tilde{m}}{\partial\tilde{\mu}^2} \right)_{\mu=0}^2 \\ &\quad + 5n^2 F^{(2)}(\epsilon_0) \left(\frac{\tilde{m}_0}{\epsilon_0} \left(\frac{\partial^4\tilde{m}}{\partial\tilde{\mu}^4} \right)_{\mu=0} \right) \\ &\quad \left. + \frac{3x^2}{x^2 + \tilde{m}_0^2} \left(\frac{\partial^2\tilde{m}}{\partial\tilde{\mu}^2} \right)_{\mu=0}^2 \right]. \end{aligned} \quad (14)$$

In the above equations we have used $\epsilon_0 = \sqrt{x^2 + \tilde{m}_0^2}$ and $\tilde{m}_0(T) = M(T, \mu = 0)/T$, and also $F^{(i)}(y)$ means the i th derivative of the function $F(y)$.

The model used in the BKS paper assumes some hard thermal loop based perturbative form for the T, μ -dependent mass with the coupling $g^2(T, \mu)$ running in a complicated fashion fitted to reproduce the susceptibilities we discuss in this work. However, we do not see why any assumptions about the mass dependence are actually needed⁸ at this point.

We thus suggest a generalization of what was done in [11]. Assuming a simple ideal gas model of quark quasiparticles, one has their mass to be the only input needed. With the lattice data on $d_2(T)$, $d_4(T)$, and $d_6(T)$ used as input, one can simply solve for the three functions of T which would ideally fit them. We have chosen those to be:

⁸There is of course no reason to trust any perturbative formula near T_c at all, where the coupling becomes as strong as it was found by BKS themselves.

(i) the quark mass $M(T, \mu = 0)$ and its two lowest non-zero⁹ derivatives over μ ; (ii) $M'' = \frac{\partial^2 M}{\partial \mu^2}(T, \mu = 0)$, and (iii) $M'''' = \frac{\partial^4 M}{\partial \mu^4}(T, \mu = 0)$. With these at hand, of course, we are able to develop the Taylor's expansion for quark mass as a function of $|\frac{\mu}{T}| < 1$:

$$\begin{aligned} M\left(T, \frac{\mu}{T}\right) &= M(T, \mu = 0) + \frac{1}{2!} \frac{\partial^2 M}{\partial \mu^2}(T, \mu \\ &= 0) \cdot \left(\frac{\mu}{T}\right)^2 + \frac{1}{4!} \frac{\partial^4 M}{\partial \mu^4}(T, \mu = 0) \left(\frac{\mu}{T}\right)^4. \end{aligned} \quad (15)$$

The procedure is iterative. First we used $c_2(T)$ data to solve for the mass \tilde{m}_0 as unknown. Then we go to c_4 , the equation of which includes both \tilde{m}_0 and $\frac{\partial^2 \tilde{m}}{\partial \mu^2}|_{\mu=0}$, but since we have already solved \tilde{m}_0 from c_2 now the only unknown term is $\frac{\partial^2 \tilde{m}}{\partial \mu^2}|_{\mu=0}$, which could be solved out from lattice results of c_4 . Finally, we can obtain $\frac{\partial^4 \tilde{m}}{\partial \mu^4}|_{\mu=0}$ from c_6 with \tilde{m}_0 and $\frac{\partial^2 \tilde{m}}{\partial \mu^2}|_{\mu=0}$ already being solved from c_2 and c_4 . The results for these three functions are shown in Fig. 2. (The error bars in \tilde{m}_0 are determined from uncertainty in c_2 . While for $\frac{\partial^2 \tilde{m}}{\partial \mu^2}|_{\mu=0}$ the errors should come from both c_4 and \tilde{m}_0 , the error bars in the figure only include those from c_4 , and also for $\frac{\partial^4 \tilde{m}}{\partial \mu^4}|_{\mu=0}$ the error bars solely include that originated from c_6 .)

As an independent check, we also have extracted the same three quantities, $M(T, \mu = 0)$, $\frac{\partial^2 M}{\partial \mu^2}(T, \mu = 0)$, and $\frac{\partial^4 M}{\partial \mu^4}(T, \mu = 0)$ from the lattice data set for $c_2^I(T)$, $c_4^I(T)$, and $c_6^I(T)$ from [7] by the same strategy (but starting with isospin densities).

The results are shown in Fig. 2. As can be seen, two sets of parameters we extracted from both data sets are well consistent with each other at $T > T_c$, while for $T < T_c$ they do not agree. It is a good feature, as the quark gas model is not supposed to work there, in the domain of the baryon resonance gas.

Let us summarize these results. The most important lessons are (i) the mass $M(T)$ strongly increases when cooling down toward the critical point T_c ; (ii) large and negative $\frac{\partial^2 M}{\partial \mu^2}(T, \mu = 0)$ close to T_c ; (iii) the fourth derivative is positive: so this decrease of the mass due to the second derivative will stop at about $\mu/T \sim 1$; see (15).

The first two points are the trends already emphasized by BKS [11]. In their approach these two features are related with each other because of the assumed perturbative origin of the effective quark mass:

$$M = g^2(T, \mu) T^2 \left(1 + N_f/6 + \frac{1}{2\pi^2 T^2} \sum_f \mu_f^2 \right), \quad (16)$$

⁹The quasiparticle masses and other quantities obviously can depend only quadratically on μ because of $\mu \rightarrow -\mu$ symmetry based on $\bar{C}P$ invariance.

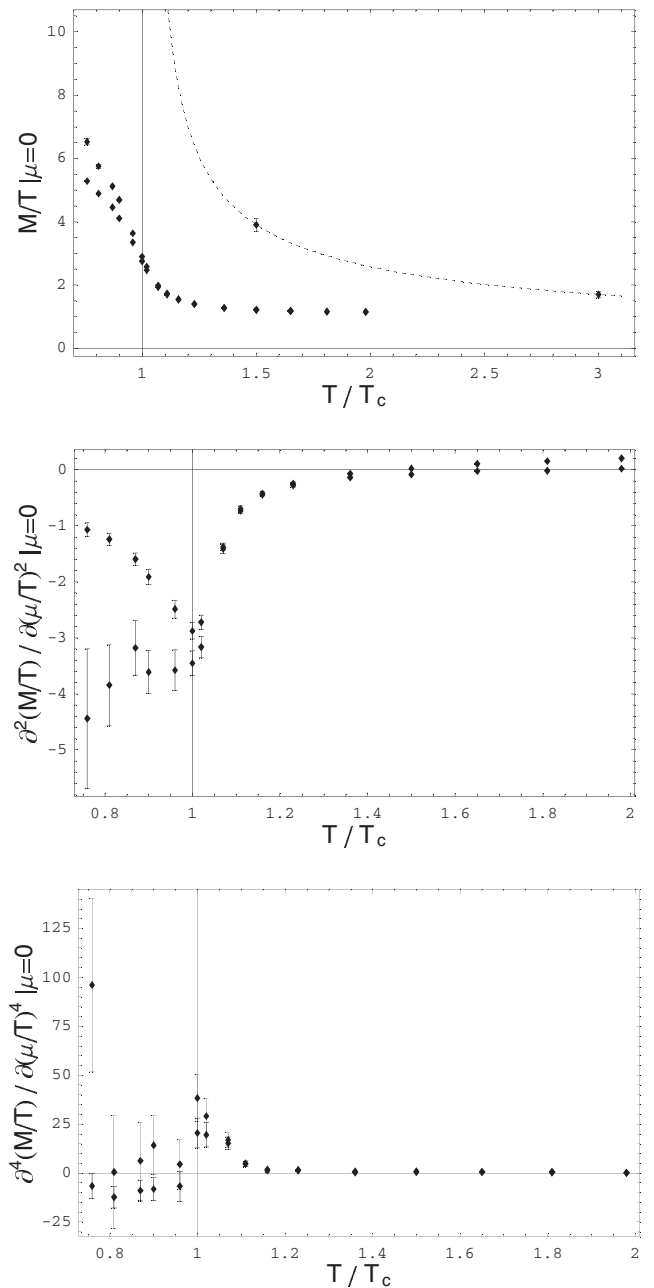


FIG. 2. Quark quasiparticle mass and its second and fourth derivatives over μ as a function of temperature T , extracted from lattice data for susceptibilities. There are two sets of points in each figure that are obtained from c_2, c_4, c_6 and from c_2^I, c_4^I, c_6^I , respectively. In the top figure for quark mass, we also plotted the two points with error bars measured by lattice via propagator, and the mass given by (20) as well (the dashed line).

where the sum runs over all flavors f . Ignoring for a moment a (rather complicated) running of the coupling, the BKS mass is thus constant at the particular ellipsoids in the $T - \mu$ plane, thus the derivatives over T and μ are related.

We would like to propose another reasoning that leads to a similar effect but is free from perturbative assumptions. Its idea can be described as follows: the quark mass should

be getting large not only near the critical point $T \rightarrow T_c$, $\mu = 0$, but also near the *whole critical line* at all μ . It is needed to ensure that quark degrees of freedom do not contribute in the confined phase, at any μ .

The critical line at nonzero μ is schematically shown in Fig. 3; its shape at not-too-large μ/T can be described by an ellipsoid, or an unit circle, if the units are chosen appropriately. One may further think that the mass dependence on the radial coordinate R on such a plot is much more important than on the angular one ϕ since the “lines of constant mass” should be nearly parallel to the critical line, at least in its vicinity where the discussed effect takes place.

So, the proposed extension of the T dependence of the mass to its μ dependence is based on a substitution

$$M(T, \mu = 0) \rightarrow M(R(T, \mu)), \quad (17)$$

$$R^2 = \frac{T^2}{T_c^2} + \frac{\mu^2}{\mu_c^2}. \quad (18)$$

We have introduced here a new parameter μ_c : its value can be readily obtained from the experimental freeze-out curve measured in heavy ion collisions at small μ , believed to represent the critical line. If so, the value of this parameter is

$$\mu_c/T_c = 1.7, \quad (19)$$

which is quite different from the value given by “perturbative scaling” (16): $\sqrt{(1 + \frac{N_f}{6})\pi^2} \approx 3.63$ which is not

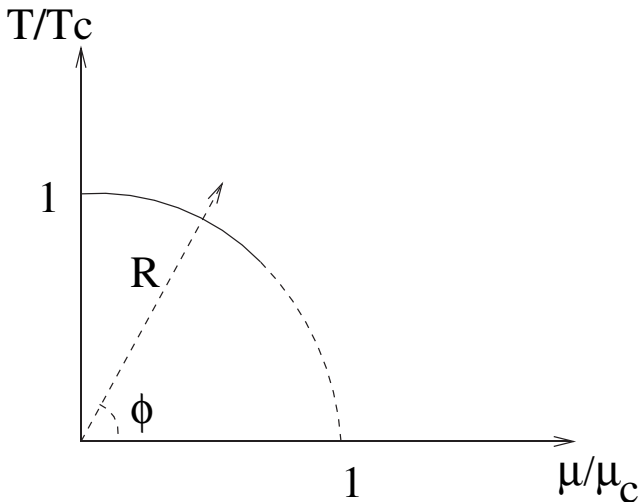


FIG. 3. In the plane of temperature T -baryonic chemical potential μ , both appropriately normalized, the phase boundary looks like a part of a circle. (At least for the part marked by the solid line, studied well at SPS and RHIC, with quite well established chemical freeze out. The dashed line is a continuation of the freeze-out line where its association with the critical line is questionable.) The polar coordinates to be used are the radial distance R and the angle ϕ .

supposed to work in the nonperturbative regime near T_c . It should be pointed out that though no criticality in the strict sense was observed in lattice data—no susceptibility gets divergent—there are, however, peaks (e.g. in d_4), the maximum of which can be taken as “practical definition” of the critical line. The semicircle shape (17) and (18) of it on the phase diagram has been widely used in other applications, and at not-too-large μ it is certainly a good parametrization.

III. MODEL II: THE CONSTRAINED QUARK GAS

The “unconstrained model I” discussed above, although consistent with both data sets $d_n(T)$, $d_n^l(T)$, is unfortunately unacceptable, for two main reasons: (i) it contradicts direct lattice measurements of the quasiparticle masses; (ii) it implies that quark degrees of freedom still significantly contribute in the confining phase at $T < T_c$. In this section we will show what happens if one tries to modify the unconstrained model to make it compatible with both.

One feature of the model I is the relatively light quark mass $M(T, \mu = 0)$ in region $1 - 2T_c$ ranging from about $1.7T_c$ to $2.2T_c$. Such mass conflicts with another lattice data about quark quasiparticle mass at $1.5T_c$ and $3T_c$, see [13], which are $m_q/T = 3.9 \pm 0.2$ and $m_q/T = 1.7 \pm 0.1$, respectively, and are shown in Fig. 2 by two crosses with the error bars. Although these results are based on only one paper and have not been systematically studied by other lattice groups so far, they nevertheless represent direct measurements from the quark propagators. Furthermore, such large masses correspond to the interparticle potentials at large distances measured in separate lattice study [14].

Although the mass extracted via the model I grows toward T_c , this effect is still not robust enough to make quark contribution near-zero (or negligible) at $T = T_c$. (In fact, BKS proceeded to fit equally well some region below T_c .) This is unacceptable, since we know that there are no propagating quark degrees of freedom in the confining phase.

Both these reasons force us to reconsider model I, basically by increasing the quark mass significantly to meet both constraints. This can be achieved by a quark mass formula similar to that used in [5]

$$M(T) = \frac{0.9}{T-1} + 3.45 + 0.4T \quad (20)$$

with all units in proper powers of T_c . [This and subsequent mass formulas would then be generalized to finite μ according to (17).] The coefficients are chosen so that the curve goes through the two lattice-measured points for quark mass at $T = 1.5, 3.0T_c$; see the dashed line in Fig. 2.

We show what happens then to the susceptibilities; see the medium-thickness solid lines in Fig. 7. In short, good description of $c_2(T)$ is definitely ruined. The issue is the same as for pressure in [5] and perhaps can be cured by qg and other bound states. But this is not the only problem of

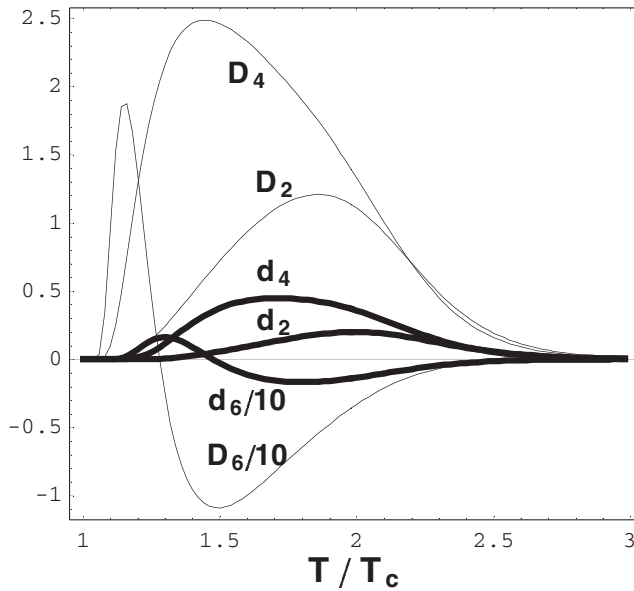


FIG. 4. Comparison of susceptibilities from quark-gluon states with two limiting case, zero binding ($d_{2,4,6}$) and “full-compensation” binding ($D_{2,4,6}$).

the constrained model: although it can produce a peak in $d_4(T)$ and a wiggle in $d_6(T)$, given large enough derivatives over μ , those get displaced toward larger T as compared to the data. It is an inevitable consequence of the second constraint, insisting that quark effect be effectively zero at T_c .¹⁰

Completing our discussion of purely quark models, we now proceed to the possible role of their bound states, diquarks and baryons.

IV. THE EFFECT OF DIQUARKS AND BARYONS

We will now proceed to contributions of the bound states to the baryonic susceptibilities. Let us remind the reader that the particular reason to focus on diquarks and especially baryons is that the role of diquarks and baryons relative to quarks grows with μ because of their larger baryon charges. Alternatively, their contribution to the susceptibility d_n grows exponentially with n : by the factors 2^n for the diquarks and 3^n for baryons. For example, the contribution of N, Δ is enhanced by a factor 81 for d_4 and 729 for d_6 relative to pressure; estimates of pressure given above one may then expect to see their contribution there. On the other hand, for lower derivative d_2 we expect quark-gluon bound states, which are more numerous and more tightly bound, to contribute significantly. We summarized all bound states, together with their multiplicities, in the Table I.

¹⁰The very heavy mass due to the constraints significantly decrease quark contribution to thermodynamics and hence disfavor quark-only model, yet on the other hand, it strongly favors the formation of bound states.

TABLE I. Summary of states with baryon number at $T > T_c$ studied in this paper.

State	Spin	Flavor	Color	Multiplicity
q	2	2	3	12
$(qg)_3$	4	2	3	24
$(qg)_6$	4	2	6	48
$(qq)_3^{J=I=0}$	1	1	3	3
$(qq)_3^{J=I=1}$	3	3	3	27
N	2	2	1	4
Δ	4	4	1	16

[In passing, let us comment about the numbers in the real world with strangeness, $N_f = 3$. The number of diquark flavor states is increased to be 3 times larger, for baryons the total spin-flavor multiplicity increases from $4 + 16 = 20$ to 56 (an octet $J = 1/2$ and a decuplet $J = 3/2$) which is roughly enhanced by 3 times, so the numbers both diquarks and baryons states are increased by the factor 3. The quark number increases as $3/2$, so the overall enhancement of the ratios we will discuss below from $N_f = 2$ to $N_f = 3$ is the factor 2.]

Quark-gluon bound states.—Before we proceed to actual calculation, let us make simple estimates of the relative weighing in pressure of quark-gluon bound states. The 2-body states qg are thermodynamically suppressed by an additional Boltzmann factor, $\exp(-M/T) \sim 0.02 - 0.04$ (by including their considerable binding). However, due to their relatively large multiplicity (6 times the number of the quark states) they contribute to the pressure and susceptibilities at the level of about $1/10$ or more.

To get a more quantitative answer one has to know the binding energy of these states. While the binding of the category three states $(qg)_6$ can be reasonably neglected, the category two $(qg)_3$ states have considerable binding at the same order as meson states. The potential model calculations in [6] lead to $(qg)_3$ binding up to $|\delta E|/T \approx 1.4$ at $T = T_c$, which means their contribution increases relative to the simple estimate above by an extra factor 2–3.

However there are many reason to doubt that close to T_c this calculation can be trusted quantitatively. In particular, the potential used is measured on the lattice for *static* charges only, and the corresponding calculations are supposed to be reliable only when the binding is small: near T_c more complicated dynamics beyond the potential model will contribute as well.

Let us thus just suggest an *upper limit* for the qg states’ contribution. Since the qg states are colored, they should get infinitely heavy at T_c , together with all other colored states. Furthermore, as (the more tightly bound) qg_3 states have the total charge of one quark, their mass should not be smaller than that of one charge $|\delta E| \leq M_q(T \approx T_c)$. So we expect $M(qg)$ to interpolate between $M_q + M_g \approx 2M_q$ at zero binding to a single M_q at $T \approx T_c$. The contribution of these states to susceptibilities in the two limiting cases,

namely the zero-binding case (labeled in the figure by d_i) and "full-compensation" binding case (labeled by D_i) are shown in Fig. 4. We conclude that large uncertainty, of the order of factor 3, remains in the contributions of such states.

[These results are calculated with $(qg)_6$ always having twice quark mass and melting at $1.4T_c$ while with $(qg)_3$ having twice quark mass in the former case and the same mass as quark in the latter, both melting at $2.1T_c$. The actual contribution of quark-gluon bound states should be somewhere in between, near to D_i around T_c while rapidly decreasing to d_i for higher temperatures.]

One may also ask what is the contribution of the various polymerlike qg states $qg, qgg, qggg, \dots$ which, according to [6], has the same binding energy per bond. The effect of these states can be easily evaluated via a geometric series: the resulting enhancement factor is

$$f_{\text{polymers}} = \frac{1}{1 - 6 \exp[(|\delta E| - M_g)/T]}, \quad (21)$$

where 6 is the color and spin degeneracy added by each link. For small binding this is just a few percent correction, but if it may get to be strong enough to drive the denominator toward zero, a total "polymerization" of sQGP would occur.

Diquarks.—For $N_f = 2$ gauge theory corresponding to the UKB data at hand there is only one attractive diquark channel, the antisymmetric color triplet¹¹ $(qq)_3$. Because of Fermi statistics, it means that the product of spin and flavor should be symmetric, and thus there are two options: (i) spin-0 isospin-0 ud diquark $(qq)_3^{J=I=0}$, and (ii) spin-1 isospin-1 one $(qq)_3^{J=I=1}$. These are the diquarks which are familiar in hadronic spectroscopy; the former appears inside the N , the latter inside Δ (octet and decuplet members, for 3 flavors). The lesson from this spectroscopy (at $T = 0$, of course) is that while the former is well bound, by about 300 MeV, the latter is not. In view of the rather marginal character of diquark binding, we expect only the former one able to be seriously considered as bound state above T_c . Nevertheless to confirm the point that diquarks will not play any role in all susceptibilities measured, we include both of them in calculation of Fig. 7. If we only use antisymmetric states, then the contribution will be reduced to only 1/10 of that.

The diquark-to-quark pressure ratio can be estimated as follows:

$$\frac{(qq)_3}{q} \approx \frac{3 + 27}{12} \exp\left(\frac{\mu - M + |\delta E|}{T}\right) 2^{3/2}, \quad (22)$$

where the binding $|\delta E|$ is negligible [actually only the 3 $(qq)_3^{J=I=0}$ states are very likely bound].¹² At small μ

¹¹In the talks [9] Karsch mentioned "hundreds of diquark bound states" in our model: it must be some misunderstanding.

¹²The last factor comes from $M^{3/2}$ in the preexponent, originated from momentum integral.

where the data under consideration are calculated, $M/T \approx 5$ and their contribution is at a few percent level, negligible compared to uncertainties.

Baryons.—As we found in [6] they are bound till about $T = 1.6T_c$. In the 2-flavor theory they are the N, Δ 3-quark states. Only the s -wave basic states survive above T_c , while all other resonances (used in the first section at $T < T_c$) which are orbital or radial excitations of N, Δ families are "melted."

The baryons are also numerous (20) but the suppression factor due to mass is much smaller

$$\frac{(qqq)}{q} \approx \frac{20}{12} \exp\left(\frac{2\mu - 2M + |\delta E|}{T}\right) 3^{3/2}. \quad (23)$$

Near the "end point" of baryons with zero binding (which according to [6] is at $T = 1.6T_c$) their mass is $3M_q$, expected to be in the range of 2.5–3 GeV. As it is an order of magnitude larger than T , one would not be expected to contribute to pressure etc.

However, unlike the quark, quark-gluon, and diquarks (which after all are colored objects existing only above T_c), N, Δ baryons are colorless and thus survive on both sides of the boundary of (a continuous) phase transition (a crossover, more accurately), thus the masses of baryons at $T \rightarrow T_c$ are expected to join continuously to their known values at lower T . This of course implies that the binding energy near T_c gets very large due to some deeper yet poorly known mechanism, and the potential model used in [6] to evaluate this binding will not be applicable. The situation is basically the same as with mesons: as emphasized in [5] the pion mass must (by definition of chiral breaking) vanish (in the chiral limit) at $T \rightarrow T_c$, which potential model also cannot reproduce. We will use below the following parameterization (in T_c units)

$$M_N = 9.5 + 4.6 * \tanh[3.8 * (T - 1.4)], \quad (24)$$

$$M_\Delta = 10.25 + 3.85 * \tanh[3.8 * (T - 1.4)] \quad (25)$$

interpolating between the nucleon and Δ vacuum masses at low T , while approaching the same value $3M_q$ at high temperatures. We nevertheless point out that the above equations are certainly not the unique choice. Instead we think by adopting more complicated formulas the data could be even better fitted. But that is not the purpose of this paper, and thus we use the simplest form highlighting the essential features needed for understanding those susceptibilities. We plot it in Fig. 5, together with the masses of various other states to be used later in Fig. 7 for susceptibilities. The main feature is fundamentally again enforcing confinement: when going from the QGP side toward T_c , all colored degrees of freedom get extremely heavy and drop out from the system, while all colorless degrees of freedom get more tightly bound and eventually dominate.

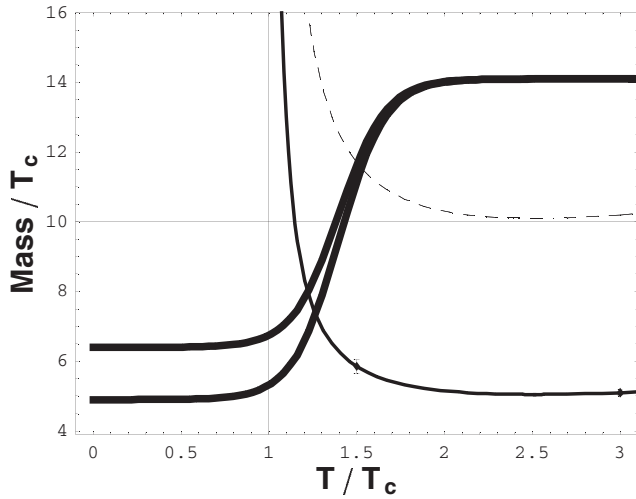


FIG. 5. Masses of various states studied in this work. The thin solid line is for quark and the dashed line is twice quark mass which is roughly for quark-gluon and diquark. The lower thick solid line is for nucleon states and the upper one for Δ states. These masses are used for calculation of Fig. 7.

We now approach the central point of the paper: the baryon contribution provides a natural interpretation of the structures observed in susceptibilities measured on the lattice, the large peak near T_c in $d_4(T)$ and a more complicated wiggle structure is seen in $d_6(T)$. This happens because the expected mass dependence of baryons on T , μ , shown in Fig. 5, should have a characteristic shape with an inflection point, separating the region in which the second derivative M_B'' is negative (above T_c) and positive (below T_c). That is why the contributions of the baryons to d_6 show a wiggle as seen from the corresponding curves in Fig. 7. Note also, that there is a less pronounced wiggle of the same origin in baryonic d_4 : we think its negative part is the reason why the qg and qq contributions above T_c can get compensated and by coincidence the d_4/d_2 ratio gets close to 1 there.

One additional argument for baryonic nature of the structures seen in d_4 , d_6 is the following one. Each derivative over μ_q leads to factor 3, so 2 of them give 9. If instead one has two derivatives over μ_l the factor obtained is $(2I_3)^2$, which is 1 for p, n, Δ^+, Δ^0 and 9 for Δ^{++}, Δ^- . As a result, if one ignores the mass difference between these states, one finds that baryonic contribution to both should have the ratio $d_n^l/d_n = (1/9) * (4/20) + (1 + 1/9)(8/20) = 0.467$, while this ratio should be 1 for ideal quark gas. The actual ratio of these quantities according to UKB data are shown in Fig. 6. We see near T_c the data obviously favor the existence of baryons, especially for d_6^l/d_6 , and the quark asymptotic end is arrived at about $1.4T_c$ for d_4^l/d_4 while only after $1.8T_c$ for d_6^l/d_6 . These evidences strengthen the necessity of baryonic interpretation of the higher susceptibilities.

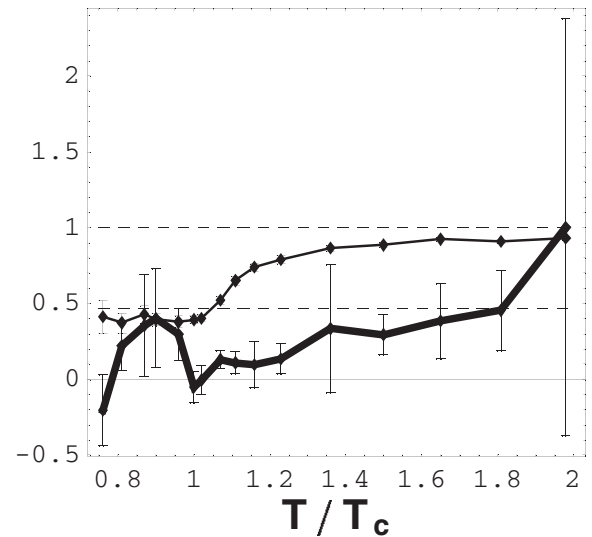


FIG. 6. The susceptibilities ratios d_4^l/d_4 (the thin solid line) and d_6^l/d_6 (the thick solid line). The dashed lines correspond to ideal quark gas (upper) and ideal baryonic gas (lower).

(Having considered baryons, one may ask if there are objects with even higher baryonic numbers, say dibaryons, thus contributing even more to high susceptibilities. Note however the baryons, as color singlets, have at most only the higher-order dipole-dipole attraction making them much less bound than qg or gg . They are not even bound at $T = 0$, apart of really weakly bound nuclearlike states. So we do not expect that any of those clusters based on baryons can withstand $T > T_c$.)

Taking everything together.—Including quarks, quark-gluons, diquarks, and baryons, we arrived at summary plots shown in Fig. 7. We repeat that all masses used are as shown in Fig. 5 and their μ dependence is introduced in the same way according to (17).

The bound states' end points are set to be $2.1T_c$ for $(qg)_3$ quark-gluons, $1.4T_c$ for $(qg)_6$, $1.4T_c$ for diquarks, and $1.6T_c$ for baryons, according to [6]. The gradual removal near melting point is done by similar means as in [5]. The results are shown in Fig. 7, where the overall values as well as the contributions of each kind of states are all present. It is noticed that there are few “spurious wiggles.” These undesired structures rise up as a consequence of the cancellation of two large contributions with opposite signs: the negative derivatives of baryons on μ and those positive of quarks. Consequently many spurious structures may appear, depending on the details of the parameterizations. Also the exact way to remove various bound states above their melting temperatures is partly responsible for undesired structures, see discussions in [5].

We focus on the $T > T_c$ side. The conclusions are (i) as expected the diquark contribution is negligible for all three quantities even after including the suspect $(qq)_3^{J=I=1}$ states, but it is clearly growing as getting to higher derivatives;

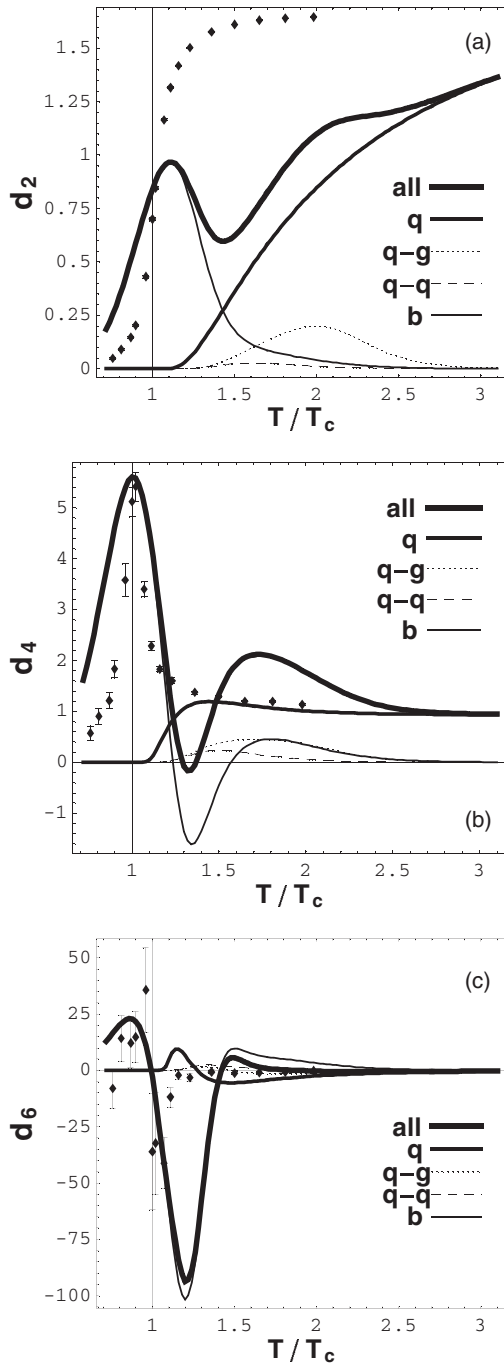


FIG. 7. The contributions of different states to (a) d_2 , (b) d_4 , and (c) d_6 , as well as the summed total values. The thickest solid lines are for taking all together, while the medium solid lines for quark, the thin solid lines for baryon, the dotted lines for quark-gluon, and the dashed lines for diquark, respectively.

(ii) for d_2 quark provides the main contribution, and we emphasize the fitting will be much better if the quark mass is finely tuned and particularly if we include the large binding of qg states near T_c . We have demonstrated above that large uncertainty in its binding, including polymers [see (21)] would very hopefully allow for a good fit, which

we decided not to do here. As has been emphasized many times, we really want to center on the role of baryons which are completely dominant in d_4 and d_6 . (iii) In d_4 it is precisely the baryons that produce the desired large peak near T_c till about $1.3T_c$ where quarks become important; (iv) the baryons' contribution extremely dominant the behavior of d_6 , especially the wiggle shape.

We conclude that two prominent structures, a peak in $d_4(T)$ and a wiggle in $d_6(T)$ are naturally reproduced by baryons.

V. SUMMARY

In one sentence, the main lessons from the UKB susceptibilities is that the baryons $N\Delta$ do survive the QCD phase transition, but are rapidly becoming quite heavy across it.

More generally, the discussed data set on the baryonic and isospin susceptibilities at $T > T_c$ can be described in two different scenarios. (i) The first is a quark quasiparticle gas, with the effective mass which is strongly *decreasing* near the phase boundary into the QGP phase; (ii) the second is a picture including baryons with the mass rapidly *increasing* across the phase boundary toward QGP, to about $3M_q$.

The first scenario was already pointed out by BKS [11], while our discussion makes it a bit more general. Its attractive features notwithstanding, it suggests the values of the mass not large enough to accommodate the existing constraints from other lattice measurements. We also think it is not possible to have quark degrees of freedom in a hadronic phase. Thus we conclude that success of such a scenario is unlikely.

The second scenario, based on baryons, can provide another explanation of the main features of the data, namely, the observed peak in $d_4(T)$ and a wiggle in $d_6(T)$. It also naturally explains the flavor-changing d_4^{ud} , d_6^{ud} , which are not small relative to flavor-diagonal ones. Last but not least, this scenario provides a desired continuity to the baryon resonance gas picture at $T < T_c$.

Although the susceptibilities $d_n(T)$ we used in this work are highly sensitive tools, they are quite indirect. Thermodynamical observable in general cannot tell the difference between “melting” baryons (getting unbound) and baryons remaining well bound but just getting too heavy: in both cases all one finds is that their contribution to thermodynamics effectively disappears. Besides, the ideal gas models used in these studies are probably too naïve to claim a really quantitative description of the data. One should instead study directly the spectral densities of the correlators of the appropriate baryonic currents (qqq) and see if there are baryonic peaks there, like what has been done for charmonium and light mesonic channels. Only such direct measurements would tell us which scenario is the correct one.

Speaking about experimental confirmation of the “bound state” scenario, we think the best chance could

be observation of the vector mesons. As described in detail in [15], vector mesons ρ , ω , ϕ are expected to become heavy near their disappearance point, like the baryons discussed above, reaching the mass $\approx 2M_q = 1.5\text{--}2$ GeV. The next generation of RHIC dilepton experiments have a chance to see if this is indeed what is happening in QGP.

ACKNOWLEDGMENTS

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