PHYSICAL REVIEW D **73,** 014026 (2006)

Status of the observed and predicted $b\bar{b}$ production at the Fermilab Tevatron

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(Received 29 September 2005; published 25 January 2006)

We review the experimental status of the b-quark production at the Fermilab Tevatron. We compare all available measurements to perturbative QCD predictions (NLO and FONLL) and also to the parton-level cross section evaluated with parton-shower Monte Carlo generators. We examine both the single b cross section and the so called $b\bar{b}$ correlations. The review shows that the experimental situation is quite complicated because the measurements appear to be inconsistent among themselves. In this situation, there is no solid basis to either claim that perturbative QCD is challenged by these measurements or, in contrast, that long-standing discrepancies between data and theory have been resolved by incrementally improving the measurements and the theoretical prediction.

DOI: 10.1103/PhysRevD.73.014026 PACS numbers: 13.85.Lg, 14.65.Fy

I. INTRODUCTION

The bottom quark production at the Fermilab Tevatron has been called one of the few instances in which experimental results appear to challenge the ability of perturbative QCD to accurately predict absolute rates in hadronic collisions. In general, the data are underestimated by the exact next-to-leading-order (NLO) QCD prediction [1,2]. The most recent measurement from the Tevatron [3] is however in very good agreement with an improved QCD calculation (FONLL [4]), and has prompted a number of studies [5–7] suggesting that the apparent discrepancy has been resolved with incremental improvements of the measurements and predictions.

Because of the experimental difficulty inherent to each result, in Sec. II we review all measurements of the single b cross section performed at the Tevatron, and then compare their average to the standard and to the improved QCD predictions. In Sec. III we review a number of measurements that compare the heavy-flavor production at the Tevatron to the prediction of parton-shower Monte Carlo generators [8,9]. Section IV compares cross sections for producing both b and $\bar{b}$ quarks—centrally and above a given transverse momentum cut—to theoretical predictions. Our conclusions are presented in Sec. V.

II. SINGLE B-QUARK PRODUCTION CROSS SECTION AT THE TEVATRON

The single b-quark cross section is inferred from the measurement of the production rate as a function of the transverse momentum, p_T, of: B hadrons; or some of their decay products (leptons or ψ mesons); or jets produced by the hadronization of b quarks. Most of the Tevatron measurements correspond to b quarks produced centrally (rapidity $|y^b| \leq 1$) and with $p_T \geq 6$ GeV/c (up to $p_T \simeq 100$ GeV/c). The measured cross sections are tabulated as a function of the transverse momentum of different objects such as the parent b quark, the B hadron, or a B-hadron decay prong (a lepton or a ψ meson). This makes the comparison of different measurements quite difficult, and usually only a few of them are presented together in review articles [6,7,10] and compared to the same theoretical prediction. Therefore, we start this review with a consistency check of all available data. For that purpose, we use the value of the single b-quark cross section extracted from the data and integrated from the p_T threshold of each experiment. We determine the ratio R of each measurement to the same theoretical prediction. We then evaluate the average R and its dispersion. As benchmark prediction of the b-quark parton-level cross section, we choose the exact NLO calculation [1] implemented with old but consistent sets[1] of parton distribution functions (PDF) that have been used in most published works[2]. There are 10 measurements of the single b cross section performed by the CDF and DØ collaborations at the Tevatron[3]. With the exception of one measurement, the b-quark cross section is extracted from the data using a fragmentation model based on the Peterson fragmentation function [16] with the ϵ parameter set to 0.006 according to fits to e^+e^- data [17]. With the exception of those cases in which the full B-hadron decay is reconstructed, the fragmentation model is convoluted through Monte Carlo calculations with the efficiency of the selection criteria used to identify b quarks. Different experiments have different sensitivity to the theoretical uncertainty of the fragmentation function. This systematic uncertainty has been evaluated by each experiment and it is part of the measurement error.

The extraction of the b-quark cross section from the data also requires the Monte Carlo simulation of B-hadron

[1]We use the $MRSD_0$ [11] or MRSA′ [12] fits.

[2]In the calculation all publications use a b-quark mass of $m_b = 4.75$ GeV/c^2 and renormalization and factorization scales $\mu_R = \mu_F = \sqrt{p_T^2 + m_b^2}$.

[3]We do not include the measurements in Refs. [13,14] because they are based on a handful of events. The measurement in Ref. [15] is not included because prompt ψ mesons are not separated from those produced by b-quark decays.

decays. In the Monte Carlo simulation, the distributions of the B-decay prongs are convoluted with the efficiency of the selection criteria used to identify b quarks. With one exception, all publications model B-hadron decays with the same version of the QQ Monte Carlo generator program [18]. More accurate experimental data are presently available for modeling these decays [19] and this might affect differently the result of different experiments. However, systematic uncertainties due to the decay model uncertainty[4] have been evaluated by each experiment and are included in the measurement error.

The b cross section derived from the production and decay of B hadrons also depends on the value of f_u, the fragmentation fraction of b quarks into B_u hadrons, and the branching fractions of the B decay available at the time of publication. The value of these parameters has changed appreciably over time; we use the same parameters, the value of which will be specified in the following, for all measurements and correct accordingly the published cross sections. We evaluate the ratio R to the standard theory for the few cases in which it is not provided in the publication. The measurements are based upon different b-quark signatures:

The measurement in Ref. [20] uses B mesons reconstructed through the decay $B \to J/\psi K$ with $J/\psi \to \mu^+\mu^-$. The ratio $R = 3.5 \pm 15\%$[5] for b quarks with $p_T^{\min} \geq 9$ GeV/c is derived using a fragmentation fraction $f_u = 0.375$ and a branching fraction of 5.88×10^{-5} [22].

Reference [23] is an earlier CDF measurement that uses the same decay mode and the same kinematic selection. Using the same fragmentation and branching fractions of the previous measurement, we derive $R = 2.9 \pm 23\%$[6].

Reference [24] presents a measurement based on the process $p\bar{p} \to \mu X$. The contribution of misidentified muons and of c quarks is evaluated using Monte Carlo simulations. Using a b-quark semileptonic branching fraction of 11.2%, the measurement yields $R = 2.5 \pm 26\%$ for b quarks with $p_T^{\min} \geq 21$ GeV/c; for $p_T^{\min} \geq 29$ GeV/c, the ratio is $R = 1.9 \pm 35\%$[7].

Reference [26] reports two complementary measurements that use the processes $p\bar{p} \to eD^0X$, with $D^0 \to K^+\pi^-$, and $p\bar{p} \to eX$; while the first channel is almost background free, the second has large background contributions of misidentified electrons and of electrons from c-quark decays. These background contributions are determined by studying the distribution of $p_T^{\rm rel}$, the transverse momentum of the electron with respect to the direction of the momentum of all tracks around the electron direction. Using a 11.2% semileptonic branching fraction, the inclusive electron channel yields $R = 2.4 \pm 27\%$ for b quarks with $p_T^{\min} \geq 15$ GeV/c. Using a branching fraction of 3.14×10^{-3}, the eD^0 channel yields $R = 2.1 \pm 34\%$ for b quarks with $p_T^{\min} \geq 19$ GeV/c[8].

The study in Ref. [27] uses the decay $B \to J/\psi X$ with $J/\psi \to \mu^+\mu^-$. The B contribution is separated from prompt J/ψ production by studying lifetime distributions. By using the fragmentation fraction $f_u = 0.375$ and a branching fraction of 6.74×10^{-4}, the measurement yields $R = 2.0 \pm 10\%$ for b quarks with $p_T^{\min} \geq 9$ GeV/c; for $p_T^{\min} \geq 14$ GeV/c, R decreases to $1.7 \pm 15\%$.

Reference [3] reports the first measurement at $\sqrt{s} = 1.96$ TeV[9] through the decay $B \to J/\psi X$ with $J/\psi \to \mu^+\mu^-$. This measurement extends the differential cross section to $p_T \simeq 0$ GeV/c, and the data are compared only to an improved QCD calculation [5]. We compare to the standard theory using the information that the observed cross section is 85% of that reported in Ref. [27], whereas it should have been 10% larger [3]. We derive a ratio $R = 1.5 \pm 9\%$ for b quarks with $p_T^{\min} \geq 9$ GeV/c; for $p_T^{\min} \geq 14$ GeV/c, R decreases to $1.3 \pm 9\%$.

The study in Ref. [28] uses the channel $p\bar{p} \to \mu X$. The b contribution is separated from backgrounds due to misidentified muons or c-quark decays by looking at the distribution of $p_T^{\rm rel}$, the transverse momentum of the muon with respect to the direction of a jet with $E_T \geq 8$ GeV that includes the muon. Using a semileptonic branching fraction of 11.2% and the MRSD_0 set of parton distribution functions, the measurement yields $R = 2.1 \pm 27\%$ for b quarks with $p_T^{\min} \geq 6$ GeV/c; the ratio is $1.7 \pm 30\%$ for $p_T^{\min} \geq 12$ GeV/c.

Reference [29] is a repetition of the previous measurement that uses slightly different kinematic cuts and an improved simulation of the b hadronization and decay. The publication uses the MRSR2 fits [30]. We correct for the fact that the theoretical cross sections are 36% and 18% higher than those obtained using the MRSD_0 fits for $p_T^{\min} \geq 6$ GeV/c and $p_T^{\min} \geq 12$ GeV/c, respectively. The measurement yields $R = 2.5 \pm 25\%$ for b quarks

[4] According to the publications, the systematic uncertainty due to the decay spectra is not very large, and ranges between 1 and 20%.

[5] The paper quotes a discrepancy of $2.9 \pm 15\%$ with respect to the standard theory that uses the MRST [21] set of parton distribution functions. This discrepancy is evaluated by fitting the ratio of the data to the standard theory as a function of the B p_T. This procedure underestimates the ratio of the observed to predicted integrated cross sections that is $3.2 \pm 15\%$; this ratio becomes $R = 3.5 \pm 15\%$ when using the MRSD_0 set of structure functions as in the measurement described next.

[6] This ratio is larger than that quoted in the publication ($1.9 \pm 15\%$) and derived by fitting the ratio of the data to the standard theory as a function of the B p_T.

[7] The published result uses the DFLM fits [25]. This old set of parton distribution functions is quite similar to the most recent PDF fits and yields theoretical cross sections that are 17% (for $p_T^{\min} \geq 21$ GeV/c) and 11% (for $p_T^{\min} \geq 29$ GeV/c) higher than those obtained using the MRSD_0 fits.

[8] The published ratios are based on the use the DFLM fits. We correct for the fact that these fits yield a theoretical prediction that is 22% and 18% larger, respectively, than that based on the MRSD_0 fits.

[9] All other measurements considered in this review are performed at $\sqrt{s} = 1.8$ TeV.

TABLE I. Ratio R of measured single b cross sections to a prediction based on the exact NLO calculation (see text). The cross sections are for producing b quarks above a given transverse momentum $p_T^{\min}$. The ratios in parentheses highlight those cases in which data and theory appear to have different transverse momentum distributions and are not used in deriving $<R>$. Each measurement covers b-quark momenta as large as $4-5$ times the $p_T^{\min}$ threshold. The measurement in the seventh row also covers small transverse momenta down to $p_T \simeq 0$ GeV/c.

channel	(experiment)	R for $p_T^{\min}$ (GeV/c) =					
		6	$8-10$	$12-15$	$19-21$	$\simeq 29$	$\simeq 40$
$J/\psi K^+$	(CDF [20])		$3.5 \pm 15\%$	(3)			
$J/\psi K^+$	(CDF [23])		$2.9 \pm 23\%$	(1.9)			
μX	(CDF [24])				$2.5 \pm 26\%$	(1.9)	
eX	(CDF [26])			$2.4 \pm 27\%$			
eD^0	(CDF [26])				$2.1 \pm 34\%$		
$J/\psi X$	(CDF [27])		$2.0 \pm 10\%$	(1.7)			
$J/\psi X$	(CDF [3])		$1.5 \pm 9\%$	(1.3)			
μX	(DØ [28])	$2.1 \pm 27\%$		(1.7)			
μX	(DØ [29])	$2.5 \pm 25\%$		(3.5)			
b jets (μ)	(DØ [31])				$2.4 \pm 20\%$		(2.0)

with $p_T^{\min} \geq 6$ GeV/c and $R = 3.5 \pm 25\%$ for b quark with $p_T^{\min} \geq 12$ GeV/c.

Reference [31] compares the production of central b jets to the prediction of the standard theory. The measurement requires the presence of a muon within the jets and uses its $p_T^{\rm rel}$ distribution to separate the b-quark contribution from the background. The publication uses the MRSA$'$ fits, and reports $R = 2.4 \pm 20\%$ for b quarks with $p_T^{\min} \geq 20$ GeV/c; the ratio decreases to $R \simeq 2.0 \pm 30\%$ for $p_T^{\min} \geq 40$ GeV/c.

The ratios of the data to the standard theory are summarized in Table I. Using the measurements listed in Table I, we derive an average ratio of the data to the standard theory that is $\langle R \rangle = 2.39$; the RMS deviation of the 10 measurements in Table I is 0.54 that in turn yields a 0.17 error on $\langle R \rangle$. Before comparing the data to the improved QCD calculation, the summary of the experimental situation in Table I prompts a few additional remarks.

The 0.54 RMS deviation is much larger than the measurement uncertainties (these uncertainties are dominated by systematic errors that are generally quoted as conservative estimates). When not using the four measurements based on detection of J/ψ mesons, the average ratio becomes $\langle R \rangle = 2.33$ with a 0.19 RMS deviation that, as expected, is smaller than the measurement uncertainties. The remaining measurements based on detection of J/ψ mesons (first, second, sixth, and seventh line in Table I) yield $\langle R \rangle = 2.5$ with a RMS deviation of 0.9. These four measurements are experimentally the cleanest and easiest to perform, and have the smallest systematic errors; however, they appear to be inconsistent among themselves. For the latter reason, it does not seem judicious to use only these four measurements as benchmarks of theoretical progresses [5,32]. Additional data by CDF and DØ are certainly needed to clarify this situation.

In most, but not all, measurements the shape of the observed transverse momentum distribution is different from that of the standard theory (see values in parentheses in Table I). In general, data and theory tend to agree better with increasing p_T; in one case (ninth line of Table I) they disagree more. It could be a real effect, but it remains open the possibility that some measurements do not model correctly the background contribution as a function of the b-quark transverse momentum.

As noted in Refs. [5,6], the measurement with b jets, listed in the last row of Table I, depends little on the modeling of the b-quark fragmentation. This measurement yields $R = 2.4 \pm 20\%$, whereas $<R> = 2.39$, and does not provide, in contrast to what claimed in Refs. [6,7], any evidence that the b fragmentation function is a major cause of discrepancy between data and standard theory[10].

The NLO prediction depends strongly on the choice of the factorization and normalization scales; by changing the scales by a factor of 2 the prediction changes by approximately 40% [1,2,10]. At perturbation level, the large scale dependence of the NLO prediction is generally taken as a symptom of large higher-order corrections[11]. In addition, there are logarithmic corrections that are present at all

[10]This result is confirmed by a recent measurement [33] of the single b cross that uses central jets with transverse energy $E_T \geq 40$ GeV; b jets are selected identifying the presence of displaced secondary vertices. This study finds that the ratio of the observed cross section to that predicted by the PYTHIA Monte Carlo generator is $1.2 \pm 20\%$; this implies that the ratio of these data to the standard NLO prediction is approximately 2.3 (see the discussion in the next section).

[11]It is well known that there are new partonic processes that appear first at NLO, such as gluons branching into b and $\bar{b}$ quarks (gluon splitting) in the final or initial state of the hard scattering.

orders of perturbation theory [34–37]. The resummation of the logarithms of (p_T/m_b) with next-to-leading logarithmic accuracy (NLL), and the matching with the fixed-order NLO calculation (FONLL) for massive quark has been performed in Ref. [4]. A calculation with the same level of accuracy, available for the production of b quarks at e^+e^- colliders [38], has been used to extract nonperturbative fragmentation functions from LEP and SLC data [39]. These new fragmentation functions have been consistently[12] convoluted with the FONLL calculation to predict the B cross section at the Tevatron [32]. The inclusion of NLL logarithms has a modest effect in the p_T range considered in this review; the new fragmentation functions are harder than the Peterson fragmentation function and explain most of the 30% increase of the FONLL prediction with respect to the standard theory [32]. In the p_T range considered in this study, the latest PDF fits, that include HERA data [40,41] and a more accurate value of α_s at the Z mass, increase by 20% the predicted b-quark cross section with respect to the PDF fits used for the comparisons in Table I [6]. By also using $f_u = 0.39$ in place of 0.375, the final FONLL prediction is approximately 60% higher than the standard NLO prediction. In conclusion, the ratio of the average single b cross section measured at the Tevatron with respect to the FONLL prediction is approximately 1.5. The uncertainty of the FONLL prediction is estimated to be approximately 40% [5][13]. Therefore, the average single b cross section measured at the Tevatron is within the range of values predicted by the FONLL calculation.

Exact NLO predictions do not easily allow the full simulation of events produced at the Tevatron. Therefore, studies that involve b-quark production such as top quark studies or searches for new physics, make use of parton-shower Monte Carlo programs [8,9]. Parton-level cross sections, evaluated by these generators using the leading-log (LL) approximation, also have large uncertainties, comparable to that of the NLO or FONLL prediction: gluon splitting to heavy quarks in the final state has a 30% uncertainty [42] whereas gluon splitting in the initial state (flavor excitation diagrams) depends on the PDF choice and can vary by as much as $\pm 40\%$ when using a wide range of structure functions in the PDF library [43]. Since, as correctly noted in Ref. [6], studies searching for new physics should not depend on the prediction of a QCD calculation, a significant effort has been put in calibrating the parton-level cross section predicted by parton-shower Monte Carlo programs by using jet data [44–46]. Buried in top quark studies or dubious hints of new physics, the significance of this calibration has been overlooked. Therefore, we review it in detail in the next section.

III. COMPARISONS WITH THE HERWIG AND PYTHIA PREDICTIONS

It was first reported in Ref. [47] that parton-shower Monte Carlos, such as the PYTHIA and HERWIG generators, predict a parton-level single b cross section that approximately matches the Tevatron measurements for b quarks with $p_T \geq 6$ GeV/c and $|y| \leq 1$. The parton-level cross section estimated with LL generators is approximately a factor of 2 larger than the exact NLO prediction because the contribution of terms of order higher than α_s^2 is a factor of 2 larger than the contribution of α_s^3 terms estimated with the exact NLO calculation [2].

Leading-order (LO) and higher-than-LO terms are sources of b and $\bar{b}$ quarks with quite different topological structure. The production of events with both a b and $\bar{b}$ quark with $p_T \geq 6$ GeV/c and $|y| \leq 1$ is dominated by LO diagrams and the parton-level cross sections predicted by the exact NLO calculation is comparable to that predicted by LL Monte Carlo generators. At the time, both LL and NLO predictions underestimated by a factor of 2 the available measurements [29,48,49][14]. Therefore, the fact that LL generators model correctly the single b cross section was considered merely accidental, and the source of the discrepancy between data and NLO prediction was searched in nonperturbative fragmentation effects that enhance equally LO and NLO terms.

In Refs. [44,46], the heavy-flavor cross section evaluated with the HERWIG Monte Carlo generator has been tuned by using jet data collected by the CDF experiment at the Tevatron. This study uses four samples of data consisting of events with two or more jets, one of which is central and has transverse energy E_T larger than 20, 50, 70, and 100 GeV, respectively, and a data sample, richer in heavy flavor, collected requiring two or more central jets with $E_T \geq 15$ GeV, one of which contains a lepton with $p_T \geq 8$ GeV/c from heavy-flavor decays. Jets containing heavy flavor are identified by finding displaced secondary vertices produced by the decay of b and c hadrons inside a jet; an additional algorithm uses track impact parameters to select jets with a small probability of originating from the primary event vertex. In the data, the b- and c-quark contributions are separated because both algorithms have the same tagging efficiency for b jets, whereas for c jets the efficiency of the second algorithm is approximately 2.5 times larger than that of the first algorithm. The tagging rates in the data are compared to those of samples simu-

[12] As correctly noted in Ref. [32], the Peterson form of the fragmentation function used in the standard NLO calculation has been tuned in conjunction with a parton-level cross section evaluated with the leading-log (LL) approximation of parton-shower Monte Carlo programs and should not be convoluted with a NLO prediction.

[13] The uncertainty is estimated by changing the normalization and factorization scales by a factor of 2 ($\pm 35\%$) and m_b by ± 0.25 GeV/c^2 ($\pm 16\%$).

[14] These measurement identify b quarks through their semileptonic decays.

lated using the HERWIG Monte Carlo program[15]. The study compares momentum distributions of leptons or of the system of tracks forming a secondary vertex (decay products of the B hadron inside the jet) in the data and simulation. This comparison shows that the hadronization of heavy quarks at the Tevatron is modeled correctly by HERWIG tuned with e^+e^- data. Therefore, one is allowed to tune the parton-level cross section predicted by the Monte Carlo generator to match the heavy-flavor content—or the tagging rate—of the data. The contribution of LO and higher-order terms can be separated because the 90% of the LO contribution consists of events which contain two jets with heavy flavor inside the kinematic cuts. In contrast, only 10% of the events due to higher-order terms contains two jets with heavy flavor in the detector acceptance. The higher-order contributions due to gluons splitting into heavy quarks in the initial and final state are disentangled by studying the $\delta R = \sqrt{(\delta\phi)^2 + (\delta\eta)^2}$ distribution between two jets with heavy flavor (gluon splitting in the final state clusters at small values of δR).

References [44,46] show that the data can be modeled by tuning the parton-level cross section predicted by the HERWIG generator within the theoretical and experimental uncertainties[16]. In the tuned LL generator, the contribution of higher-order terms to the single b cross section is approximately 4 times larger than the LO contribution. In contrast, for the same kinematics, the exact NLO calculation with standard scales returns α_s^3 contributions that are only a factor of 2 larger than the α_s^2 contribution. As discussed in the next section, the study of $b\bar{b}$ correlations can be used to assess the correct ratio of higher-than-LO to LO contributions.

IV. MEASUREMENT OF THE $b\bar{b}$ CORRELATIONS

The cross section for producing both b and $\bar{b}$ quarks centrally and above a given p_T threshold, $\sigma_{b\bar{b}}$ or $b\bar{b}$ correlation, is dominated by LO terms, and the LL and NLO predictions are quite close[17]. In addition, the exact NLO prediction of $\sigma_{b\bar{b}}$ depends little on the choice of the normalization and factorization scales as well as on the b-quark mass[18] and appears to be a robust prediction of perturbative QCD.

Therefore, it is important to determine precisely the value of R_{2b}, the ratio of $\sigma_{b\bar{b}}$ measured at the Tevatron to the exact NLO prediction (or to the LL prediction that is very close). A ratio $R_{2b} \simeq 1$ would imply that the parton-level cross section predicted by LL Monte Carlo generators is correct and that the contribution of higher-than-LO terms has to be a factor of 2 larger than in the present NLO or FONLL prediction. If the ratio R_{2b} is much larger than 1, then the agreement between the observed single b cross section and the prediction of LL Monte Carlo generators is fortuitous. Since the NLO prediction of $\sigma_{b\bar{b}}$ is robust, agreement with the data may be found by using harder fragmentation functions as in the FONLL calculation. Unfortunately, the status of the $\sigma_{b\bar{b}}$ measurements at the Tevatron is quite disconcerting.

The study in Ref. [44] (CDF) uses two central jets with $E_T \geq 15$ GeV, each containing a secondary vertex due to b- or $\bar{b}$-quark decays. The LL prediction, tuned to fit the data, yields $R_{2b} = 1.2$[17] with a 25% uncertainty mostly due to the efficiency for finding a secondary vertex in a jet.

A recent measurement [50] (CDF) supports the conclusion of Ref. [44]. The study in Ref. [50] uses events containing two central jets with $E_T \geq 30$ and 20 GeV, respectively; pairs of b jets are also identified by requiring the presence of displaced secondary vertices. This study finds that the ratio of $\sigma_{b\bar{b}}$ to the LL PYTHIA prediction is $0.9 \pm 31\%$, while the ratio of $\sigma_{b\bar{b}}$ to the NLO prediction[19] is $R_{2b} = 1.0 \pm 32\%$.

In contrast, discrepancies between data and the NLO prediction of $\sigma_{b\bar{b}}$ are observed when identifying b quarks through their semileptonic decay into muons.

The study in Ref. [48] (CDF) uses events with a muon recoiling against a jet that contains tracks with large impact parameter (b jet). Using the average branching fraction $\mathrm{BR} = 10.3\%$ for $b \to \mu X$ decays and 10.2% for $b \to cX \to \mu Y$ sequential decays [52], the ratio of $\sigma_{b\bar{b}}$ to the exact NLO prediction is measured to be $R_{2b} = 1.5 \pm 10\%$ for b and $\bar{b}$ quarks produced centrally and with $p_T^{\min} \geq 12$ GeV/c.

Reference [49] (CDF) reports a measurement that uses two centrally produced muons. By using the square of the semileptonic branching fraction quoted above, the study yields $R_{2b} = 3.0 \pm 20\%$ for central b and $\bar{b}$ quarks with $p_T^{\min} \geq 6$ GeV/c.

Reference [29] (DØ) reports an analogous measurement that also uses two centrally produced muons. The square of the semileptonic branching fraction is evaluated with the ISAJET generator [53] implemented with the QQ decay table and is consistent with the value used by CDF. The study yields a ratio $R_{2b} = 2.3 \pm 33\%$ for central b and $\bar{b}$ quarks with $p_T^{\min} \geq 7$ GeV/c.

[15]The study uses option 1500 of version 5.6 with the MRS(G) set of parton distribution functions [12].

[16]The gluon splitting in the final state predicted by HERWIG has to be increased by $(40 \pm 20)\%$. Before tuning the simulation, the size of gluon splitting in the final state predicted by HERWIG is 1/2 of that in the initial state.

[17]For example, in Ref [44] the tuned LL generator predicts a modest contribution of higher-than-LO order terms to $\sigma_{b\bar{b}}$ ($\simeq$ 30%); for the same kinematics, the exact NLO calculation predicts a $\simeq 15\%$ contribution of higher-than-LO order terms. In this case, the LL and NLO predictions of $\sigma_{b\bar{b}}$ are within 20%.

[18]The prediction changes by no more than 15% by changing the scales by a factor of 2 and m_b by ± 0.25 GeV/c^2 [44].

[19]In this case the NLO prediction has been evaluated using the MC@NLO Monte Carlo generator [51].

TABLE II. Ratio R_{2b} of $\sigma_{b\bar{b}}$, the observed cross section for producing both b and $\bar{b}$ quarks, centrally and above a given $p_T^{\min}$ threshold, to the exact NLO prediction (see text). Each measurement covers b-quark momenta as large as $4-5$ times the $p_T^{\min}$ threshold. Jets produced by b and $\bar{b}$ quarks are identified by the presence of displaced secondary vertex or tracks with a large impact parameter. Muons from b and $\bar{b}$ decays are separated from the background by studying impact parameter [49] or p_T^{rel} [29] distributions.

channel	(experiment)	R_{2b} for $p_T^{\min}$ (GeV/c) =			
		$6-7$	10	15	$\simeq 20$
$b+\bar{b}$ jets	(CDF [44])			$1.2 \pm 25\%$	
$b+\bar{b}$ jets	(CDF [50])				$1.0 \pm 32\%$
$\mu + b$ jet	(CDF [48])		$1.5 \pm 10\%$		
$\mu^+ + \mu^-$	(CDF [49])	$3.0 \pm 20\%$			
$\mu^+ + \mu^-$	(DØ [29])	$2.3 \pm 33\%$			

These five measurements, listed in Table II, yield $<R_{2b}> = 1.8$ with a 0.8 RMS deviation. Such a large RMS deviation indicates that the experimental results are inconsistent among themselves. Additional measurements are certainly needed to clarify the experimental situation. However, it is quite obvious that the present discrepancies are reduced if the rate of observed semileptonic decays is approximately 50% higher than what is expected because: (a) lepton identification efficiencies are underestimated by approximately 50% or (b) additional objects with a 100% semileptonic branching ratio and a cross section of the order of 1/10 of the b cross section are produced [44]. Reference [44] has investigated these hypotheses by comparing the rate of observed and predicted leptons from b-quark decays in jets that recoil against a generic jet or a jet that also contains a lepton (the jets are central with $E_T \geq 15$ GeV). This study finds that in the second case the rate of jets containing a lepton from presumed b decays is 50% higher than in the first case. The magnitude of the effect is consistent with hypothesis (b). In light of this observation, it is worth to go back to Table I. If there was a reason to disregard the first two measurements using the $B \to J/\psi K$ channel, one would see a completely different picture. Six measurements identify b quarks through their semileptonic decay and yield $<R> = 2.33 \pm 0.06$; the measurements with inclusive J/ψ mesons (sixth and seventh line) do not use b semileptonic decays and yield $<R> = 1.7 \pm 0.1$; this conjecture very well highlights the need for additional cross-checks of the measurements based on $B \to J/\psi K$ decays and on the inclusive J/ψ production.

V. CONCLUSIONS

We review all measurements of the single b cross section performed at the Tevatron and compare them to an exact NLO perturbative QCD prediction, that uses pre-HERA sets of parton distribution functions and the Peterson fragmentation function, in order to test their consistency. We also compare the data to an improved QCD calculation (FONLL) and to the prediction of LL Monte Carlo generators. The average ratio of the data to the NLO prediction is $<R> = 2.39$ with a 0.54 RMS deviation. The RMS deviation is much larger than the quoted measurement uncertainties, and indicates that experimental results are inconsistent among themselves. With this caveat, the average of the data is found to be in agreement with the parton-level cross section evaluated with parton-shower Monte Carlo generators and is within the range of uncertainty of the FONLL prediction that in turn is 60% higher than the NLO prediction. The increase of the FONLL prediction with respect to the NLO calculation is mostly due to PDF improvements and the usage of a harder fragmentation function, whereas the parton-level cross section is the same in both predictions. On the contrary, the contribution of higher-than-LO terms returned by LL Monte Carlo generators fitted to the data is approximately a factor of 2 larger than that in the FONLL or NLO calculations. The measurement of $\sigma_{b\bar{b}}$, the cross section for producing both b and $\bar{b}$ quarks centrally and above the same p_T threshold, has a decisive role in assessing the correct parton-level cross section. In fact, the higher-than-LO contribution to $\sigma_{b\bar{b}}$ is quite modest in all theoretical approaches. Because of the use of harder fragmentation functions, the FONLL calculation yields a prediction appreciably larger than that of LL generators or NLO generators convoluted with the Peterson fragmentation function. Unfortunately, the experimental situation is quite disconcerting and only raises additional questions. The average ratio of the $\sigma_{b\bar{b}}$ measurements to the NLO prediction is $<R_{2b}> = 1.8$ with a 0.8 RMS deviation, and suggests that these measurements are also inconsistent among themselves. The $<R_{2b}>$ value supports the FONLL approach. However, the level of agreement between data and theory appears to be a function of the number of semileptonic decays used to identify b quarks. In this situation, it cannot be excluded that the b parton-level cross section is correctly described by LL Monte Carlo generators, and that measurements using b semileptonic decays are affected by new physics.

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