$J^P = 1^+$ $ud\bar{s}\,\bar{s}$ tetraquark

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(Received 15 November 2005; published 20 January 2006)

Using the color-magnetic interaction Hamiltonian with SU(3) flavor symmetry breaking, we perform a schematic study of the masses of the $J^P = 1^+$ tetraquarks in the antidecuplet representation. After diagonalizing the mass matrix, we find the $ud\bar{s}\,\bar{s}$ tetraquark could lie as low as about 1350 MeV. It decays into $K^+K^0\pi^0$, $K^+K^+\pi^-$, $K^0K^0\pi^+$ via P-wave. The dual suppression from the not-so-big three-body phase space and P-wave decay barrier may render this exotic state rather narrow. Future experimental exclusion of this state will cast doubt on the validity of applying the simple color-magnetic Hamiltonian to the multiquark system.

DOI: 10.1103/PhysRevD.73.014018 PACS numbers: 12.39.Mk, 12.39.Jh

I. INTRODUCTION

The identification and interpretation of scalar mesons below 1 GeV have been a difficult issue in hadron spectroscopy for decades. Especially their underlying structure and decay patterns have been challenging. Jaffe gave a fairly good description of scalar mesons below 1 GeV assuming they are tetraquarks [1–3]. It is important to note there does not exist a scalar tetraquark with exotic quantum numbers in Jaffe's "good" diquark scheme.

Recently, Karliner and Lipkin argued that the $(ud\bar{s})$ triquark cluster and antistrange can form a narrow resonance with double strangeness [4]. Now no P-wave is introduced. This state has $I = 0$, $J^P = 0^+$. Isospin symmetry and parity conservation forbids it to decay into either K^+K^0 or $KK\pi$. Hence, the only allowed decay mode is $KK\pi\pi$ if it lies above the four-body threshold. Because of strong phase space suppression, the width of this tetraquark should be small according to Ref. [4]. Burns, Close, and Dudek pointed out there would exist a $J^P = 1^-$ $ud\bar{s}\,\bar{s}$ tetraquark around 1.6 GeV if the $J^P = \frac{1}{2}+$ Θ^+ exists [5].

It will be very interesting to confirm its existence both theoretically and experimentally. A fully dynamical calculation of the tetraquark spectrum based on the first principle is still very demanding, although there have appeared some preliminary results on tetraquarks from the lattice QCD approach [6].

Recently, in the framework of the flux-tube quark model Kanada-En'yo, Morimatsu, and Nishikawa argued that the scalar tetraquark with double strangeness does not exist [7]. Instead they pointed out that the $J^P = 1^+$ $ud\bar{s}\,\bar{s}$ is stable and low lying. Its mass is around 1.4 GeV and decays into $K^*\pi$ via S-wave with its width around 20–50 MeV [7].

On the other hand, there has accumulated good evidence of the $J^P = 1^{-+}$ exotic mesons which cannot be $q\bar{q}$ mesons. There are two candidates $\Pi_1(1400)$, $\Pi_1(1600)$ [8]. Lattice QCD simulation, QCD sum rule approach, and the flux-tube model predict the lowest 1^{-+} hybrid meson around 1.9 GeV. Their masses are too low as a hybrid meson. The assignment of $\Pi_1(1400)$, $\Pi_1(1600)$ as tetraquarks with one orbital excitation is quite attractive now. P-wave tetraquarks were studied extensively in Ref. [9].

In this work we perform a schematic study of the mass splitting of the $J^P = 1^+$ tetraquarks in the antidecuplet representation, of which $ud\bar{s}\,\bar{s}$ is a member. In Sec. II we present the color-magnetic interaction Hamiltonian. Then we construct the wave functions of $J^P = 1^+$ tetraquarks in Sec. III. The formalism of calculating SU(3) flavor symmetry breaking corrections to the color-magnetic interaction energy is presented in Sec. IV. Section V discusses the extraction of the parameter and numerical analysis. The last section is a short summary.

II. COLOR-MAGNETIC INTERACTION FOR THE MULTIQUARK SYSTEM

The constituent quark model (CQM) is quite successful in the description of the meson and baryon spectrum. Within CQM the color-magnetic interaction arising from one-gluon exchange is responsible for the mass splitting between the octet and decuplet baryons as first pointed out by De Rujula, Georgi, and Glashow [10]. The Hamiltonian describing the color-spin hyperfine interaction of a multiquark system reads [2,11]

$$H_{\rm CM} = -\sum_{i>j} v_{ij}\vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j, \tag{1}$$

where $\vec{\sigma}_i$ is the quark spin operator and $\vec{\lambda}_i$ the color operator. For the antiquark, $\vec{\lambda}_{\bar{q}} = -\vec{\lambda}^*$ and $\vec{\sigma}_{\bar{q}} = -\vec{\sigma}^*$. The value of the coefficient v_{ij} depends on the multiquark system and specific models. In the present convention, v_{ij} is positive. For example, v_{ij} takes different values for $q\bar{q}$, qqq, and $q\bar{q}q\bar{q}$ systems. In the bag model, v_{ij} depends on the bag radius and the constituent quark mass $m_{i,j}$. In CQM, we have $v_{ij} = [m_u^2/(m_i m_j)]\bar{v}(m_i, m_j)$. $\bar{v}(m_i, m_j)$ still depends on the constituent quark mass through the spatial wave function. For example, $\bar{v}$ for the J/Ψ and η_c

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system is very different from $\bar{v}$ for the rho and pion system. However, $\bar{v}$ can be treated as a constant for different members within a single flavor-spin representation.

Under exact $SU(3)_f$ flavor symmetry, $v_{ij} = v$,

$$H_{CM} = -v\sum_{i>j}\vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j. \tag{2}$$

We use the notation $|D_6, D_{3c}, S, N, D_{3f}\rangle$ to denote a particular multiquark configuration, where D_6, D_{3c}, and D_{3f} are SU(6) color spin, $SU(3)_c$ color, and $SU(3)_f$ flavor representations of the multiquark system, respectively. S is the spin of the system, and N is the total number of quarks and antiquarks. The $SU(6)_{cs}$ generators are $\alpha = \sqrt{2/3}\sigma^k,\ \lambda^a,\ \sigma^k\lambda^a,\ k = 1, 2, 3,\ a = 1, 2, \ldots, 8$. For the antiquark $\alpha_{\bar{q}} = -\alpha^*$. The Casimir operators of $SU(6)_{cs}$ and $SU(3)_f$ groups are defined as $C_6 = \sum_{u=1}^{35}(\sum_{i=1}^{N}\alpha_i^u)^2$, $C_3 = \sum_{a=1}^{8}(\sum_{i=1}^{N}\lambda_i^a)^2$. The color-magnetic interaction energy of the multiquark system can be expressed in terms of the quadratic Casimir operators of $SU(2)_s$, $SU(3)_c$, and $SU(6)_{cs}$:

$$H_{CM} = \frac{v}{2}[\overline{C}(\text{total}) - 2\overline{C}(Q) - 2\overline{C}(\overline{Q}) + 16N], \tag{3}$$

where $\overline{C} = C_6 - C_3 - \frac{8}{3}S(S+1)$ and $\overline{C}(\text{total})$, $\overline{C}(Q)$, and $\overline{C}(\overline{Q})$ denote the $\overline{C}$ of the whole multiquark system, the subsystem of quarks and antiquarks, respectively.

III. 1^+ TETRAQUARKS

We use diquarks to construct the 1^+ teraquark wave function. Nominally there are four types of diquarks: $|21, \bar{3}_c, 0, 2, \bar{3}_f\rangle$, $|21, 6_c, 1, 2, \bar{3}_f\rangle$, $|15, \bar{3}_c, 1, 2, 6_f\rangle$, $|15, 6_c, 0, 2, 6_f\rangle$. The spin and color wave functions of the first two type of diquarks are simultaneously symmetric or antisymmetric. Using the master formula Eq. (3),

$$V_{CM}(Q) = -\frac{v}{2}[\overline{C}(Q) - 16N], \tag{4}$$

it is easy to show that the color-magnetic interaction is attractive for the first two types of diquarks and repulsive for the latter two types of diquarks. Numerically, we have $V_{CM} = -8v, -\frac{4}{3}v, \frac{8}{3}v, 4v$ for four types of diquarks, respectively. The first type of diquark is what Jaffe called the good diquark since its color-magnetic (CM) interaction is the strongest. Jaffe used only the good diquark as the building block of scalar tetraquarks. Our present calculation does not rely on any specific clustering. We do not truncate the basis and limit to the good diquarks. Instead we diagonalize the chromomagnetic interaction in the whole basis.

We want to emphasize that diquarks are not pointlike particles. Instead they are extended objects in space. There also exists color-magnetic interaction between quarks inside different diquarks. We can construct six types of 1^+ tetraquarks. Their underlying diquark-diquark structures were presented in Ref. [2]. In the exact SU(6) symmetry limit,

$$|35, 1_c, 1, 4, (1+8)_f\rangle = |21, 6_c, 1, 2, \bar{3}_f\rangle \otimes |\overline{21}, \bar{6}_c, 1, 2, 3_f\rangle, \tag{5}$$

$$|35, 1_c, 1, 4, (1+8+27)_f\rangle = |15, \bar{3}_c, 1, 2, 6_f\rangle \otimes |\overline{15}, 3_c, 1, 2, \bar{6}_f\rangle, \tag{6}$$

$$\begin{aligned}|35, 1_c, 1, 4, (8+\overline{10})_f\rangle = {}& \sqrt{1/3}|21, \bar{3}_c, 0, 2, \bar{3}_f\rangle \otimes |\overline{15}, 3_c, 1, 2, \bar{6}_f\rangle \\ & - \sqrt{2/3}|21, 6_c, 1, 2, \bar{3}_f\rangle \otimes |\overline{15}, \bar{6}_c, 0, 2, \bar{6}_f\rangle,\end{aligned} \tag{7}$$

$$\begin{aligned}|280, 1_c, 1, 4, (8+\overline{10})_f\rangle = {}& \sqrt{2/3}|21, \bar{3}_c, 0, 2, \bar{3}_f\rangle \otimes |\overline{15}, 3_c, 1, 2, \bar{6}_f\rangle \\ & + \sqrt{1/3}|21, 6_c, 1, 2, \bar{3}_f\rangle \otimes |\overline{15}, \bar{6}_c, 0, 2, \bar{6}_f\rangle,\end{aligned} \tag{8}$$

$$\begin{aligned}|35, 1_c, 1, 4, (8+10)_f\rangle = {}& \sqrt{1/3}|\overline{21}, 3_c, 0, 2, 3_f\rangle \otimes |15, \bar{3}_c, 1, 2, 6_f\rangle \\ & - \sqrt{2/3}|\overline{21}, \bar{6}_c, 1, 2, 3_f\rangle \otimes |15, 6_c, 0, 2, 6_f\rangle,\end{aligned} \tag{9}$$

$$\begin{aligned}|280, 1_c, 1, 4, (8+10)_f\rangle = {}& \sqrt{2/3}|\overline{21}, 3_c, 0, 2, 3_f\rangle \otimes |15, \bar{3}_c, 1, 2, 6_f\rangle \\ & + \sqrt{1/3}|\overline{21}, \bar{6}_c, 1, 2, 3_f\rangle \otimes |15, 6_c, 0, 2, 6_f\rangle.\end{aligned} \tag{10}$$

In the presence of CM interaction H_{CM} in Eq. (2) with exact flavor symmetry, the first two kinds of tetraquarks in Eqs. (5) and (6) are still mass eigenstates:

$$\begin{aligned}|1^+(1+8)_f\rangle &= |35, 1_c, 1, 4, (1+8)_f\rangle, \\ |1^+(1+8+27)_f\rangle &= |35, 1_c, 1, 4, (1+8+27)_f\rangle.\end{aligned} \tag{11}$$

CM interaction H_{CM} mixes the two flavor eigenstates in Eqs. (7) and (8) to yield the mass eigenstates:

$$|1^+(8+\overline{10})_f\rangle = \left(\frac{2\sqrt{2}}{3}\right)|35, 1_c, 1, 4, (8+\overline{10})_f\rangle + \left(\frac{1}{3}\right)|280, 1_c, 1, 4, (8+\overline{10})_f\rangle,$$
$$|1^+(8+\overline{10})'_f\rangle = \left(\frac{1}{3}\right)|35, 1_c, 1, 4, (8+\overline{10})_f\rangle - \left(\frac{2\sqrt{2}}{3}\right)|280, 1_c, 1, 4, (8+\overline{10})_f\rangle. \quad (12)$$

$H_{\rm CM}$ also mixes the fifth and sixth states in Eqs. (9) and (10) to yield the mass eigenstates $|1^+(8+10)_f\rangle$ and $|1^+(8+10)'_f\rangle$.

The CM interaction energy $V_{\rm CM} = -16v, 0, -\frac{40}{3}v, \frac{32}{3}v, -\frac{40}{3}v, \frac{32}{3}v$ for $|1^+(1+8)_f\rangle$, $|1^+(1+8+27)_f\rangle$, $|1^+(8+\overline{10})_f\rangle$, $|1^+(8+\overline{10})'_f\rangle$, $|1^+(8+10)_f\rangle$, and $|1^+(8+10)'_f\rangle$, respectively, among which three are unbound with $V_{\rm CM} \geq 0$. Although $|1^+, (1+8)_f\rangle$ and the octet part of $|1^+(8+\overline{10})_f\rangle$ are bound, they do not carry exotic quantum numbers. They lie around ~1.1–1.4 GeV and mix strongly with conventional $L = 1$ $q\bar{q}$ states such as $b_1(1235)$, $a_1(1260)$, $f_1(1285)$, etc. No symmetry forbids them fall apart into two mesons. Hence these states are very broad. Experimental identification of these broad bumps above background is nearly impossible. We do not discuss them further in this work.

From now on, we focus on the antidecuplet part of the $|1^+(8+\overline{10})_f\rangle$. $|1^+(8+10)_f\rangle$ is its charge conjugate representation. If 1^+ tetraquarks *really* exist, they should belong to this category. In fact, the antidecuplet contains several 1^+ tetraquarks which are exotic in flavor and useful for the experimental search. Their flavor wave functions are presented in Table I. As will be shown below, the $ud\bar{s}\,\bar{s}$ 1^+ tetraquark is expected to be rather narrow from symmetry considerations.

IV. 1^+ TETRAQUARK MASSES

For the tetraquark, the Hamiltonian reads

$$H = \sum m(q_i) + H_{\rm CM}, \quad (13)$$

where $m(q_i)$ is the mass of ith constituent quark. The $SU(3)_f$ flavor symmetry is badly broken since $m_s > m_u$. Besides the constituent quark mass difference, we need to consider the symmetry breaking corrections to the CM interaction energy in Eq. (1). The explicit expression of its matrix element between two states $|k\rangle$ and $|l\rangle$, $V_{\rm CM} = \langle k \mid H_{\rm CM} \mid l\rangle$ with given flavor context $q_1q_2\bar{q}_3\bar{q}_4$ ($q_i = u, d, s$), reads

$$V_{\rm CM}(q_1q_2\bar{q}_3\bar{q}_4) = V_{12}(q_1q_2) + V_{13}(q_1\bar{q}_3) + V_{14}(q_1\bar{q}_4) + V_{23}(q_2\bar{q}_3) + V_{24}(q_2\bar{q}_4) + V_{34}(\bar{q}_3\bar{q}_4) \quad (14)$$

with $V_{ij} = -v_{ij}\langle k|\vec{\lambda}_i\cdot\vec{\lambda}_j\vec{\sigma}_i\cdot\vec{\sigma}_j|l\rangle$, where $v_{ij} = v$ in the limit of exact flavor symmetry and $v_{ij} = v[m_u^2/(m_im_j)]$ with our assumption for symmetry breaking. $|1^+(8+\overline{10})_f\rangle$ and $|1^+(8+\overline{10})'_f\rangle$ are mass eigenstates of $H_{\rm CM}$ in the exact SU(3) flavor symmetry. Clearly, they are not eigenstates of the CM interaction with SU(3) flavor symmetry breaking. They will mix each other and also with the $|1^+(1+8)_f\rangle$, $|1^+(1+8+27)_f\rangle$, and their conjugate representation.

In order to get the physical mass eigenstates, we need to calculate every individual term in Eq. (14) and diagonalize the new 6×6 mass matrix. For the purpose of calculating $V(q\bar{q})$ terms, we need to do some recouplings [12,13]:

$$\{|q_1q_2D_6(Q)D_3(Q)S(Q);\bar{q}_3\bar{q}_4D_6(\bar{Q})D_3(\bar{Q})S(\bar{Q})\rangle\}_{(D_3,S)} = \sum R(D_3(Q)D_3(\bar{Q});D_3(13)D_3(24);D)R(S(Q)S(\bar{Q});S(13)S(24);S) \times\{|q_1\bar{q}_3D_6(13)D_3(13)S(13);q_2\bar{q}_4D_6(24)D_3(24)S(24)\rangle\}_{(D_3,S)}, \quad (15)$$

TABLE I. The flavor wave function and mass of the antidecuplet tetraquarks. The CM energy is in unit of v.

(Y, I, I_3)	Quark content	$V_{\rm CM}(v)$	Mass (MeV)
(2, 0, 0)	$[ud]\bar{s}\,\bar{s}$	−11.4	1347
$(1, \frac{1}{2}, -\frac{1}{2})$	$\sqrt{1/3}([ds]\bar{s}\,\bar{s}) + \sqrt{2/3}([ud]\{\bar{u}\,\bar{s}\})$	−8.4	1351
$(1, \frac{1}{2}, \frac{1}{2})$	$\sqrt{1/3}([us]\bar{s}\,\bar{s}) + \sqrt{2/3}([ud]\{\bar{d}\,\bar{s}\})$	−8.4	1351
(0, 1, −1)	$\sqrt{1/3}([ud]\bar{u}\,\bar{u}) + \sqrt{2/3}([ds]\{\bar{u}\,\bar{s}\})$	−10.1	1256
(0, 1, 0)	$\sqrt{1/3}([ud]\{\bar{u}\,\bar{d}\}) + \sqrt{2/3}\sqrt{1/2}([ds]\{\bar{d}\,\bar{s}\} + [us]\{\bar{u}\,\bar{s}\})$	−10.1	1256
(0, 1, 1)	$\sqrt{1/3}([ud]\bar{d}\,\bar{d}) + \sqrt{2/3}([us]\bar{d}\,\bar{s})$	−10.1	1256
$(-1, \frac{3}{2}, -\frac{3}{2})$	$[ds]\bar{u}\,\bar{u}$	−10.1	1197
$(-1, \frac{3}{2}, -\frac{1}{2})$	$\sqrt{1/3}([us]\bar{u}\,\bar{u}) + \sqrt{2/3}([ds]\{\bar{u}\,\bar{d}\})$	−10.1	1197
$(-1, \frac{3}{2}, \frac{1}{2})$	$\sqrt{1/3}([ds]\bar{d}\,\bar{d}) + \sqrt{2/3}([us]\{\bar{u}\,\bar{d}\})$	−10.1	1197
$(-1, \frac{3}{2}, \frac{3}{2})$	$[us]\bar{d}\,\bar{d}$	−10.1	1197

where the recoupling coefficients are

$$R(S(Q)S(\bar{Q}); S(13)S(24); S) = \sqrt{(2S(Q)+1)(2S(\bar{Q})+1)(2S(13)+1)(2S(24)+1)} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & S(Q) \\ \frac{1}{2} & \frac{1}{2} & S(\bar{Q}) \\ S(13) & S(24) & S \end{pmatrix}, \tag{16}$$

$$R((\lambda_Q\mu_Q)(\lambda_{\bar{Q}}\mu_{\bar{Q}}); (\lambda_{13}\mu_{13})(\lambda_{24}\mu_{24})) = (-1)^{\lambda_Q+\mu_Q+\lambda_{13}+\mu_{13}} U((10)(10)(10)(01); (\lambda_Q\mu_Q)(\lambda_{13}\mu_{13})). \tag{17}$$

With SU(3) Racah coefficients $U((10)(10)(10)(01);(20)(00)) = \sqrt{2/3}$, $U((10)(10)(10)(01);(20)(11)) = \sqrt{1/3}$, $U((10)\times(10)(10)(01);(01)(00)) = -\sqrt{1/3}$, $U((10)(10)(10)(01);(01)(11)) = \sqrt{2/3}$, we have

$$\begin{aligned}\{|q_1q_2 21, \bar{3}, S=0; \bar{q}_3\bar{q}_4 15, 3, S=1\rangle\}_{(1,1)} &= \frac{\sqrt{3}}{6}|q_1\bar{q}_3 1,1,0; q_2\bar{q}_4 35,1,1\rangle - \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35,8,0; q_2\bar{q}_4 35,8,1\rangle - \frac{\sqrt{3}}{6}|q_1\bar{q}_3 35,1,1; q_2\bar{q}_4 1,1,0\rangle \\ &\quad + \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35,8,1; q_2\bar{q}_4 35,8,0\rangle - \frac{\sqrt{3}}{3}|q_1\bar{q}_3 35,8,1; q_2\bar{q}_4 35,8,1\rangle + \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35,1,1; q_2\bar{q}_4 35,1,1\rangle,\end{aligned} \tag{18}$$

$$\begin{aligned}\{|q_1q_2 21, 6, 1; \bar{q}_3\bar{q}_4 15, \bar{6}, 0\rangle\}_{(1,1)} &= -\frac{\sqrt{6}}{6}|q_1\bar{q}_3 1,1,0; q_2\bar{q}_4 35,1,1\rangle - \frac{\sqrt{3}}{6}|q_1\bar{q}_3 35,8,0; q_2\bar{q}_4 35,8,1\rangle \\ &\quad + \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35,1,1; q_2\bar{q}_4 1,1,0\rangle + \frac{\sqrt{3}}{6}|q_1\bar{q}_3 35,8,1; q_2\bar{q}_4 35,8,0\rangle \\ &\quad + \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35,8,1; q_2\bar{q}_4 35,8,1\rangle + \frac{\sqrt{3}}{3}|q_1\bar{q}_3 35,1,1; q_2\bar{q}_4 35,1,1\rangle.\end{aligned} \tag{19}$$

Then we can rewrite the SU(6) flavor eigenstates in terms of two pairs of $q_1\bar{q}_3 \otimes q_2\bar{q}_4$:

$$\begin{aligned}|35, 1_c, 1, (8+\bar{10})_f\rangle &= \frac{1}{2}|q_1\bar{q}_3 1,1,0; q_2\bar{q}_4 35,1,1\rangle - \frac{1}{2}|q_1\bar{q}_3 35,1,1; q_2\bar{q}_4 1,1,0\rangle - \frac{2}{3}|q_1\bar{q}_3 35,8,1; q_2\bar{q}_4 35,8,1\rangle \\ &\quad - \frac{\sqrt{2}}{6}|q_1\bar{q}_3 35,1,1; q_2\bar{q}_4 35,1,1\rangle,\end{aligned} \tag{20}$$

$$\begin{aligned}|280, 1_c, 1, (8+\bar{10})_f\rangle &= -\frac{1}{2}|q_1\bar{q}_3 35,8,0; q_2\bar{q}_4 35,8,1\rangle + \frac{1}{2}|q_1\bar{q}_3 35,8,1; q_2\bar{q}_4 35,8,0\rangle - \frac{\sqrt{2}}{6}|q_1\bar{q}_3 35,8,1; q_2\bar{q}_4 35,8,1\rangle \\ &\quad + \frac{2}{3}|q_1\bar{q}_3 35,1,1; q_2\bar{q}_4 35,1,1\rangle.\end{aligned} \tag{21}$$

With the $SU_c(3)$ and $SU_s(2)$ symmetries, the recoupling to $q_1\bar{q}_4 \otimes q_2\bar{q}_3$ can be easily obtained. With the help of the above formulas, we can calculate the CM energy of any tetraquark states. The results are collected in Table I in units of v.

V. THE EXTRACTION OF v

In order to get the numerical values of 1^+ tetraquark mass, we need to determine the constituent quark mass and the value of the parameter v in Eq. (1) for the 1^+ tetraquark system. With the experimental values of π, ρ, K, K^* mesons as inputs, we extract the constituent quark mass consistently:

$$\begin{cases} M(\pi) = 2m_u - 16v' \\ M(K) = m_u + m_s - 16\zeta v' \\ M(\rho) = 2m_u + \frac{16}{3}v' \\ M(K^*) = m_u + m_s + \frac{16}{3}\zeta v' \end{cases} \Rightarrow \begin{cases} m_u = 308\ \text{MeV} \\ m_s = 486\ \text{MeV} \\ v' = 29.9\ \text{MeV} \\ \zeta = \frac{m_u}{m_s} = 0.63 \end{cases}$$

where v' is the CM interaction parameter in Eq. (1) for the $q\bar{q}$ meson system without the orbital and radial excitation.

In order to extract v for the tetraquark system, we follow Jaffe's assumption that scalar mesons below 1 GeV are mainly composed of a pair of good diquark and antidiquark [1–3]. We are not arguing this is the only interpretation. For example, $f_0/a_0(980)$ could be the KK molecule states as suggested by Weinstein and Isgur [14]. They could also be conventional $q\bar{q}$ states or mixture of $q\bar{q}$ and tetraquarks. We use this *working* assumption to constrain the 1^+ tetraquark mass only.

Under this working assumption, there exists a scalar tetraquark nonet [1]. Moreover, strong mixing between the SU(3) singlet scalar tetraquark σ' and the octet member f_0' will split the spectrum, leading to the physical $f_0(980)$ $[(us\bar{u}\,\bar{s}+ds\bar{d}\,\bar{s})/\sqrt{2}]$ and σ $(ud\bar{u}\,\bar{d})$. This mixing mechanism violates Okubo-Zweig-Iizuka rule and cannot be described by the color-magnetic interaction Hamiltonian. Therefore we avoid $f_0(980)$ and σ mesons. Instead we use $a_0(980)$ meson mass to extract v. The quark content of $a_0(980)$ with positive charge is $us\bar{d}\,\bar{s}$. Using similar techniques, we derive its CM interaction eigenstate:

$$\begin{aligned}|0^+(1+8)_f\rangle_{(Y=0,I=1)} &= 0.974|1, 1_c, 0, 4, (1+8)_f\rangle \\ &\quad - 0.225|405, 1_c, 0, 4, (1+8)_f\rangle \\ &\quad + \cdots, \end{aligned} \tag{22}$$

where the ellipse denotes the other four components from other representations with tiny coefficients and

$$\begin{aligned}|1, 1_c, 0, 4, (1+8)_f\rangle &= \sqrt{6/7}|21, 6_c, 1, 2, \bar{3}_f\rangle \\ &\quad \otimes |\overline{21}, \bar{6}_c, 1, 2, 3_f\rangle \\ &\quad + \sqrt{1/7}|21, \bar{3}_c, 0, 2, \bar{3}_f\rangle \\ &\quad \otimes |\overline{21}, 3_c, 0, 2, 3_f\rangle \\ |405, 1_c, 0, 4, (1+8)_f\rangle &= \sqrt{1/7}|21, 6_c, 1, 2, \bar{3}_f\rangle \\ &\quad \otimes |\overline{21}, \bar{6}_c, 1, 2, 3_f\rangle \\ &\quad - \sqrt{6/7}|21, \bar{3}_c, 0, 2, \bar{3}_f\rangle \\ &\quad \otimes |\overline{21}, 3_c, 0, 2, 3_f\rangle. \end{aligned} \tag{23}$$

Similarly, the SU(6) flavor eigenstates can be expressed in terms of two pairs of $q\bar{q}$.

$$\begin{aligned}|1, 1_c, 0, 4, (1+8)_f\rangle &= \frac{\sqrt{21}}{6}|q_1\bar{q}_3 1, 1, 0; q_2\bar{q}_4 1, 1, 0\rangle \\ &\quad - \frac{\sqrt{7}}{14}|q_1\bar{q}_3 35, 1, 1; q_2\bar{q}_4 35, 1, 1\rangle \\ &\quad + \frac{\sqrt{42}}{21}|q_1\bar{q}_3 35, 8, 0; q_2\bar{q}_4 35, 8, 0\rangle \\ &\quad - \frac{\sqrt{14}}{7}|q_1\bar{q}_3 35, 8, 1; q_2\bar{q}_4 35, 8, 1\rangle, \end{aligned} \tag{24}$$

$$\begin{aligned}|405, 1_c, 0, 4, (1+8)_f\rangle &= -\frac{2\sqrt{42}}{21}|q_1\bar{q}_3 35, 1, 1; q_2\bar{q}_4 35, 1, 1\rangle \\ &\quad + \frac{3\sqrt{7}}{14}|q_1\bar{q}_3 35, 8, 0; q_2\bar{q}_4 35, 8, 0\rangle \\ &\quad + \frac{5\sqrt{21}}{42}|q_1\bar{q}_3 35, 8, 1; q_2\bar{q}_4 35, 8, 1\rangle. \end{aligned} \tag{25}$$

Its CM interaction energy and mass are

$$V_{CM}(a_0(980)) = -28.8v,$$
$$M(a_0(980)) = 2m_s + 2m_u - 28.8v.$$

Using the experimental value of $a_0(980)$ mass as input, we get

$$v \approx 21.1 \text{ MeV}. \tag{26}$$

The V_{CM} of κ, $f_0(980)$, and σ mesons are $-35.4v$, $-28.8v$, and $-43.4v$, respectively. Thus, we can predict the masses of κ, $f_0(980)$, and σ mesons: $m_\kappa = 663$ MeV, $m_{f_0(980)} = 980$ MeV, $m_\sigma = 316$ MeV. As pointed out by Jaffe decades ago [1–3], strong correlations between light quarks lead to low-lying scalar tetraquark mesons. Mixing between flavor eigenstates within different representations further lowers σ meson mass. Since it is above $\pi\pi$ threshold, it falls apart via S-wave easily and becomes very broad and buried by background. Moreover, one should be cautious that other complicated mechanisms will alter the sigma meson mass from the color-magnetic interaction Hamiltonian significantly.

Note the same v' is assumed for $L^P = 0^-$ and 1^- $q\bar{q}$ mesons within this model. In our case neither 0^+ nor 1^+ tetraquarks have orbital excitation. Hence, the parameter v should be the same for both 0^+ and 1^+ tetraquark systems. Now we can estimate the 1^+ tetraquark mass. For example, the mass of the A_{1333} member of the antidecuplet $(ud\bar{s}\,\bar{s})$ reads

$$\begin{aligned} M(A_{1333}) &= \sum m_q + V_{CM}(A_{1333}) \\ &= m(u) + m(d) + 2m(s) + V_{CM}(A_{1333}) \\ &= 308 \times 2 + 486 \times 2 + (-11.4 \times 21.1) \\ &\approx 1347 \text{ MeV}. \end{aligned} \tag{27}$$

The antidecuplet 1^+ tetraquark masses are collected in Table I.

VI. DISCUSSIONS

In short summary, we have performed a schematic study of the masses of 1^+ tetraquarks in the decuplet representation. We have paid special attention to the SU(3) flavor symmetry breaking corrections to the color-magnetic interaction energy. The SU(3) flavor symmetry breaking color-magnetic Hamiltonian mixes $J^P = 1^+$ states in different SU(6) representations. We have diagonalized the 6×6 mass matrix to obtain the masses of 1^+ tetraquarks in the decuplet representation.

Only three flavor-exotic 1^+ tetraquarks, $ud\bar{s}\,\bar{s}$, $us\bar{d}\,\bar{d}$, and $ds\bar{u}\,\bar{u}$, are potentially interesting. All the other seven states will easily fall apart into two mesons via S-wave and become completely buried by background. Among the three flavor exotics, both $us\bar{d}\,\bar{d}$ and $ds\bar{u}\,\bar{u}$ 1^+ tetraquarks will unfortunately fall apart into $K^*\pi$ very easily via S-wave. Hence they are too broad also. It is impossible to measure these states experimentally.

Now let us focus on 1^+ $ud\bar{s}\,\bar{s}$ tetraquark. Let us call it $\mathcal{T}^+$. Its quantum numbers are $Y = +2$, $Q = +1$, $I = 0$. With a mass of 1347 MeV, $ud\bar{s}\,\bar{s}$ cannot decay into K^*K final states since $m_{K^*} + m_K = 1386$ MeV. Parity and angular momentum conservation forbid it decay into K^+K^0, $K^+K^+\pi^-\pi^0$, $K^+K^0\pi^0\pi^0$, $K^0K^0\pi^+\pi^0$. Its decay modes, which are allowed by both kinematics and symmetry, are $K^+K^0\pi^0$, $K^+K^+\pi^-$, $K^0K^0\pi^+$. These decays occur through P-wave. The dual suppression from the not-so-big three-body phase space and P-wave decay barrier may render this exotic state rather narrow. The value of 1347 MeV is clearly a lower bound, as the parameters leading to this estimate predict too low a mass for κ and σ before introducing mixing which will further lower their mass. However, a higher lying, and thus broader, KK^* resonance would still be very interesting by its exotic character.

We strongly call for experimentalists to search for this interesting state (i) in the double-strangeness-exchange reactions on the proton or deuteron target: $K^+d \rightarrow p + p + K^- + \mathcal{T}^+$; or (ii) in the J/ψ or Υ decays: $J/\psi(\Upsilon) \rightarrow K^-\bar{K}^0\mathcal{T}^+$. The existence of this state is a generic feature of the color-magnetic interaction model. The future experimental exclusion of this state will cast doubt on the validity of the application of the simple color-magnetic Hamiltonian to the multiquark system.

ACKNOWLEDGMENTS

This project was supported by the National Natural Science Foundation of China under Grants No. 10375003 and No. 10421003, Ministry of Education of China, FANEDD, Key Grant Project of Chinese Ministry of Education (No. 305001) and SRF for ROCS, SEM. C. Y. thanks Y. R. Liu and W. Wei for helpful discussions.

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