PHYSICAL REVIEW D **73,** 014014 (2006)

# New states above charm threshold

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(Received 17 November 2005; published 19 January 2006)

We revise and extend expectations for the properties of charmonium states that lie above charm threshold, in light of new experimental information. We refine the Cornell coupled-channel model for the coupling of $c\bar{c}$ levels to two-meson states, defining resonance masses and widths by pole positions in the complex energy plane, and suggest new targets for experiment.

DOI: 10.1103/PhysRevD.73.014014 PACS numbers: 14.40.Gx, 13.25.Gv, 14.40.Lb

## I. INTRODUCTION

In the short time since the Belle Collaboration reported sighting the "missing" charmonium state $\eta_c'(2^1S_0)$ in exclusive $B \to KK_S K^- \pi^+$ decays [1], new states associated with charmonium have appeared in great profusion. Apart from the long-sought $h_c(1^1P_1)$, recently observed by the CLEO Collaboration in the decay $\psi(2S) \to \pi^0 h_c \to (\gamma\gamma)(\gamma\eta_c)$ [2,3], all the newly observed states lie near or above charm threshold. The identification of levels above the $D\bar{D}$ flavor threshold has renewed the importance of theoretical studies that couple the $c\bar{c}$ spectrum to charmed-meson–anticharmed-meson channels.

Motivated by the Belle Collaboration's discovery [4] of the narrow state $X(3872) \to \pi^+\pi^- J/\psi$, confirmed in short order by the CDF [5], D0 [6], and *BABAR* [7] experiments, we explored the influence of open-charm channels on charmonium properties and profiled the $1^3D_2$, $1^3D_3$, and $2^1P_1$ charmonium candidates for $X(3872)$ [8]. We reaffirmed the single-channel potential-model expectation [9,10] that the favored $1^3D_2$ and $1^3D_3$ candidates both should have prominent radiative decays, and we noted that the $1^3D_2$ might be visible in the $D^0\bar{D}^{*0}$ channel, while the dominant decay of the $1^3D_3$ state should be into $D\bar{D}$. We proposed that additional discrete charmonium levels were ripe for discovery as narrow resonances of charmed and anticharmed mesons. Barnes, Godfrey, and Swanson have reached similar conclusions using a different model for the coupling to open charm [11].

Experiments have enriched our knowledge of $X(3872)$, but its precise nature remains open to debate. On current evidence, $X(3872)$ is likely to be a $J^{PC} = 1^{++}$ state. If so, the surviving $2^3P_1$ charmonium candidate is problematic, as we discuss in Sec. IV B 2. Not every state associated with charmonium need be identified as a $c\bar{c}$ state; we will mention below what some of the interlopers might be. An important goal of predicting the properties of charmonium states above flavor threshold is to tag new states that do not fit the template, and so might represent new spectroscopies.

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Four other new particles require confirmation, for each has been seen only in a single experiment so far. Belle [12] has reported $Y(3943 \pm 11 \pm 13)$ in the decay $B \to K\omega J/\psi$; it is a relatively broad state, with total width $\Gamma = 87 \pm 22 \pm 26$ MeV. Belle also reports the state $X(3943 \pm 9)$, seen in $e^+e^- \to J/\psi + X$ [13]. It is observed to decay into $D\bar{D}^*$, but not $D\bar{D}$, which suggests an unnatural parity assignment. The total width is $\Gamma < 52$ MeV.

Belle has observed a narrow ($\Gamma \approx 20$ MeV) state in $\gamma\gamma \to D\bar{D}$ that they call $Z(3929 \pm 5 \pm 2)$ [14]. The production and decay characteristics are consistent with a $2^{++}$ assignment, and this state is a plausible $\chi_{c2}'$ $(2^3P_2)$ candidate. The most recent addition to the collection is $Y(4260)$, a broad ($\approx 88$ MeV) $1^{--}$ level seen by *BABAR* in $e^+e^- \to \gamma\pi^+\pi^- J/\psi$ [15], with supporting evidence from $B \to K^- J/\psi\pi\pi$ [16].

In this paper, we fix the masses of the $3^1S_0$ and $2^3P_2$ charmonium states at the positions of $Y(3943)$ and $Z(3930)$, in order to sharpen our expectations for the properties of other states above charm threshold. We also extend our previous calculations to include additional states at higher masses. We highlight the search for narrow structures in charmed-meson–anticharmed-meson pairs, and we remark on analogue states to be expected in the $b\bar{b}$ spectrum.

## II. EXPERIMENTAL STATUS

Let us briefly summarize what experiment has taught us about the new levels associated with charmonium, and how the experimental facts square with theoretical expectations.

### A. Properties of $\eta_c'$

The hyperfine partner of $\psi(2S)$ now seems convincingly established, thanks to observations by the Belle [1,17], CLEO [18], and *BABAR* [19] experiments. The world-average values of the mass and width are $M(\eta_c') = 3638 \pm 5$ MeV and $\Gamma(\eta_c') = 14 \pm 7$ MeV [20].

The proximity of $\psi'$ and $\eta_c'$ to charm threshold means that communication with open-charm channels will alter the properties of these two narrow levels. As we elaborated in Ref. [8], the basic coupled-channel interaction is spin-

 

independent, but the unequal masses and quantum numbers of the $2^3S_1$ and $2^1S_0$ potential-model eigenstates mean that their interaction with different charmed-meson–anticharmed-meson channels induces spin-dependent forces that affect the charmonium states. These spin-dependent forces give rise to S-D mixing that contributes to the $\psi(3770)$ electronic width, for example, and are a source of additional spin splitting.

In a single-channel nonrelativistic potential picture, the 2S hyperfine splitting is given by $M(\psi') - M(\eta_c') = 32\pi\alpha_s|\Psi(0)|^2/9m_c^2$. Normalizing to the observed 1S hyperfine splitting, $M(J/\psi) - M(\eta_c) = 117$ MeV, we would find $M(\psi') - M(\eta_c') = 67$ MeV, to be compared with the observed $48 \pm 5$ MeV separation. The 2S induced shifts calculated in Ref. [8] draw $\psi'$ and $\eta_c'$ closer by 20.9 MeV, substantially improving the agreement between theory and experiment. It is tempting to conclude that the $\psi'$-$\eta_c'$ splitting reflects the influence of virtual decay channels.

### B. CLEO's $h_c$ candidate

Like the $\eta_c'$, the spin-singlet $L = 1$ level of charmonium has been an object of desire for three decades. The barrier to observation has been the absence of allowed E1 or M1 transitions from the $J^{PC} = 1^{--}$ states readily formed in $e^+e^-$ annihilations. The $1^1P_1$ state holds a special interest, beyond the tidiness that would come from completing the spectrum of narrow states. In the nonrelativistic potential-model descriptions that have provided such a reliable guide to quarkonium spectroscopy, the central potential is generally taken to consist of a Coulomb piece with vector Lorentz structure (arising from one-gluon exchange) plus a confining term with scalar Lorentz structure. For such a potential, the hyperfine splitting between spin-singlet and spin-triplet orbital excitations is very small. A significant deviation from the expectation that $M(1^1P_1) \approx \langle M(1^3P_J)\rangle$ could offer evidence for an unexpected Lorentz structure, or for the importance of effects not included in the nonrelativistic potential-model description.

An earlier observation of $\bar{p}p \to h_c \to \pi^0\eta_c$ near the $(1^3P_J)$ centroid in Fermilab Experiment 760 [21] is not sustained by two additional data runs with $2\times$ and $3\times$ the luminosity in the successor experiment E835 [22,23]. E835 does report a narrow ($\Gamma < 1$ MeV) event excess in the $\bar{p}p \to h_c \to \gamma\eta_c$ channel at a mass $M(h_c) = 3525.8 \pm 0.2 \pm 0.2$ MeV [22]. The signal bin contains 13 signal events on a 3-event background, and the signal strength is $\Gamma(h_c \to \bar{p}p)\mathcal{B}(h_c \to \gamma\eta_c) \approx 11$ eV.

The CLEO Collaboration has examined $3.08 \times 10^6$ $\psi(2S)$ decays, in search of examples of the isospin-violating decay to $\pi^0 h_c$, followed by $\pi^0 \to \gamma\gamma$ and $h_c \to \gamma\eta_c$ [2,3]. They find $150 \pm 40$ signal counts for the inclusive channel, in which $\eta_c$ is not reconstructed, and $17.5 \pm 4.5$ signal counts spread over seven hadronic decay channels of $\eta_c$, for a combined significance exceeding $5\sigma$. In the inclusive sample, the angular distribution of photons in the $h_c \to \gamma\eta_c$ cascade follows the $1 + \cos^2\theta$ pattern characteristic of E1 emission from a spin-1 state. The CLEO experimenters infer the $1^1P_1$ mass $M(h_c) = 3524.4 \pm 0.6 \pm 0.4$ MeV, with a product branching fraction $\mathcal{B}(\psi(2S) \to \pi^0 h_c)\mathcal{B}(h_c \to \gamma\eta_c) = (4.0 \pm 0.8 \pm 0.7) \times 10^{-4}$.

The $1^1P_1$ level lies $1.0 \pm 0.6 \pm 0.4$ MeV below the spin-triplet centroid, $\langle M(1^3P_J)\rangle = 3525.36 \pm 0.06$ MeV [20], consistent with theoretical expectations.

### C. Rare decays of $\psi(3770)$

The color-multipole expansion [24–26] yields symmetry relations for hadronic transition rates between charmonium states but does not enable calculations of rates from first principles. By the Wigner-Eckart theorem for E1-E1 transitions, all the $1^3D_J \to \pi\pi J/\psi$ rates should be equal (for degenerate $^3D_J$ states), so a reliable determination of the rate $\Gamma(\psi(3770) \to \pi\pi J/\psi)$ is important for anticipating the properties of $\psi_2$ and $\psi_3$. In our survey of $B$-meson gateways to missing charmonium states [9], we adopted the value $\Gamma(1^3D_1 \to \pi\pi J/\psi) \approx 45$ keV.

The CLEO Collaboration has just reported the first high-significance ($11.6\sigma$) observation of the decay $\psi(3770) \to \pi^+\pi^- J/\psi$ [27]. Their measured branching fraction, $\mathcal{B}(\psi(3770) \to \pi^+\pi^- J/\psi) = (1.89 \pm 0.20 \pm 0.20) \times 10^{-3}$, corresponds to a partial width $\Gamma(1^3D_1 \to \pi^+\pi^- J/\psi) \approx (45 \pm 9)$ keV. [The CLEO rate is consistent with, but much more precise than, the Beijing Spectrometer observation [28], $\mathcal{B}(\psi(3770) \to \pi^+\pi^- J/\psi) = (3.4 \pm 1.4 \pm 0.9) \times 10^{-3}$.] A less significant ($3.4\sigma$) observation of the decay $\psi(3770) \to \pi^0\pi^0 J/\psi$ is consistent with the expectation that the $\pi^0\pi^0 J/\psi$ rate should be half the $\pi^+\pi^- J/\psi$ rate. Accordingly, we shall take $\Gamma(1^3D_J \to \pi\pi J/\psi) = (68 \pm 15)$ keV as an improved estimate of the dipion decay rates of the $\psi_2$ and $\psi_3$. The increase from our earlier estimate does not alter the expectation that the most prominent decay of the $1^3D_2$ state should be $\psi_2 \to \gamma\chi_{c1}$.

CLEO has also searched for rare radiative decays of $\psi(3770)$, reporting the preliminary values $\Gamma(\psi(3770) \to \gamma\chi_{c1}) = (78 \pm 19)$ keV and $\Gamma(\psi(3770) \to \gamma\chi_{c2}) < 20$ keV [29]. These observations are in line with expectations ($\Gamma(1^3D_1 \to \gamma\chi_{c1}) \approx 59$ keV and $\Gamma(1^3D_1 \to \gamma\chi_{c2}) \approx 4$ keV [8]) and give us no reason to question the estimated E1 transition rates for the other 1D levels (see Sec. III D).

### D. Properties of $X(3782)$

As the best studied of the new $c\bar{c}$-associated states, $X(3872)$ has been subjected to a broad range of diagnostic tests. Upon discovery, $X(3872)$ seemed a likely—though somewhat heavy—candidate for $\psi_2$ (or perhaps $\psi_3$), but the expected radiative transitions to $\chi_c$ states have never been seen. For a state that lies approximately 50 MeV

above charm threshold, the narrow width [$\Gamma(X(3872) < 2.3$ MeV] [4] and the absence of a $D\bar{D}$ signal [30] suggest unnatural parity $P = (-1)^{J+1}$. The *BABAR* Collaboration has set limits on the existence of a charged partner [31]. In 1.96-TeV $\bar{p}p$ collisions, the production characteristics of $X(3872)$ are similar to those of $\psi(2S)$ [6]. CDF has determined the fraction of $X(3872)$ arising from $B$ decays as $16.9 \pm 4.9 \pm 2.0\%$ [32].

The $\pi\pi$ mass spectrum favors high dipion masses [33], suggesting a $J/\psi\rho$ decay that would be incompatible with the identification of $X(3872) \to \pi^+\pi^- J/\psi$ as the strong decay of a pure isoscalar state. See, however, the interesting discussion of alternative interpretations in Ref. [34]. Suzuki has suggested [35] that the $\rho$-like behavior of the $\pi^+\pi^-$ mass spectrum in these decays may result from a dominant decay to $\omega J/\psi$ slightly off mass shell and the small (isospin breaking) $\omega$-$\rho$ mixing. In this case the $X(3872)$ could be an isoscalar state as expected in a charmonium interpretation.

Observing—or limiting—the $\pi^0\pi^0 J/\psi$ decay remains an important goal [10]. An observed $J/\psi\pi^+\pi^-\pi^0$ decay suggests an appreciable transition rate to $J/\psi\omega$ [36]. Belle's 4.4-$\sigma$ observation of the decay $X(3872) \to J/\psi\gamma$ [36] determines $C = +$, opposite to the charge-conjugation of the leading charmonium candidates. Finally, an analysis of angular distributions [37] supports the assignment $J^{PC} = 1^{++}$ [38], but the mass of $X(3872)$ is too low to be gracefully identified with the $2^3\mathrm{P}_1$ charmonium state, especially if $Z(3931)$ (cf. Sec. II E 3) is to be identified as the $2^3\mathrm{P}_2$ level. The $\chi'_{c1}$ also lacks an isospin-allowed $\pi\pi J/\psi$ decay mode and acquires a substantial $D\bar{D}^*$ width if $M(\chi'_{c1}) > M(D^0) + M(D^{*0})$. We shall examine this hypothesis further in Sec. IV B 2.

If $X(3872)$ is not a charmonium level, what might it be [39]? Three interpretations take the near coincidence of the new state's mass and the $D^0\bar{D}^{*0}$ to be a decisive clue: a $s$-wave cusp at $D^0\bar{D}^{*0}$ threshold [40], a $D^0$-$\bar{D}^{*0}$ "molecule" bound by pion exchange [41–44], and a diquark-antidiquark "tetraquark" state $[cq][\bar{c}\,\bar{q}]$ [45,46]. According to a different four-quark interpretation, $X(3872)$ is a $c\bar{c}q\bar{q}$ state organized by the chromomagnetic (color-hyperfine) interaction into a $(c\bar{c})_{\mathbf{8}}$-$(q\bar{q})_{\mathbf{8}}$ pair [47].

What distinctive predictions might allow us to put these interpretations to the test? On the threshold enhancement interpretation, we should expect bumps at many thresholds, but no radial or orbital excitations. If pion exchange is decisive, then there should be no analogue molecule at $D_s\bar{D}_s^*$ threshold. The tetraquark interpretation suggests that $X(3872)$ should be split into two levels, because $[cu][\bar{c}\,\bar{u}]$ and $[cd][\bar{c}\,\bar{d}]$ would be displaced by about 7 MeV. A first experimental test by the *BABAR* Collaboration is inconclusive [16]. In $61.2 \pm 15.3$ events that fit the hypothesis $B^- \to K^- X(3872)$, they determine a mass of $3871.3 \pm 0.6 \pm 0.1$ MeV, whereas $8.3 \pm 4.5$ $B^0 \to K^0 X(3872)$ events yield $3868.6 \pm 1.2 \pm 0.2$ MeV. The mass difference, $2.7 \pm 1.3 \pm 0.2$ MeV, does not yet distinguish between one $X$ and two. If diquarks are useful dynamical objects, there should be a sequence of excited states as well. On the color-hyperfine $c\bar{c}q\bar{q}$ interpretation, the only analogous level should be a $(b\bar{b})_{\mathbf{8}}$-$(q\bar{q})_{\mathbf{8}}$ state near $B\bar{B}^*$ threshold.

What if the $D^0\bar{D}^{*0}$ threshold is not the decisive element? Hybrid $c\bar{c}$-gluon states might appear in the charmonium spectrum [48]. The mass and $1^{++}$ quantum numbers of $X(3872)$ do not match lattice-QCD expectations [49–51]. The valence gluon in the hybrid charmonium wave function leads to speculation that the $\eta J/\psi$ decay mode might be quite prominent. The *BABAR* experiment [52] has found no sign of $X(3872) \to \eta J/\psi$. More generally, it is plausible that the gluonic degrees of freedom should manifest themselves as "vibrational states" in the charmonium spectrum [53,54]. While theoretical estimates of the spectrum of string-vibration modes do not reproduce the characteristics of $X(3872)$, we should be alert to the appearance of such states elsewhere.

### E. Evidence for states beyond $X(3782)$

The other new states associated with charmonium require confirmation and elaboration. For the moment, experiments have identified four distinct candidates.

#### 1. $Y(3940)$

Exploring the $J/\psi\omega$ mass spectrum in decays $B \to KJ/\psi\omega$, the Belle Collaboration observed an enhancement of $58 \pm 11$ events near threshold [12]. Treated as an $s$-wave resonance, this state, named $Y(3940)$, has a mass $M(Y) = 3943 \pm 11 \pm 13$ MeV and a total width $\Gamma = 87 \pm 22 \pm 26$ MeV. The mass of this object lies well above $DD^*$ threshold, so if $Y(3940)$ is a charmonium state, it would be expected to decay dominantly into $D\bar{D}$ or $D\bar{D}^*$. A small $J/\psi\omega$ decay rate would complicate the interpretation of the product branching fraction, $\mathcal{B}(B \to KY(3940))\mathcal{B}(Y(3940) \to J/\psi\omega) = (7.1 \pm 1.3 \pm 3.1) \times 10^{-5}$. For comparison, the branching fractions for the decays $B^+ \to K^+(c\bar{c})$, with $(c\bar{c}) = \eta_c, J/\psi, \chi_{c0}, \chi_{c1}, \psi'$, are between $6\text{–}10 \times 10^{-4}$ [20]. This suggests a branching ratio of about 10% for $Y(3940) \to \omega J/\psi$ if $Y(3940)$ is a conventional charmonium state. In contrast, a $c\bar{c}$-gluon hybrid state might decay preferentially into $J/\psi$ or $\psi'$ plus light hadrons [55,56]. To settle the identity of $Y(3940)$ will require confirming that it is a single state and distinguishing it from the $X(3940)$, to which we now turn.

#### 2. $X(3940)$

The Belle [17,57] and *BABAR* collaborations [58] have observed copious double charmonium production in $e^+e^-$ annihilations, with strong signals for $J/\psi\eta_c$, $J/\psi\chi_{c0}$, and $J/\psi\eta'_c$. Searching in the region above $D\bar{D}$ threshold, Belle

has observed a new peak at $3943 \pm 6 \pm 6$ MeV, with a width $\Gamma < 52$ MeV at 90% C.L. [13].

The new structure, given the temporary name $X(3940)$, is seen to decay into $D\bar{D}^*$, with a branching fraction $\mathcal{B}(X(3940) \to D\bar{D}^*) = 0.96^{+0.45}_{-0.32} \pm 0.22$. The absence of any signal in the $D\bar{D}$ channel (for which the branching fraction is $<0.41$ at 90% C.L.) suggests that $X(3940)$ is an unnatural parity state. Its properties roughly match those expected for $\eta_c(3S)$, and we shall examine that hypothesis further in Sec. IV A.

#### 3. $Z(3930)$

The Belle Collaboration has examined the $D\bar{D}$ invariant-mass distribution in $\gamma\gamma$-fusion events in which the transverse momentum of the $D\bar{D}$ pair is small. A narrow peak of $64 \pm 18$ events appears at $M(D\bar{D}) = 3929 \pm 5 \pm 2$ MeV [14]. The state, provisionally named $Z(3930)$, has a total width $\Gamma(Z(3930)) = 29 \pm 10 \pm 2$ MeV, and $\Gamma(Z(3930) \to \gamma\gamma)\mathcal{B}(Z(3930) \to D\bar{D}) = 0.18 \pm 0.05 \pm 0.03$ keV, assuming $J = 2$. The (helicity) angular distribution strongly prefers $\sin^4\theta^*$, which corresponds to a spin-2 resonance, over the flat angular distribution characteristic of spin-0. Belle accordingly proposes to identify $Z(3930)$ as the $2^3P_2$ $\chi'_{c2}$. The measured properties are in reasonable accord with those anticipated in Ref. [8]; we probe the $2^3P_2$ assignment further in Sec. IV B 1.

#### 4. $Y(4260)$

In a study of initial-state radiation events, $e^+e^- \to (\gamma)\pi^+\pi^- J/\psi$, the *BABAR* Collaboration has observed an accumulation of events near 4260 MeV in the $\pi^+\pi^- J/\psi$ invariant-mass distribution [15]. The excess of $125 \pm 23$ events can be characterized as a single $J^{PC} = 1^{--}$ resonance with mass $4259 \pm 8^{+2}_{-6}$ MeV and total width $\Gamma(Y(4260)) = 88 \pm 23^{+6}_{-4}$ MeV, but a more complicated structure has not been ruled out. It is perhaps significant that $Y(4260)$ lies near the $D_s^{*+}\bar{D}_s^{*-}$ threshold at 4224 MeV. *BABAR* also reports a $3.1\sigma$ signal for $Y(4260)$ in exclusive $B^\pm \to K^\pm\pi^+\pi^- J/\psi$ decays [16]. An excess of $128 \pm 42$ signal events corresponds to a product branching fraction $\mathcal{B}(B^- \to K^- Y(4260))\mathcal{B}(Y(4260) \to \pi^+\pi^- J/\psi) = (2.0 \pm 0.7 \pm 0.2) \times 10^{-5}$.

The new state has variously been interpreted as $\psi(4S)$, a $c\bar{c}g$ hybrid state [56,59,60], and the radial excitation of a diquark-antidiquark state identified with $X(3872)$ [46]. Curiously, the $\pi^+\pi^- J/\psi$ enhancement occurs in the neighborhood of a local minimum in the cross section for electron-positron annihilation into hadrons. We will have more to say about possible interpretations in Sec. IV C.

## III. STRONG DYNAMICS NEAR THRESHOLD

Near the threshold for open heavy-flavor pair production, light-quark pairs induce significant nonperturbative contributions to the masses, wave functions, and decay properties of physical $Q\bar{Q}$ states. Lattice QCD calculations, once extended into the flavor-threshold region, should provide firm theoretical predictions. At present, only a phenomenological approach can offer a detailed description of these effects.

The effects of light-quark pairs can be described by coupling the potential-model $Q\bar{Q}$ states to nearby physical multibody states. In this threshold picture, the strong interactions are broken into sectors defined by the number of valence quarks. We decompose the full Hamiltonian $\mathcal{H}$ as

$$\mathcal{H} = \sum_n \mathcal{H}_n + \mathcal{H}_I, \tag{1}$$

where $\mathcal{H}_n$ is the Hamiltonian that governs the $(c\bar{c}) + n$-light-quark sector, and $\mathcal{H}_I$ is the interaction that couples the various sectors.

The dynamics of the $Q\bar{Q}$ states (with no valence light quarks, $q$) is described by the interaction $\mathcal{H}_0$. In principle, excited $Q\bar{Q}$ states with valence or vibrational gluonic degrees of freedom also would be contained in the spectrum of $\mathcal{H}_0$. In practice, a simple nonrelativistic potential model is used to determine the properties of the bound states in this sector.

The two-meson sector $Q\bar{q} + q\bar{Q}$ is described by the Hamiltonian $\mathcal{H}_2$. As a first approximation, we shall assume $\mathcal{H}_2$ to be represented by the low-lying spectrum of two free heavy-light mesons. The physical situation is more complex. At large separation between the two mesons the interactions are dominated by $t$-channel pion exchanges, when they are allowed. For states very near threshold such as $X(3872)$, such pion exchange in attractive channels might have significant effects on properties of the physical states [61]. At somewhat shorter distances, more complicated interactions exist and new bound states might arise, e.g. molecular states [41–44,62–64]. Furthermore, at small $Q\bar{Q}$ separation the $Q\bar{Q} + q\bar{q}$ sector may contribute significantly to $\mathcal{H}_2$. In particular, as Swanson has emphasized [65], the $\rho J/\psi$ and $\omega J/\psi$ thresholds lie in the $D\bar{D}$ threshold region. The main landmarks throughout the region we consider are shown in Table I.

Because current mastery of nonperturbative quantum chromodynamics does not suffice to derive a realistic description of the interactions that link the $Q\bar{Q}$ and $Q\bar{q} + q\bar{Q}$ sectors, we must resort to a phenomenological *ansatz* for $\mathcal{H}_I$. Following our earlier work [8], we consider here the Cornell coupled-channel ($C^3$) model [66–68] for the creation of $q\bar{q}$ pairs. A brief discussion of various other models for $\mathcal{H}_I$ is contained in the Quarkonium Working Group's CERN Yellow Report [69]. The open-charm threshold region occupies our interest in this study. However, analogous effects are present in the $b\bar{b}$ states near $B\bar{B}$ threshold and $c\bar{b}$ states near $DB$ threshold. A detailed comparison of different heavy-quark systems could provide valuable insight into the correct form for the coupling to light-quark pairs.

TABLE I. Thresholds for decay into open-charm and nearby hidden-charm thresholds.

| Channel | Threshold energy (MeV) |
|---|---|
| $D^0\bar{D}^0$ | 3729.4 |
| $D^+D^-$ | 3738.8 |
| $D^0\bar{D}^{*0}$ or $D^{*0}\bar{D}^0$ | 3871.2 |
| $\rho^0 J/\psi$ | 3872.7 |
| $D^{\pm}D^{*\mp}$ | 3879.5 |
| $\omega^0 J/\psi$ | 3879.6 |
| $D_s^+D_s^-$ | 3936.2 |
| $D^{*0}\bar{D}^{*0}$ | 4013.6 |
| $D^{*+}D^{*-}$ | 4020.2 |
| $\eta' J/\psi$ | 4054.7 |
| $f^0 J/\psi$ | $\approx 4077$ |
| $D_s^+\bar{D}_s^{*-}$ or $D_s^{*+}\bar{D}_s^-$ | 4080.0 |
| $a^0 J/\psi$ | 4081.6 |
| $\varphi^0 J/\psi$ | 4116.4 |
| $D_s^{*+}D_s^{*-}$ | 4223.8 |
| $\Lambda_c\bar{\Lambda}_c$ | 4569.8 |

The $C^3$ formalism generalizes the $c\bar{c}$ model without introducing new parameters, writing the interaction Hamiltonian in second-quantized form as

$$\mathcal{H}_I = \frac{3}{8}\sum_{a=1}^{8}\int :\rho_a(\mathbf{r})V(\mathbf{r}-\mathbf{r}')\rho_a(\mathbf{r}'): d^3r d^3r', \qquad (2)$$

where $V$ is the charmonium potential and $\rho_a(\mathbf{r}) = \frac{1}{2}\psi^\dagger(\mathbf{r})\lambda_a\psi(\mathbf{r})$ is the color current density, with $\psi$ the quark field operator and $\lambda_a$ the octet of SU(3) matrices. To generate the relevant interactions, $\psi$ is expanded in creation and annihilation operators (for charm, up, down, and strange quarks), but transitions from two mesons to three mesons and all transitions that violate the Zweig rule are omitted. It is a good approximation to neglect all effects of the Coulomb piece of the potential in (2). This simple model for the coupling of charmonium to charmed-meson decay channels gives a qualitative understanding of the structures observed above threshold while preserving the successes of the single-channel $c\bar{c}$ analysis below threshold [67,68].

### A. Mass shifts

In the presence of coupling to two-light-quark decay channels, the mass $\omega$ of the quarkonium state $\Psi$ is defined by the eigenvalue equation

$$[\mathcal{H}_0 + \mathcal{H}_2 + \mathcal{H}_I]\Psi = \omega\Psi. \qquad (3)$$

Above the flavor threshold, $\omega$ is a complex eigenvalue.

The basic coupled-channel interaction $\mathcal{H}_I$ given by (2) is independent of the heavy quark's spin, but the hyperfine splittings of $D$ and $D^*$, $D_s$ and $D_s^*$, induce spin-dependent forces that affect the charmonium states. These spin-dependent forces give rise to S-D mixing that contributes to the electronic widths of $^3D_1$ states and induces additional spin splitting among the physical states.

The masses that result from the full coupled-channel analysis are shown in the second column of Table II, which revises and extends our previously published results [8]. The new version presented here includes the 3S levels and takes account of Belle's evidence [14] for $Z(3930)$, interpreted as a $2^3P_2$ state (cf. Sec. II E 3). As in our earlier analysis, the parameters of the potential-model sector governed by $\mathcal{H}_0$ must be readjusted to fit the physical masses, $\omega$, to the observed experimental values. The centroids of the 1D and 2P spin-triplet masses are pegged to the observed masses of $1^3D_1$ [$\psi(3770)$] and $2^3P_2$ [$Z(3930)$], respectively. The assumed spin splittings in the single-channel potential model are shown in the penultimate column and the induced coupled-channel spin splittings for initially unsplit multiplets are presented in the rightmost column of Table II. The shifts induced in the low-

TABLE II. Charmonium spectrum, including the influence of open-charm channels. All masses are in MeV. The penultimate column holds an estimate of the spin splitting due to tensor and spin-orbit forces in a single-channel potential model. The last column gives the spin splitting induced by communication with open-charm states, for an initially unsplit multiplet.

| State | Mass | Centroid | Splitting (potential) | Splitting (induced) |
|---|---|---|---|---|
| $1^1S_0$ | 2979.9[a] | 3067.6[b] | −90.5[e] | +2.8 |
| $1^3S_1$ | 3096.9[a] | | +30.2[e] | −0.9 |
| $1^3P_0$ | 3415.3[a] | 3525.3[c] | −114.9[e] | +5.9 |
| $1^3P_1$ | 3510.5[a] | | −11.6[e] | −2.0 |
| $1^1P_1$ | 3524.4[f] | | +0.6[e] | +0.5 |
| $1^3P_2$ | 3556.2[a] | | +31.9[e] | −0.3 |
| $2^1S_0$ | 3638[a] | 3674[b] | −50.1[e] | +15.7 |
| $2^3S_1$ | 3686.0[a] | | +16.7[e] | −5.2 |
| $1^3D_1$ | 3769.9[a] | (3815)[d] | −40 | −39.9 |
| $1^3D_2$ | 3830.6 | | 0 | −2.7 |
| $1^1D_2$ | 3838.0 | | 0 | +4.2 |
| $1^3D_3$ | 3868.3 | | +20 | +19.0 |
| $2^3P_0$ | 3881.4 | (3922)[d] | −90 | +27.9 |
| $2^3P_1$ | 3920.5 | | −8 | +6.7 |
| $2^1P_1$ | 3919.0 | | 0 | −5.4 |
| $2^3P_2$ | 3931[g] | | +25 | −9.6 |
| $3^1S_0$ | 3943[h] | (4015) | −66[e] | −3.1 |
| $3^3S_1$ | 4040[a] | (4015) | +22[e] | +1.0 |

[a]Observed mass, from *Review of Particle Physics*, Ref. [20].
[b]Input to potential determination.
[c]Observed $1^3P_J$ centroid.
[d]Computed centroid.
[e]Required to reproduce observed masses.
[f]Observed mass from CLEO [3].
[g]Observed mass from Belle [14].
[h]Observed mass from Belle [13].
[i]Observed 3S centroid.

lying 1S and 1P levels are small. For all the other states, coupled-channel effects are noticeable and interesting.

An important consequence of coupling to the open-charm threshold is that the $\psi'$ receives a downward shift through its communication with the nearby $D\bar{D}$ channel; the unnatural parity $\eta_c'$ does not couple to $D\bar{D}$, and so is not depressed in the same degree. This effect is implicitly present in the early Cornell papers [67,68], but the shift of spin-singlet states was not calculated there. The first explicit mention—and the first calculation—of the unequal effects on the masses of the 2S hyperfine partners is due to Martin and Richard [70]. In the framework of the $C^3$ model, we found [8,9] (cf. Table II) that the induced shifts draw $\psi'$ and $\eta_c'$ closer by 20.9 MeV, substantially improving the agreement between theory and experiment. This suggests that the $\psi'$-$\eta_c'$ splitting reflects the influence of virtual decay channels. In the case of the 3S system, both the $3^1S_0$ $\eta_c''$ and the $3^3S_1$ $\psi(4040)$ communicate with open decay channels, and the $C^3$ model leads to a modest 9 MeV *increase* in the interval between them, as seen in Table II.

### B. Mixing and properties of physical states

The physical states are not pure potential-model eigenstates but include components with two virtual (real, above threshold) open-flavor meson states. Separating the physical state $\Psi$ into $Q\bar{Q}$ ($\Psi_0$) and two-charmed-meson components ($\Psi_2$), the resulting decomposition of $\mathcal{H}$ by sector leads to an effective Hamiltonian for the $Q\bar{Q}$ sector given by

$$\left[\mathcal{H}_0 + \mathcal{H}_I^\dagger \frac{1}{\omega - \mathcal{H}_2 + i\epsilon}\mathcal{H}_I\right]\Psi_0 = \omega\Psi_0. \quad (4)$$

Solving Eq. (4) in the $Q\bar{Q}$ sector determines the mixing among the potential-model states and the coupling to decay channels. Calculational procedures for the $C^3$ model based on the coupling Hamiltonian (2) have been described in detail elsewhere [8,67,68]. Even above threshold for strong decays, the coupled-channel Hamiltonian approach (4) allows a definition of the mass and width of the resonances in terms of the complex eigenvalues and a decomposition of the state into the $c\bar{c}$ and $D\bar{D}$ components via the associated eigenvector. Of course, the width and mass so defined will differ somewhat from the resonance mass and width inferred from the observed enhancement in any particular open-charm–meson-pair channel.

The two-meson contributions to the wave functions of the low-lying $c\bar{c}$ states are shown in Table III for the $C^3$ model. The overall probability for the physical state to be in what we have called the $\Psi_0$ ($c\bar{c}$) sector, denoted $\mathcal{Z}_{c\bar{c}}$, decreases as charm threshold is approached. For states above threshold the mixing coefficients become complex, as shown in Table IV. Configuration mixing effects modify radiative transition rates [8,71,72], in addition to their influence on S-D mixing and spin splittings we have already mentioned.

TABLE III. Wave function fractions (in percent) in the $\Psi_0$ ($c\bar{c}$) and $\Psi_2$ [$D^{(*)}\bar{D}^{(*)}$] sectors for near-threshold states.

| State | $\Psi_0$ fraction $\mathcal{Z}_{c\bar{c}}$ | | $\Psi_2$ fraction $D\bar{D}$ | $D\bar{D}^* + D^*\bar{D}$ | $D^*\bar{D}^*$ |
|---|---|---|---|---|---|
| $1^3D_1$ | 96.83 | $u$ | 0.14 | 0.46 | 0.67 |
| | | $d$ | 0.14 | 0.46 | 0.66 |
| | | $s$ | 0.07 | 0.23 | 0.34 |
| $1^3D_2$ | 61.10 | $u$ | 0.00 | 15.10 | 3.30 |
| | | $d$ | 0.00 | 13.94 | 3.21 |
| | | $s$ | 0.00 | 2.38 | 0.96 |
| $1^1D_2$ | 63.36 | $u$ | 0.00 | 12.33 | 4.88 |
| | | $d$ | 0.00 | 11.38 | 4.74 |
| | | $s$ | 0.00 | 2.00 | 1.31 |
| $1^3D_3$ | 50.06 | $u$ | 16.51 | 1.59 | 7.47 |
| | | $d$ | 13.17 | 1.54 | 7.20 |
| | | $s$ | 0.25 | 0.43 | 1.77 |
| $2^3P_0$ | 50.03 | $u$ | 16.48 | 0.00 | 5.23 |
| | | $d$ | 17.74 | 0.00 | 5.04 |
| | | $s$ | 4.41 | 0.00 | 1.06 |
| $2^1P_1$ | 50.04 | $u$ | 0.00 | 19.40 | 4.95 |
| | | $d$ | 0.00 | 18.73 | 4.68 |
| | | $s$ | 0.00 | 1.43 | 0.78 |
| $2^3P_1$ | 50.06 | $u$ | 0.00 | 20.82 | 3.10 |
| | | $d$ | 0.00 | 20.41 | 2.99 |
| | | $s$ | 0.00 | 2.00 | 0.63 |
| $2^3P_2$ | 49.95 | $u$ | 4.47 | 12.80 | 7.74 |
| | | $d$ | 4.32 | 11.47 | 7.17 |
| | | $s$ | 0.24 | 0.87 | 0.97 |
| $3^1S_0$ | 50.05 | $u$ | 0.00 | 20.78 | 4.02 |
| | | $d$ | 0.00 | 19.82 | 3.78 |
| | | $s$ | 0.00 | 1.01 | 0.55 |
| $3^3S_1$ | 50.04 | $u$ | 1.42 | 10.12 | 11.86 |
| | | $d$ | 1.45 | 10.24 | 11.24 |
| | | $s$ | 0.98 | 1.58 | 1.08 |
| $2^3D_1$ | 71.80 | $u$ | 4.43 | 4.16 | 4.49 |
| | | $d$ | 3.94 | 3.98 | 4.38 |
| | | $s$ | 0.55 | 0.96 | 1.31 |

### C. Zweig-allowed strong decays

Once the mass of a resonance is given, the $C^3$ formalism yields reasonable predictions for the other resonance properties. We present in Table V our estimates of the strong decay rates for all the $s$-, $p$-, and $d$-wave charmonium levels that populate the threshold region below 4200 MeV, together with what is known from experiment. No new experimental information has come to light about $L \geq 3$ levels; the predictions given in Ref. [8] remain current.

The $1^3D_1$ state $\psi''(3770)$, which lies some 40 MeV above charm threshold, calibrates the reasonableness of our calculated widths. As we noted in Ref. [8], our value

TABLE IV. Charmonium content of states near $c\bar{c}$ threshold. The wave function $\Psi$ reflects mixing induced through communication with open-charm channels. Unmixed potential-model eigenstates are denoted by $|n^{2s+1}L_J\rangle$. The coefficient of the dominant unmixed state is chosen real and positive. Physical states are evaluated at masses given in Table II.

| State | | Principal $\Psi_0$ ($c\bar{c}$) components |
|---|---|---|
| $\Psi(1^3D_1)$ | = | $0.10e^{+0.59i\pi}\|2S\rangle + 0.01e^{+0.81i\pi}\|3S\rangle$ $+0.69\|1D\rangle + 0.10e^{+0.86i\pi}\|2D\rangle$ |
| $\Psi(1^3D_2)$ | = | $0.77\|1D\rangle - 0.10\|2D\rangle - 0.02\|3D\rangle$ |
| $\Psi(1^1D_2)$ | = | $0.79\|1D\rangle - 0.10\|2D\rangle - 0.02\|3D\rangle$ |
| $\Psi(1^3D_3)$ | = | $0.70\|1D\rangle - 0.09\|2D\rangle + 0.02e^{-0.98i\pi}\|3D\rangle$ |
| $\Psi(2^3P_0)$ | = | $0.11e^{-0.38i\pi}\|1P\rangle + 0.70\|2P\rangle + 0.03e^{+0.56i\pi}\|3P\rangle$ |
| $\Psi(2^1P_1)$ | = | $0.18e^{-0.19i\pi}\|1P\rangle + 0.68\|2P\rangle + 0.07e^{+0.77i\pi}\|3P\rangle$ |
| $\Psi(2^3P_1)$ | = | $0.18e^{-0.27i\pi}\|1P\rangle + 0.68\|2P\rangle + 0.07e^{+0.69i\pi}\|3P\rangle$ |
| $\Psi(2^3P_2)$ | = | $0.18e^{-0.09i\pi}\|1P\rangle + 0.66\|2P\rangle + 0.07e^{+0.85i\pi}\|3P\rangle$ |
| $\Psi(3^1S_0)$ | = | $0.02e^{-0.06i\pi}\|1S\rangle + 0.20e^{-0.18i\pi}\|2S\rangle + 0.68\|3S\rangle$ $+0.05e^{+0.67i\pi}\|4S\rangle$ |
| $\Psi(3^3S_1)$ | = | $0.02e^{-0.05i\pi}\|1S\rangle + 0.19e^{-0.30i\pi}\|2S\rangle + 0.67\|3S\rangle$ $+0.07e^{+0.54i\pi}\|4S\rangle$ $+0.04e^{+0.59i\pi}\|1D\rangle + 0.04e^{+0.59i\pi}\|2D\rangle$ |
| $\Psi(2^3D_1)$ | = | $0.02e^{+0.60i\pi}\|2S\rangle + 0.03e^{-0.71i\pi}\|3S\rangle$ $+0.14e^{-0.50i\pi}\|1D\rangle + 0.69\|2D\rangle$ |

of 20.1 MeV is in excellent agreement with the world average, $\Gamma(\psi(3770)) = 23.6 \pm 2.7$ MeV [20].

The results presented here differ in two respects from those of Ref. [8]. First, the $\chi'_{c2}$ and $\eta''_c$ masses have been fixed, if only provisionally, by experiment. This results in shifts of the masses and properties of all the 2P and 3S states as shown in Tables II, V, and VI. Second, the present approach allows a more detailed extraction of the composition of the charmonium states above threshold. These results are shown in Tables III and IV.

Along with the current PDG values for the total widths of higher $1^{--}$ $c\bar{c}$ resonances, we show in Table V a reanalysis of the existing experimental data by Seth [73].

The natural-parity $1^3D_3$ state can decay into $D\bar{D}$, but its $f$-wave decay is suppressed by the centrifugal barrier factor. Thus the $1^3D_3$ may be discovered as a narrow $D\bar{D}$ resonance up to a mass of about 4000 MeV.

Barnes, Godfrey, and Swanson [11] have recently reported extensive calculations of decay widths of higher charmonium states in the framework of the $^3P_0$ model for quark-pair production. [Shortcomings of both the $C^3$ and $^3P_0$ models are assessed in Ref. [69]. Detailed comparisons (e.g. Ackleh, Barnes, and Swanson [74]) between various light-quark pair creation models are highly desirable.] Unlike the analysis presented here, their calculation does not resum the effects of coupling to decay channels. The general scale of resulting decay rates is similar to those displayed in Table V. For example, Barnes *et al.* determine $\Gamma(3^3S_1) = 80$ MeV and $\Gamma(2^3D_1) = 74$ MeV.

TABLE V. Open-charm strong decay modes of the charmonium states near threshold. The theoretical widths using the $C^3$ model [8] are shown.

| State | $n^{2s+1}L_J$ | Mode | Decay width (MeV) Experiment | Computed |
|---|---|---|---|---|
| $\psi(3770)$ | $1^3D_1$ | $D^0\bar{D}^0$ | | 11.8 |
| | | $D^+D^-$ | | 8.3 |
| | | total | $23.6 \pm 2.7$[b] | 20.1 |
| $\psi(3868)$ | $1^3D_3$ | $D\bar{D}$ | | 0.82 |
| | | total | | 0.82 |
| $\chi'_{c0}(3881)$ | $2^3P_0$ | $D\bar{D}$ | | 61.5 |
| | | total | | 61.5 |
| $h'_{c1}(3919)$ | $2^1P_1$ | $D\bar{D}^*$ | | 59.8 |
| | | total | | 59.8 |
| $\chi'_{c1}(3920)$ | $2^3P_1$ | $D\bar{D}^*$ | | 81.0 |
| | | total | | 81.0 |
| $\chi'_{c2}(3931)$ | $2^3P_2$ | $D\bar{D}$ | | 21.5 |
| | | $D\bar{D}^*$ | | 7.1 |
| | | total | $29 \pm 10(\text{stat}) \pm 2(\text{sys})$[d] | 28.6 |
| $\eta''_c(3943)$ | $3^1S_0$ | $D\bar{D}^*$ | | 49.8 |
| | | total | $<52$[e] | 49.8 |
| $\psi(4040)$ | $3^3S_1$ | $D\bar{D}$ | | 0.1 |
| | | $D\bar{D}^*$ | | 33.0 |
| | | $D_s\bar{D}_s$ | | 8.0 |
| | | $D^*\bar{D}^*$ | | 33.0 |
| | | total | $52 \pm 10$[b] | 74.0 |
| | | | $88 \pm 5$[c] | |
| $\psi(4159)$ | $2^3D_1$ | $D\bar{D}$ | | 3.2 |
| | | $D\bar{D}^*$ | | 6.9 |
| | | $D^*\bar{D}^*$ | | 41.9 |
| | | $D_s\bar{D}_s$ | | 5.6 |
| | | $D_s\bar{D}^*_s$ | | 11.0 |
| | | total | $78 \pm 20$[b] | 69.2 |
| | | | $107 \pm 8$[c] | |

[a]Computed from CLEO branching fractions [13].
[b]*Review of Particle Physics* [20].
[c]Reanalysis by Seth [73].
[d]Belle [14].
[e]Belle [13].

### D. Radiative transitions

As Tables III and IV show, the physical charmonium states are not pure potential-model eigenstates. To compute the E1 radiative transition rates, we must take into account both the standard $(c\bar{c}) \to (c\bar{c})\gamma$ transitions and the transitions between (virtual) decay channels in the initial and final states. Details of the calculational procedure are given in Sec. IV.B of Ref. [68]. There we also illuminated the differences between single-channel and coupled-channel expectations for the radiative rates.

Our expectations for E1 transition rates among spin-triplet levels are shown in Table VI. Again the observed properties of $\psi(3770)$ confirm the reasonableness of the $C^3$

TABLE VI. Calculated and observed rates for E1 radiative transitions among charmonium levels. $C^3$ model rates include the influence of open-charm channels. Photon energies, in MeV, in parentheses.

| | Partial width (keV) | | |
|---|---|---|---|
| $1^3D_1(3770) \to$ | $\chi_{c2}\gamma(208)$ | $\chi_{c1}\gamma(251)$ | $\chi_{c0}\gamma(338)$ |
| model | 3.9 | 59 | 225 |
| CLEO [75] | $<40$ | $75 \pm 18$ | $<1100$ |
| $1^3D_2(3831) \to$ | $\chi_{c2}\gamma(266)$ | $\chi_{c1}\gamma(308)$ | |
| model | 45 | 212 | |
| $1^3D_3(3868) \to$ | $\chi_{c2}\gamma(303)$ | | |
| model | 286 | | |
| $2^3P_0(3881) \to$ | $J/\psi\gamma(704)$ | $\psi'\gamma(190)$ | $1^3D_1\gamma(110)$ |
| model | 39 | 17 | 1.6 |
| $2^3P_1(3920) \to$ | $1^3D_1\gamma(147)$ | $1^3D_2\gamma(86)$ | |
| model | 5.2 | 2.6 | |
| $2^3P_1(3920) \to$ | $J/\psi\gamma(737)$ | $\psi'\gamma(227)$ | |
| model | 15 | 75 | |
| $2^3P_2(3931) \to$ | $1^3D_1\gamma(157)$ | $1^3D_2\gamma(95)$ | $1^3D_3\gamma(62)$ |
| model | 0.4 | 3.0 | 3.6 |
| $2^3P_2(3931) \to$ | $J/\psi\gamma(775)$ | $\psi'\gamma(272)$ | |
| model | 14 | 120 | |
| $\psi(4040) \to$ | $2^3P_2\gamma(84)$ | $2^3P_1\gamma(132)$ | $2^3P_0\gamma(156)$ |
| model | 18 | 5.4 | 20 |
| $\psi(4040) \to$ | $1^3P_2\gamma(456)$ | $1^3P_1\gamma(495)$ | $1^3P_0\gamma(577)$ |
| model | 12 | 0.4 | 0.03 |

framework. The radiative decay rates for the $2^3P_J$ and $3^3S_1$ levels are calculated at the predicted masses. All these rates are small compared to the expected open-charm decay rates.

## IV. IMPLICATIONS FOR NEWFOUND STATES

### A. $X(3940)$ as $\eta_c(3S)$

The observation by Belle [13] of a state $X(3940)$ recoiling against the $J/\psi$ system in continuum production at the $\Upsilon(4S)$ region fits the general behavior expected for the $\eta_c''$ charmonium level. This state is not seen to decay into $D\bar{D}$ but does decay into $D\bar{D}^*$, suggesting that it has unnatural parity. Figure 1 shows the complex pole position of the $\eta_c''$ in the $C^3$ model as its bare mass is varied in 10-MeV steps. At 3943 MeV, we estimate a $D\bar{D}^*$ width of 50 MeV, which is quite close to the experimental upper bound on the total width, $\Gamma(X(3940)) < 52$ MeV.

If $\psi(4040)$ is assigned to the $3^3S_1$ level, it is somewhat problematic to identify $X(3940)$ as its hyperfine partner. We show in Fig. 2 the behavior of the physical mass and width of the triplet 3S state as the bare mass is varied in 10-MeV steps. To reproduce the observed masses, we would require the spin splitting in the charmonium sector to be 88 MeV, which is considerably larger than the 2S splitting, and larger than expected in naïve potential models.

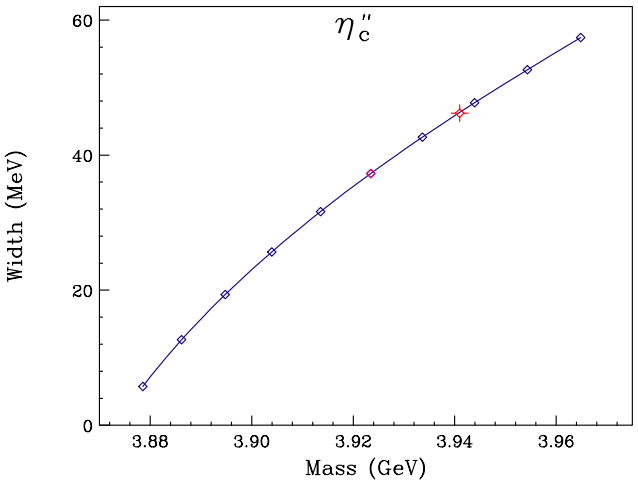


FIG. 1 (color online). Variation of the $\eta_c''$ width with mass in the $C^3$ model. The physical mass of the $X(3940)$ is denoted by a crossed dot.

If, for the moment, we take the existence, mass, and unnatural parity of $X(3940)$ as well established, then two possible sources of the discrepancy deserve further investigation.

First, it is difficult to determine the true pole position for the $3^3S_1$ state from measurements of the step in $R_{c\bar{c}} \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ [73]. The resonance decay widths are determined from fitting measurements of $\Delta R$ in $e^+e^-$ annihilation to a model for each resonance including radiative corrections. This whole procedure is complicated by its dependence on the resonance shape, i.e., the expected non-Breit-Wigner nature of the partial widths for radially excited resonances. It may be more useful to produce a model of $\Delta R$ for direct compari-

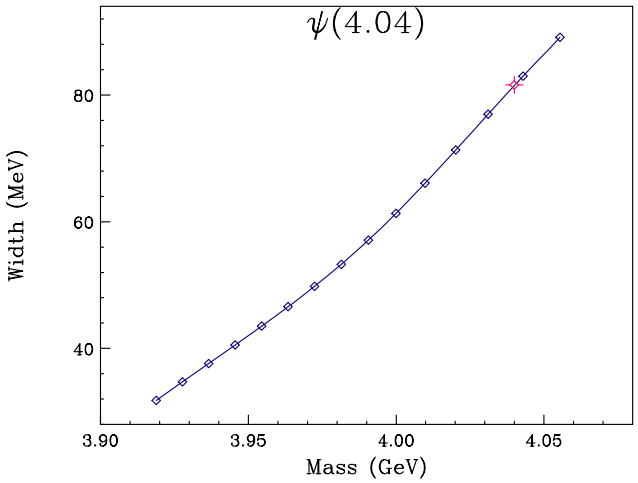


FIG. 2 (color online). Width of the $3^3S_1$ state as a function of its mass. The physical mass of the $\psi(4040)$ is denoted by a crossed dot.

son with data. Greater resolving power between models is possible if the contribution from each individual open heavy-flavor final state is separately reported.

For the $C^3$ model, the structure of $\Delta R(c\bar{c})$ in the threshold region was studied in the original Cornell group works [66–68] and later extended to the $\Delta R(b\bar{b})$ in the threshold region [76]. The structure of $\Delta R(c\bar{c})$ and $\Delta R(b\bar{b})$ has been studied also in $^3P_0$ models of quark-pair creation [77]. There are also some attempts to compare the different models [78,79].

Second, thresholds corresponding to a $s$-wave plus a $p$-wave charmed meson are nearby. Unlike the two $s$-wave charm meson channels, which are $p$-wave decays for the 3S states, these are $s$-wave decays and thus have stronger threshold effects. In particular the lowest mass channel $D(J=0; j_l^P = 1/2^-)D(J=0; j_l^P = 1/2^+)$ couples to the $3^1S_1$ $\eta_c''$ state but not the $3^3S_1$ $\psi(4040)$ state. This will increase the induced spin splitting (see Table II, column 5) between the states. Thus the observed $\psi'' - \eta_c''$ separation will correspond to a smaller potential-model splitting (column 4).

### B. The 2P states

#### 1. $Z(3930)$ as $\chi_{c2}'$

Belle has also observed the new state $Z(3930)$ in two-photon production of $D\bar{D}$ pairs [14]. The $2^3P_2$ and $2^3P_0$ levels are natural charmonium candidates; experiment favors the $J=2$ assignment. In constructing Table II, we adjusted the 2P centroid to give the mass of the $J=2$ state in agreement with observation. At the observed mass, we compute $\Gamma(2^3P_2 \to D\bar{D}) = 21.5$ MeV and $\Gamma(2^3P_2 \to D\bar{D}^*) = 7.1$ MeV, in reasonable agreement with experiment (see Table V). We show in Fig. 3 the variation of the $2^3P_2$ pole position with bare mass in 10-MeV steps, using Eq. (4) to fully account for finite-width effects.

#### 2. $X(3872)$ as $2^3P_1$

If $X(3872)$ is a $1^{++}$ state, the unique surviving $c\bar{c}$ candidate would be the $2^3P_1$ level [35]. One problem with this interpretation is clear: the expected mass (cf. Table II) is 48 MeV above the observed $X$ mass. If $Z(3930)$ is indeed the $2^3P_2$ level, it is very unlikely that its $J=1$ partner should lie at 3872 MeV.

If the mass were not a problem, the $2^3P_1$ interpretation would still be difficult to sustain. The E1 radiative transition to $\gamma J/\psi$ is greatly suppressed because of a cancellation between the $c\bar{c}$ and $D\bar{D}^*$ contributions. We obtain 35 and 21 keV for the transitions to $J/\psi$ and $\psi'$ respectively from the $c\bar{c}$ contributions alone. When the effects of the charmed-meson virtual channels are included, the rates change dramatically to 2.4 and 90 keV, respectively. This would make the reported observation of the $\gamma J/\psi$ mode highly unlikely. Of course, within the $C^3$ model the isospin breaking between $D^0\bar{D}^{*0}$ and $D^+D^{*-}$ virtual states is relatively small. Even if we assume it to be large, so that only the virtual $D^0\bar{D}^{*0}$ component is important, our analysis suggests that a search for the radiative decay to $\psi'$ would be valuable.

Finally, the decay rate of $2^3P_1 \to D\bar{D}^*$ increases very rapidly above threshold as shown in Fig. 4. Since the total width of $X(3872)$ is less than 2.3 MeV, the possibility that the $2^3P_1$ level lies more than 22 keV above the $D^0D^{*0}$ threshold is precluded. When the additional partial width $\Gamma_{\rm other}$ of the $2^3P_1$ into other modes ($ggg$, $\gamma J/\psi$, etc.) is taken into account, the largest tenable mass is lowered to $M(D^0) + M(D^{*0}) - \Gamma_{\rm other}$. Combining this result with the large ratio of $\Gamma(X(3872) \to D^0\pi^0\bar{D}^0) \approx 10\Gamma(X(3872) \to \pi\pi J/\psi)$ reported by Belle [80] renders the $2^3P_1$ interpretation very improbable.

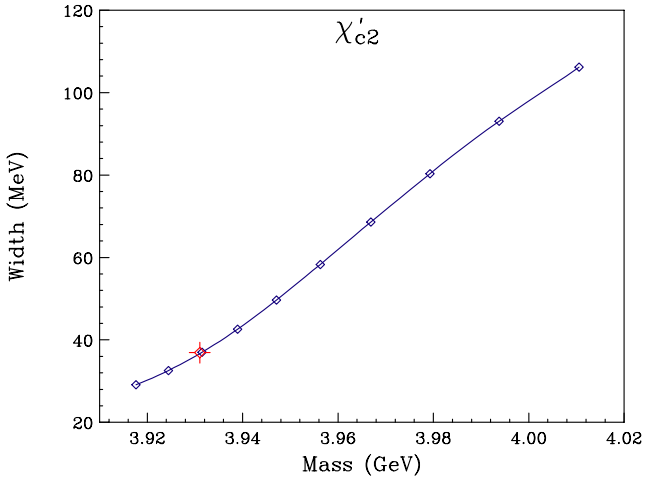


FIG. 3 (color online). Mass dependence of $\Gamma(2^3P_2)$.

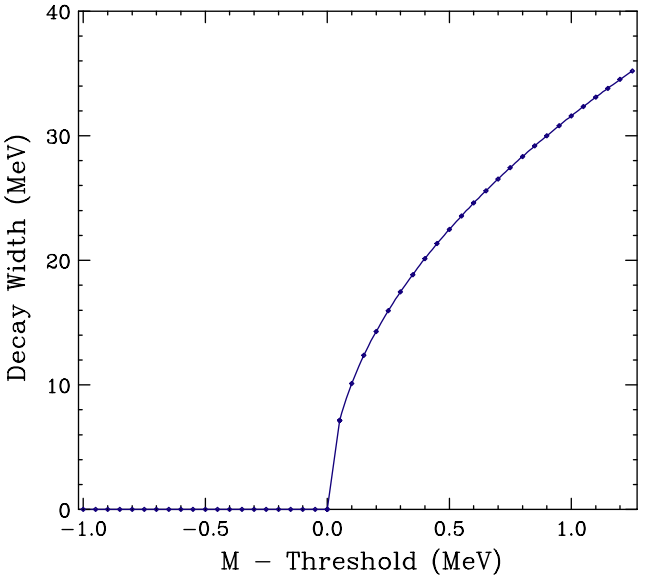


FIG. 4 (color online). Mass dependence of $\Gamma(2^3P_1) \to D\bar{D}^*$.

### C. $Y(4260)$ and other states

In initial-state radiation events, *BABAR* [15] has recently reported a $\pi\pi J/\psi$ resonance, $Y(4260)$, with a width of approximately 88 MeV. This state is hard to assign within the $c\bar{c}$ spectrum. To be directly produced from a virtual photon, it must be a $J^{PC} = 1^{--}$ S or D state. The small contribution to $\Delta R$ discourages the 4S interpretation, but leaves open the $2^3D_1$ identification. However the $C^3$ analysis requires that state at 4.16 GeV in order to reproduce the structure of $\Delta R$. The identification of the $Y(4260)$ with the $2^3D_1$ state is ruled out in the $C^3$ model because the partial decay width of the 2D state (at 4260 MeV) into charm meson pairs is 125 MeV. This is not the discovery mode for $Y(4260)$ but exceeds the reported total width. Therefore, this state does not have a conventional charmonium interpretation. One plausible explanation for this state is that it is a hybrid state [56,59]. The lattice calculations of Juge and collaborators [49,51] for the $Q\bar{Q}$ potentials with excited gluonic modes give a lowest multiplet ($H_1$ in their notation) consisting of a quark spin-singlet $1^{--}$ state and a spin-triplet set of states ($0^{-+}$, $1^{-+}$, and $2^{-+}$). These states would be degenerate in the absence of relativistic spin-dependent mass corrections. If this interpretation were correct, then analogous narrow states would be expected in the $b\bar{b}$ system.

## V. SUMMARY

The computed properties of the $2^3P_2$ and $3^1S_0 c\bar{c}$ levels are in rough agreement with the $Z(3930)$ and $X(3943)$ states observed at Belle. We await more statistics and confirmation of these results to make a definitive statement on the identification of these states.

It is extremely important to solidify what appears to be known about $X(3872)$, the best studied of the new states, beginning with its $1^{++}$ quantum numbers. We emphasize the need to confirm the $\gamma J/\psi$ decay mode, which fixes the $C = +$ assignment, and to determine the $X(3872) \to \pi^0\pi^0 J/\psi$ branching fraction.

Experiments should continue to search for additional narrow charmonium states in neutral combinations of charmed mesons and anticharmed mesons. The most likely candidates correspond to the $1^3D_3$, $1^3D_2$, and $1^3F_4$ levels [8]. The analysis we have carried out can be extended to the $b\bar{b}$ system, where it may be possible to see discrete threshold-region states as well. For any of these new states that are threshold bound states their mass is primarily determined by the positions of open-flavor channels. For example, if the $X(3872)$ arises through additional interaction in the two-meson sector, then in the $b\bar{b}$ system we would expect an analog state not far below $B^*\bar{B}$ threshold.

## ACKNOWLEDGMENTS

We thank our experimental colleagues for stimulating discussions. K. L.'s research was supported by the Department of Energy under Grant No. DE-FG02-91ER40676. Fermilab is operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the U.S. Department of Energy.

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