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CP asymmetry in $B^0(t) \to K_S\pi^0\gamma$ in the standard model

Benjamín Grinstein
Department of Physics, University of California at San Diego, La Jolla, California 92093, USA

Dan Pirjol
Center for Theoretical Physics, Massachusetts Institute for Technology, Cambridge, Massachusetts 02139, USA
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The time-dependent CP asymmetry in exclusive $B^0(t) \to K^{*0}\gamma$ decays has been proposed as a probe of new physics in B decays. Recently, this method was extended to radiative decays into multibody hadronic final states such as $B^0(t) \to K_S\pi^0\gamma$ and $B^0(t) \to \pi^+\pi^-\gamma$. The CP asymmetry in these decays vanishes to the extent that the photon is completely polarized. In the standard model, the photon emitted in $b \to s\gamma$ has high left-handed polarization, but right-handed contamination enters already at leading order in Λ/m_b even for vanishing light quark masses. We compute here the magnitude of this effect and the time-dependent CP asymmetry parameter $S_{K_S\pi^0\gamma}$. We find that the standard model can easily accommodate values of S as large as 10%, but a precise value cannot be obtained at present because of strong interactions uncertainties.

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I. INTRODUCTION

The standard model (SM) predicts that photons are predominantly left-handed in $b \to q\gamma$ $(q = s, d)$ decay (and right-handed in $\bar{b} \to \bar{q}\gamma$). In the presence of new physics this prediction can be changed, and a significant right-handed photon amplitude can appear in $b \to s\gamma$ decays. Several methods have been suggested for testing this prediction in radiative B decays [1,2].

One of these methods makes use of time-dependent CP violation in $B^0 \to f\gamma$ with f a CP eigenstate [1]. Since γ_L and γ_R cannot interfere, the time-dependent CP asymmetry

$$\frac{\Gamma[\bar{B}^0(t) \to f\gamma] - \Gamma[B^0(t) \to f\gamma]}{\Gamma[\bar{B}^0(t) \to f\gamma] + \Gamma[B^0(t) \to f\gamma]} = S_{f\gamma}\sin(\Delta m t) - C_{f\gamma}\cos(\Delta m t), \quad (1)$$

is sensitive to the ratio of the right-/left-handed photon amplitudes. These can be parametrized as

$$r_f e^{i\delta_f} = \eta_{\rm CP}(f)\frac{A(\bar{B}^0 \to f\gamma_R)}{A(\bar{B}^0 \to f\gamma_L)} \quad (2)$$

where $\eta_{\rm CP}(f)$ is the CP eigenvalue of the state f. This method has been extended in Ref. [3] also to decays into multibody final states, such as for example $B \to K_S\pi^0\gamma$.

Summing over the unobserved photon polarization, the CP violating parameter in Eq. (1) is given by

$$S_{f\gamma} = -\frac{2r_f}{1 + r_f^2}\cos\delta_f \sin 2\beta. \quad (3)$$

In the SM, it is usually assumed (incorrectly, see [9]) that $r_q \sim m_q/m_b$, which leads to a small CP asymmetry $S \sim 2\%$ in the $b \to s\gamma$ transition. We used in this estimate $\sin 2\beta = 0.685 \pm 0.032$ [4].

Measurements of the CP asymmetries in $B \to K^*\gamma$ were reported by *BABAR* [5] and BELLE [6], see Table I. In addition, these two collaborations measured the CP asymmetry in the nonresonant mode $S_{K_S\pi^0\gamma}$ in two different ways: (a) *BABAR* excludes the K^* resonance by integrating over the $K_S\pi^0$ invariant mass range $1.1\ \mathrm{GeV} < M_{K_S\pi^0} < 1.8\ \mathrm{GeV}$; (b) BELLE includes both resonant and nonresonant modes by integrating over the range $m_K + m_\pi \le M_{K_S\pi^0} \le 1.8$ GeV. The error in these determinations is still too large to allow a meaningful comparison with the SM prediction. With a view to improving the statistics of such measurements, we would like to assess the feasibility of combining the resonant and nonresonant measurements. Also, it is clear that searching for new physics with such measurements requires a reliable estimate of the standard model background.

At leading order in $1/m_b$, the photon emitted in $B \to K^*\gamma$ is always left-handed polarized, to all orders in α_s [9]. However, in multibody decays such as $B \to K_S\pi^0\gamma$ a right-handed component appears, already at leading order in $1/m_b$, in the kinematical region with an energetic kaon and a soft pion. This is mediated by a B^* pole diagram,

TABLE I. Experimental results for the CP asymmetry parameter S_f for $f = K^{*0}\gamma$ and $f = K_S\pi^0\gamma$ from *BABAR* and BELLE. The *BABAR* nonresonant region includes all states with $1.1\ \mathrm{GeV} < m_{K_S\pi^0} < 1.8$ GeV, while BELLE uses the range $1.0\ \mathrm{GeV} < m_{K_S\pi^0} < 1.8$ GeV. The errors shown are statistical and systematic, respectively.

f	S_f	
	BABAR	BELLE
$K^{*0}\gamma$	$-0.21 \pm 0.40 \pm 0.05$	$+0.01 \pm 0.52 \pm 0.11$
$[K_S\pi^0]_{\rm nonres}\gamma$	$0.9 \pm 1.0 \pm 0.2$	$+0.20 \pm 0.66$
$K_S\pi^0\gamma$	-0.06 ± 0.37	$+0.08 \pm 0.41 \pm 0.10$

with the $B^* \to K\gamma_R$ amplitude calculable at leading order in Λ/m_b using factorization in QCD as following from Soft Collinear Effective Theory (SCET) [20–22]. The presence of two hadrons in the final state evades the helicity argument which forbids a right-handed photon in $\bar{B} \to \bar{K}^*\gamma$. Note that a right-handed photon is present in inclusive $B \to X_s\gamma$ at leading order [9]. The right-handed photon couples to the charm quark loop induced by the 4-quark operator $O_2 = (\bar{s}c)(\bar{c}b)$, which gives equal rates for $b \to sg\gamma_L$ and $b \to sg\gamma_R$. In addition to this leading order effect, a significant right-handed photon amplitude in $B \to K^*\gamma$ can appear at subleading order in Λ/m_b from graphs with photon emission from the charm quark loop.

The purpose of this paper is to study in more detail the magnitude of the leading order effects described above on the time-dependent CP asymmetry in $B \to K_S\pi^0\gamma$. In Sec. II we introduce the effective theory formalism used in our computation. This is a combination of the soft-collinear effective theory (SCET) with the chiral perturbation theory recently proposed in Ref. [37]. In Sec. III the helicity amplitudes are written down, and used to compute decay distributions for $\bar{B} \to K_S\pi^0\gamma$ with a right-handed photon. Sec. IV gives the results for the time-dependent CP violation $S_{K_S\pi^0\gamma}$ in the kinematical region with an energetic kaon and a soft pion. This has a significant overlap with the region used in the BELLE and *BABAR* measurements of the time-dependent CP asymmetry into a nonresonant $K_S\pi^0$ state. Sec. V summarizes our results. Readers interested in the phenomenology of the decay can skip the formalism and proceed directly to Sec. III.

II. EFFECTIVE THEORY FORMALISM

The exclusive radiative decays $B \to K^*\gamma$ can be described in the large recoil region in factorization. At leading order in Λ/m_b with $\Lambda \sim 500$ MeV, the existence of such factorization relations has been demonstrated in [11–15] at lowest order in α_s, and proven to all orders in α_s using the soft-collinear effective theory [24,26–29].

The $b \to s\gamma$ transitions with an energetic s quark are mediated in $\mathrm{SCET_I}$ by the effective Lagrangian

$$H_{\rm eff} = N_0[m_b c(\omega)\bar{s}_{n,\omega}\mathcal{A}^\perp P_L b_v + b_{1L}(\omega_i)O^{(1L)}(\omega_i) + b_{1R}(\omega_i)O^{(1R)}(\omega_i) + \mathcal{O}(\lambda^2)], \quad (4)$$

with $N_0 = \frac{G_F V_{tb}V_{ts}^* e}{\sqrt{2}\pi^2}E_\gamma$. The relevant modes are soft quarks and gluons with momenta $k_s \sim \Lambda$ and collinear quarks and gluons along n [28]. We use everywhere the SCET notations in Ref. [25]. We choose the photon momentum to move along the $-\vec{e}_3$ direction $q_\mu = E_\gamma\bar{n}_\mu$, such that the hadronic system has a large momentum along the opposite direction n_μ. The hard scale in this problem is $Q \equiv \bar{n}\cdot p_X \sim m_b$, and the expansion parameter in SCET is $\lambda^2 \sim \Lambda/Q$. We denoted $n_\mu, \bar{n}_\mu$ unite light-cone vectors satisfying $n^2 = \bar{n}^2 = 0, n\cdot\bar{n} = 2$. The transverse photon field is $\mathcal{A}_\mu^\perp$, and its polarization vectors are $\varepsilon_+ = (0, \frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0)$ (right-handed photon) and $\varepsilon_- = (0, \frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}, 0)$ (left-handed photon).

We neglect here and in the following s quark mass effects, which can be included straightforwardly [23]. The photon coupling to the spectator quark in the B introduces new factorizable operators containing collinear modes along the photon momentum [28–30]. These spectator effects do not contribute to the right-handed photon amplitude, and we return to them below (see the discussion around Eq. (14)).

The first operator in Eq. (4) scales like $O(\lambda^0)$ and couples only to left-handed photons. The $O(\lambda)$ operators $O^{(1L,R)}$ couple to left- and right-handed photons, respectively, and are defined as

$$O^{(1L)}(\omega_1, \omega_2) = \bar{s}_{n,\omega_1}\mathcal{A}^\perp\Big[\frac{1}{\bar{n}\cdot\mathcal{P}}ig\mathcal{B}_n^\perp\Big]_{\omega_2}P_R b_v,$$
$$O^{(1R)}(\omega_1, \omega_2) = \bar{s}_{n,\omega_1}\Big[\frac{1}{\bar{n}\cdot\mathcal{P}}ig\mathcal{B}_n^\perp\Big]_{\omega_2}\mathcal{A}^\perp P_R b_v. \quad (5)$$

At lowest order in matching, the Wilson coefficients can be extracted from the computations of [12–15,19] and are given by [28]

$$c(\omega) = C_7^{\rm eff} + O(\alpha_s(m_b)) \quad (6)$$

$$b_{1L}(\omega_1, \omega_2) = C_7^{\rm eff} + \frac{2C_2}{3}\kappa\Big(\frac{-2E_\gamma\omega_2}{m_c^2}\Big) + \mathcal{O}[C_{3-6,8}, \alpha_s(m_b)],$$
$$b_{1R}(\omega_1, \omega_2) = -\frac{2C_2}{3}\kappa\Big(\frac{-2E_\gamma\omega_2}{m_c^2}\Big) + \mathcal{O}[C_{3-6,8}, \alpha_s(m_b)]. \quad (7)$$

with

$$C_7^{\rm eff} = C_7 - \frac{4}{9}C_3 - \frac{4}{3}C_4 + \frac{1}{9}C_5 + \frac{1}{3}C_6 \quad (8)$$

in the operator basis of Ref. [17]. Beyond tree level the Wilson coefficients c, b_{1L} and b_{1R} receive hard corrections $\sim\alpha_s(m_b)$ from charm loops [18] and from matching onto $\mathrm{SCET_I}$. The $O(\alpha_s(m_b))$ matching corrections are known only for $c(\omega)$ [21], so for consistency we do not include them in any of the coefficients. The function $\kappa(x)$ appears in the 3-point function with a charm loop, and is given by [19]

$$\kappa(x) = \begin{cases} \frac{1}{2} - \frac{2}{x}\arctan^2[\sqrt{\frac{x}{4-x}}] & x < 4 \\ \frac{1}{2} + \frac{2}{x}(\log(\frac{\sqrt{x}+\sqrt{x-4}}{2}) - \frac{\pi i}{2})^2 & x > 4 \end{cases} \quad (9)$$

In the Wilson coefficients of the $O(\lambda)$ operators b_{1L} and b_{1R} we neglect small contributions from the penguin operators O_{3-6} and the gluon dipole operator O_8.

After matching onto $\mathrm{SCET_{II}}$ [24], the $b \to s\gamma$ effective Lagrangian (4) contains both factorizable and nonfactorizable operators

$$H_{\rm eff} \to (O_\mu^{\rm nf} + O_\mu^{\rm fact} O_\mu^{\rm sp}) \mathcal{A}^\mu + \cdots \quad (10)$$

where the ellipses stand for higher dimension operators. The details of this matching are given in Refs. [24–26], and we give here only the points essential in the following.

The nonfactorizable operators couple only to the left-handed photon field, $\varepsilon_+^{*\mu} O_\mu^{\rm nf} = 0$, while the factorizable operators couple to both left- and right-handed photons. Working at tree level in matching $\mathrm{SCET_I} \to \mathrm{SCET_{II}}$, the factorizable operators are

$$\begin{aligned} O_\mu^{\rm fact} = N_0 \bigg[& -\frac{1}{2\omega} \int_0^1 {\rm d}z {\rm d}x {\rm d}k_+ b_{1L}(z) J_\perp(x,z,k_+) \\ & \times (\bar{q}_{k_+} \not{n} \gamma_\mu^\perp \gamma_\perp^\lambda P_R b_v) \Big(\bar{s}_{n,\omega_1} \frac{\bar{\not{n}}}{2} \gamma_\lambda^\perp q_{n,\omega_2} \Big) \\ & - \frac{1}{2\omega} \int_0^1 {\rm d}z {\rm d}x {\rm d}k_+ b_{1R}(z) J_\parallel(x,z,k_+) \\ & \times (\bar{q}_{k_+} \not{n} \gamma_\mu^\perp P_R b_v) \Big(\bar{s}_{n,\omega_1} \frac{\bar{\not{n}}}{2} P_L q_{n,\omega_2} \Big) \bigg] \end{aligned} \quad (11)$$

where we used a momentum space notation for the soft nonlocal operators

$$\bar{q}^{\,i}_{k_+} b_v^j = \int \frac{d\lambda}{4\pi} e^{-(i/2)\lambda k_+} \bar{q}^i(\lambda n/2) Y_n(\lambda, 0) b_v^j(0). \quad (12)$$

The functions $b_{1L}(z)$ and $b_{1R}(z)$ appearing here are related to the Wilson coefficients in Eq. (4) as $b_i(z) = b_i((1-z)\omega, z\omega)$. The momentum labels of the collinear fields are parametrized as $\omega_1 = x\omega$, $\omega_2 = -(1-x)\omega$. We denoted here with $J_{\perp,\parallel}$ jet functions defined as in Ref. [33]. They have perturbative expansions in $\alpha_s(\mu_c)$ with $\mu_c^2 \sim Q\Lambda$. At leading order they are given by

$$J_\parallel(x,z,k_+) = J_\perp(x,z,k_+) = \frac{\pi\alpha_s(\mu_c) C_F}{N_c} \frac{1}{\bar{x} k_+} \delta(x-z) \quad (13)$$

The $O(\alpha_s^2)$ corrections to the jet functions have been recently obtained in Ref. [27].

Another class of factorizable operators not present in Eq. (11) arise from the photon coupling to the spectator quarks [28–30]. (Photon coupling to the final state quarks leads to power suppressed operators [31].) After matching onto $\mathrm{SCET_{II}}$, they are given at leading order in $\alpha_s(m_b)$ by

$$\begin{aligned} O_\mu^{\rm sp} = & \frac{G_F}{2\sqrt{2}} e \sum_{q=u,d,s} b_{\rm sp}(\omega_i) \\ & \times \int dk_- J_{\rm sp}(k_-) e_q (\bar{q}_{k_-} \gamma_\mu \bar{\not{n}} \not{n} P_L b_v) \\ & \times \Big(\bar{s}_{n,\omega_1} \frac{\bar{\not{n}}}{2} P_L q_{n,\omega_2} \Big) \end{aligned} \quad (14)$$

with $b_{\rm spec}(\omega_i) = V_{ub} V_{us}^*(C_1 + C_2/N_c)\delta_{qu} - V_{tb}V_{ts}^*(C_4 + C_3/N_c) + O(\alpha_s(m_b))$ and

$$J_{\rm sp}(k_-) = \frac{1}{k_- + i\epsilon} \Big(1 + \frac{\alpha_s C_F}{4\pi} \Big(L^2 - 1 - \frac{\pi^2}{6} \Big) \Big) \quad (15)$$

with $L = \log[(-2E_\gamma k_- - i\epsilon)/\mu^2]$ a jet function known to $O(\alpha_s(\mu_c))$ [32]. The spectator operator couples only to left-handed photons. For consistency with the other factorizable operators included, we work to $O(\alpha_s^0(Q))$, but keep terms of $O(\alpha_s(\mu_c))$ in the factorized amplitude. The dominant term $\sim C_{1,2}$ contributes only to $\bar{B}^0 \to K^- \pi^+ \gamma$, but not to $\bar{B}^0 \to K_S \pi^0 \gamma$. In Eqs. (6) we neglected the contributions from $O_{3-6,8}$ to the Wilson coefficients $c(\omega)$, $b_{1L,1R}(\omega_i)$, so for consistency we neglect such terms also in Eq. (14).

The SCET formalism introduced above has been applied to prove factorization relations for exclusive semileptonic $B \to M\ell\nu$ and radiative $B \to M\gamma$, $M\ell^+\ell^-$ decays into one energetic light hadron, with M a light pseudoscalar or vector meson [9,24–29]. In all these cases, the transition matrix element is written as the sum of a soft (nonfactorizable) and hard-scattering (factorizable) terms, as follows.

The matrix elements of the nonfactorizable operators are parametrized in terms of soft form factors. In our calculation we require only the matrix element

$$\langle K^*(p,\eta) | \bar{s}_n \not{\varepsilon}_-^* P_L b_v | \bar{B}(v) \rangle = (\varepsilon_-^* \cdot \eta^*) \bar{n} \cdot p_{K^*} \zeta_\perp^{BK^*} \quad (16)$$

where we use the $\mathrm{SCET_I}$ notation for the operators obtained from them by matching onto $\mathrm{SCET_{II}}$.

The matrix elements of the factorizable operators $O_\mu^{\rm fact}$ given in Eq. (11) are given by convolutions of soft and collinear matrix elements with the Wilson coefficients. The matrix elements are parametrized in terms of light-cone wave functions of the B and $K^{(*)}$ mesons [28]. In the $\bar{B} \to \bar{K}^*$ transition, only the left-handed photon amplitude is nonvanishing [9,28], and is given by

$$\begin{aligned} H_-^{\rm fact}(\bar{B} \to \bar{K}^* \gamma_L) &= \langle \bar{K}^* \gamma_L | \mathcal{H}_{\rm eff} | \bar{B} \rangle \\ &= N_0 m_b m_B \int_0^1 {\rm d}z b_{1L}(z) \zeta_{J\perp}^{BK^*}(z) \end{aligned} \quad (17)$$

where the factorizable function $\zeta_{J\perp}^{BK^*}(z)$ is defined as

$$\zeta_{J\perp}^{BK^*}(z) = \frac{f_B f_{K^*}^\perp}{m_B} \int dx dk_+ J_\perp(x,z,k_+) \phi_+^B(k_+) \phi_{K^*}^\perp(x) \quad (18)$$

In Ref. [37] it was proposed to extend the application of this formalism also to multibody B decays to final states containing one energetic meson and one soft hadron. We summarize briefly the main points of this approach, before proceeding with the details of the computation.

The matrix elements of the nonfactorizable operators is parametrized in a manner similar to Eq. (16) in terms of a new soft function

$$H_-^{\rm nf}(\bar B \to K_{\rm n}\pi\gamma_L) = N_0 m_b c(\bar n\cdot {\rm p}_{K^*})\zeta_\perp^{BK\pi}(E_K, E_\pi).$$

This nonfactorizable amplitude couples only to left-handed photons, just as in the case of the $B\to K^*\gamma$ transition. Furthermore, the soft function $\zeta_\perp^{BK\pi}$ is related by a symmetry relation to a similar function appearing in multibody semileptonic decay $B\to \pi\pi\ell\bar\nu$ [37], analogous to the appearance of a common nonfactorizable amplitude ζ^{BM} in both rare and semileptonic form factors [10,11,24].

The matrix elements of the factorizable operators in Eq. (11) are also given by convolutions of hard, jet and soft factors, as in the case of the $B\to K^*$ transition discussed above. At leading order in Λ, Q, soft and collinear modes decouple in the SCET Lagrangian [22], which is the statement of soft-collinear factorization. This fact has two important implications. First, the soft pion does not couple to the collinear meson in the final state M. Second, the matrix elements of factorizable operators corresponding to the transition $B\to M_n\pi$ factor as

$$\langle K_n\pi|O_{\rm fact}|\bar B\rangle = \langle K_n|O_{\rm C}|0\rangle\langle\pi|O_S|\bar B\rangle,$$

and are calculable in terms of the kaon light-cone wave functions and a new soft matrix element of the O_S operator in the $B\to\pi$ transition.

The soft operator O_S required here appears in the b_{1R} factorizable operator. We define it as

$$O_S(k_+) = \int\frac{d\lambda}{4\pi}e^{(-i/2)\lambda k_+}\bar q\left(\lambda\frac{n}{2}\right)Y_n\left(\lambda\frac{n}{2},0\right)\not\epsilon_+\frac{\not n}{2}P_R b_v(0).$$

Its $B\to\pi$ matrix element is parametrized in terms of one soft function $S(k_+, t^2, \zeta)$, defined as

$$\langle\pi(p_\pi)|O_S(k_+)|\bar B(v)\rangle = -2(\varepsilon_+\cdot p_\pi)S(k_+, t^2, \zeta)$$

with $t = m_B v - p_\pi$ and $\zeta = n\cdot p_\pi/(n\cdot v)$. The support of this function is $-n\cdot p_\pi \le k_+ \le \infty$. This is the analog for B physics of the generalized parton distributions (GPD), commonly encountered in nucleon physics.

The complete factorization relation for the right-handed photon amplitude can now be written down as

$$\begin{aligned} &H_+(\bar B\to\bar K\pi\gamma_R)\\ &\quad = 2N_0 f_K(\varepsilon_+\cdot p_\pi)\int_0^1 {\rm d}z{\rm d}x b_{1R}(z)\\ &\qquad\times\int_{-p_\pi^+}^\infty {\rm d}k_+ J_\parallel(x,z,k_+)S(k_+,\zeta)\phi_K(x),\end{aligned}$$

The predictive power of such relations depends on the existence of reliable information about the soft function S. Eventually the function S should be extracted using B decays data, or constrained by lattice QCD computations. In the soft pion region, the soft function S is fixed by chiral symmetry in terms of one of the B meson light-cone wave functions [37]. We will use in this paper the result for S predicted at leading order in chiral perturbation theory.

The predictions of chiral symmetry for the couplings of Goldstone bosons are most conveniently derived using chiral perturbation theory. The applicability of this approach is limited to problems describing only soft hadrons. The extension to heavy hadrons is possible, provided that the large scale m_b is eliminated by going over to HQET. The corresponding chiral effective theory is the heavy hadron chiral perturbation theory (HHChPT), and its degrees of freedom are heavy meson spin doublets $H = (B, B^*)$ and the Goldstone bosons [34–36].

The effective Lagrangian that describes the strong interactions of the Goldstone bosons with the ground state heavy mesons is [34–36]

$$\begin{aligned}\mathcal{L} &= \frac{f_\pi^2}{8}{\rm Tr}[\partial^\mu\Sigma\partial_\mu\Sigma^\dagger] - i{\rm Tr}[\bar H^a v_\mu\partial^\mu H_a]\\ &\quad + \frac{i}{2}{\rm Tr}[\bar H^a H_b]v^\mu[\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger]_{ab}\\ &\quad + \frac{1}{2}ig{\rm Tr}[\bar H^a H_b\gamma_\nu\gamma_5][\xi^\dagger\partial^\nu\xi - \xi\partial^\nu\xi^\dagger]_{ab} + \cdots \end{aligned}\tag{19}$$

where the ellipsis denote light quark mass terms, $O(1/m_b)$ operators associated with the breaking of heavy quark spin symmetry, and terms of higher order in the derivative expansion. The pseudo-Goldstone bosons appear in the Lagrangian through $\xi = e^{i\Pi/f_\pi}$ ($\Sigma = \xi^2$) where

$$\Pi = \begin{pmatrix} \frac{1}{\sqrt2}\pi^0 + \frac{1}{\sqrt6}\eta & \pi^+ & K^+\\ \pi^- & -\frac{1}{\sqrt2}\pi^0 + \frac{1}{\sqrt6}\eta & K^0\\ K^- & \bar K^0 & -\frac{2}{\sqrt6}\eta\end{pmatrix}\tag{20}$$

with the pion decay constant $f_\pi \simeq 135$ MeV. These fields transform as

$$\xi\to L\xi U^\dagger = U\xi R^\dagger \tag{21}$$

under chiral $SU(3)_L\times SU(3)_R$ transformations. The superfield H_a contains the pseudoscalar and vector heavy meson fields $\bar B_a$ and $\bar B^*_{a\mu}$ with velocity label v_μ

$$H_a = \frac{1+\not v}{2}[\bar B^*_{a\mu}\gamma^\mu - \bar B_a\gamma_5].\tag{22}$$

The flavor index runs over $a = 1, 2, 3$ corresponding to $\bar B_a = (B^-, \bar B^0, \bar B_s)$. Under chiral $SU(3)_L\times SU(3)_R$, the superfield H_a transforms as

$$H_a\to H_b U^\dagger_{ba}.\tag{23}$$

The numerical value of the coupling $g = 0.5 \pm 0.1$ is taken to cover a range compatible with its determination from $D^*\to D\pi$ decays $g = 0.59\pm 0.08$ [39] and lattice QCD $g = 0.48\pm 0.03\pm 0.11$ [40], $g = 0.42\pm 0.04\pm 0.08$ [41]

We consider next the matrix element $\langle\pi|O_S|\bar B\rangle$ in chiral perturbation theory. This requires the chiral representation of the nonlocal soft operators O_S. Consider light-cone nonlocal heavy-light bilinears of the form

$$O^a_{L,R}(k_+) = \int \frac{dx_-}{4\pi} e^{-\frac{i}{2}k_+x_-} \bar{q}^a(x_-)Y_n(x_-,0)P_{R,L}\Gamma b_v(0). \tag{24}$$

Under the chiral group they transform as $(\bar{\mathbf{3}}_L, \mathbf{1}_R)$ and $(\mathbf{1}_L, \bar{\mathbf{3}}_R)$, respectively. For each case, there is a unique operator in the effective theory with the correct transformation properties [37]

$$O^a_L(k_+) = \frac{i}{4}\mathrm{Tr}[\hat{\alpha}_L(k_+)P_R\Gamma H_b \xi^\dagger_{ba}], \tag{25}$$

$$O^a_R(k_+) = \frac{i}{4}\mathrm{Tr}[\hat{\alpha}_R(k_+)P_L\Gamma H_b \xi_{ba}] \tag{26}$$

The common matrix $\hat{\alpha}_L(k_+) = \hat{\alpha}_R(k_+) = \hat{\alpha}(k_+)$ is given by

$$\hat{\alpha}(k_+) = f_B\sqrt{m_B}[\bar{\not{n}}\not{n}\phi^B_+(k_+) + \not{n}\bar{\not{n}}\phi^B_-(k_+)] \tag{27}$$

where $\phi^B_\pm(k_+)$ are the usual light-cone wave functions of a B meson, defined by [11]

$$\int \frac{dz_-}{2\pi} e^{-(i/2)k_+z_-}\langle 0|\bar{q}_i(z_-)Y_n(z_-,0)b^j_v(0)|\bar{B}(v)\rangle = -\frac{i}{4}f_Bm_B\left\{\frac{1+\not{v}}{2}[\bar{\not{n}}n\cdot v\phi^B_+(k_+) + \not{n}\bar{n}\cdot v\phi^B_-(k_+)]\gamma_5\right\}_{ji} \tag{28}$$

The operators in Eqs. (25) and (26) with (27) can be used to compute the matrix elements of $O_{L,R}$ on states with a B meson and any number of pseudo-Goldstone bosons. In particular, they give the following prediction for the soft function $S(k_+, t^2, \zeta)$defined by the $B \to \pi$ matrix element of $O_S(k_+)$ at leading order in the chiral expansion [37].

$$S(k_+, t^2, \zeta) = \frac{gf_Bm_B}{f_\pi}\frac{1}{v\cdot p_\pi + \Delta}\phi^B_+(k_+).$$

We will use this result in the numerical evaluations of this paper. No information can be obtained using chiral symmetry about the S function for $-n\cdot p_\pi < k_+ < 0$. For soft pions this contribution is likely to be small, so it will be neglected in the following.

Finally, we comment briefly on previous applications [38] of the HHChPT to B decays into multibody final states containing soft Goldstone bosons. These applications involve the generalization to B^* decays of the factorization formula for nonleptonic B decays, with the B^* appearing in intermediate states of pole diagrams. The usual HHChPT [29–31] methods are applied to compute the pion coupling in both pole diagrams and in contact diagrams.

There are several issues with such a simplified approach: (i) the application of chiral perturbation theory to the nonfactorizable operators $O_{\rm nf}$ contributing to the $B \to \pi$ transition with an energetic pion is problematic. Since these operators couple to both soft and collinear modes, loop corrections to their matrix elements do not have a well-behaved power counting. In our approach these contributions are simply parametrized by new soft functions $\zeta^{BK\pi}_\perp$, which are related by symmetry relations to similar matrix elements appearing in other processes. (ii) the pion contact terms, such as that in Fig. 2(b), can be computed only for the factorizable operators, (but not for the entire weak vertex), and are given by factorization relations.

In the next section we will combine the pieces of the factorizable amplitudes, add in the nonfactorizable amplitude and write down the complete result for multibody $B \to K\pi\gamma$ amplitudes.

III. HELICITY AMPLITUDES AND DECAY RATES

We will use the formalism described in Sec. II to compute the amplitude for the decays $\bar{B} \to K_S\pi^0\gamma$ and $\bar{B} \to K^-\pi^+\gamma$ in the kinematical region with one energetic (collinear) kaon and one soft pion. To establish the region of validity of our computation, we show in Fig. 1 the phase space for this decay, in variables $(M_{K\pi}, E_\pi)$, with $M^2_{K\pi} = (p_K + p_\pi)^2$.

We distinguish three distinct regions for the pion and kaon energies in $\bar{B} \to K\pi\gamma$ decay (see Fig. 1):
(I) $E_\pi \sim \Lambda$, $E_K \sim Q$
(II) $E_\pi \sim Q$, $E_K \sim Q$
(III) $E_\pi \sim Q$, $E_K \sim \Lambda$

These three regions are treated differently in the SCET, and the heavy quark mass scaling of the decay amplitudes is correspondingly different in each of them, as follows.

The region (I) contains a soft pion and an energetic kaon. Part of this region, but not all, can be treated using the SCET + ChPT combination considered in this paper. We

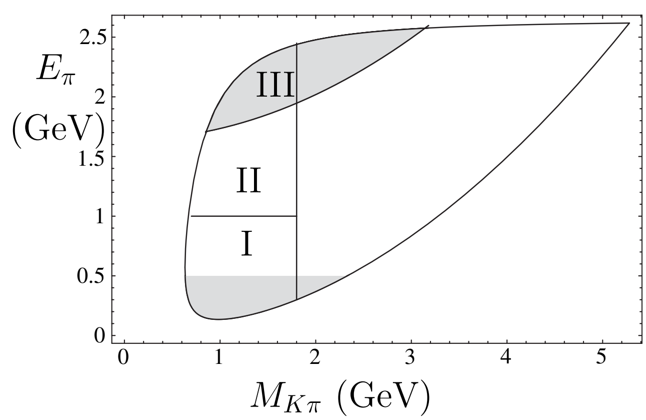

FIG. 1. The phase space of the $\bar{B}^0 \to K_S\pi^0\gamma$ decay in variables $(M_{K\pi}, E_\pi)$. The vertical line denotes the maximum $K\pi$ invariant mass used in the *BABAR* and BELLE measurements. The 3 regions shown correspond to (I) soft pion $E_\pi \sim \Lambda$; the shaded region $E_\pi \le 0.5$ GeV (implying $E_K > 2.18$ GeV) shows the region of applicability of ChPT; (II) collinear pion and kaon $E_\pi \sim Q$, $E_K > 1$ GeV; (III) soft kaon $E_K < 1$ GeV (implying $E_\pi > 1.7$ GeV).

subdivide it into the soft pion region with $E_\pi < 0.5$ GeV, where chiral perturbation theory is valid, and the intermediate pion region $0.5\ \text{GeV} < E_\pi < 1.0 - 1.5$ GeV (which we will call, for lack of a better name, the hard-soft pion region).

The region (II) includes collinear pion and kaons. In general this configuration can have the kaon and pion momenta moving in different directions forming a large angle in the B rest frame $\theta_{\pi K} \sim O(1)$. In our case, the experimental constraint $M_{K\pi} < 1.8$ GeV forces the angle to be small $\theta_{\pi K} \sim O(\Lambda/Q)$ (valid for $M_{K\pi}^2 \sim \Lambda Q$), such that the π, K constituent partons can be described by collinear fields with a common n.

The region (III) contains an energetic pion and a soft kaon $E_K \sim \Lambda$, $E_\pi \sim Q$. This region is not described by the leading order SCET operators in Sec. II, and the corresponding amplitudes are suppressed by at least one power of Λ/Q relative to those in regions (I) and (II).

We start by writing down the helicity amplitudes for the $B \to K^*\gamma$ decay at leading order in Λ/m_b, by combining the partial results in Sec. II. At this order, only the left-handed photon amplitude is nonvanishing

$$H_+(\bar{B} \to \bar{K}^*\gamma_R) = 0 \tag{29}$$

$$\begin{aligned} H_-(\bar{B} \to \bar{K}^*\gamma_L) &= N_0 m_b m_B (c(m_B)\zeta_\perp^{BK^*} \\ &\quad + \int_0^1 \mathrm{d}z b_{1L}(z)\zeta_{J\perp}^{BK^*}(z)) \\ &\equiv N_0 m_b m_B C_7^{\text{eff}} g_+^{\text{eff}}(0). \end{aligned} \tag{30}$$

We defined here the effective tensor form factor $g_+^{\text{eff}}(0)$, which absorbs the contributions of the operators other than O_7. Similar factorization relations are expected to hold also for the $\bar{B} \to K\pi\gamma$ transition in region (II), with the K^* light-cone wave function replaced by a two-body $K\pi$ light-cone wave function. We do not pursue this further here, but note only that the vanishing of the right-handed photon amplitude at LO observed in Eq. (29) should hold also in the multibody case. This follows from the vanishing of the $\bar{B} \to 0$ matrix element of the soft operator in Eq. (11) multiplying b_{1R}.

The amplitudes for the multibody transition $\bar{B} \to K_n\pi\gamma$ in region (I) are given at leading order by the graphs in Fig. 2

$$\text{(I)}: H_+(\bar{B} \to \bar{K}\pi\gamma_R) = N_0 \frac{1}{2} m_B^2 S_R(p_\pi) \int_0^1 dz b_{1R}(z) \zeta_{J\parallel}^{BK}(z) \tag{31}$$

$$H_-(\bar{B} \to \bar{K}\pi\gamma_L) = N_0 \bar{n} \cdot p_K c(m_B) \zeta_\perp^{BK\pi}(E_K, E_\pi) \tag{32}$$

The nonfactorizable operators O_{nf}^μ contribute only to the left-handed photon amplitude, and the corresponding matrix element is parametrized by $\zeta_\perp^{BK\pi}$. On the other hand, the factorizable operators O_{fact}^μ contribute only to the right-handed photon amplitude. (We consider only $\bar{B}^0 \to K_S\pi^0\gamma$ decays, for which the spectator factorizable operators O_{spec} do not contribute in the approximation used here of neglecting O_{3-6}.)

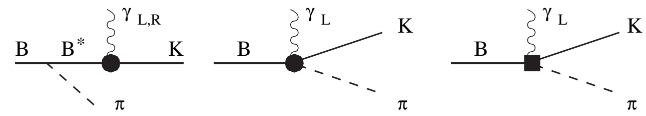

FIG. 2. Diagrams showing leading order contributions to the decay $\bar{B} \to K\pi\gamma$ with one collinear kaon and a soft pion. The filled circle in (a) and (b) represents a factorizable operator O_{fact}, while the filled square in (c) represents a nonfactorizable operator O_{nf}.

Note the appearance of a nonvanishing right-handed photon amplitude at leading order in the region (I). This amplitude is factorizable and can be computed as explained in Sec. II. The HHChPT diagrams required for its computation are shown in Fig. 2(a). The factorizable function $\zeta_{J\parallel}^{BK}(z)$, appearing here is defined in analogy with the function in Eq. (18), with the replacements $f_{K^*}^\perp \to f_K$, $\phi_{K^*}^\perp(x) \to \phi_K(x)$, $J_\perp \to J_\parallel$. The dependence on the pion momentum is contained in the soft functions $S_R(p_\pi)$ given by

$$S_R(p_\pi) = \frac{g}{f_\pi} \frac{\varepsilon_+ \cdot p_\pi}{E_\pi + \Delta} \tag{33}$$

with $\Delta = m_{B^*} - m_B = 50$ MeV.

Finally, in the kinematical region (III) with one soft kaon and an energetic pion, the effective Lagrangian equation (4) does not apply. The leading SCET_I operator mediating such a transition contains the $\bar{s}\Gamma b_v$ soft current, with at least two insertions of the soft-collinear Lagrangian, acting on the spectator quark

$$T\{(\bar{s}\Gamma b_v), i\mathcal{L}_{q\xi}^{(1)}, i\mathcal{L}_{q\xi}^{(1)}\} \tag{34}$$

This is suppressed by at least $\lambda^2 \sim \Lambda/Q$ relative to the operators in Eq. (4), which implies that the decay amplitudes in this region must be power suppressed relative to those in regions (I) and (II).

We summarize the different contributions enumerated above by showing them in graphical form in Fig. 2. We emphasize our different treatment of the factorizable and nonfactorizable operators: the matrix elements of the factorizable operators are computed in chiral perturbation theory, and include the B^* pole and contact terms [Fig. 2 and 2(b)]. The matrix element of the nonfactorizable operator [Fig. 2(c)] is parametrized in terms of a new soft function $\zeta_\perp^{BK\pi}$. This new soft function appears only in the left-handed photon amplitude, and is related by symmetry relations to a similar soft function which can be determined in principle from $\bar{B} \to \pi_n\pi\ell\bar{\nu}$ [37].

The only region where the right-handed photon amplitude $\bar{B}^0 \to K_S\pi^0\gamma$ contributes at leading order in Λ/Q is the region (I) with a soft pion. In region (II) the right-handed amplitude is suppressed by Λ/Q relative to the left-handed amplitude, and in region (III) both amplitudes are suppressed by at least Λ/Q. The contribution of the region (I) with a hard-soft pion $0.5\ \mathrm{GeV} < E_\pi < 1.0 - 1.5\ \mathrm{GeV}$ will be estimated by assuming the validity of HHChPT in the entire region (I). We proceed to compute the right-handed photon effect in region (I) on the decay rates and time-dependent CP asymmetry.

The left-handed photon amplitude $H_-(\bar{B} \to \bar{K}\pi\gamma_L)$ in Eq. (30) (region (I)) does not include the K^* pole contribution, although this likely dominates numerically in the resonant region $M_{K\pi} \sim M_{K^*}$. This contribution is parametrically suppressed by Λ/m_b in the soft pion kinematical region. The reason for this is that by soft-collinear factorization at leading order in Λ/m_b, the coupling of a soft pion to two collinear hadrons, $K_n^* K_n \pi_S$, must be power suppressed. On the other hand, the K^* pole contribution is numerically enhanced by the K^* propagator, so we will include it in our computation, despite being formally of higher order in Λ/m_b relative to the latter.

In the absence of data on $\zeta_\perp^{BK\pi}$, we will model it by a K^* pole contribution. We introduce the following model for the $\bar{B}^0 \to \bar{K}\pi\gamma$ decay amplitudes in the kinematical region (I) with one soft pion and an energetic kaon (this is similar to the model used in Ref. [42]). In the left-handed photon amplitude we neglect the nonresonant contribution and keep only the K^* pole term (Fig. 2(c))

$$H_-^{\mathrm{model}}(\bar{B} \to K^-\pi^+\gamma_L) = H_-^{BK^*} g_{K^*K\pi}(\varepsilon_- \cdot p_\pi)\mathrm{BW}_{K^*}(M_{K\pi}) \tag{35}$$

with $H_-^{BK^*} = A(\bar{B} \to K^*\gamma_L)$ the 1-body helicity amplitude given in Eq. (32), and $\mathrm{BW}_{K^*}(M_{K\pi}) = [M_{K\pi}^2 - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}]^{-1}$ the Breit-Wigner function for a K^* resonance. The amplitude for $\bar{B}^0 \to K_S\pi^0\gamma_L$ is given by Eq. (35) multiplied by $1/2$. The $K^{*0}K^+\pi^-$ coupling with a charged pion can be extracted from the total $K^* \to K\pi$ decay width, $\Gamma = g_{K^*K\pi}^2 p_\pi^3/(16\pi m_{K^*}^2)$, which gives $g_{K^*K\pi} = 9.1$.

The right-handed photon amplitude is given by the sum of the K^* and B^* resonant terms

$$\begin{aligned} H_+^{\mathrm{model}}&(\bar{B}^0 \to K^-\pi^+\gamma_R) \\ &= N_0 \frac{g m_B^2}{2f_\pi}\frac{\varepsilon_+ \cdot p_\pi}{E_\pi + \Delta}\int_0^1 dz b_{1R}(z)\zeta_J^{BK}(z) \\ &\quad + H_+^{BK^*} g_{K^*K\pi}(\varepsilon_+ \cdot p_\pi)\mathrm{BW}_{K^*}(M_{K\pi}) \end{aligned} \tag{36}$$

The $\bar{B}^0 \to K_S\pi^0\gamma_R$ amplitude has an additional factor of $1/2$. We included here also a nonvanishing resonant $\bar{B} \to K^*\gamma_R$ right-handed photon amplitude, which is introduced by a nonvanishing strange quark mass, and by power suppressed contributions neglected in Eq. (31). We will parametrize it as

$$\frac{H_+^{BK^*}}{H_-^{BK^*}} = \frac{m_s}{m_b} + h_s e^{i\phi_s} \tag{37}$$

The potentially leading mechanism contributing to h_s has been identified in Ref. [9], and it arises from charm loops coupling to the B and K^* through soft gluons. Although no first principles calculation of this parameter is yet available, it can be estimated from a simple power counting argument as

$$h_s \sim \frac{1}{3}\frac{C_2}{C_7}\frac{\Lambda}{m_b} \sim 0.09 \tag{38}$$

In our numerical evaluation we will use $h_s = 5\%$, keeping in mind that this estimate is on the lower side of the dimensional estimate.

In the remainder of the paper we will use the model described by Eqs. (35)–(37) to compute distributions and decay rates for the $B \to K_S\pi^0\gamma$ decay. We start by computing the right-handed photon rate; although not directly observable, this quantity will illustrate the relative importance of the different mechanisms contributing to the wrong-helicity photon amplitude.

The $\bar{B} \to \bar{K}\pi\gamma$ decay rate is given by

$$\frac{d^2\Gamma(B \to K\pi\gamma)}{dE_\pi dM_{K\pi}^2} = \frac{1}{2(4\pi)^3 m_B^2}(|H_+|^2 + |H_-|^2) \tag{39}$$

where $H_\pm$ are given in Eqs. (35) and (36), respectively. In the limit of a very narrow K^* the integrations over $(M_{K\pi}, E_\pi)$ can be performed exactly, and the well-known result for the $\bar{B} \to \bar{K}^*\gamma$ rate is recovered

$$\begin{aligned}\int dE_\pi dM_{K\pi}^2 \frac{d^2\Gamma(\bar{B} \to \bar{K}\pi\gamma_L)}{dE_\pi dM_{K\pi}^2} &= \frac{E_\gamma^{(0)}}{8\pi m_B^2}|H_-(\bar{B} \to K^*\gamma_L)|^2 \\ &\equiv \Gamma_0 \end{aligned} \tag{40}$$

Here $E_\gamma^{(0)} = (m_B^2 - m_{K^*}^2)/(2m_B)$ denotes the photon energy corresponding to the 2-body kinematics.

It is convenient to express the $\bar{B} \to K^-\pi^+\gamma_R$ decay rate by normalizing it to the $\bar{B} \to K^*\gamma_L$ decay rate (a factor of $1/4$ has to be added for $\bar{B}^0 \to K_S\pi^0\gamma_R$)

$$\begin{aligned}\frac{1}{\Gamma_0}\frac{d^2\Gamma_R}{dM_{K\pi}^2 dE_\pi} = \frac{|\vec{p}_\pi^{\perp}|^2}{2(4\pi)^2 E_\gamma^{(0)}}\bigg| &\frac{E_\gamma}{E_\gamma^{(0)}}\frac{g\kappa e^{i\phi}}{2f_\pi(E_\pi + \Delta)} \\ &+ \frac{g_{K^*K\pi}(\frac{m_s}{m_b} + h_s e^{i\phi_s})}{M_{K\pi}^2 - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}}\bigg|^2 \end{aligned} \tag{41}$$

with $M_{K\pi}^2 = m_B^2 - 2m_B E_\gamma$. The two terms give the contributions of the B^* resonant pole, and that of the K^* resonant right-handed amplitude. The hadronic dynamics in the B^* resonant contribution enters through the RG invariant ratio

$$\begin{aligned}\kappa e^{i\phi} &= \frac{m_B}{m_b C_7^{\mathrm{eff}} g_+^{\mathrm{eff}}(0)}\int_0^1 dz b_{1R}(z)\zeta_J^{BK}(z) \\ &= -0.013 - 0.045i \end{aligned} \tag{42}$$

In the numerical evaluation of this parameter we used the tree level result for the jet functions Eq. (13) and the lowest

TABLE II. Input parameters used in the numerical computation, and results for the effective Wilson coefficients and factorizable matrix elements. The values of the effective Wilson coefficients are quoted at the scale $\mu = 4.8$ GeV. The strange quark mass is taken from [41].

$m_b^{\rm pole}$	4.8 GeV	C_2	1.107
$m_c(m_c)$	1.4 GeV	C_7	-0.343
$m_s(2\ {\rm GeV})$	78 ± 10 MeV	f_B	200 MeV
$g_+^{\rm eff}(0)$	0.3	f_K	170 MeV
$\langle k_+^{-1}\rangle_B^{-1}$	350 MeV	$g_{K^*K\pi}$	9.1
g	0.5 ± 0.1	$\kappa e^{i\phi}$	$-0.013 - 0.045i$

order matching result for b_{1R} from Eq. (6). The remaining required parameters are listed in Table II.

The result equation (41) can be used to compute the energy spectrum and integrated rate with a right-handed photon, with an upper cut-off on the pion energy. The interference of the two terms depends sensitively on the unknown strong phase ϕ_s. For this reason, we will give only an upper bound on this rate, obtained by assuming that the two terms have the same strong phase and interfere constructively. The resulting photon energy spectrum and its components are shown in Fig. 3, for the central values of the parameters. For completeness, we quote also the fraction of events which survive a pion energy cut $n_{\rm cut}(E_\pi^{\rm max})$. This can be computed from the K^* pole contribution to $H_-(\bar B \to \bar K\pi\gamma)$ and is: $n_{\rm cut}(0.5\ {\rm GeV}) = 11.4\%$, $n_{\rm cut}(1\ {\rm GeV}) = 50.5\%$, $n_{\rm cut}(1.5\ {\rm GeV}) = 88.4\%$.

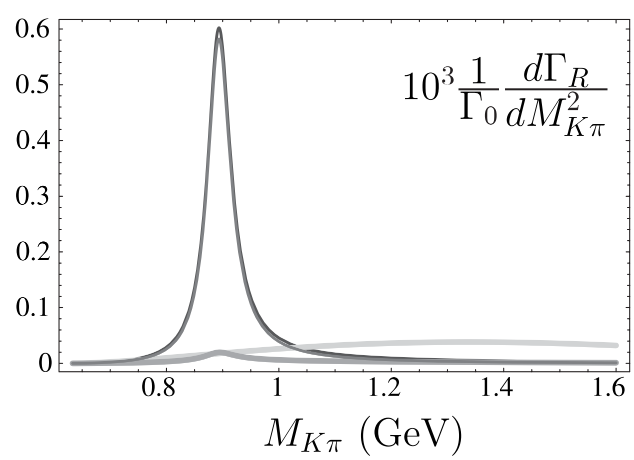

FIG. 3. The decay rate ($\times 10^3$) with a right-handed photon $\bar B^0 \to K_S\pi^0\gamma_R$, normalized to the $B \to K^*\gamma_L$ rate, with a cut on the pion energy $E_\pi \le 0.5$ GeV, and its components, computed as described in the text. The black curve gives an upper bound on the total decay rate. The dark gray curve shows the $B^* - K^*$ interference term, and the light gray curve shows the contribution of the B^* pole graph (magnified by a factor of 100 relative to the other curves).

IV. TIME-DEPENDENT *CP* ASYMMETRY

We compute in this section the mixing-induced CP violating parameter $S_{K_S\pi^0\gamma}$ in the standard model. We start by defining the time-independent amplitudes

$$\bar A_L = H(\bar B^0 \to K_S\pi^0\gamma_L) \tag{43}$$

$$\bar A_R = H(\bar B^0 \to K_S\pi^0\gamma_R) \tag{44}$$

$$A_L = H(B^0 \to K_S\pi^0\gamma_L) \tag{45}$$

$$A_R = H(B^0 \to K_S\pi^0\gamma_R) \tag{46}$$

Since the $b \to s$ transition is CP conserving, there are relations among these amplitudes, such that only two of them are independent. We choose the independent amplitudes to be $\bar A_L, \bar A_R$, and obtain the remaining two amplitudes from them by CP transformations. Charge conjugation exchanges particles and antiparticles, and parity takes $\gamma_L \leftrightarrow \gamma_R$ and changes the directions of momenta. We apply a rotation by 180° around the x axis, which restores the momenta to their original directions. The effect of the rotation is to multiply the amplitudes with $\Pi_i \pi_i(-)^{J_i} = +1$ (with $J_i^{\pi_i}$ the spin-parity of the particles), and exchange $\varepsilon_+ \leftrightarrow \varepsilon_-$. This gives

$$A_L = \frac{\varepsilon_+\cdot p_\pi}{\varepsilon_-\cdot p_\pi}\bar A_R, \qquad A_R = \frac{\varepsilon_-\cdot p_\pi}{\varepsilon_+\cdot p_\pi}\bar A_L \tag{47}$$

The time-dependent differential rate has a form similar to Eq. (40) (with $i = L, R$)

$$\frac{d^2\Gamma(B^0(t) \to K_S\pi^0\gamma_i)}{dE_\pi dM^2_{K\pi}} = \frac{1}{2(4\pi)^3 m_B^2}(|A_i|^2 + |\bar A_i|^2) \times \frac{1}{2}e^{-\bar\Gamma t}\{1 + C_i\cos\Delta mt - S_i\sin\Delta mt\} \tag{48}$$

with

$$C_i(E_\pi, M_{K\pi}) = \frac{|A_i|^2 - |\bar A_i|^2}{|A_i|^2 + |\bar A_i|^2} \tag{49}$$

$$S_i(E_\pi, M_{K\pi}) = 2\frac{{\rm Im}(e^{-2i\beta}\bar A_i A_i^*)}{|A_i|^2 + |\bar A_i|^2} \tag{50}$$

From this expression, results for the CP violating coefficients integrated over parts of the phase space can be straightforwardly obtained.

The BELLE and *BABAR* Collaborations measured the S and C parameters integrated over all E_π and a range of $M_{K\pi}$. We compute the SM values of these parameters, by integrating the time-dependent distribution Eq. (48) with appropriate cuts. Applicability of the chiral perturbation theory computation of the $\bar A_R$ amplitude requires that we restrict the pion energy in the B rest frame by $E_\pi < E_\pi^{\rm max}$, with $E_\pi^{\rm max} = 500$ MeV.

Integrating over E_π with an upper cut-off $E_\pi^{\max}$, and summing over the photon polarizations gives the mixing-induced CP asymmetry parameter $S_{K_S\pi^0\gamma}(M_{K\pi})$

$$S_{K_S\pi^0\gamma}(M_{K\pi}) = -2\sin2\beta\bigg\{\frac{m_s}{m_b} + h_s\cos\phi_s + \frac{gI(M_{K\pi})}{2f_\pi g_{K^*K\pi}}\frac{E_\gamma}{E_\gamma^{(0)}}[(M_{K\pi}^2 - M_{K^*}^2)\mathrm{Re}\kappa - M_{K^*}\Gamma_{K^*}\mathrm{Im}\kappa]\bigg\} \quad (51)$$

The first two terms represent the resonant $B \to K^*\gamma$ effect, and the last term is the nonresonant contribution. The dependence on the pion energy cut-off is contained in

$$I(M_{K\pi}, E_\pi^{\max}) = \frac{\int^{E_\pi^{\max}} dE_\pi \frac{|\vec{p}_\pi^{\perp}|^2}{E_\pi+\Delta}}{\int^{E_\pi^{\max}} dE_\pi|\vec{p}_\pi^{\perp}|^2} \quad (52)$$

We show in Fig. 4 results for the $S_{K_S\pi^0\gamma}$ parameter as a function of $M_{K\pi}$, integrated with an upper pion energy cut-off $E_\pi^{\max} = 0.5$ GeV. We used in this computation the central value for $\sin2\beta = 0.685 \pm 0.032$ as measured in the charmonium system [5]. The effect of the nonresonant contribution is to introduce a mild dependence of the asymmetry on $M_{K\pi}$.

Finally, we integrate also over $M_{K\pi} = (m_K + m_\pi, 1.8)$ GeV to obtain the inclusive CP asymmetry parameter (for an upper pion energy cut)

$$S_{K_S\pi^0\gamma} = -2\sin2\beta\bigg\{\frac{m_s}{m_b} + h_s\cos\phi_s + \frac{g}{2f_\pi g_{K^*K\pi}}\mathrm{Re}[\kappa I_2(E_\pi^{\max})]\bigg\} \quad (53)$$

where the phase space factor $I_2(E_\pi^{\max})$ is defined as

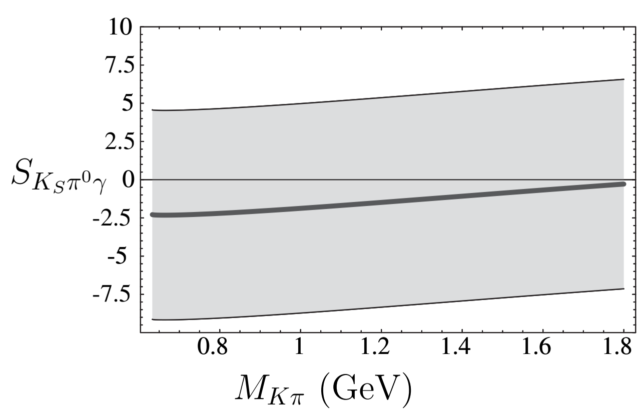

FIG. 4. The time-dependent CP asymmetry parameter $S_{K_S\pi^0\gamma}$ (in percent) as a function of $M_{K\pi}$. The three lines correspond to (from bottom to top): $h_s\cos\phi_s = -0.05, 0, 0.05$. We used here $E_\pi^{\max} = 0.5$ GeV.

$$I_2(E_\pi^{\max}) = \frac{\int dM_{K\pi}^2 \mathrm{BW}_{K^*}^*(M_{K\pi})\frac{E_\gamma}{E_\gamma^{(0)}}\int^{E_\pi^{\max}} dE_\pi\frac{|\vec{p}_\pi^{\perp}|^2}{E_\pi+\Delta}}{\int dM_{K\pi}^2|\mathrm{BW}_{K^*}(M_{K\pi})|^2\int^{E_\pi^{\max}} dE_\pi|\vec{p}_\pi^{\perp}|^2} = 0.20 + 0.11i \text{ GeV}, \quad (54)$$

and the numerical value corresponds to $E_\pi^{\max} = 0.5$ GeV. As mentioned, for pion energies above $1.0 - 1.5$ GeV, the right-handed photon amplitude is power suppressed, so it can be expected to be numerically small. We estimate the contribution from the hard-soft region 0.5 GeV $< E_\pi < 1.0 - 1.5$ GeV by assuming the validity of the low energy expression for the decay amplitudes over this range. Taking $E_\pi^{\max} = 1.5$ GeV replaces the numerical value in Eq. (54) with $I_2 = 0.14 + 0.05i$ GeV. In both cases discussed above, the contribution of the nonresonant (third) term in the braces in Eq. (53) is less than 0.5%, and is thus negligible.

We neglected in this computation the presence of higher K^* resonances. The Particle Data Book [43] lists four K^* resonances in the region $m_K + m_\pi \le M_{K\pi} \le 1.8$ GeV, which can appear in the $K_S\pi^0$ invariant mass spectrum (with quantum numbers $J^P = 1^-, 2^+, 3^-, \ldots$). Their inclusion does not change the leading order nonresonant right-handed photon amplitude computed here, but introduces additional power suppressed effects similar to those parametrized by (h_s, ϕ_s). In the narrow width limit, their effect is to replace $h_s\cos\phi_s$ in Eq. (53) with

$$h_s\cos\phi_s \to \frac{1}{1+\sum_i x_i}(h_s\cos\phi_s + \sum_i x_i h_s^i\cos\phi_s^i) \quad (55)$$

with $x_i = Br(B \to K_i^*\gamma)Br(K_i^* \to K\pi)/Br(B \to K^*\gamma)$, and (h_s^i, ϕ_s^i) new parameters for $B \to K_i^*\gamma$ defined analogously to Eq. (40). Of the four kaon resonances contributing to the sum, only two of them decay into $K\pi$ with a branching fraction larger than 30%: $K_2^*(1430)$ and $K^*(1680)$, with $x_i = 0.15$ and 0.01, respectively. This shows that the contributions of the higher kaon resonances are likely very small and can be neglected.

Our results demonstrate that the nonresonant contribution to the mixing-induced CP asymmetry is negligibly small, and the SM contamination is dominated by the right-handed photon amplitude in $\bar{B} \to K^*\gamma_R$, parameterized by $h_s\cos\phi_s$. Our results show that averaging the nonresonant and resonant measurements of the S parameter is a justified procedure.

V. CONCLUSION

We studied in this paper the standard model prediction for the mixing-induced CP asymmetry parameter in $B^0 \to K_S\pi^0\gamma$ decay. This decay is important as a probe for new physics manifested through a right-handed photon in $b \to s\gamma$ decay. The naive expectation [1] for the S parameter in the SM is $S = -2\sin2\beta(m_s/m_b) \sim 2\%$. We computed the corrections to this prediction introduced by strong interaction effects.

In the kinematical region with $M_{K_S\pi^0} \sim M_{K^*}$ and a soft pion in the rest frame of the B meson, there is a unique SM mechanism contributing to the S parameter at leading order in Λ/m_b, arising from the B^* pole diagrams. These effects are factorizable and calculable using a combination of SCET and heavy hadron chiral perturbation theory [37]. In addition, power suppressed effects can introduce a potentially sizeable contamination from nonfactorizable graphs with the photon coupling to the charm quark loop [9]. These are difficult to compute in a reliable way, and a simple power counting estimate allows a right-handed photon amplitude as large as ~9%.

We performed a detailed numerical study of the nonresonant effects. We find that the leading order B^* pole effect is numerically small. It introduces a weak dependence of the CP asymmetry $S_{K_S\pi^0\gamma}$ on $M_{K\pi}$. The dominant SM contamination is from power suppressed effects in the $\bar{B} \to K^*\gamma_R$ resonant amplitude, and our best estimate in the $M_{K\pi}$ dependent asymmetry is $|S^{\rm SM}_{K_S\pi^0\gamma}| \leq 8\%$ (see Fig. 4).

When integrated over $M_{K\pi}$, the nonresonant effect is practically negligible, and the CP asymmetry $S_{K_S\pi^0\gamma}$ is dominated by the resonant $\bar{B} \to \bar{K}^*\gamma_R$ amplitude. This means that averaging the results of the resonant and nonresonant measurements, as currently done at B factories, is a justified procedure. If improved measurements of the CP asymmetry confirm the present average $|S| \sim 8\%$, this would be consistent with a power suppressed correction in the SM. We reiterate that the naive estimate $S \sim -2(m_s/m_b)\sin 2\beta$ seriously underestimates the value of the S parameter in the SM. Furthermore, one would also expect the agreement between resonant and nonresonant measurements to improve.

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[1] D. Atwood, M. Gronau, and A. Soni, Phys. Rev. Lett. **79**, 185 (1997).
[2] Y. Grossman and D. Pirjol, J. High Energy Phys. 06 (2000) 029; M. Gronau, Y. Grossman, D. Pirjol, and A. Ryd, Phys. Rev. Lett. **88**, 051802 (2002); G. Hiller and A. Kagan, Phys. Rev. D **65**, 074038 (2002); M. Gronau and D. Pirjol, Phys. Rev. D **66**, 054008 (2002); M. Knecht and T. Schietinger, hep-ph/0509030.
[3] D. Atwood, T. Gershon, M. Hazumi, and A. Soni, Phys. Rev. D **71**, 076003 (2005).
[4] Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag/.
[5] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **93**, 201801 (2004); B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **72**, 051103 (2005).
[6] Y. Ushiroda *et al.*, Phys. Rev. Lett. **94**, 231601 (2005); K. Abe *et al.* (BELLE Collaboration), hep-ex/0507059.
[7] A. G. Akeroyd *et al.* (SuperKEKB Physics Working Group Collaboration), hep-ex/0406071.
[8] K. Abe *et al.* (Belle Collaboration), hep-ex/0411056.
[9] B. Grinstein, Y. Grossman, Z. Ligeti, and D. Pirjol, Phys. Rev. D **71**, 011504 (2005).
[10] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Phys. Rev. D **60**, 014001 (1999); G. Burdman and G. Hiller, Phys. Rev. D **63**, 113008 (2001).
[11] M. Beneke and T. Feldmann, Nucl. Phys. **B592**, 3 (2001).
[12] A. Ali and A. Y. Parkhomenko, Eur. Phys. J. C **23**, 89 (2002).
[13] M. Beneke, T. Feldmann, and D. Seidel, Nucl. Phys. **B612**, 25 (2001).
[14] S. W. Bosch and G. Buchalla, Nucl. Phys. **B621**, 459 (2002)
[15] S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. **B693**, 103 (2004).
[16] C. D. Lu, M. Matsumori, A. I. Sanda, and M. Z. Yang, Phys. Rev. D **72**, 094005 (2005).
[17] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
[18] C. Greub, T. Hurth, and D. Wyler, Phys. Rev. D **54**, 3350 (1996); A. J. Buras, A. Czarnecki, M. Misiak, and J. Urban, Nucl. Phys. **B611**, 488 (2001).
[19] N. Pott, Phys. Rev. D **54**, 938 (1996).
[20] C. W. Bauer, S. Fleming, and M. E. Luke, Phys. Rev. D **63**, 014006 (2001).
[21] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001).
[22] C. W. Bauer, D. Pirjol, and I. W. Stewart, Phys. Rev. D **65**, 054022 (2002).
[23] A. K. Leibovich, Z. Ligeti, and M. B. Wise, Phys. Lett. B **564**, 231 (2003).
[24] C. W. Bauer, D. Pirjol, and I. W. Stewart, Phys. Rev. D **67**, 071502 (2003)
[25] D. Pirjol and I. W. Stewart, Phys. Rev. D **67**, 094005 (2003); **69**, 019903 (2004); D. Pirjol and I. W. Stewart, hep-ph/0309053.
[26] M. Beneke and T. Feldmann, Nucl. Phys. **B685**, 249 (2004); B. O. Lange and M. Neubert, Nucl. Phys. **B690**, 249 (2004).
[27] R. J. Hill, T. Becher, S. J. Lee, and M. Neubert, J. High Energy Phys. 07 (2004) 081.
[28] J. g. Chay and C. Kim, Phys. Rev. D **68**, 034013 (2003).
[29] T. Becher, R. J. Hill, and M. Neubert, Phys. Rev. D **72**,

094017 (2005).
[30] B. Grinstein and D. Pirjol, Phys. Rev. D **62**, 093002 (2000).
[31] A. Hardmeier, E. Lunghi, D. Pirjol, and D. Wyler, Nucl. Phys. **B682**, 150 (2004).
[32] E. Lunghi, D. Pirjol, and D. Wyler, Nucl. Phys. **B649**, 349 (2003).
[33] C. W. Bauer, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D **70**, 054015 (2004).
[34] M. B. Wise, Phys. Rev. D **45**, R2188 (1992).
[35] G. Burdman and J. F. Donoghue, Phys. Lett. B **280**, 287 (1992).
[36] T. M. Yan *et al.*, Phys. Rev. D **46**, 1148 (1992); **55**, 5851 (1997).
[37] B. Grinstein and D. Pirjol, Phys. Lett. B **615**, 213 (2005).
[38] N. G. Deshpande, G. Eilam, X. G. He, and J. Trampetic, Phys. Rev. D **52**, 5354 (1995); S. Fajfer, R. J. Oakes, and T. N. Pham, Phys. Rev. D **60**, 054029 (1999); H. Y. Cheng and K. C. Yang, Phys. Rev. D **66**, 054015 (2002); H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D **72**, 094003 (2005).
[39] S. Ahmed *et al.* (CLEO Collaboration), Phys. Rev. Lett. **87**, 251801 (2001).
[40] A. Abada, D. Becirevic, P. Boucaud, G. Herdoiza, J. P. Leroy, A. Le Yaouanc, and O. Pene, J. High Energy Phys. 02 (2004) 016.
[41] S. Hashimoto and T. Onogi, Annu. Rev. Nucl. Part. Sci. **54**, 451 (2004); S. Hashimoto, Annu. Rev. Nucl. Part. Sci. **54**, 451 (2004).
[42] B. Grinstein and D. Pirjol, hep-ph/0505155.
[43] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).