Broken flavor $2 \leftrightarrow 3$ symmetry and phenomenological approach for universal quark and lepton mass matrices

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A phenomenological approach for the universal mass matrix model with a broken flavor $2 \leftrightarrow 3$ symmetry is explored by introducing the $2 \leftrightarrow 3$ antisymmetric parts of mass matrices for quarks and charged leptons. We present explicit texture components of the mass matrices, which are consistent with all the neutrino oscillation experiments and quark mixing data. The mass matrices have a common structure for quarks and leptons, while the large lepton mixings and the small quark mixings are derived with no fine-tuning due to the difference of the phase factors. The model predicts a value 2.4×10^{-3} for the lepton mixing matrix element square $|U_{13}|^2$, and also $\langle m_\nu \rangle = (0.89 - 1.4) \times 10^{-4}$ eV for the averaged neutrino mass which appears in the neutrinoless double beta decay.

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I. INTRODUCTION

It has been established through the discovery of neutrino oscillation [1] that neutrinos have finite masses and mix one another with near bimaximal lepton mixings ($\sin^2 2\theta_{12} \sim 1$, $\sin^2 2\theta_{23} \simeq 1$) which are in contrast to small quark mixings. In order to explain the large lepton mixing and small quark mixing, mass matrix models with various structures have been investigated in the literature [2–12]. For example, it is argued that the large lepton mixing can be explained by mass matrices with a flavor $2 \leftrightarrow 3$ symmetry [13–28]. We think that quarks and leptons should be unified. Therefore, it is interesting to investigate a possibility that all the mass matrices of the quarks and leptons have the same matrix form, which leads to large lepton mixings and small quark mixings. The mass matrix model with the universal form for quarks and leptons is also useful when it is embedded into a grand unified theory (GUT).

In this paper, we discuss a Hermit mass matrix model with a universal form given by

$$M = \begin{pmatrix} 0 & ae^{-i\phi} & ae^{-i\phi''} \\ ae^{i\phi} & b & ce^{-i\phi'} \\ ae^{i\phi''} & ce^{i\phi'} & b \end{pmatrix}, \tag{1.1}$$

where a, b, and c are real parameters and ϕ, ϕ', and ϕ'' are phase parameters. It is important from a phenomenological point of view to parametrize the texture components of the mass matrix as the first step to make a GUT scenario. Assuming that neutrinos are the Majorana particles, we present the texture components of the universal mass matrices which will lead to the Cabibbo–Kobayashi–Maskawa (CKM) [29] quark mixing and the Maki–Nakagawa–Sakata (MNS) [30] lepton mixing which are consistent with the present experimental data. Here we explore a phenomenological mass matrix model base on the flavor $2 \leftrightarrow 3$ symmetry. Our mass matrices have a broken flavor $2 \leftrightarrow 3$ symmetry for quarks and charged leptons by introducing the $2 \leftrightarrow 3$ antisymmetric parts of their mass matrices. We assume that this broken flavor $2 \leftrightarrow 3$ symmetry is due to the **120** Higgs scalar in the SO(10) GUT model, while mass matrices contributed from **10** and **126** Higgs scalars are $2 \leftrightarrow 3$ symmetric.

This article is organized as follows. In Sec. II, our mass matrix model is presented. In Sec. III, we discuss the diagonalization of the mass matrix of our model. The analytical expressions of the quark and lepton mixings of the model are given in Sec. IV. Section V is devoted to a summary.

II. MASS MATRIX MODEL

In this paper, we propose the following mass matrices:

$$M_u = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}e^{-i\phi_u}A_u & \frac{1}{\sqrt{2}}e^{-i\phi_u}A_u \\ \frac{1}{\sqrt{2}}e^{i\phi_u}A_u & \frac{B_u+D_u}{2} & \frac{B_u-D_u}{2} \\ \frac{1}{\sqrt{2}}e^{i\phi_u}A_u & \frac{B_u-D_u}{2} & \frac{B_u+D_u}{2} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & iC_u \\ 0 & -iC_u & 0 \end{pmatrix}, \tag{2.1}$$

$$M_d = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}e^{-i\phi_d}A_d & \frac{1}{\sqrt{2}}e^{-i\phi_d}A_d \\ \frac{1}{\sqrt{2}}e^{i\phi_d}A_d & \frac{B_d+D_d}{2} & \frac{B_d-D_d}{2} \\ \frac{1}{\sqrt{2}}e^{i\phi_d}A_d & \frac{B_d-D_d}{2} & \frac{B_d+D_d}{2} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & iC_d \\ 0 & -iC_d & 0 \end{pmatrix}, \tag{2.2}$$

$$M_e = \begin{pmatrix} 0 & \frac{1}{2}A_e & \frac{1}{2}A_e \\ \frac{1}{2}A_e & \frac{B_e+D_e}{2} & -C_e \\ \frac{1}{2}A_e & -C_e & \frac{B_e+D_e}{2} \end{pmatrix} + \begin{pmatrix} 0 & -i\frac{1}{2}A_e & i\frac{1}{2}A_e \\ i\frac{1}{2}A_e & 0 & i\frac{B_e-D_e}{2} \\ -i\frac{1}{2}A_e & -i\frac{B_e-D_e}{2} & 0 \end{pmatrix}, \tag{2.3}$$

$$M_\nu = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}A_\nu & \frac{1}{\sqrt{2}}A_\nu \\ \frac{1}{\sqrt{2}}A_\nu & \frac{B_\nu-D_\nu}{2} & \frac{B_\nu+D_\nu}{2} \\ \frac{1}{\sqrt{2}}A_\nu & \frac{B_\nu+D_\nu}{2} & \frac{B_\nu-D_\nu}{2} \end{pmatrix}, \tag{2.4}$$

$$M_D = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}A_D & \frac{1}{\sqrt{2}}A_D \\ \frac{1}{\sqrt{2}}A_D & \frac{B_D+D_D}{2} & \frac{B_D-D_D}{2} \\ \frac{1}{\sqrt{2}}A_D & \frac{B_D-D_D}{2} & \frac{B_D+D_D}{2} \end{pmatrix}, \tag{2.5}$$

$$M_R = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}A_R & \frac{1}{\sqrt{2}}A_R \\ \frac{1}{\sqrt{2}}A_R & \frac{B_R+D_R}{2} & \frac{B_R-D_R}{2} \\ \frac{1}{\sqrt{2}}A_R & \frac{B_R-D_R}{2} & \frac{B_R+D_R}{2} \end{pmatrix}, \tag{2.6}$$

where M_u, M_d, M_e, and M_ν are mass matrices for up-quarks (u, c, t), down-quarks (d, s, b), charged leptons (e, μ, τ), and neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$, respectively. The mass matrices M_D and M_R are, respectively, the Dirac and the right-handed Majorana type neutrino mass matrices, from which with the seesaw mechanism [31] we derive M_ν. Here A_f, B_f, C_f, and D_f are real parameters and ϕ_u and ϕ_d are phase parameters with $f = u, d, e$, and ν.

Let us mention a particular feature of these mass matrices with respect to the flavor $2 \leftrightarrow 3$ symmetry. We assume that the neutrino mass matrix has only a $2 \leftrightarrow 3$ symmetric part. In the mass matrices for quarks and charged leptons, the $2 \leftrightarrow 3$ antisymmetric terms (the second terms) are added as broken $2 \leftrightarrow 3$ symmetric parts, in addition to the $2 \leftrightarrow 3$ symmetric terms (the first terms). This structure is motivated by the SO(10) GUT model in which **10**, **120**, and **126** Higgs scalars contribute to the fermion mass matrices, together with the following assumptions: (i) The contribution from the **120** Higgs scalar is $2 \leftrightarrow 3$ antisymmetric, while those from **10** and **126** Higgs scalars are $2 \leftrightarrow 3$ symmetric for quarks and charged leptons. (ii) There exists the contribution to the Dirac type neutrino mass matrix M_D from only the **10** and **126** Higgs scalars. and (iii) The texture components of the broken $2 \leftrightarrow 3$ symmetric parts are assumed to have different forms between quarks and charged leptons, which derives a difference between the small quark mixing and the large lepton mixing. Namely, we assume that the mass matrices M_u and M_d are superpositions of the common real symmetric matrices S and S' and pure imaginary antisymmetric one $\mathcal{A}$ and that M_e, M_D, and M_R consist of the common real symmetric matrices S'' and S''' and pure imaginary antisymmetric one $\mathcal{A}'$, as follows.

$$M_u = \alpha_u S + \beta_u S' + \gamma_u \mathcal{A}, \tag{2.7}$$

$$M_d = \alpha_d S + \beta_d S' + \gamma_d \mathcal{A}, \tag{2.8}$$

$$M_e = \alpha_e S'' + \beta_e S''' + \gamma_e \mathcal{A}', \tag{2.9}$$

$$M_D = \alpha_D S'' + \beta_D S''', \tag{2.10}$$

$$M_R = \beta_R S''', \tag{2.11}$$

$$M_\nu = -M_D^T M_R^{-1} M_D, \tag{2.12}$$

where the matrices S, S', S'' and S''' are $2 \leftrightarrow 3$ symmetric too, and $\mathcal{A}$ and $\mathcal{A}'$ are $2 \leftrightarrow 3$ antisymmetric too. Here α_i, β_i, γ_i $(i = u, d, e)$, α_D, β_D, and β_R are real coefficient parameters. Note that the $2 \leftrightarrow 3$ symmetry of the model is broken through only $\mathcal{A}$ in the quark sector and $\mathcal{A}'$ in the lepton sector.

Some semiempirical approaches for mass matrices with the similar structure to the above Eqs. (2.7), (2.8), (2.9), (2.10), (2.11), and (2.12) have been proposed in the literature. For example, Gronau, Johnson, and Schechter [4] have discussed a model which consists of combining the Fritzch [2] and Stech [3] ansatz for quarks. They use the combination of the symmetric mass matrix with an antisymmetric one, although they do not use the $2 \leftrightarrow 3$ symmetry. An extension to leptons based on a SO(10) GUT model has been investigated with use of the type I and type II seesaw mechanism for neutrino masses [7,8]. In the present paper, we use the $2 \leftrightarrow 3$ symmetry for a common origin of the small quark and the large lepton mixings. This is the large difference between our model and the other $2 \leftrightarrow 3$ symmetry models [13–21].

The mass matrix M_f $(f = u, d, e$, and $\nu)$ given in Eqs. (2.1), (2.2), (2.3), and (2.4) has a common structure when it is expressed with a unitary matrix Q_f as follows:

$$\begin{aligned} M_f &= Q_f \widehat{M}_f Q_f^\dagger, \quad \text{for} \quad f = u, d, \quad \text{and } e, \\ M_f &= Q_f \widehat{M}_f Q_f^T, \quad \text{for} \quad f = \nu \end{aligned} \tag{2.13}$$

where $\widehat{M}_f$ $(f = u, d, e$, and $\nu)$ is one of the seesaw-invariant type of mass matrix defined by [32]

$$\widehat{M}_f = \begin{pmatrix} 0 & A_f & 0 \\ A_f & B_f & C_f \\ 0 & C_f & D_f \end{pmatrix}. \tag{2.14}$$

Here the unitary matrices Q_f are given by

$$Q_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}e^{i\phi_u} & \frac{1}{\sqrt{2}}ie^{i\phi_u} \\ 0 & \frac{1}{\sqrt{2}}e^{i\phi_u} & -\frac{1}{\sqrt{2}}ie^{i\phi_u} \end{pmatrix}, \tag{2.15}$$

$$Q_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}e^{i\phi_d} & \frac{1}{\sqrt{2}}ie^{i\phi_d} \\ 0 & \frac{1}{\sqrt{2}}e^{i\phi_d} & -\frac{1}{\sqrt{2}}ie^{i\phi_d} \end{pmatrix}, \tag{2.16}$$

$$Q_e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}e^{i\pi/4} & \frac{1}{\sqrt{2}}ie^{i\pi/4} \\ 0 & \frac{1}{\sqrt{2}}e^{-i\pi/4} & -\frac{1}{\sqrt{2}}ie^{-i\pi/4} \end{pmatrix}, \tag{2.17}$$

$$Q_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}i \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}i \end{pmatrix}. \tag{2.18}$$

Note that the structure of Q_f mentioned above is the same for all the quarks and leptons except for the phase factors in it. It should be also noted that Eq. (2.13) implies that the mass matrix M_f is transformed to $\widehat{M_f}$ by using a rebasing of the quark and lepton fields, respectively.

III. DIAGONALIZATION OF THE MASS MATRIX

We now discuss a diagonalization of the mass matrix M_f given in Eq. (2.13). First let us discuss the diagonalization of the mass matrix $\widehat{M_f}$ given in Eq. (2.14), which appears as a part of M_f. This $\widehat{M_f}$ is diagonalized by an orthogonal matrix O_f as discussed in Refs. [23,24];

$$O_f^T \begin{pmatrix} 0 & A_f & 0 \\ A_f & B_f & C_f \\ 0 & C_f & D_f \end{pmatrix} O_f = \begin{pmatrix} -m_{1f} & & \\ & m_{2f} & \\ & & m_{3f} \end{pmatrix}. \tag{3.1}$$

Here m_{1f}, m_{2f}, and m_{3f} are eigenvalues of M_f. Explicit expressions of the orthogonal matrix O_f, and components A_f, B_f, C_f, and D_f in terms of m_{1f}, m_{2f}, and m_{3f} are presented in Appendix A. Namely, the mass matrix M_f is diagonalized as

$$U_{Lf}^\dagger M_f U_{Lf} = \begin{pmatrix} -m_{1f} & & \\ & m_{2f} & \\ & & m_{3f} \end{pmatrix} \quad \text{for } f = u, d, \quad \text{and } e, \tag{3.2}$$

$$U_{Lf}^\dagger M_f U_{Lf}^* = \begin{pmatrix} -m_{1f} & & \\ & m_{2f} & \\ & & m_{3f} \end{pmatrix} \quad \text{for } f = \nu. \tag{3.3}$$

where the unitary matrix U_{Lf} is given by

$$U_{Lf} = Q_f O_f. \tag{3.4}$$

Here we list the expressions for O_f and Q_f in order:

$$O_f \simeq \begin{pmatrix} 1 & \sqrt{\frac{m_{1f}}{m_{2f}}} & \sqrt{\frac{m_{1f}m_{2f}^2}{m_{3f}^3}} \\ -\sqrt{\frac{m_{1f}}{m_{2f}}} & 1 & \sqrt{\frac{m_{1f}}{m_{3f}}} \\ \sqrt{\frac{m_{1f}^2}{m_{2f}m_{3f}}} & -\sqrt{\frac{m_{1f}}{m_{3f}}} & 1 \end{pmatrix} \quad \text{for } f = u, d, \quad \text{and } e, \tag{3.5}$$

$$O_\nu = \begin{pmatrix} \sqrt{\frac{m_2}{m_2+m_1}} & \sqrt{\frac{m_1}{m_2+m_1}} & 0 \\ -\sqrt{\frac{m_1}{m_2+m_1}} & \sqrt{\frac{m_2}{m_2+m_1}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.6}$$

and

$$Q_f = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}e^{i\phi_f} & \frac{1}{\sqrt{2}}ie^{i\phi_f} \\ 0 & \frac{1}{\sqrt{2}}e^{i\phi_f} & -\frac{1}{\sqrt{2}}ie^{i\phi_f} \end{pmatrix} \quad \text{for } f = u \quad \text{and} \quad d, \tag{3.7}$$

$$Q_e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}e^{i\pi/4} & \frac{1}{\sqrt{2}}ie^{i\pi/4} \\ 0 & \frac{1}{\sqrt{2}}e^{-i\pi/4} & -\frac{1}{\sqrt{2}}ie^{-i\pi/4} \end{pmatrix}, \tag{3.8}$$

$$Q_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}i \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}i \end{pmatrix}. \tag{3.9}$$

Here, m_{iu}, m_{id}, m_{ie}, and $m_{i\nu}(i = 1, 2, 3)$ are, respectively, the masses of up-quarks, down-quarks, charged leptons, and neutrinos, which we shall denote as (m_u, m_c, m_t), (m_d, m_s, m_b), (m_e, m_μ, m_τ) and (m_1, m_2, m_3).

Furthermore, the neutrino mass matrix is diagonalized as

$$U_{L\nu}^{\prime\dagger} M_f U_{L\nu}^{\prime *} = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}, \tag{3.10}$$

where the unitary matrix $U'_{L\nu}$ is given by

$$U'_{L\nu} = U_{L\nu} P_\nu = Q_f O_f P_\nu. \tag{3.11}$$

Here, in order to make the neutrino masses to be real positive, we introduced a diagonal phase matrix P_ν defined by

$$P_\nu = \mathrm{diag}(i, 1, 1). \tag{3.12}$$

IV. CKM QUARK AND MNS LEPTON MIXING MATRICES

Next we discuss the CKM quark mixing matrix V and the MNS lepton mixing matrix U of the model, which are given by

$$V = U_{Lu}^\dagger U_{Ld} = O_u^T Q_u^\dagger Q_d O_d, \tag{4.1}$$

$$U = U_{Le}^{\dagger} U_{L\nu}' = O_e^T Q_e^{\dagger} Q_\nu O_\nu P_\nu. \tag{4.2}$$

From Eqs. (3.7), (3.8), and (3.9), we obtain

$$Q_u^{\dagger} Q_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i(\phi_d - \phi_u)} & 0 \\ 0 & 0 & e^{i(\phi_d - \phi_u)} \end{pmatrix}, \tag{4.3}$$

$$Q_e^{\dagger} Q_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{4.4}$$

It should be noted that $Q_e^{\dagger} Q_\nu$ takes quite a different structure from that of $Q_u^{\dagger} Q_d$ in our model. Namely, $Q_u^{\dagger} Q_d$ is a diagonal phase matrix, while $Q_e^{\dagger} Q_\nu$ represents a mixing matrix with a maximal lepton mixing between the second and third generations. Therefore, the large lepton mixing is realized with no fine-tuning in our model.

Let us discuss the quark and lepton mixing matrices in detail.

A. CKM quark mixing matrix

We obtain the CKM quark mixing matrix V as follows:

$$V = O_u^T Q_u^{\dagger} Q_d O_d \tag{4.5}$$

$$= \begin{pmatrix} 1 & \sqrt{\frac{m_u}{m_c}} & \sqrt{\frac{m_u m_c^2}{m_t^3}} \\ -\sqrt{\frac{m_u}{m_c}} & 1 & \sqrt{\frac{m_u}{m_t}} \\ \sqrt{\frac{m_u^2}{m_c m_t}} & -\sqrt{\frac{m_u}{m_t}} & 1 \end{pmatrix}^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i(\phi_d - \phi_u)} & 0 \\ 0 & 0 & e^{i(\phi_d - \phi_u)} \end{pmatrix} \times \begin{pmatrix} 1 & \sqrt{\frac{m_d}{m_s}} & \sqrt{\frac{m_d m_s^2}{m_b^3}} \\ -\sqrt{\frac{m_d}{m_s}} & 1 & \sqrt{\frac{m_d}{m_b}} \\ \sqrt{\frac{m_d^2}{m_s m_b}} & -\sqrt{\frac{m_d}{m_b}} & 1 \end{pmatrix}. \tag{4.6}$$

The explicit magnitudes of (i, j) elements of V are obtained as

$$|V_{12}| \simeq \left| \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} e^{i(\phi_d - \phi_u)} \right| = |0.224 - 0.06 e^{i(\phi_d - \phi_u)}|, \tag{4.7}$$

$$|V_{23}| \simeq \left| \sqrt{\frac{m_d}{m_b}} - \sqrt{\frac{m_u}{m_t}} \right| = 0.0336, \tag{4.8}$$

$$|V_{13}| \simeq \left| \sqrt{\frac{m_d m_s^2}{m_b^3}} - \sqrt{\frac{m_u m_d}{m_c m_b}} e^{i(\phi_d - \phi_u)} \right| = |0.000\,22 - 0.0021 e^{i(\phi_d - \phi_u)}|. \tag{4.9}$$

Here we have used the following numerical values for the quark masses estimated at the unification scale $\mu = M_X$, which are presented in Appendix B.

$$\begin{aligned} &m_u(M_X) = 1.04^{+0.19}_{-0.20}\,\text{MeV}, \quad m_d(M_X) = 1.33^{+0.17}_{-0.19}\,\text{MeV}, \\ &m_c(M_X) = 302^{+25}_{-27}\,\text{MeV}, \quad m_s(M_X) = 26.5^{+3.3}_{-3.7}\,\text{MeV}, \\ &m_t(M_X) = 129^{+196}_{-40}\,\text{GeV}, \quad m_b(M_X) = 1.00 \pm 0.04\,\text{GeV}. \end{aligned} \tag{4.10}$$

By using the rephasing of the up- and down-quarks, Eq. (4.6) is changed to the standard representation of the CKM quark mixing matrix,

$$V_{\rm std} = \mathrm{diag}(e^{i\zeta_1^u}, e^{i\zeta_2^u}, e^{i\zeta_2^u}) V \mathrm{diag}(e^{i\zeta_1^d}, e^{i\zeta_2^d}, e^{i\zeta_2^d}) = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{4.11}$$

Here ζ_i^q comes from the rephasing in the quark fields to make the choice of phase convention. The CP violating phase δ in Eq. (4.11) is predicted with the expression of V in Eq. (4.6) as

$$\delta = \arg\left[\left(\frac{V_{us}V_{cs}^*}{V_{ub}V_{cb}^*}\right) + \frac{|V_{us}|^2}{1 - |V_{ub}|^2}\right] \simeq \phi_u - \phi_d + \pi. \tag{4.12}$$

The predicted values of $|V_{12}|$, $|V_{23}|$, $|V_{13}|$, and δ are functions of a free parameter $\phi_u - \phi_d$ as shown in Eqs. (4.7), (4.8), (4.9), and (4.12). They are roughly consistent with the following numerical values at $\mu = M_X$, which are estimated from the experimental data observed at the electroweak scale $\mu = M_Z$ by using the renormalization group equation and presented in Appendix B:

$$|V_{12}^0| = 0.2226 - 0.2259, \qquad |V_{23}^0| = 0.0295 - 0.0387, \tag{4.13}$$

$$|V_{13}^0| = 0.0024 - 0.0038, \qquad \delta^0 = 46° - 74°. \tag{4.14}$$

B. MNS lepton mixing matrix

We obtain the MNS lepton mixing matrix U as follows:

$$U = O_e^T Q_e^\dagger Q_\nu O_\nu P_\nu \tag{4.15}$$

$$= \begin{pmatrix} 1 & \sqrt{\frac{m_e}{m_\mu}} & \sqrt{\frac{m_e m_\mu^2}{m_\tau^3}} \\ -\sqrt{\frac{m_e}{m_\mu}} & 1 & \sqrt{\frac{m_e}{m_\tau}} \\ \sqrt{\frac{m_e^2}{m_\mu m_\tau}} & -\sqrt{\frac{m_e}{m_\tau}} & 1 \end{pmatrix}^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\times \begin{pmatrix} \sqrt{\frac{m_2}{m_2+m_1}} & \sqrt{\frac{m_1}{m_2+m_1}} & 0 \\ -\sqrt{\frac{m_1}{m_2+m_1}} & \sqrt{\frac{m_2}{m_2+m_1}} & 0 \\ 0 & 0 & 1 \end{pmatrix} P_\nu$$

$$\simeq \begin{pmatrix} c_1 i & s_1 & -\frac{1}{\sqrt{2}}\sqrt{\frac{m_e}{m_\mu}} \\ -\frac{1}{\sqrt{2}} s_1 i & \frac{1}{\sqrt{2}} c_1 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} s_1 i & -\frac{1}{\sqrt{2}} c_1 & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{4.16}$$

with

$$s_1 \equiv \sqrt{\frac{m_1}{m_2 + m_1}}, \quad c_1 \equiv \sqrt{\frac{m_2}{m_2 + m_1}}. \tag{4.17}$$

The explicit magnitudes of (i, j) elements of U are

$$|U| \simeq \begin{pmatrix} \sqrt{\frac{m_2}{m_2+m_1}} & \sqrt{\frac{m_1}{m_2+m_1}} & \frac{1}{\sqrt{2}}\sqrt{\frac{m_e}{m_\mu}} \\ \frac{1}{\sqrt{2}}\sqrt{\frac{m_1}{m_2+m_1}} & \frac{1}{\sqrt{2}}\sqrt{\frac{m_2}{m_2+m_1}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\sqrt{\frac{m_1}{m_2+m_1}} & \frac{1}{\sqrt{2}}\sqrt{\frac{m_2}{m_2+m_1}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{4.18}$$

Therefore, we obtain

$$\tan^2\theta_{\text{solar}} = \frac{|U_{12}|^2}{|U_{11}|^2} \simeq \frac{m_1}{m_2}, \tag{4.19}$$

$$\sin^2 2\theta_{\text{atm}} = 4|U_{23}|^2|U_{33}|^2 \simeq 1, \tag{4.20}$$

$$|U_{13}|^2 \simeq \frac{m_e}{2m_\mu}. \tag{4.21}$$

In the following discussions we consider the normal mass hierarchy $m_1 < m_2 \ll m_3$ for the neutrino masses. Then the evolution effects which only give negligibly small correction effects can be ignored. Scenarios in which the neutrino masses have the quasidegenerate or the inverse hierarchy will be denied from Eqs. (4.19) and (4.24).

It can be seen from Eq. (4.16) that the large lepton mixing angle between the second and third generation is well realized with no fine-tuning in the model. It should be noted that the present model leads to the same results for θ_{solar} and θ_{atm} as the model in Ref. [25], while a different feature for $|U_{13}|^2$ is derived.

On the other hand, we have [33] a experimental bound for $|U_{13}|^2_{\text{exp}}$ from the CHOOZ [34], solar [35], and atmospheric neutrino experiments [1]. From the global analysis of the SNO solar neutrino experiment [33,35], we have Δm_{12}^2 and $\tan^2\theta_{12}$ for the large mixing angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) solution. From the atmospheric neutrino experiment [1,33] , we also have Δm_{23}^2 and $\tan^2\theta_{23}$. These experimental data with 3σ range are given by

$$|U_{13}|^2_{\text{exp}} < 0.054, \tag{4.22}$$

$$\Delta m_{12}^2 = m_2^2 - m_1^2 = \Delta m_{\text{sol}}^2 = (5.2 - 9.8) \times 10^{-5} \text{ eV}^2, \tag{4.23}$$

$$\tan^2\theta_{12} = \tan^2\theta_{\text{sol}} = 0.29 - 0.64, \tag{4.24}$$

$$\Delta m_{23}^2 = m_3^2 - m_2^2 \simeq \Delta m_{\text{atm}}^2 = (1.4 - 3.4) \times 10^{-3} \text{ eV}^2, \tag{4.25}$$

$$\tan^2\theta_{23} \simeq \tan^2\theta_{\text{atm}} = 0.49 - 2.2. \tag{4.26}$$

Hereafter, for simplicity, we take $\tan^2\theta_{\text{atm}} \simeq 1$. Thus, by combining the present model with the mixing angle θ_{sol}, we have

$$\frac{m_1}{m_2} \simeq \tan^2\theta_{\text{sol}} = 0.29 - 0.64. \tag{4.27}$$

Therefore we predict the neutrino masses as follows.

$$\begin{aligned} m_1^2 &= (0.48 - 6.8) \times 10^{-5} \text{ eV}^2, \\ m_2^2 &= (5.7 - 16.6) \times 10^{-5} \text{ eV}^2, \\ m_3^2 &= (1.4 - 3.4) \times 10^{-3} \text{ eV}^2. \end{aligned} \tag{4.28}$$

Let us mention other predictions in our model. Our model imposes a restriction on $|U_{13}|$ as

$$|U_{13}|^2 \simeq \frac{m_e}{2m_\mu} = 2.4 \times 10^{-3}. \tag{4.29}$$

Here we have used the running charged lepton masses at the unification scale $\mu = \Lambda_X$ [36]: $m_e(\Lambda_X) = 0.325$ MeV, $m_\mu(\Lambda_X) = 68.6$ MeV, and $m_\tau(\Lambda_X) = 1171.4 \pm 0.2$ MeV. The value in Eq. (4.29) is consistent with the present experimental constraints Eq. (4.22).

Next let us discuss the CP-violation phases in the lepton mixing matrix. The Majorana neutrino fields do not have the freedom of rephasing invariance, so that we can use only the rephasing freedom of M_e to transform Eq. (4.16) to the standard form

$$U_{\rm std} = {\rm diag}(e^{i\alpha_1^e}, e^{i\alpha_2^e}, e^{i\alpha_2^e})U$$
$$= \begin{pmatrix} c_{\nu 13}c_{\nu 12} & c_{\nu 13}s_{\nu 12}e^{i\beta} & s_{\nu 13}e^{i(\gamma-\delta_\nu)} \\ (-c_{\nu 23}s_{\nu 12} - s_{\nu 23}c_{\nu 23}s_{\nu 13}e^{i\delta_\nu})e^{-i\beta} & c_{\nu 23}c_{\nu 12} - s_{\nu 23}s_{\nu 12}s_{\nu 13}e^{i\delta_\nu} & s_{\nu 23}c_{\nu 13}e^{i(\gamma-\beta)} \\ (s_{\nu 23}s_{\nu 12} - c_{\nu 23}c_{\nu 12}s_{\nu 13}e^{i\delta_\nu})e^{-i\gamma} & (-s_{\nu 23}c_{\nu 12} - c_{\nu 23}s_{\nu 12}s_{\nu 13}e^{i\delta_\nu})e^{-i(\gamma-\beta)} & c_{\nu 23}c_{\nu 13} \end{pmatrix}. \tag{4.30}$$

Here, α_i^e comes from the rephasing in the charged lepton fields to make the choice of phase convention. The CP-violating phase δ_ν, the additional Majorana phase β and γ [37,38] in the representation Eq. (4.30) are calculable and obtained as

$$\delta_\nu = \arg\left[\frac{U_{12}U_{22}^*}{U_{13}U_{23}^*} + \frac{|U_{12}|^2}{1-|U_{13}|^2}\right] \simeq \pi,$$
$$\beta = \arg\left(\frac{U_{12}}{U_{11}}\right) \simeq -\pi/2, \qquad \gamma = \arg\left(\frac{U_{13}}{U_{11}}e^{i\delta_\nu}\right) \simeq \pi/2, \tag{4.31}$$

by using the relation $m_e \ll m_\mu \ll m_\tau$.

We also predict the averaged neutrino mass $\langle m_\nu\rangle$ which appears in the neutrinoless double beta decay [38] as follows:

$$\langle m_\nu\rangle \equiv |m_1U_{11}^2 + m_2U_{12}^2 + m_3U_{13}^2| = \frac{m_em_3}{2m_\mu}$$
$$= (0.89 - 1.4)\times 10^{-4}\ {\rm eV}. \tag{4.32}$$

This value of $\langle m_\nu\rangle$ is too small to be observed in near future experiments [39].

V. SUMMARY

We have investigated a Hermite mass matrix model given in Eqs. (2.1), (2.2), (2.3), (2.4), (2.5), and (2.6). In this model, the mass matrices for quarks and charged leptons are assumed to have a term in which the $2 \leftrightarrow 3$ symmetry is maximally broken. The mass matrices for up-quarks, down-quarks, charged leptons, and neutrinos have a common structure as shown by $\widehat{M_f}$ in Eq. (2.7) when it is expressed after rebasing of the quark and lepton fields. The large lepton mixing angle between the second and third generation is realized with no fine-tuning in our model. The model is almost consistent with the present data in the quark as well as lepton sectors. The model also predicts $|U_{13}|^2 \simeq \frac{m_e}{2m_\mu} = 2.4\times 10^{-3}$ for the lepton mixing matrix element U_{13}, and neutrino masses shown in Eq. (4.28) are obtained from the neutrino oscillation data for $\theta_{\rm sol}$, Δm_{23}^2, and Δm_{12}^2. We also predict $\langle m_\nu\rangle = (0.89 - 1.4)\times 10^{-4}$ eV for the averaged neutrino mass which appears in the neutrinoless double beta decay.

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APPENDIX A: DIAGONALIZATION OF MASS MATRIX $\widehat{M_f}$

For the purpose of making this paper self-contained, here we summarize the diagonalization of mass matrix $\widehat{M_f}$ ($f = u, d, e$ and ν) defined by

$$\widehat{M_f} = \begin{pmatrix} 0 & A_f & 0 \\ A_f & B_f & C_f \\ 0 & C_f & D_f \end{pmatrix}, \tag{A1}$$

for up-quarks, down-quarks, charged leptons, and neutrinos.

1. Mass matrix $\widehat{M_f}$ for quarks and charged leptons

For quarks and charged leptons ($f = u$, d, and e), let us take a following choice for $\widehat{M_f}$:

$$\widehat{M_f} = \begin{pmatrix} 0 & A_f & 0 \\ A_f & B_f & C_f \\ 0 & C_f & D_f \end{pmatrix} = \begin{pmatrix} 0 & \sqrt{\frac{m_{1f}m_{2f}m_{3f}}{m_{3f}-m_{1f}}} & 0 \\ \sqrt{\frac{m_{1f}m_{2f}m_{3f}}{m_{3f}-m_{1f}}} & m_{2f} & \sqrt{\frac{m_{1f}m_{3f}(m_{3f}-m_{2f}-m_{1f})}{m_{3f}-m_{1f}}} \\ 0 & \sqrt{\frac{m_{1f}m_{3f}(m_{3f}-m_{2f}-m_{1f})}{m_{3f}-m_{1f}}} & m_{3f}-m_{1f} \end{pmatrix}. \tag{A2}$$

This is diagonalized by an orthogonal matrix O_f as (see Refs. [23,24])

$$O_f^T\begin{pmatrix} 0 & A_f & 0 \\ A_f & B_f & C_f \\ 0 & C_f & D_f \end{pmatrix}O_f = \begin{pmatrix} -m_{1f} & & \\ & m_{2f} & \\ & & m_{3f} \end{pmatrix}. \tag{A3}$$

Here $m_{if}(i = 1, 2, 3)$ are eigenmasses and O_f is given by

$$O_f = \begin{pmatrix} \sqrt{\frac{m_{2f}m_{3f}^2}{(m_{2f}+m_{1f})(m_{3f}^2-m_{1f}^2)}} & \sqrt{\frac{m_{1f}m_{3f}(m_{3f}-m_{2f}-m_{1f})}{(m_{2f}+m_{1f})(m_{3f}-m_{2f})(m_{3f}-m_{1f})}} & \sqrt{\frac{m_{1f}^2 m_{2f}}{(m_{3f}-m_{2f})(m_{3f}^2-m_{1f}^2)}} \\ -\sqrt{\frac{m_{1f}m_{3f}}{(m_{2f}+m_{1f})(m_{3f}+m_{1f})}} & \sqrt{\frac{m_{2f}(m_{3f}-m_{2f}-m_{1f})}{(m_{2f}+m_{1f})(m_{3f}-m_{2f})}} & \sqrt{\frac{m_{1f}m_{3f}}{(m_{3f}-m_{2f})(m_{3f}+m_{1f})}} \\ \sqrt{\frac{m_{1f}^2(m_{3f}-m_{2f}-m_{1f})}{(m_{2f}+m_{1f})(m_{3f}^2-m_{2f}^2)}} & -\sqrt{\frac{m_{1f}m_{2f}m_{3f}}{(m_{3f}-m_{2f})(m_{2f}+m_{1f})(m_{3f}-m_{1f})}} & \sqrt{\frac{(m_{3f})^2(m_{3f}-m_{2f}-m_{1f})}{(m_{3f}^2-m_{2f}^2)(m_{3f}-m_{2f})}} \end{pmatrix} \simeq \begin{pmatrix} 1 & \sqrt{\frac{m_{1f}}{m_{2f}}} & \sqrt{\frac{m_{1f}m_{2f}^2}{m_{3f}^3}} \\ -\sqrt{\frac{m_{1f}}{m_{2f}}} & 1 & \sqrt{\frac{m_{1f}}{m_{3f}}} \\ \sqrt{\frac{m_{1f}^2}{m_{2f}m_{3f}}} & -\sqrt{\frac{m_{1f}}{m_{3f}}} & 1 \end{pmatrix}$$

$$(\text{for} \quad m_{3f} \gg m_{2f} \gg m_{1f}). \tag{A4}$$

Here m_{iu}, m_{id}, and $m_{ie}(i = 1, 2, 3)$ are, respectively, masses of up-quarks, down-quarks, charged leptons, and neutrinos, which we shall denoted as (m_u, m_c, m_t), (m_d, m_s, m_b), and (m_e, m_μ, m_τ).

2. Mass matrix $\widehat{M_\nu}$ for neutrinos

For neutrinos ($f = \nu$) we choose :

$$\widehat{M_\nu} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & 0 \\ 0 & 0 & D_\nu \end{pmatrix} = \begin{pmatrix} 0 & \sqrt{m_1 m_2} & 0 \\ \sqrt{m_1 m_2} & m_2 - m_1 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{A5}$$

Note we take $C_\nu = 0$. This $\widehat{M_\nu}$ is diagonalized as

$$O_\nu^T \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & 0 \\ 0 & 0 & D_\nu \end{pmatrix} O_\nu = \begin{pmatrix} -m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}, \tag{A6}$$

where $m_i(i = 1, 2, 3)$ are neutrino masses and the orthogonal matrix O_ν is given by

$$O_\nu = \begin{pmatrix} \sqrt{\frac{m_2}{m_2+m_1}} & \sqrt{\frac{m_1}{m_2+m_1}} & 0 \\ -\sqrt{\frac{m_1}{m_2+m_1}} & \sqrt{\frac{m_2}{m_2+m_1}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{A7}$$

APPENDIX B: EVOLUTION EFFECT

We have estimated the evolution effects for the CKM matrix elements from the electroweak scale $\mu = m_Z$ to the unification scale $\mu = M_X$ by using the two-loop renormalization group equation (RGE) [minimal supersymmetric standard model with $\tan\beta = 10$ case] for the Yukawa coupling constants. In the numerical calculations, we have used the following running quark masses at $\mu = m_Z$ and at $\mu = M_X$ [36]:

$$\begin{aligned} m_u(m_Z) &= 2.33^{+0.42}_{-0.45}\ \text{MeV}, \\ m_c(m_Z) &= 677^{+56}_{-61}\ \text{MeV}, \\ m_t(m_Z) &= 181 \pm 13\ \text{GeV}, \\ m_d(m_Z) &= 4.69^{+0.60}_{-0.66}\ \text{MeV}, \\ m_s(m_Z) &= 93.4^{+11.8}_{-13.0}\ \text{MeV}, \\ m_b(m_Z) &= 3.00 \pm 0.11\ \text{GeV}. \end{aligned} \tag{B1}$$

$$\begin{aligned} m_u(M_X) &= 1.04^{+0.19}_{-0.20}\ \text{MeV}, \\ m_c(M_X) &= 302^{+25}_{-27}\ \text{MeV}, \\ m_t(M_X) &= 129^{+196}_{-40}\ \text{GeV}, \\ m_d(M_X) &= 1.33^{+0.17}_{-0.19}\ \text{MeV}, \\ m_s(M_X) &= 26.5^{+3.3}_{-3.7}\ \text{MeV}, \\ m_b(M_X) &= 1.00 \pm 0.04\ \text{GeV}. \end{aligned} \tag{B2}$$

We have calculated numerical values of the CKM mixing matrix elements at $\mu = M_X$ from their observed values at $\mu = m_Z$. Namely using as inputs the observed quark mixing angles and the CP violating phase at $\mu = m_Z$ given by

$$\begin{aligned} \sin\theta_{12}(m_Z) &= 0.2243 \pm 0.0016, \\ \sin\theta_{23}(m_Z) &= 0.0413 \pm 0.0015, \\ \sin\theta_{13}(m_Z) &= 0.0037 \pm 0.0005, \\ \delta(m_Z) &= 60° \pm 14°, \end{aligned} \tag{B3}$$

we obtain the following numerical values for the mixing angles and the magnitude of the mixing matrix elements at $\mu = M_X$ [28]:

$$\begin{aligned} \sin\theta_{12}^0 &= 0.2226 - 0.2259, \\ \sin\theta_{23}^0 &= 0.0295 - 0.0383, \\ \sin\theta_{13}^0 &= 0.0024 - 0.0038, \\ \delta^0 &= 46° - 74°, \end{aligned} \tag{B4}$$

$$|V^0| = \begin{pmatrix} 0.9741 - 0.9749 & 0.2226 - 0.2259 & 0.0024 - 0.0038 \\ 0.2225 - 0.2259 & 0.9734 - 0.9745 & 0.0295 - 0.0387 \\ 0.0048 - 0.0084 & 0.0289 - 0.0379 & 0.9993 - 0.9996 \end{pmatrix}. \tag{B5}$$

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