Photon-neutrino scattering in noncommutative space

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We extend the noncommutative standard model based on the minimal $SU(3) \times SU(2) \times U(1)$ gauge group to include the interaction of photon with neutrinos. We show that, in the gauge invariant manner, only the right-handed neutrino can directly couple to the photon. Consequently, we obtain the Feynman rule for the $\gamma\nu\bar{\nu}$-vertex which does not exist in the minimal extension of the noncommutative standard model (mNCSM). We calculate the amplitude for $\gamma\nu \to \gamma\nu$ in both the nonminimal noncommutative standard model (nmNCSM) and the extended version of mNCSM. The obtained cross section grows in the center of mass frame, respectively, as $(\theta_{NC})^2 M_Z^{-4} E^6$ and $(\theta_{NC})^4 E^6$ which can exceed the cross section for $\gamma\nu \to \gamma\gamma\nu$ and $\gamma\nu \to \gamma\nu$ in the high energy limit in the commutative space.

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I. INTRODUCTION

High energy photons and neutrinos and their scattering channels based on the standard model are currently of interest to many authors in astrophysics and cosmology [1–4]. In the low energy limit the elastic photon-neutrino scattering is strongly suppressed by Yang's theorem in the lowest order [5]. Meanwhile, the inelastic scattering of photon-neutrino such as $\gamma\nu \to \gamma\gamma\nu$ and its crossed processes are not subject to this suppression i.e. $\sigma_{\gamma\nu\to\gamma\gamma\nu}(1\ \text{MeV}) \sim 10^{-52}\ \text{cm}^2$ in comparison with $\sigma_{\gamma\nu\to\gamma\nu}(1\ \text{MeV}) \sim 10^{-65}\ \text{cm}^2$ [2]. Nevertheless, in the high energy limit it is shown that [3]

$$\sigma_{\gamma\nu\to\gamma\nu} = 6.7 \times 10^{-33}\left(\frac{E}{m_e}\right)^6 pb, \tag{1}$$

while [4]

$$\sigma_{\gamma\nu\to\gamma\gamma\nu} = 1.74 \times 10^{-16}\left(\frac{E}{m_e}\right)^2 pb, \tag{2}$$

in which the photon energy E in the center of mass frame satisfies $m_e \ll E \ll M_W$. Obviously, with increasing E the elastic cross section exceeds the inelastic one and it can be easily seen that the crossover occurs at $E \sim 7$ GeV. In the high energy limit the noncommutativity effects seem to be significant and therefore the new interactions of photon and neutrino in the noncommutative space and time can be potentially important to astrophysics. However, noncommutative field theory and its phenomenological aspects have been recently considered by many authors [6–10]. Such theories are mostly characterized on a noncommutative space-time with the noncommutativity parameter $\theta_{\mu\nu}$. In the canonical version of the noncommutative space-time one has

$$\theta^{\mu\nu} = -i[\hat{x}^\mu, \hat{x}^\nu], \tag{3}$$

where a hat indicates a noncommutative coordinate and $\theta_{\mu\nu}$ is a real, constant, and antisymmetric matrix. The action for field theories on noncommutative spaces is then obtained by using the Weyl-Moyal correspondence; accordingly, the usual product of fields should be replaced by the star product:

$$f \star g(x, \theta) = f(x, \theta)\exp\left(\frac{i}{2}\overleftarrow{\partial}_\mu \theta^{\mu\nu}\overrightarrow{\partial}_\nu\right)g(x, \theta). \tag{4}$$

In replacing the ordinary product with the star product there is an ambiguity in transcribing $gA\psi$, where g, A, and ψ are coupling constant, gauge, and particle fields, respectively, into noncommutative form that is: $gA \star \psi$, $g\psi \star A$ or $g_1 A \star \psi + g_2 \psi \star A$. In the commutative limit all the terms can be reduced to the same term while for the neutral particles, for example, in QED, the third term in the noncommutative limit is essentially different from the other two. In fact this can bring about direct interaction of photon and neutral particles.

In Sec. II we give a brief review on the direct interaction of neutral particles with photons in the noncommutative QED and subsequently extend the noncommutative standard model (NCSM) based on the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group to incorporate the direct interaction of photons with neutrinos. In Sec. III we explore the photon-neutrino elastic scattering in the extended minimal NCSM as well as the nonminimal NCSM at the lowest order. Finally, we compare our results with the results on the photon-neutrino scattering given in the literature.

II. NONCOMMUTATIVE STANDARD MODEL

In the frame work of NCQED it is shown that the neutral particles interact with photons if they transform under U(1) in a similar way as in the adjoint representation of a non-Abelian gauge theory. In fact, for this purpose $eA \star \psi - e\psi \star A$ should be added to ordinary derivative to construct the covariant derivative [11,12]. In the limit of $\theta \to 0$, we have

$$eA \star \psi - e\psi \star A = 0 + \mathcal{O}(\theta), \tag{5}$$

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therefore the covariant derivative to the lowest order can be obtained as follows

$$\hat{D}_\mu \hat{\psi} = \partial_\mu \hat{\psi} + e\theta^{\nu\rho}\partial_\nu \hat{A}_\mu \partial_\rho \hat{\psi}. \tag{6}$$

The fields themselves in the noncommutative space can be expanded by the Seiberg-Witten (SW) map [6] up to the lowest order as

$$\hat{\psi} = \psi + e\theta^{\nu\rho}A_\rho \partial_\nu \psi, \tag{7}$$

$$\hat{A}_\mu = A_\mu + e\theta^{\nu\rho}A_\rho[\partial_\nu A_\mu - \tfrac{1}{2}\partial_\mu A_\nu]. \tag{8}$$

Therefore the interaction term in terms of commutative fields is

$$-\frac{e}{2}F_{\mu\nu}(i\theta^{\mu\nu\rho}\partial_\rho)\psi, \tag{9}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and

$$\theta^{\mu\nu\rho} = \theta^{\mu\nu}\gamma^\rho + \theta^{\nu\rho}\gamma^\mu + \theta^{\rho\mu}\gamma^\nu. \tag{10}$$

It should be noted that for the neutrino as a neutral particle in the NCQED, as well as QED, in contrast with the standard model, there is not any constraint on the mass or even the chirality of the neutrino. In the standard model, the neutrino is massless and only the left-handed one has weak interaction while the right-handed neutrino, if existing, has an expectator role in all reactions. However, there are two approaches to construct the standard model in the noncommutative space. In the minimal extension the gauge group is $SU(3)_c \times SU(2)_L \times U(1)_Y$ in which the number of gauge fields, couplings and particles are the same as the ordinary one [13]. Although in this extension new interactions will appear due to the star product and the SW map, the photon-neutrino vertex is absent. In the second approach the gauge group is $U(3) \times U(2) \times U(1)$ which is reduced to $SU(3)_c \times SU(2)_L \times U(1)_Y$ by an appropriate symmetry breaking [14]. However, in the latter approach, besides many new interaction like the former one, photon can interact with the left-handed neutrino.

To introduce the neutrino-photon interaction in the minimal NCSM, one can define the adjoint representation in the covariant derivative for the neutral particle as is done in the NCQED. The main difference in the SM is $U(1)_Y$ instead of $U(1)_{EM}$. Therefore the neutral hypercharge particle can only couple to the hypergauge field in a gauge invariant manner. The only particle with zero hypercharge in the SM is the right-handed neutrino therefore the covariant derivative for this particle can be written as follows

$$\hat{D}_\mu \hat{\psi}_{\nu_R} = \partial_\mu \hat{\psi}_{\nu_R} + e\theta^{\nu\rho}\partial_\nu \hat{B}_\mu \partial_\rho \hat{\psi}_{\nu_R}, \tag{11}$$

in which $\hat{\psi}_{\nu_R}$ and $\hat{B}$, respectively, denote the NC-fields of the right-handed neutrino and the hypercharge with their own expansion in the NC-space as are given in Eqs. (7) and (8). Consequently, the Lagrangian density for the right-handed neutrino part of NCSM can be written as follows

$$\mathcal{L}_{\nu_R} = i\bar{\psi}\partial\!\!\!/\psi + ie\theta^{\mu\nu}[\partial_\mu\bar{\psi}B_\nu\gamma^\rho(\partial_\rho\psi) - \partial_\rho\bar{\psi}B_\nu\gamma^\rho(\partial_\mu\psi) + \bar{\psi}(\partial_\mu B_\rho)\gamma^\rho(\partial_\nu\psi)], \tag{12}$$

where B in terms of the photon and the Z-gauge boson fields is

$$B = \cos\theta_W A - \sin\theta_W Z. \tag{13}$$

Therefore, the Feynman rules for $\gamma\nu\bar{\nu}$ and $Z\nu\bar{\nu}$ vertices can be obtained from the Lagrangian (12) as:

$$\Gamma^\mu_{\gamma\nu\bar{\nu}} = i\frac{e}{2}\cos\theta_W(1+\gamma_5)(\theta^{\mu\nu}k_\nu q\!\!\!/ + \theta^{\rho\mu}q_\rho k\!\!\!/ + \theta^{\nu\rho}k_\nu q_\rho\gamma^\mu), \tag{14}$$

and

$$\Gamma^\mu_{Z\nu\bar{\nu}} = -i\frac{e}{2}\sin\theta_W(1+\gamma_5)(\theta^{\mu\nu}k_\nu q\!\!\!/ + \theta^{\rho\mu}q_\rho k\!\!\!/ + \theta^{\nu\rho}k_\nu q_\rho\gamma^\mu). \tag{15}$$

It should be noted here that in the minimal extension of the standard model to the noncommutative space-time (mNCSM) there is not any $\gamma\nu\bar{\nu}$-vertex while the $Z\nu\bar{\nu}$-vertex has already existed for the left-handed neutrino, therefore $\Gamma^\mu_{Z\nu\bar{\nu}}$ for the right handed neutrino can be considered as a correction to the same vertex in the mNCSM. Since the other particles in the SM, even the left-handed neutrino, all have nonzero hypercharge, the remaining parts of the SM in the noncommutative space do not change.

III. PHOTON-NEUTRINO INTERACTION IN NCSM

In the minimal extension of the standard model to the noncommutative space-time due to the different choices for representations of the gauge group the trace in the kinetic terms for gauge bosons is not unique. In fact the freedom in the choice of the traces can be used to construct a new version of the NCSM which is called nmNCSM. Neutral triple-gauge boson vertices such as $\gamma\gamma\gamma$ and $Z\gamma\gamma$ in contrast to the mNCSM as well as the SM can arise within the framework of the nmNCSM. These vertices can be extracted from the Lagrangian of nmNCSM which are given in [13] as follows

$$\mathcal{L}_{\gamma\gamma\gamma} = \frac{e}{4}\sin 2\theta_W K_{\gamma\gamma\gamma}\theta^{\rho\sigma}A^{\mu\nu}(A_{\mu\nu}A_{\rho\sigma} - 4A_{\mu\rho}A_{\nu\sigma}), \tag{16}$$

$$\mathcal{L}_{Z\gamma\gamma} = \frac{e}{4}\sin 2\theta_W K_{Z\gamma\gamma}\theta^{\rho\sigma}[2Z^{\mu\nu}(2A_{\mu\rho}A_{\nu\sigma} - A_{\mu\nu}A_{\rho\sigma}) + 8Z_{\mu\rho}A^{\mu\nu}A_{\nu\sigma} - Z_{\rho\sigma}A_{\mu\nu}A^{\mu\nu}], \tag{17}$$

and

$$\mathcal{L}_{ZZ\gamma} = \mathcal{L}_{Z\gamma\gamma}(A_\mu \leftrightarrow Z_\mu), \tag{18}$$

$$\mathcal{L}_{ZZZ} = \mathcal{L}_{\gamma\gamma\gamma}(A_\mu \rightarrow Z_\mu), \tag{19}$$

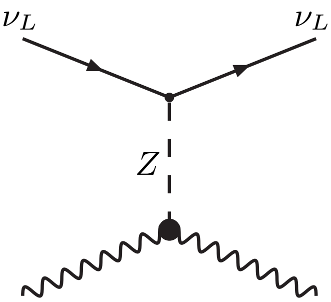


FIG. 1. Feynman diagram for the process $\gamma\nu \rightarrow \gamma\nu$ in nmNCSM at the order θ. The bold dot represents the noncommutative vertex $\Gamma^{\mu\nu\rho}$.

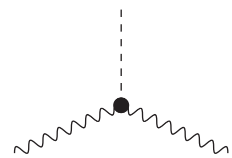
where

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{20}$$

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu. \tag{21}$$

The constants $K_{\gamma\gamma\gamma}$, $K_{Z\gamma\gamma}$, and so on are functions of the coupling constants of the noncommutative electroweak sector up to the first order of θ. They can be obtained by matching the NCSM action with the SM action and their allowed range of values is given in [15]. However, up to the first order of θ in the nmNCSM, there is a Feynman diagram which is shown in Fig. 1. The Feynman rule for the $Z\gamma\gamma$ vertex in the nmNCSM can be easily derived from the Lagrangian of Eq. (17) as follows

$$\Gamma^{\mu\nu\rho} = -2e\sin\theta_W K_{Z\gamma\gamma}\Theta((\mu, k_1), (\nu, k_2), (\rho, k_3)),$$



in which $K_{Z\gamma\gamma}$ is the strength of the $Z\gamma\gamma$ triple-gauge bosons and

$$\Theta((\mu,k_1),(\nu,k_2),(\rho,k_3)) = -\theta^{\mu\nu}(k_1^\rho(k_2\cdot k_3) - k_2^\rho(k_1\cdot k_3)) + \theta^{\mu\alpha}k_{1\alpha}(g^{\nu\rho}(k_2\cdot k_3) - k_2^\rho k_3^\nu) - \theta^{\nu\alpha}k_{1\alpha}(g^{\rho\mu}(k_2\cdot k_3) - k_2^\rho k_3^\mu) - \theta^{\rho\alpha}k_{1\alpha}(g^{\mu\nu}(k_2\cdot k_3) - k_2^\mu k_3^\nu) + k_1\cdot\theta\cdot k_2(k_3^\mu g^{\nu\rho} - k_3^\nu g^{\rho\mu}) + \text{cycl. permut. of } (\mu_i, k_i), \tag{22}$$

where $\mu_1 = \mu$, $\mu_2 = \nu$, and $\mu_3 = \rho$. Therefore, the invariant amplitude for the reaction

$$\gamma(k_1, \varepsilon_\mu) + \nu(k_3) \rightarrow \gamma(p_2, \varepsilon_\rho) + \nu(p_1) \tag{23}$$

can be easily written as

$$\begin{aligned} -i\mathcal{M} &= \varepsilon_\mu(k_1)\varepsilon_\rho(p_2)T^{\mu\rho} \\ &= \varepsilon_\mu(k_1)\varepsilon_\rho(p_2)\overline{u}(p_1)\frac{-ig}{2\cos\theta_W}\gamma_\nu\frac{1}{2}(1-\gamma^5)u(k_3) \\ &\quad\times\frac{i(-2e\sin2\theta_W k_{Z\gamma\gamma})}{M_Z^2 - k_2^2}\Theta^{\mu\nu\rho}, \end{aligned} \tag{24}$$

where, after some algebra $\Theta^{\mu\nu\rho}$ in the center of mass frame, can be obtained as:

$$\begin{aligned} \Theta^{\mu\nu\rho} = \{&2(k_1\cdot p_2)\theta^{\mu\alpha}p_{2_\alpha}g^{\nu\rho} - p_2^\rho p_2^\nu\theta^{\mu\alpha}k_{2_\alpha} \\ &+ k_1^\rho k_1^\mu\theta^{\nu\alpha}p_{2_\alpha} - p_2^\mu p_2^\rho\theta^{\nu\alpha}k_{1_\alpha} - k_1^\nu k_1^\mu\theta^{\rho\alpha}k_{2_\alpha} \\ &- 2(k_1\cdot p_2)\theta^{\rho\alpha}k_{1_\alpha}g^{\mu\nu} - (k_1\cdot p_2)\theta^{\mu\nu}p_2^\rho \\ &- (k_1\cdot p_2)\theta^{\nu\rho}k_1^\mu + 2(k_1\cdot p_2)\theta^{\rho\mu}k_1^\nu \\ &+ (k_1\cdot\theta\cdot p_2)(k_1^\mu - 2p_2^\mu)g^{\nu\rho} + 2(k_1\cdot\theta\cdot p_2)k_1^\nu g^{\rho\mu} \\ &+ (k_1\cdot\theta\cdot p_2)(p_2^\rho - 2k_1^\rho)g^{\mu\nu}\}, \end{aligned} \tag{25}$$

and as a natural consequence of gauge symmetry one can easily show that $T^{\mu\rho}$ satisfies the Ward identity as

$$k_{1\mu}T^{\mu\rho} = p_{2\rho}T^{\mu\rho} = 0. \tag{26}$$

It therefore follows that if $E \ll M_Z$ then, after a little algebra, the spin-averaged amplitude is

$$\begin{aligned} \overline{|\mathcal{M}|}^2 = \left(\frac{4\pi\alpha}{M_Z^2}\right)^2 |k_{Z\gamma\gamma}|^2 2^5\{&(k_1\cdot p_2)^3(p_2\cdot\theta\cdot\theta\cdot p_2 + k_1\cdot\theta\cdot\theta\cdot k_1) + (k_1\cdot p_2)^2(p_1\cdot\theta\cdot\theta\cdot p_2(k_1\cdot k_3) \\ &+ k_3\cdot\theta\cdot\theta\cdot p_2(p_1\cdot k_1) - k_1\cdot\theta\cdot\theta\cdot p_2(k_1\cdot p_2) + p_1\cdot\theta\cdot\theta\cdot k_1(k_1\cdot k_3) + k_3\cdot\theta\cdot\theta\cdot k_1(p_1\cdot k_1) \\ &- k_1\cdot\theta\cdot\theta\cdot k_1(k_1\cdot p_2)) - (k_1\cdot p_2)^2((p_1\cdot\theta\cdot k_1)(k_3\cdot\theta\cdot p_2) + k_1\cdot\theta\cdot\theta\cdot p_2(k_1\cdot p_2)) \\ &+ (k_1\cdot p_1)(k_1\cdot k_3)((k_1\cdot p_2)^2|\vec{\theta}|^2 + 2(k_1\cdot\theta\cdot p_2)^2)\}, \end{aligned} \tag{27}$$

thus by doing some manipulation the total cross section for $\gamma\nu \rightarrow \gamma\nu$ in nmNCSM results in

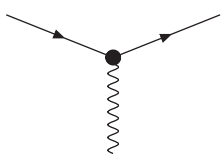
$$\sigma \cong 11.5\,|k_{Z\gamma\gamma}|^2\frac{\alpha^2E^6}{\Lambda^4M_Z^4}, \tag{28}$$

where the scale of noncommutativity Λ is defined as

$$\Lambda = \frac{1}{\sqrt{|\vec{\theta}|}}. \tag{29}$$

The constant $K_{Z\gamma\gamma}$ varies in the range $-0.3 \leq K_{Z\gamma\gamma} \leq 0.1$ and for $K_{Z\gamma\gamma} \sim 0.1$ the cross section varies in the range 10^{-42}–10^{-46} cm^2 for $\Lambda \sim 100$–1000 GeV and $E \sim 0.1 M_Z$ which is comparable with its counterpart in the commutative space, see Table I. Although, the triple-gauge boson coupling constants simultaneously do not vanish the $K_{Z\gamma\gamma}$ is the only coupling which is appeared in the cross section and it may be even zero. Since the values of the triple-gauge boson coupling constants cannot be uniquely obtained in the nmNCSM, to be certain, we may restrict ourselves to the mNCSM where there is not such a freedom. In contrast to the nmNCSM in the mNCSM there is not any triple-gauge boson vertex in the electroweak sector therefore we have not any diagram at the tree level for the elastic photon-neutrino scattering. But in the extended version of mNCSM which is introduced in Sec. II the photon can interact directly with right-handed neutrino therefore at the tree level there are two Feynman diagrams for the photon-neutrino elastic scattering which is shown in Fig. 2. The Feynman rule for the $\gamma\nu\bar{\nu}$ vertex in the extended mNCSM is given in Eq. (14) as

$$\Gamma^{\mu}_{\gamma\nu\bar{\nu}} = i\frac{e}{2}\cos\theta_W(1+\gamma_5) \times (\theta^{\mu\nu}k_\nu \not{q} + \theta^{\rho\mu}q_\rho \not{k} + \theta^{\nu\rho}k_\nu q_\rho \gamma^\mu).$$



Therefore, the invariant amplitude for the first diagram of Fig. 2 in the center of mass frame can be written as

$$-i\mathcal{M}_1 = \varepsilon_\mu \varepsilon'_\nu \bar{u}(p')(ie\cos\theta_W)\frac{1}{2}(1+\gamma^5)[k'\cdot\theta\cdot(k+p)\gamma^\nu + \theta^{\nu\alpha}k'_\alpha(\not{k}+\not{p}) - \theta^{\nu\alpha}(k_\alpha+p_\alpha)\not{k}']\frac{(-i)(\not{k}+\not{p})}{(k+p)^2} \times(-ie\cos\theta_W)\frac{1}{2}(1+\gamma^5)[k\cdot\theta\cdot p\gamma^\mu + k_\beta\theta^{\mu\beta}\not{p} - p_\beta\theta^{\mu\beta}\not{k}]u(p), \tag{30}$$

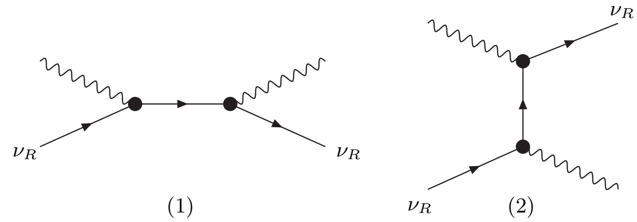


FIG. 2. Feynman diagrams for the process $\gamma\nu \to \gamma\nu$ in mNCSM. The bold dot represents the noncommutative vertex $\Gamma^{\mu}_{\gamma\nu\bar{\nu}}$.

which, because of the momentum conservation $k+p = k'+p'$, the Dirac equations $\not{p}u(p) = 0$, $\bar{u}(p')\not{p}' = 0$ and the following identity

$$A_\mu\theta^{\mu\nu}B_\nu = A\cdot\theta\cdot B = \vec{\theta}\cdot(A\times B), \tag{31}$$

where $\vec{\theta} = (\theta_{23}, \theta_{31}, \theta_{12})$, results in

$$\mathcal{M}_1 = -e^2\cos^2\theta_W \varepsilon'_\nu\varepsilon_\mu p'_\alpha p_\beta \theta^{\nu\alpha}\theta^{\mu\beta}\bar{u}(p') \times\frac{1}{2}(1+\gamma^5)\not{k}'u(p). \tag{32}$$

For the second diagram one similarly has

$$-i\mathcal{M}_2 = \varepsilon_\mu\varepsilon'_\nu\bar{u}(p')(-ie\cos\theta_W)\frac{1}{2}(1+\gamma^5)[k\cdot\theta\cdot(p'-k)\gamma^\mu + \theta^{\mu\beta}k_\beta(\not{p}'-\not{k}) - \theta^{\mu\beta}(p'_\beta - k_\beta)\not{k}]\frac{-i(\not{p}'-\not{k})}{(p'-k)^2} \times(ie\cos\theta_W)\frac{1}{2}(1+\gamma^5)[k'\cdot\theta\cdot p\gamma^\nu + \theta^{\nu\alpha}k'_\alpha\not{p} - \theta^{\nu\alpha}p_\alpha\not{k}']u(p), \tag{33}$$

which after some manipulation yields

$$\mathcal{M}_2 = -e^2\cos^2\theta_W\varepsilon_\mu\varepsilon'_\nu\bar{u}(p')\frac{1}{2}(1+\gamma^5) \times\left[\frac{(k\cdot\theta\cdot p')(k'\cdot\theta\cdot p)}{(p'-k)^2}\gamma^\mu(\not{p}'-\not{k})\gamma^\nu - p_\alpha p'_\beta\theta^{\mu\beta}\theta^{\nu\alpha}\not{k}' - (k\cdot\theta\cdot p')(p_\alpha\theta^{\nu\alpha}\gamma^\mu - p'_\alpha\theta^{\mu\alpha}\gamma^\nu)\right]u(p). \tag{34}$$

Therefore by introducing the appropriate tensor $T^{\mu\nu}$ in terms of the total amplitude $\mathcal{M}_{\rm tot} = \mathcal{M}_1 + \mathcal{M}_2$ one can show that

TABLE I. The total cross section for $\gamma\nu \to \gamma\nu$ in the nmNCSM given in Eq. (28) for $K_{Z\gamma\gamma} = 0.1$, the mNCSM given in Eq. (51) and in the standard model (SM) obtained in [3].

$\sigma(\nu\gamma \to \nu\gamma)$ (cm^2)	nmNCSM ($\Lambda \sim 100$–1000 GeV)	mNCSM ($\Lambda \sim 100$–1000 GeV)	SM
$E = 1$ MeV	3.4×10^{-67}–3.4×10^{-71}	1×10^{-66}–1×10^{-74}	4×10^{-67}
$E = 10$ GeV	3.4×10^{-43}–3.4×10^{-47}	1×10^{-42}–1×10^{-50}	2×10^{-43}

$$k_\mu T^{\mu\nu} = k'_\nu T^{\mu\nu} = 0. \tag{35}$$

Thus, the spin-averaged amplitude for $\gamma\nu \to \gamma\nu$ scattering can be evaluated as

$$\overline{|\mathcal{M}_{\rm tot}|}^2 = \frac{1}{2} e^4 \cos^4\theta_W g_{\nu\delta} g_{\mu\eta} \,\mathrm{Tr}\Big(\not p' \frac{1}{2}(1-\gamma^5) \times \sum_{i=1}^{3} I_i^{\mu\nu} \not p \sum_{j=1}^{3} J_j^{\eta\delta} \Big) \tag{36}$$

where

$$\begin{aligned} I_1^{\mu\nu} &\equiv \frac{(k\cdot\theta\cdot p')(k'\cdot\theta\cdot p)}{(p'-k)^2}\gamma^\mu(\not p' - \not k)\gamma^\nu, \\ I_2^{\mu\nu} &\equiv (k\cdot\theta\cdot p')(p_\alpha\theta^{\nu\alpha}\gamma^\mu - p'_\alpha\theta^{\mu\alpha}\gamma^\nu), \\ I_3^{\mu\nu} &\equiv (p_\beta p'_\alpha - p_\alpha p'_\beta)\theta^{\mu\beta}\theta^{\nu\alpha}\not k', \qquad J_1^{\eta\delta} = I_1^{\delta\eta}, \\ J_2^{\eta\delta} &= I_2^{\eta\delta}, \qquad J_3^{\eta\delta} = I_3^{\eta\delta}, \end{aligned} \tag{37}$$

which, using the trace theorems, implies

$$\begin{aligned} \overline{|\mathcal{M}_{\rm tot}|}^2 = \frac{e^4\cos^4\theta_W}{2}\Big[& 8(k\cdot\theta\cdot p')^4 \frac{p\cdot p'}{(p'-k)^2} + 8(k\cdot p) \\ & \times (k'\cdot p)((p'\cdot\theta\cdot\theta\cdot p')(p\cdot\theta\cdot\theta\cdot p) \\ & - (p'\cdot\theta\cdot\theta\cdot p)^2) + 4(k\cdot\theta\cdot p')^2 \\ & \times ((k'\cdot p)(p\cdot\theta\cdot\theta\cdot p' - p\cdot\theta\cdot\theta\cdot p \\ & - p'\cdot\theta\cdot\theta\cdot p') - 3(k\cdot p)(p\cdot\theta\cdot\theta\cdot p'))\Big]. \end{aligned} \tag{38}$$

To evaluate the total cross section the particle momenta are shown in Fig. 3 and the differential cross section is given by

$$d\sigma = \frac{\overline{|\mathcal{M}_{\rm tot}|}^2}{4\pi^2 \times 4k\cdot p}\frac{d^3P'}{2E'_\nu}\frac{d^3K'}{2E'_\gamma}\delta^4(k'+p'-k-p). \tag{39}$$

Now by introducing:

$$p = (E, P), \tag{40}$$

$$k = (E, -P), \tag{41}$$

$$p' = (E'_\nu, P'), \tag{42}$$

$$k' = (E'_\gamma, K'), \tag{43}$$

the differential cross section can be cast into

$$d\sigma = \frac{\overline{|\mathcal{M}_{\rm tot}|}^2}{4\pi^2 \times 4k\cdot p}\frac{d^3P'}{4E'^2}\delta(2E' - 2E), \tag{44}$$

where in the center of mass frame $E' \equiv E'_\gamma = E'_\nu$. In the relativistic limit d^3P' is equal to $E'^2 dE' d\beta d\cos\alpha$, therefore, in this limit one has

$$d\sigma = \frac{\overline{|\mathcal{M}_{\rm tot}|}^2_{E'=E}}{2^7 \times \pi^2 \times k.p} d\beta d\cos\alpha. \tag{45}$$

Now by using the invariant quantities:

$$k\cdot p = 2E^2, \tag{46}$$

$$p\cdot p' = k\cdot k' = E^2(1-\cos\alpha), \tag{47}$$

$$p\cdot k' = k\cdot p' = E^2(1+\cos\alpha), \tag{48}$$

and also the identity given in Eq. (31) and

$$\begin{aligned} A_\mu\theta^{\mu\nu}\theta_\nu^\beta B_\beta &= A\cdot\theta\cdot\theta\cdot B \\ &= |\theta|^2(A\cdot B) - (A\cdot\vec\theta)(B\cdot\vec\theta), \end{aligned} \tag{49}$$

which leads to

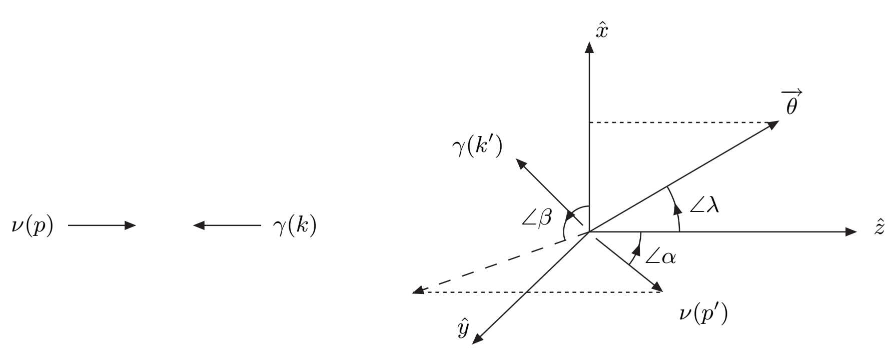


FIG. 3. The process $\gamma\nu \to \gamma\nu$ in the center of mass frame.

$$(k\cdot\theta\cdot p')^2 = E^4\,|\vec{\theta}\,|^2\sin^2\alpha\sin^2\lambda\sin^2\beta,$$
$$p\cdot\theta\cdot\theta\cdot p = E^2\,|\vec{\theta}\,|^2\sin^2\lambda,$$
$$p'\cdot\theta\cdot\theta\cdot p' = E^2\,|\vec{\theta}\,|^2(1-\cos^2\alpha\cos^2\lambda - \sin^2\alpha\sin^2\lambda\cos^2\beta - 0.5\sin 2\alpha\sin 2\lambda\cos\beta),$$
$$p\cdot\theta\cdot\theta\cdot p' = E^2\,|\vec{\theta}\,|^2(\sin^2\lambda\cos\alpha - 0.5\sin 2\lambda\sin\alpha\cos\beta), \tag{50}$$

one can easily perform the β and the α integration of (45) to find

$$\sigma = 0.5\cos^4\theta_W\alpha^2\frac{E^6}{\Lambda^8}, \tag{51}$$

or

$$\sigma = 3.8\times 10^{-32}\left(\frac{M_Z}{\Lambda}\right)^8\left(\frac{E}{m_e}\right)^6\ \text{pb}. \tag{52}$$

By choosing $\Lambda = 113$ GeV in Eq. (52) one has

$$\sigma = 6.7\times 10^{-33}\left(\frac{E}{m_e}\right)^6\ \text{pb}, \tag{53}$$

which is equal to the cross section of photon-neutrino elastic scattering in the range $m_e \ll E \ll M_W$ in the commutative standard model given in Eq. (1) while for the cross section of Eq. (53) there is not such a constraint.

IV. SUMMARY

In this paper , we extended the noncommutative standard model based on the minimal SU(3) × SU(2) × U(1) gauge group to include the interaction of the neutral gauge bosons with the neutrino. Since in the gauge invariant manner only the particle with neutral hypercharge can couple to the hypergauge field, the right-handed neutrino part of the NCSM Lagrangian density changes as is given in Eq. (12). Consequently, we obtained the Feynman rule for the $\gamma\nu\bar{\nu}$-vertex which does not exist in the minimal extension of the noncommutative standard model introduced in [13], while for the $Z\nu\bar{\nu}$-vertex we find some corrections given in Eqs. (14) and (15). We explored the photon-neutrino elastic scattering in both the nmNCSM and the extended version of mNCSM. In the former model, the left-handed neutrino at the tree level can interact with photon via Z-gauge boson exchange as is shown in Fig. 1. We showed that the cross section grows as E^6 in the center of mass and depends on the new undetermined constant, $K_{Z\gamma\gamma}$, as well as the parameter of noncommutativity, see Eq. (28). The cross section for $K_{Z\gamma\gamma} = 0.1$ varies in the range 10^{-42}–10^{-46} cm^2 for $\Lambda \sim 100$–1000 GeV and $E \sim 0.1M_Z$ which is comparable with its counterpart in the commutative space though $K_{Z\gamma\gamma}$ varies in the range $-0.3 \le K_{Z\gamma\gamma} \le 0.1$ and it may be zero. Nevertheless, the photon-neutrino elastic scattering is also examined in the extended version of mNCSM where the photon can interact directly with the neutrino. In this case there are two Feynman diagrams at the tree level which are presented in Fig. 2. Since the parameter of noncommutativity is the only mass scale, the cross section should be proportional to $\alpha^2\Lambda^{-8}E^6$ which is explicitly obtained in Eq. (51). Comparison of Eq. (53) and (1) with Eq. (2) shows that the three cross sections are equal for $E = 6.5$ GeV while the value of the photon-neutrino elastic scattering cross section in the noncommutative space at $E = 10$ GeV is about 2 times the value of its counterpart in the commutative space. Therefore, at sufficiently high energies the process $\nu\gamma \to \nu\gamma$ in the noncommutative space dominates the processes $\nu\gamma \to \nu\gamma$ and $\nu\gamma \to \nu\gamma\gamma$ in the commutative space. Nonetheless, for the higher values of Λ the elastic cross section in the NC-space will be comparable with the elastic one in the commutative space at the higher energies. For example, for $\Lambda = 1000$ GeV at $E = 500$ GeV it is still 1% of the cross section of $\nu\gamma \to \nu\gamma$ in the SM while they are equal at $E \sim 1000$ GeV. Therefore, in the high energy limit the right-handed neutrino has the same contribution to the photon-neutrino scattering as the left-handed one and is not the expectator particle.

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