Inclusive production of the $X(3872)$

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If the $X(3872)$ is a loosely-bound $D^{*0}\bar{D}^0/D^0\bar{D}^{*0}$ molecule, its inclusive production rate can be described by the NRQCD factorization formalism that applies to inclusive quarkonium production. We argue that if the molecule has quantum numbers $J^{PC} = 1^{++}$, the most important term in the factorization formula should be the color-octet $^3S_1$ term. This is also one of the two most important terms in the factorization formulas for χ_{cJ}. Since the color-octet $^3S_1$ term dominates χ_{cJ} production for many processes, the ratio of the inclusive direct production rates for X and χ_{cJ} should be roughly the same for these processes. The assumption that the ratio of the production rates for X and χ_{cJ} is the same for all processes is used to estimate the inclusive production rate of X in B meson decays, Z^0 decays, and in $p\bar{p}$ collisions.

The $X(3872)$ is a mysterious charmoniumlike state discovered by the Belle collaboration in 2003 [1]. It was discovered through the exclusive decay $B^+ \rightarrow XK^+$ followed by the decay $X \rightarrow J/\psi\pi^+\pi^-$. The discovery was confirmed by the CDF collaboration [2], which observed the inclusive production of the X meson in $p\bar{p}$ collisions. The X has also been observed in the discovery mode by the *BABAR* collaboration [3] and in $p\bar{p}$ collisions by the D0 collaboration [4].

The interpretation of the X meson remains a mystery [5–14]. The decay mode $J/\psi\pi^+\pi^-$ suggests that it is a charmonium state, but its decay properties seem to be incompatible with its identification with any charmonium state [11,13,15]. The Belle collaboration found that the width of X is small compared to that of other charmonium states above the $D\bar{D}$ threshold: $\Gamma_X < 2$ MeV at the 90% confidence level [1]. In addition to the discovery decay mode $J/\psi\pi^+\pi^-$, the $X(3872)$ has been observed in the decay modes $J/\psi\pi^+\pi^-\pi^0$ and $J/\psi\gamma$ [16]. Upper limits have been placed on the branching fractions for several other decay modes of the X, including $D^0\bar{D}^0$, D^+D^-, $D^0\bar{D}^0\pi^0$ [17], $\chi_{c1}\gamma$, $\chi_{c2}\gamma$, $J/\psi\pi^0\pi^0$ [18], and $J/\psi\eta$ [19].

Another possibility is that the X meson is a $D^{*0}\bar{D}^0/D^0\bar{D}^{*0}$ molecule [5,8,10,12]. This possibility is motivated by the fact that its mass is extremely close to the $D^{*0}\bar{D}^0$ threshold. The mass obtained by combining the measurements of the four experiments is $M_X = 3871.9 \pm 0.5$ MeV [15]. Subtracting the $D^{*0}\bar{D}^0$ threshold energy, we obtain $M_X - M_{D^0} - M_{D^{*0}} = +0.6 \pm 1.1$ MeV. For this molecule to have a substantial branching fraction into $J/\psi\pi^+\pi^-$, it must be a bound state, which requires that its binding energy $E_X = M_{D^0} + M_{D^{*0}} - M_X$ be positive. (Note that the central value of E_X from the mass measurement is negative.) If the binding of the $D^{*0}\bar{D}^0$ molecule arises from pion exchange, the most likely channel is S-wave with quantum numbers $J^{PC} = 1^{++}$ [5]. The observed binding energy E_X is small compared to the natural energy scale associated with pion-exchange interactions:

$E_X \ll m_\pi^2/2\mu \approx 10$ MeV, where μ is the reduced mass of the D^{*0} and $\bar{D}^0$. The small binding energy implies that the $D^{*0}\bar{D}^0$ scattering length a is large compared to the natural length scale $1/m_\pi \approx 1.4$ fm. As a consequence of the large scattering length, the molecule has universal properties that are determined by a and insensitive to the shorter length scales of QCD [10]. The simplest example is the binding energy itself: $E_X = 1/(2\mu a^2)$. Another example is the rate and momentum spectrum for the decays $X \rightarrow D^0\bar{D}^0\pi^0$ and $X \rightarrow D^0\bar{D}^0\gamma$ [8]. A third example is the rate for the decay $\Upsilon(4S) \rightarrow Xh^+h^-$, where h^+ and h^- are light charged hadrons [20]. Another universal feature associated with the large scattering length is an enhancement in the $D^{*0}\bar{D}^0$ invariant mass distribution near the $D^{*0}\bar{D}^0$ threshold in the decay $B^+ \rightarrow D^{*0}\bar{D}^0K^+$ [21].

A challenge to any interpretation of the $X(3872)$ is to understand its production rate. The Belle and *BABAR* collaborations have measured the product of the branching fractions for the two decays associated with its discovery [1,3]:

$$\text{Br}[B^+ \rightarrow XK^+]\text{Br}[X \rightarrow J/\psi\pi^+\pi^-] = (1.3 \pm 0.3) \times 10^{-5}. \tag{1}$$

The only other information about the production of X that is available is that its inclusive production has been observed in $p\bar{p}$ collisions.

In this paper, we will discuss the inclusive production of the $X(3872)$. We will point out that if the X meson is a loosely-bound S-wave $D^{*0}\bar{D}^0/D^0\bar{D}^{*0}$ molecule, its inclusive production rate can be analyzed using the NRQCD factorization formalism [22]. We will argue that if its quantum numbers are 1^{++}, the most important term in the NRQCD factorization formula is the color-octet $^3S_1$ term. This is also one of the two most important terms in the NRQCD factorization formulas for χ_{cJ}. Thus for all processes in which χ_{cJ} production is dominated by the color-octet $^3S_1$ term, the ratio of the inclusive production

rates for X and χ_{cJ} should be roughly the same. The assumption that the ratio of the inclusive production rates for X and χ_{cJ} should be the same will be used to estimate the inclusive production rate of X in B meson decays, Z^0 decays, and in $p\bar{p}$ collisions.

The production of the X meson involves many length scales. The creation of the $c\bar{c}$ pair with small relative momentum requires a hard-scattering process with a short-distance length scale $1/m_c$. The evolution of the $c\bar{c}$ into a color-singlet hadronic system, either a charmonium or a pair of charmed mesons, occurs over a longer length scale $1/m_c v$ or $1/m_c v^2$. The evolution of the charmed mesons into one of the four possibilities $D\bar{D}$, $D^*\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$ occurs over an even longer length scale $1/m_\pi$. Finally, the binding of $D^{*0}\bar{D}^0$ into the molecular state X occurs over the very long length scale a. We will use the NRQCD factorization formalism to exploit the separation between the short-distance scale m_c and all the longer distance scales of order $1/m_c v$ and larger.

The NRQCD factorization formalism [22] is used to describe the inclusive production of charmonium. This formalism exploits the separation of scales between the hard-scattering process that creates the $c\bar{c}$ pair and the hadronization process by which the $c\bar{c}$ pair evolves into a color-singlet charmonium. The NRQCD factorization formula expresses the inclusive cross section for producing the charmonium state H as a sum of products of $c\bar{c}$ cross sections and NRQCD matrix elements:

$$\sigma[H + \text{anything}] = \sum_n \hat{\sigma}[c\bar{c}_n + \text{anything}]\langle\mathcal{O}_n^H\rangle. \quad (2)$$

The $c\bar{c}$ cross section $\hat{\sigma}$ is the inclusive cross section for producing a $c\bar{c}$ pair in the color and angular-momentum channel labeled by n. It involves momentum scales of order m_c and larger. The corresponding NRQCD matrix element $\langle\mathcal{O}_n^H\rangle$ is the vacuum expectation value of an operator that creates a $c\bar{c}$ pair at a point in the channel specified by n, projects onto states that in the asymptotic future include the charmonium H, and then annihilates the $c\bar{c}$ pair at the creation point in the channel specified by n. The NRQCD matrix elements involve only momentum scales of order $m_c v$ or smaller, where v is the typical relative velocity of the charm quark in the charmonium. The sum in (2) includes infinitely many terms, so it is useful only if the sum over channels can be truncated. The truncation can be justified by the velocity-scaling rules for quarkonium, which specify how each of the NRQCD matrix elements scale with v. For P-wave charmonium states, there are two NRQCD matrix elements at leading order in v: a color-singlet P-wave matrix element and a color-octet S-wave matrix element [23]. If the NRQCD expansion is truncated at this order, the inclusive cross section for χ_{cJ} can be written

$$\sigma[\chi_{cJ} + \text{anything}] \simeq \hat{\sigma}[c\bar{c}_1(^3P_J) + \text{anything}]\langle\mathcal{O}_1^{\chi_{cJ}}(^3P_J)\rangle$$
$$+ \hat{\sigma}[c\bar{c}_8(^3S_1) + \text{anything}]$$
$$\times \langle\mathcal{O}_8^{\chi_{cJ}}(^3S_1)\rangle. \quad (3)$$

The subscript on $c\bar{c}$ and on the operator $\mathcal{O}$ indicates the color channel and the argument in parentheses indicates the angular-momentum channel. The relative sizes of the two terms in Eq. (3) depends on the NRQCD renormalization scale [23]. A change in the NRQCD renormalization scale for the operator $\mathcal{O}_1(^3P_J)$ can be compensated by adding the operator $\mathcal{O}_8(^3S_1)$ with a coefficient of order α_s. For processes in which a color-octet $c\bar{c}$ pair can be created at leading order in α_s, the color-octet $^3S_1$ term in the cross section is likely to dominate for a suitable choice of the renormalization scale. Heavy quark spin symmetry implies that the matrix elements in Eq. (3) are proportional to $2J + 1$ [22]. Thus the factorization formula can be written

$$\sigma[\chi_{cJ} + \text{anything}] \simeq (2J + 1)\hat{\sigma}[c\bar{c}_1(^3P_J) + \text{anything}]$$
$$\times \langle\mathcal{O}_1^{\chi_{c0}}(^3P_0)\rangle$$
$$+ (2J + 1)\hat{\sigma}[c\bar{c}_8(^3S_1) + \text{anything}]$$
$$\times \langle\mathcal{O}_8^{\chi_{c0}}(^3S_1)\rangle. \quad (4)$$

The only dependence on J in the coefficient of $\langle\mathcal{O}_8^{\chi_{c0}}(^3S_1)\rangle$ is the explicit factor of $2J + 1$. Thus processes that are dominated by the color-octet $^3S_1$ term can be identified experimentally by the cross sections for χ_{c1} and χ_{c2} being in the proportions 3:5.

The NRQCD factorization formalism applies equally well to a pair of charm mesons, such as D^{*0} and $\bar{D}^0$, that are produced with a relative momentum of order $m_c v$ or smaller. For example, the inclusive cross section for producing a $D^{*0}\bar{D}^0$ pair with kinetic energy in its rest frame less than $E_{\max}$, where $E_{\max} \ll m_c$, can be written

$$\sigma[D^*\bar{D}(E_{\max}) + \text{anything}] = \sum_n \hat{\sigma}[c\bar{c}_n + \text{anything}]$$
$$\times \langle\mathcal{O}_n^{D^*\bar{D}(E_{\max})}\rangle. \quad (5)$$

We have used $D^*\bar{D}(E_{\max})$ as a shorthand for $D^{*0}\bar{D}^0$ states with kinetic energy less than $E_{\max}$. The $c\bar{c}$ cross sections $\hat{\sigma}$ are the same as in (2). In the matrix elements, the NRQCD operators include a projection onto states that in the asymptotic future include a $D^{*0}\bar{D}^0$ pair with kinetic energy less than $E_{\max}$. If the X meson is a loosely-bound $D^{*0}\bar{D}^0/D^0\bar{D}^{*0}$ molecule, the NRQCD factorization formula can also be applied to its inclusive production rate. The inclusive cross section for producing X can be written

$$\sigma[X + \text{anything}] = \sum_n \hat{\sigma}[c\bar{c}_n + \text{anything}]\langle\mathcal{O}_n^X\rangle. \quad (6)$$

In the matrix elements, the NRQCD operators include a projection onto states that in the asymptotic future include an $X(3872)$.

The NRQCD factorization formulas in Eqs. (5) and (6) are useful only if the expansions can be truncated. Unlike the case of quarkonium, there are no velocity-scaling rules governing the NRQCD matrix elements for $D^{*0}\bar{D}^0$ or X that can be used to justify the truncations. The velocity-scaling rules would be applicable if the $c\bar{c}$ pair evolved into a heavy hadron H with constituents c and $\bar{c}$ only through the emission of real or virtual gluons from the c or $\bar{c}$. Velocity-scaling rules do not apply if the evolution involves the emission of real or virtual gluons from light quarks or antiquarks in the hadron H. Thus the presence of $\bar{u}$ and u as valence quarks in the D^{*0} and $\bar{D}^0$ guarantees that the velocity-scaling rules are inapplicable.

Another class of $c\bar{c}$ systems for which the velocity-scaling rules are inapplicable is charmonium hybrid states, whose valence content is $c\bar{c}g$ [24]. Chiladze, Falk and Petrov pointed out that the NRQCD factorization formula could nevertheless be applied to the inclusive production of charmonium hybrid states. We can use similar arguments to justify the NRQCD factorization formula for the inclusive production of X. The NRQCD matrix elements in the NRQCD factorization formula for inclusive X production in Eq. (6) are proportional to the probabilities for the evolution of the $c\bar{c}$ pair into a color-singlet $c\bar{c}u\bar{u}$ system, its subsequent evolution into an S-wave $D^{*0}\bar{D}^0$ or $D^0\bar{D}^{*0}$ system, and finally the binding of the charm mesons into the molecular state X. Since the X is weakly bound, the charm mesons are almost at rest. The momentum scale of the c and $\bar{c}$ in the X is therefore that of the c and $\bar{c}$ in the charm mesons, which is of order Λ_{QCD}. This is small compared to the momentum scale of the hard-scattering process that creates the $c\bar{c}$ pair, which is of order m_c or larger. This guarantees that the hard-scattering amplitude can be expanded in powers of the relative momentum of the $c\bar{c}$. The convergence of this expansion must come from a suppression of the NRQCD matrix elements of higher dimension operators. The convergence suggests a hierarchy of NRQCD matrix elements according to the dimension of the NRQCD operators. In particular, matrix elements associated with $c\bar{c}$ channels with higher orbital angular momentum quantum numbers should be suppressed. This motivates a truncation to a few of the lowest dimension operators in each of the four color/spin channels: color-singlet spin-singlet, color-singlet spin-triplet, color-octet spin-singlet, and color-triplet spin-triplet. The most extreme truncation of the NRQCD factorization formula would be to include only the four S-wave matrix elements: $\langle\mathcal{O}_1^X(^1S_0)\rangle$, $\langle\mathcal{O}_1^X(^3S_1)\rangle$, $\langle\mathcal{O}_8^X(^1S_0)\rangle$, and $\langle\mathcal{O}_8^X(^3S_1)\rangle$.

Voloshin has pointed out that if the $X(3872)$ is a $D^{*0}\bar{D}^0$ molecule with $J^{PC} = 1^{++}$, the $c\bar{c}$ pair in the X is necessarily in a spin-triplet state [25]. Heavy-quark spin symmetry implies that $X(3872)$ is produced predominantly

from $c\bar{c}$ pairs that are created in spin-triplet states. Thus the spin-singlet matrix elements for the production of $X(3872)$ are suppressed. If the NRQCD factorization formula is truncated to include only the S-wave matrix elements, the leading terms are

$$\sigma[X + \text{anything}] \approx \hat{\sigma}[c\bar{c}_1(^3S_1) + \text{anything}]\langle\mathcal{O}_1^X(^3S_1)\rangle$$
$$+ \hat{\sigma}[c\bar{c}_8(^3S_1) + \text{anything}]\langle\mathcal{O}_8^X(^3S_1)\rangle. \tag{7}$$

We now argue that the color-octet term in Eq. (7) will often dominate. First we note that since the constituent charm mesons in the X are color-singlets, the probabilities for the $c\bar{c}$ pair in the X to be in a color-singlet and color-octet state are $1/9$ and $8/9$, respectively. The naive expectation is that the probabilities for forming an X from the color-singlet $c\bar{c}$ pair and from each of the 8 color-octet $c\bar{c}$ pairs should be equal. Thus unless the color-singlet parton cross section $\hat{\sigma}$ is larger than the color-octet parton cross section by a factor large enough to compensate for the color factor of 8, we would expect the color-octet term in Eq. (7) to dominate. This reduces the NRQCD factorization formula to a single term:

$$\sigma[X + \text{anything}] \approx \hat{\sigma}[c\bar{c}_8(^3S_1) + \text{anything}]\langle\mathcal{O}_8^X(^3S_1)\rangle. \tag{8}$$

We have argued that the color-octet $^3S_1$ term should be the most important term in the NRQCD factorization formula for the inclusive production of X, in which case it reduces to Eq. (8). The color-octet $^3S_1$ term is also one of the two leading terms in the NRQCD factorization formula for χ_{cJ}, which is given in Eq. (3). For many production processes, the color-octet $^3S_1$ term in the NRQCD factorization formula for χ_{cJ} will dominate for a suitable choice of the NRQCD renormalization scale. For these processes, the ratio of the inclusive cross sections for X and χ_{cJ} will be the ratio of the color-octet $^3S_1$ matrix elements:

$$\frac{\sigma[X + \text{anything}]}{\sigma[\chi_{cJ} + \text{anything}]} \approx \frac{\langle\mathcal{O}_8^X(^3S_1)\rangle}{(2J+1)\langle\mathcal{O}_8^{\chi_{c0}}(^3S_1)\rangle}. \tag{9}$$

The cross sections in Eq. (9) should be interpreted as those for direct production, which means that the feeddown contributions from decays of heavier $c\bar{c}$ states, such as $\psi(2S) \to \chi_{cJ}\gamma$, have been subtracted. We will use the assumption that Eq. (9) holds for most production processes to estimate the inclusive production rates of X in various processes. One could ignore the motivation based on NRQCD and simply take Eq. (9) as a phenomenological assumption.

We first consider the inclusive production of $X(3872)$ in B meson decays. This decay produces a mixture of B^+, B^0, and their antiparticles. The inclusive branching fraction for the direct production of χ_{c1} from the decays of this mixture of B mesons has been measured to be $(3.3 \pm 0.5) \times 10^{-3}$. (The adjective "direct" means that the feeddown from the

decay $\psi(2S) \to \chi_{c1}\gamma$ has been removed.) The inclusive branching fraction is the sum of the branching fractions for the 2-body decay modes $\chi_{c1}K$ and $\chi_{c1}K^*$, the 3-body decay modes, etc. The fraction of the inclusive decay rate that comes from the 2-body decay mode $\chi_{c1}K$ is $(16 \pm 6)\%$. The fraction of the inclusive decay rate that comes from 2-body decays is determined largely by the available hadronic energy, which does not vary dramatically among the charmonium states. For example, the 2-body decay $B \to J/\psi K$ accounts for $(11.8 \pm 1.5)\%$ of the direct inclusive rate for $B \to J/\psi +$ anything. The 2-body decay $B \to \psi(2S)K$ accounts for $(18 \pm 4)\%$ of the inclusive rate for $B \to \psi(2S) +$ anything. These fractions all differ by less than a standard deviation. We will use the central value of the observed fraction for χ_{c1} as an estimate of the fraction for X:

$$\frac{\text{Br}[B \to XK]}{\text{Br}[B \to X + \text{anything}]} \approx \frac{B[B \to \chi_{c1}K]}{B_{\text{dir}}[B \to \chi_{c1} + \text{anything}]} \approx 16\%.$$

(10)

Combining this with the Belle measurement in Eq. (1), we obtain the estimate

$$\text{Br}[B \to X + \text{anything}]\text{Br}[X \to J/\psi\pi^+\pi^-] \approx 8 \times 10^{-5}.$$

(11)

After multiplying by the 12% branching fraction of J/ψ into leptons, the product of the three branching fractions is about 10^{-5}. With the Belle and *BABAR* collaborations having each accumulated more than 10^8 $B\bar{B}$ pairs, the predicted rate is large enough that the inclusive production of the X should be observable in the data from the B factories.

Dividing the estimate in (11) by the measured inclusive branching fraction for B decay into χ_{c1}, we obtain the product of the constant in (9) and the branching fraction for the decay $X \to J/\psi\pi^+\pi^-$:

$$\frac{\sigma[X + \text{anything}]}{\sigma_{\text{dir}}[\chi_{c1} + \text{anything}]}\text{Br}[X \to J/\psi\pi^+\pi^-] \approx 0.025.$$

(12)

This can be used to estimate the inclusive production rate of X in any process for which the inclusive production rate of χ_{c1} has been measured.

In the decay of the Z^0, charmonium is produced primarily by the decay of the Z^0 into $b\bar{b}$, followed by the fragmentation of the b or $\bar{b}$ into a heavy hadron, and then the decay of the heavy hadron into charmonium. The inclusive branching fraction of Z^0 into a charmonium H is to a good approximation the sum over b hadrons of the inclusive branching fraction for the b hadron to decay into H, weighted by the probability for a b quark to fragment into the b hadron, and multiplied by 2 to account for the b and the $\bar{b}$. The inclusive branching fraction for the decay of Z^0 into χ_{c1} has been measured. The direct branching

fraction obtained by subtracting the small feeddown from $\psi(2S)$ decays is $(2.7 \pm 0.8) \times 10^{-3}$. Multiplying this branching fraction by the number in (12), we obtain the estimate

$$\text{Br}[Z^0 \to X + \text{anything}]\text{Br}[X \to J/\psi\pi^+\pi^-] \approx 7 \times 10^{-5}.$$

(13)

After multiplying by the 12% branching fraction of J/ψ into leptons, the product of the three branching fractions is about 10^{-5}. With each of the four detectors at LEP having collected millions of hadronic Z^0 decays, the predicted rate is large enough that the X may be observable in the LEP data.

We next consider the inclusive production of $X(3872)$ in $p\bar{p}$ collisions at the Tevatron. The inclusive sample of X mesons at the Tevatron consists of two components: a b-decay component that comes from inclusive decays of b hadrons into X, and a prompt component consisting of X mesons that are produced directly in the $p\bar{p}$ collisions. The two components can be separated by using a vertex detector. Those X mesons for which the tracks of the pions from the decay $X \to J/\psi\pi^+\pi^-$ and the leptons from the decay $J/\psi \to \ell^+\ell^-$ can be traced back to the $p\bar{p}$ collision point are mostly prompt, while those for which the tracks of the charged particles can be traced back to a secondary vertex come from b decay.

In Run I of the Tevatron, which consisted of $p\bar{p}$ collisions at center-of-mass energy of 1.8 TeV, the CDF collaboration measured the fraction of prompt J/ψ's that come from χ_c decay and the ratio of the prompt cross sections for producing χ_{c1} or χ_{c2}. The fraction of prompt J/ψ's with transverse momentum $p_T > 4$ GeV and pseudorapidity satisfying $|\eta| < 0.6$ that come from the decays of χ_{c1} or χ_{c2} is $0.297 \pm 0.017(\text{stat}) \pm 0.057(\text{syst})$ [26]. The ratio of the prompt cross sections for producing χ_{c1} and χ_{c2}, where the χ_c subsequently decays into $J/\psi\gamma$ with the J/ψ having $p_T < 4$ GeV, is $1.04 \pm 0.29(\text{stat}) \pm 0.12(\text{syst})$ [27]. Combining these results with the estimate in (12), we obtain the estimate

$$\frac{\sigma[X + \text{anything}]}{\sigma[J/\psi + \text{anything}]}\text{Br}[X \to J/\psi\pi^+\pi^-] \approx 4 \times 10^{-3}.$$

(14)

The left side of Eq. (14) is simply the fraction of prompt J/ψ's that come from decays of X multiplied by the ratio of the branching fractions for $X \to J/\psi\pi^+\pi^-$ and $X \to J/\psi +$ anything. The fraction of prompt J/ψ's that come from decays of $\psi(2S)$ are observed to increase with p_T from $(7 \pm 2)\%$ at $p_T = 5$ GeV to $(15 \pm 5)\%$ at $p_T = 18$ GeV [26]. In contrast, the fraction of J/ψ's that come from decays of χ_c seemed to decrease slightly with p_T [26]. In Run II of the Tevatron, improvements in the detectors will allow these fractions to be measured as a function of p_T down to $p_T = 0$. Our assumption (9) im-

plies that the fraction of prompt J/ψ's that come from decays of X should have the same p_T-dependence as the fraction of J/ψ's that come from decays of χ_{cJ}.

We have pointed out that the NRQCD factorization formalism that describes inclusive charmonium production can also be applied to the inclusive production of pairs of charmed mesons with sufficiently small invariant mass. If the X(3872) is a $D^{*0}\bar{D}^0/D^0\bar{D}^{*0}$ molecule, the NRQCD factorization formalism also applies to the inclusive production rate of X. We argued that if X has quantum numbers 1^{++}, the leading term in the NRQCD factorization formula for X should be the color-octet $^3S_1$ term. We used the assumption that the ratio of the inclusive production rates of X and χ_{cJ} should be roughly the same for all processes to estimate the inclusive production rate of X(3872) in various high energy processes. Measurements of inclusive production rates consistent with these estimates would be consistent with the interpretation of the X(3872) as a $D^{*0}\bar{D}^0/D^0\bar{D}^{*0}$ molecule with $J^{PC} = 1^{++}$.

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