Matrix orientifolding and models with four or eight supercharges

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The conditions under which matrix orientifolding and supersymmetry transformations commute are known to be stringent. Here we present the cases possessing four or eight supercharges upon $\mathbf{Z}_3$ orbifolding followed by matrix orientifolding. These cases descend from the matrix models with eight plus eight supercharges. There are 50 in total, which we enumerate.

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I. INTRODUCTION

Continuing attention has been paid to matrix models which are proposed to enable nonperturbative studies of strings beyond their perturbative and semiclassical regimes [1–6]. The objects playing a central role are, of course, discretized string coordinates represented by matrices taking values of appropriate Lie algebras. Their diagonal entries represent spacetime points, while off diagonal ones mediate interactions between blocks which may be identified as D-objects. A few ideas on the formation of our spacetime such as the one via branched polymers [7] and the one via generalized monopoles [8] have appeared and approximation schemes [9] have been devised. (See [10] for more references.)

Not only the string coordinates but also algebraic operations in the first quantized string theory have natural matrix counterparts even when the size of the matrices is kept finite. In particular, the matrix counterpart of twist operation or orientifolding is easily obtained as any $U(2k)$ Lie algebra valued matrix splits into a direct sum of the adjoint representation and the antisymmetric representation of $USp(2k)$ or $SO(2k)$ Lie algebra. Selecting one of these two representations for each of the original matrix coordinates is referred to as matrix orientifolding in this paper.

Realizing the twist operation of matrices this way has turned out to put stringent conditions on the number of supercharges [4]: the supersymmetry transformations in the Wess-Zumino gauge are nonlinear and requiring that they commute with the projectors materializing matrix orientifolding yields nontrivial algebraic conditions. In the case of $8 + 8$ supercharges, these conditions are successful in selecting the two known cases which correspond to the USp matrix model [3,4] relevant to type I superstrings and the matrix model [5,6,11] of heterotic M theory [12]. In the light of assessing these algebraic conditions further and of hoping to find principal matrix configurations leading to $\mathcal{N} = 1$ vacua in four dimensions, it is interesting to find out how many cases of matrix orientifolding one can construct which possess fewer supercharges. To put this question more concrete, consider the matrix analog of $\mathbf{C}^3/\mathbf{Z}_3$ [13], and subsequently operate matrix orientifolding. By this procedure, we can obtain the matrix model of type I superstring theory in the spacetime compactified to four dimensions. In this paper, we focus upon the problem of enumerating all possible such cases with supersymmetries, namely, the ones obtained by $\mathbf{Z}_3$ orbifolding followed by matrix orientifolding.

It may be worthwhile to elaborate here more on physics motivations for studying orientifolding in matrix models. As is well known, the major issue of string theory is how to lift an innumerable number of perturbative vacuum degeneracies. This is certainly mandatory in order for string theory to offer us the basic understanding of particle and gravitational physics at the same time. Matrix models provide a constructive framework to study nonperturbative effects of strings as well as to determine the true vacuum. Supersymmetry is certainly requested in the current framework. Therefore for sake of vacuum selection, it is interesting and important to see whether some discrete operations valid to all order in string perturbation theory are able to select string configurations once we demand that they are compatible with $\mathcal{N} = 1$ supersymmetry in four dimensions. The matrix orientifolding is one such discrete operation which we will study in this paper, and may develop more interesting physics results such as the number of spacetime dimensions, the number of generations, and Higgs physics in the future.

In the next section, we recall the two cases of matrix orientifolding with $8 + 8$ supercharges. After introducing $\mathbf{Z}_3$ orbifolding acting upon three complex matrix coordinates and its prototypical example in Section III, we carry out the matrix orientifolding of this example in Section IV. We show that there are two consistent possibilities with respect to supersymmetries and that there are in total five cases: the one possesses $4 + 0$ supersymmetries while the remaining four possess $2 + 2$ supersymmetries. The problem to enumerate all cases obtained upon an arbitrary $\mathbf{Z}_3$ orbifolding and subsequently matrix orientifolding while keeping some supersymmetries intact is addressed in Section V. $\mathbf{Z}_3$ orbifolding leaves either $4 + 4$ supersymmetries or $8 + 8$ supersymmetries intact. We show that, to

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each of the four possibilities belonging to the former, there is one case of consistent matrix orientifolding with 4 + 0 supersymmetries and four with 2 + 2 supersymmetries. As for each of the six possibilities belonging to the latter (8 + 8 supersymmetries), we show that there is also one with 8 + 0 supersymmetries and four with 4 + 4 supersymmetries. The total number of such cases is 50. This number is considered to be small in the light of an innumerable number of perturbative superstring vacua.

II. MATRIX ORIENTIFOLDING WITH 8 + 8 SUPERCHARGES

The action of the IIB matrix model is

$$S = -\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[A_N, A_M][A^N, A^M] + \frac{1}{2}\bar{\psi}\Gamma^N[A_N, \psi]\right). \tag{2.1}$$

Here ψ is a ten-dimensional Majorana-Weyl spinor, and A_I and ψ are $N \times N$ Hermitian matrices. The action has dynamical supersymmetry

$$\delta^{(1)}\psi = \frac{i}{2}[A_N, A_M]\Gamma^{NM}\epsilon, \tag{2.2}$$

$$\delta^{(1)}A_N = i\bar{\epsilon}\Gamma^N\psi, \tag{2.3}$$

and kinematical supersymmetry

$$\delta^{(2)}\psi = \xi, \tag{2.4}$$

$$\delta^{(2)}A_N = 0. \tag{2.5}$$

As is mentioned in the introduction, any $U(2k)$ Lie algebra valued matrix splits into a direct sum of the two matrices which are, respectively, the adjoint representation and the antisymmetric representation of $USp(2k)$ Lie algebra and this is schematically drawn as

$$U(2k)\ \text{adjoint}\quad \begin{array}{l} \overset{\hat{\rho}_-}{\nearrow}\ \ \text{USp adjoint} \\ \underset{\hat{\rho}_+}{\searrow}\ \ \text{USp antisymmetric} \end{array}$$

$$\text{adj}\ \ X\colon\ X^tF + FX = 0, \tag{2.6}$$

$$\text{asy}\ \ Y\colon\ Y^tF - FY = 0. \tag{2.7}$$

Here F is the matrix counterpart of the twist operation

$$F = \begin{pmatrix} 0 & I_k \\ -I_k & 0 \end{pmatrix}, \tag{2.8}$$

and $\hat{\rho}_\mp$ are the projectors

$$\hat{\rho}_\mp \bullet = \tfrac{1}{2}(\bullet \mp F^{-1}\bullet^t F). \tag{2.9}$$

Let

$$v_M \equiv \delta_M^N \hat{\rho}_{b\mp}^{(N)} A_N, \qquad \Psi_A \equiv \delta_{AB}\hat{\rho}_{f\mp}^{(B)}\psi_B, \tag{2.10}$$

where $\hat{\rho}_{b\mp}^{(N)}$ and $\hat{\rho}_{f\mp}^{(B)}$ are either $\hat{\rho}_-$ or $\hat{\rho}_+$ for each N and for each B, respectively. More explicitly

$$\begin{aligned} \hat{\rho}_{b\mp}^{(M)} &\equiv \Theta(M \in \mathcal{M}_-)\hat{\rho}_- + \Theta(M \in \mathcal{M}_+)\hat{\rho}_+, \\ \hat{\rho}_{f\mp}^{(A)} &\equiv \Theta(A \in \mathcal{A}_-)\hat{\rho}_- + \Theta(M \in \mathcal{A}_+)\hat{\rho}_+, \end{aligned} \tag{2.11}$$

where

$$\begin{aligned} &\mathcal{M}_- \cup \mathcal{M}_+ = \{\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}\}, \\ &\mathcal{M}_- \cap \mathcal{M}_+ = \emptyset, \end{aligned} \tag{2.12}$$

$$\begin{aligned} \mathcal{A}_- \cup \mathcal{A}_+ = \{\{&1, 2, 5, 6, 9, 10, 13, 14, 19, 20, 23, \\ &24, 27, 28, 31, 32\}\}, \\ \mathcal{A}_- \cap \mathcal{A}_+ = \emptyset&. \end{aligned} \tag{2.13}$$

By construction, each component of v_M and that of Ψ_A belong either to the adjoint or to the antisymmetric representation of $USp(2k)$. We impose Eq. (2.10) on A_N and ψ_B. The condition $[\hat{\rho}_{b\mp}, \delta^{(1)}]A = 0$ gives

$$\sum_A (\bar{\epsilon}\Gamma_M)_A(\hat{\rho}_{f\mp}^{(A)} - \hat{\rho}_{b\mp}^{(M)})\psi_A = 0 \tag{2.14}$$

with M not summed, while the condition $[\hat{\rho}_{f\mp}, \delta^{(1)}]\psi|_{v_M \to \hat{\rho}_{b\mp}v_M} = 0$ gives

$$(1 - \hat{\rho}_{f\mp}^{(A)})[\hat{\rho}_{b\mp}^{(M)}A_M, \hat{\rho}_{b\mp}^{(N)}A_N](\Gamma^{MN}\epsilon)_A = 0. \tag{2.15}$$

The condition $[\hat{\rho}_{b\mp}, \delta^{(1)}]A = 0$ does not give us anything new while $[\hat{\rho}_{f\mp}, \delta^{(2)}]\psi = 0$ gives

$$\xi_A \mathbf{1} = \xi_A \hat{\rho}_{f\mp}^{(A)}\mathbf{1}. \tag{2.16}$$

Equation (2.14) gives

$$(\bar{\epsilon}\Gamma_{M_-})_{A_+} = (\bar{\epsilon}\Gamma_{M_+})_{A_-} = 0, \tag{2.17}$$

while Eq. (2.15) gives

$$(\Gamma^{M_-N_+}\epsilon)_{A_-} = 0, \qquad (\Gamma^{M_-N_-}\epsilon) = (\Gamma^{M_+N_+}\epsilon)_{A_+} = 0, \tag{2.18}$$

and Eq. (2.16) gives

$$\xi_{A_-} = 0. \tag{2.19}$$

Let

$$\epsilon = (\epsilon_0, 0, \epsilon_1, 0, 0, 0, 0, 0, 0, \bar{\epsilon}_0, 0, \bar{\epsilon}_1, 0, 0, 0, 0)^t. \tag{2.20}$$

The strategy to find solutions to Eq. (2.17), (2.18) under Eq. (2.14), (2.15), namely, that of finding two pairs of nonintersecting sets $\mathcal{M}_-$ and $\mathcal{M}_+$ and $\mathcal{A}_-$ and $\mathcal{A}_+$ are fully described in [3] and we will not repeat it here. The solution is

$$\mathcal{M}_- = \{0, 1, 2, 3, 4, 7\}, \qquad \mathcal{M}_+ = \{5, 6, 8, 9\}, \tag{2.21}$$

$$\mathcal{A}_- = \{1, 2, 5, 6, 19, 20, 23, 24\},$$
$$\mathcal{A}_+ = \{9, 10, 13, 14, 27, 28, 31, 32\}, \tag{2.22}$$

and this leads to the one [3,4][1] of the two known cases of possessing $8 + 8$ supercharges. The corresponding projectors are

$$\hat{\rho}_{b\mp} = \mathrm{diag}(\hat{\rho}_-, \hat{\rho}_-, \hat{\rho}_-, \hat{\rho}_-, \hat{\rho}_-, \hat{\rho}_+, \hat{\rho}_+, \hat{\rho}_-, \hat{\rho}_+, \hat{\rho}_+,),$$
$$\hat{\rho}_{f\mp} = \hat{\rho}_- I_4 \otimes \begin{pmatrix} I_2 & & & \\ & 0 & & \\ & & I_2 & \\ & & & 0 \end{pmatrix} + \hat{\rho}_+ I_4 \otimes \begin{pmatrix} 0 & & & \\ & I_2 & & \\ & & 0 & \\ & & & I_2 \end{pmatrix}. \tag{2.23}$$

The other solution [6] with $8 + 8$ supercharges is

$$\mathcal{M}_- = \{4, 7\}, \qquad \mathcal{M}_+ = \{0, 1, 2, 3, 5, 6, 8, 9\}, \tag{2.24}$$

$$\mathcal{A}_- = \{1, 2, 5, 6, 27, 28, 31, 32\},$$
$$\mathcal{A}_+ = \{9, 10, 13, 14, 19, 20, 23, 24\}. \tag{2.25}$$

The corresponding projectors are

$$\hat{\rho}_{b\mp} = \mathrm{diag}(\hat{\rho}_+, \hat{\rho}_+, \hat{\rho}_+, \hat{\rho}_+, \hat{\rho}_-, \hat{\rho}_+, \hat{\rho}_+, \hat{\rho}_-, \hat{\rho}_+, \hat{\rho}_+,),$$
$$\hat{\rho}_{f\mp} = \hat{\rho}_- I_4 \otimes \begin{pmatrix} I_2 & & & \\ & 0 & & \\ & & 0 & \\ & & & I_2 \end{pmatrix} + \hat{\rho}_+ I_4 \otimes \begin{pmatrix} 0 & & & \\ & I_2 & & \\ & & I_2 & \\ & & & 0 \end{pmatrix}. \tag{2.26}$$

III. $\mathbf{Z}_3$ ORBIFOLDING

We now describe $\mathbf{Z}_3$ orbifolding of the IIB matrix model. Let $A_N = (A_\mu(\mu = 0, \ldots, 3), B_1 = A_4 + iA_5, B_2 = A_6 + iA_7, B_3 = A_8 + iA_9)$. The complex coordinates B_i are postulated to transform under $\mathbf{Z}_3$ as

$$B_i \to \omega^{a_i} B_i, \tag{3.1}$$

[1]For further developments of the USp matrix model, see [14,15]. The complete construction of this matrix model includes the $n_f = 16$ sectors belonging to the (anti)fundamental representation. The use of USp Lie algebra is required by the $SO(2n_f)$ Chan-Paton factor realized by open loop variables. [14,15].

where a_i are integers and ω is a cubic root of unity. We introduce the ’tHooft matrices

$$U = \begin{pmatrix} 1 & & \\ & \omega & \\ & & \omega^2 \end{pmatrix}, \qquad V = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \tag{3.2}$$

which satisfy $UV = \omega VU$. The $\mathbf{Z}_3$ transformation is given by $M \to UMU^\dagger$. The $\mathbf{Z}_3$ invariant bosonic matrices thus satisfy the conditions:

$$A_\mu = UA_\mu U^\dagger, \qquad B_i = \omega^{a_i} UB_i U^\dagger. \tag{3.3}$$

In order to find the conditions for the fermionic matrices, let us note that ten-dimensional chirality operator Γ^{10} can be thought of as the product of the lower dimensional chirality operators $\Gamma^{11} = (i\Gamma^0 \cdots \Gamma^3) \cdot (i\Gamma^4\Gamma^5) \cdot (i\Gamma^6\Gamma^7) \cdot (i\Gamma^8\Gamma^9)$ and that ψ is expanded by a set of eigenfunctions $\psi_0 \sim \psi_3$, $\psi_0^c \sim \psi_3^c$

$$\psi = \sum_{i=0}^{3} (\psi_i + (\psi_i)^c).$$

The eigenfunctions satisfy the conditions:

$$\psi_i = \omega^{b_i} U \psi_i U^\dagger, \tag{3.4}$$

where b_i are given by the table

Γ^{11}	$i\Gamma^{0123}$	$i\Gamma^{45}$	$i\Gamma^{67}$	$i\Gamma^{89}$		b_i
+	+	+	+	+	ψ_0	$-(a_1 + a_2 + a_3)/2$
	+	+	−	−	ψ_1	$-(a_1 - a_2 - a_3)/2$
	+	−	+	−	ψ_2	$-(-a_1 + a_2 - a_3)/2$
	+	−	−	+	ψ_3	$-(-a_1 - a_2 + a_3)/2$
	−	−	−	−	$(\psi_0)^c$	$-(-a_1 - a_2 - a_3)/2$
	−	−	+	+	$(\psi_1)^c$	$-(-a_1 + a_2 + a_3)/2$
	−	+	−	+	$(\psi_2)^c$	$-(a_1 - a_2 + a_3)/2$
	−	+	+	−	$(\psi_3)^c$	$-(a_1 + a_2 - a_3)/2$

The bosonic part of the action is

$$S_b = -\frac{1}{4g^2} \mathrm{Tr}\bigg([A_\mu, A_\nu]^2 + 2\sum_{i=1}^{3} [A_\mu, B_i][A^\mu, B_i^\dagger] + \tfrac{1}{2} \sum_{i,j=1}^{3} ([B_i, B_j^\dagger][B_i^\dagger, B_j] + [B_i, B_j][B_i^\dagger, B_j^\dagger])\bigg), \tag{3.5}$$

and the fermionic part is

$$S_f = -\frac{1}{2g^2} \mathrm{Tr}\bigg(\sum_{i=0}^{3} \bar{\psi}_i \Gamma^\mu [A_\mu, \psi_i] + 2\sum_{i=1}^{3} (\bar{\psi_i})^c \bar{\Gamma}^{(i)} [B_i^\dagger, \psi_0] + \sum_{i,j,k=1}^{3} |\epsilon_{ijk}| (\bar{\psi_i})^c \Gamma^{(j)} [B_j, \psi_k] + \mathrm{H.c.}\bigg), \tag{3.6}$$

where $\Gamma^{(1)} = \frac{1}{2}(\Gamma^4 - i\Gamma^5)$, $\bar{\Gamma}^{(1)} = \frac{1}{2}(\Gamma^4 + i\Gamma^5)$, and so on.

A prototypical example is

$$a_i = 2 \quad \text{for } i = 1, 2, 3,$$

and

$$b_0 = 0 \quad \text{and} \quad b_i = -2 = 1 \quad \text{mod } 3 \quad \text{for } i = 1, 2, 3.$$

Using the 'tHooft matrices, we can represent $\mathbf{Z}_3$ invariant matrices A_μ, B_i, ψ_0, and ψ_i as

$$A_\mu = \sum_{a=0}^{2} A_\mu^a \otimes U^a, \tag{3.7}$$

$$B_i = \sum_{a=0}^{2} B_i^a \otimes (U^a V), \tag{3.8}$$

$$\psi_0 = \sum_{a=0}^{2} \psi_0^a \otimes U^a, \tag{3.9}$$

$$\psi_i = \sum_{a=0}^{2} \psi_i^a \otimes (U^a V^{-1}). \tag{3.10}$$

The dynamical supersymmetry is

$$\delta^{(1)}\psi_0 = \frac{i}{2}([A_\mu, A_\nu]\Gamma^{\mu\nu}\epsilon_0 + [B_i, B_i^\dagger]\epsilon_0), \tag{3.11}$$

$$\delta^{(1)}\psi_i = \frac{i}{2}(|\epsilon_{ijk}|[B_j, B_k]\Gamma^{(j)}\Gamma^{(k)}\epsilon_0 + 2[A_\mu, B_i^\dagger]\Gamma^\mu\bar{\Gamma}^{(i)}\epsilon_0^c), \tag{3.12}$$

$$\delta^{(1)}A_\mu = i\bar{\epsilon}_0\Gamma^\mu\psi_0 + i\bar{\epsilon}_0^c\Gamma^\mu\psi_0^c, \tag{3.13}$$

$$\delta^{(1)}B_i = 2i\bar{\epsilon}_0\bar{\Gamma}^{(i)}\psi_i, \tag{3.14}$$

while the kinematical supersymmetry is

$$\delta^{(2)}\psi_0 = \xi_0, \tag{3.15}$$

and zero otherwise. This is a model with $4 + 4$ supercharges.

IV. MATRIX ORIENTIFOLDING WITH FOUR SUPERCHARGES

Having the discussion of the preceding sections in mind, we turn to constructing cases with four supercharges upon matrix orientifolding, which descends from the case leading to the USp matrix model. In this section, we restrict our attention to the prototypical example of $\mathbf{Z}_3$ orbifolding discussed in Section III. From the condition $[\hat{\rho}_{f\mp}, \delta^{(1)}]\psi_0 = 0$, we obtain

$$\begin{aligned}(\Gamma^{\mu_-\nu_-}\epsilon_0)_{A_+} &= (\Gamma^{\mu_+\nu_+}\epsilon_0)_{A_+} = 0 = (\epsilon_0)_{A_+},\\ (\Gamma^{\mu_-\nu_+}\epsilon_0)_{A_-} &= 0.\end{aligned} \tag{4.1}$$

A similar equation holds for ϵ_0^c. The condition $[\hat{\rho}_{b\mp}, \delta^{(1)}]A = 0$ leads to

$$\begin{aligned}(\bar{\epsilon}_0\Gamma^{\mu_-})_{A_+} &= 0 = (\bar{\epsilon}_0^c\Gamma^{\mu_-})_{A_+},\\ (\bar{\epsilon}_0\Gamma^{\mu_+})_{A_-} &= 0 = (\bar{\epsilon}_0^c\Gamma^{\mu_+})_{A_-}.\end{aligned} \tag{4.2}$$

Similarly $[\hat{\rho}_{b\mp}, \delta^{(1)}]B = 0$, $[\hat{\rho}_{f\mp}, \delta^{(1)}]\psi_i = 0$, and $[\hat{\rho}_{f\mp}, \delta^{(2)}]\psi_0 = 0$, respectively yield

$$(\bar{\epsilon}_0\bar{\Gamma}^{(i_-)})_{A_+} = 0 = (\bar{\epsilon}_0\bar{\Gamma}^{(i_+)})_{A_-}, \tag{4.3}$$

$$(\Gamma^{(j_-)}\Gamma^{(k_+)}\epsilon_0)_{A_-} = (\Gamma^{\mu\pm}\bar{\Gamma}^{(i\mp)}\epsilon_0^c)_{A_-} = 0, \tag{4.4}$$

$$(\Gamma^{(j_-)}\Gamma^{(k_-)}\epsilon_0)_{A_+} = (\Gamma^{(j_+)}\Gamma^{(k_+)}\epsilon_0)_{A_+} = (\Gamma^{\mu\mp}\bar{\Gamma}^{(i\mp)}\epsilon_0^c)_{A_+} = 0, \tag{4.5}$$

$$\text{and} \quad (\xi_0)_{A_-} = 0. \tag{4.6}$$

Equations (4.1), (4.2), (4.3), (4.4), (4.5), and (4.6) define a set of conditions satisfied by the anticommuting parameters ϵ_0, ξ_0.

Let us find solutions to these equations. The spinor ϵ_0 is ψ_0 type and must be of the form

$$\epsilon_0 = (a, 0, a, 0, ia, 0, -a, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \tag{4.7}$$

where $a = (\alpha, \beta)^t$. Similarly

$$\epsilon_0^c = (0, 0, 0, 0, 0, 0, 0, 0, ib, 0, ib, 0, b, 0, -ib)^t, \tag{4.8}$$

where $b = (\gamma, \delta)^t$. The spinor and vector indices are grouped into nonintersecting sets $\mathcal{A}_+$, $\mathcal{A}_-$, $\mathcal{M}_+$, $\mathcal{M}_-$, $\mathcal{I}_+$, and $\mathcal{I}_-$ such that

$$\begin{aligned}\mathcal{A} &= \mathcal{A}_+ \cup \mathcal{A}_-\\ &= \{1, 2, 5, 6, 9, 10, 13, 14, 19, 20, 23, 24, 27, 28, 31, 32\},\\ \mathcal{M} &= \mathcal{M}_+ \cup \mathcal{M}_- = \{0, 1, 2, 3\},\\ \mathcal{I} &= \mathcal{I}_+ \cup \mathcal{I}_- = \{1, 2, 3\}.\end{aligned}$$

Let us first classify the possibilities by the division of $\mathcal{M}$ into $\mathcal{M}_+$ and $\mathcal{M}_-$. This is done by using Eq. (4.2) and by following the procedure given in [4]. It turns out that there are three distinct possibilities for the division:

(1) $(\alpha \neq \beta, \alpha, \beta \neq 0; \gamma \neq \delta, \gamma, \delta \neq 0)$;

$$\{0, 1, 2, 3\}, \varnothing,$$

(2) $(\alpha = \pm\beta \neq 0; \gamma = \pm\delta \neq 0)$;

$$\{0, 1\}, \{2, 3\},$$

(3) $(\alpha \neq 0, \beta = 0$, or $\alpha = 0, \beta \neq 0$; $\gamma \neq 0, \delta = 0$, or $\gamma = 0, \delta \neq 0)$;

$$\{0, 3\}, \{1, 2\}.$$

Let us see each possibility more closely.

(1) From Eq. (4.1), we see

$$\mathcal{A}_- = \mathcal{A}, \qquad \mathcal{A}_+ = \emptyset, \tag{4.9}$$

while $(\bar{\epsilon}_0 \Gamma^{\mu_+})_{A_-} = 0$ in Eq. (4.2) implies

$$\mathcal{M}_- = \mathcal{M}, \qquad \mathcal{M}_+ = \emptyset. \tag{4.10}$$

From Eq. (4.3), we conclude

$$\mathcal{I}_- = \mathcal{I}, \qquad \mathcal{I}_+ = \emptyset. \tag{4.11}$$

Finally Eq. (4.6) tells us that the kinematical supersymmetry is broken completely:

$$\xi_0 = 0. \tag{4.12}$$

This case has 4 + 0 supersymmetries.

(2) Following the same procedure as that of poss. 1, we conclude that this possibility does not lead to a consistent solution.

(3) This possibility leads to four different solutions.

i) Choosing $a = (\alpha, 0)^t$, $b = (\gamma, 0)^t$, from Eq. (4.1), we conclude

$$\begin{aligned} \mathcal{A}_- &= \{1, 5, 9, 13, 19, 23, 27, 31\}, \\ \mathcal{A}_+ &= \{2, 6, 10, 14, 20, 24, 28, 32\}, \end{aligned} \tag{4.13}$$

while from Eq. (4.2) and from Eq. (4.3), we conclude, respectively

$$\mathcal{M}_- = \{0, 3\}, \qquad \mathcal{M}_+ = \{1, 2\}, \tag{4.14}$$

$$\text{and} \quad \mathcal{I}_- = \{1, 2, 3\}, \qquad \mathcal{I}_+ = \emptyset. \tag{4.15}$$

Finally Eq. (4.6) is solved by

$$\xi_0 = (c, 0, c, 0, ic, 0, -c, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t,$$
$$\xi_0^c = (0, 0, 0, 0, 0, 0, 0, 0, 0, id, 0, id, 0, d, 0, -id)^t,$$

where $c = (0, \beta)^t$, $d = (0, \delta)^t$.

ii) Choosing $a = (0, \alpha)^t$, $b = (0, \gamma)^t$, from Eq. (4.1), we conclude

$$\begin{aligned} \mathcal{A}_- &= \{2, 6, 10, 14, 20, 24, 28, 32\}, \\ \mathcal{A}_+ &= \{1, 5, 9, 13, 19, 23, 27, 31\}, \end{aligned} \tag{4.16}$$

while from Eq. (4.2) and from Eq. (4.3), we, respectively, conclude

$$\mathcal{M}_- = \{0, 3\}, \qquad \mathcal{M}_+ = \{1, 2\}, \tag{4.17}$$

$$\text{and} \quad \mathcal{I}_- = \{1, 2, 3\}, \qquad \mathcal{I}_+ = \emptyset. \tag{4.18}$$

Finally Eq. (4.6) is solved by choosing $c = (\beta, 0)^t$, $d = (\delta, 0)^t$.

iii) Choosing $a = (\alpha, 0)^t$, $b = (0, \gamma)^t$, from Eq. (4.1), we conclude

$$\begin{aligned} \mathcal{A}_- &= \{1, 5, 9, 13, , 20, 24, 28, 32\}, \\ \mathcal{A}_+ &= \{2, 6, 10, 14, 19, 23, 27, 31\}, \end{aligned} \tag{4.19}$$

while from Eq. (4.2) and from Eq. (4.3), we, respectively, conclude

$$\mathcal{M}_- = \{0, 3\}, \qquad \mathcal{M}_+ = \{1, 2\}, \tag{4.20}$$

$$\text{and} \quad \mathcal{I}_- = \emptyset, \qquad \mathcal{I}_+ = \{1, 2, 3\}. \tag{4.21}$$

Finally Eq. (4.6) is solved by choosing $c = (0, \beta)^t$, $d = (\delta, 0)^t$.

iv) Choosing $a = (0, \alpha)^t$, $b = (\gamma, 0)^t$, from Eq. (4.1), we conclude

$$\begin{aligned} \mathcal{A}_- &= \{2, 6, 10, 14, 19, 23, 27, 31\}, \\ \mathcal{A}_+ &= \{1, 5, 9, 13, 20, 24, 28, 32\}, \end{aligned} \tag{4.22}$$

while from Eq. (4.2) and from Eq. (4.3), we, respectively, conclude

$$\mathcal{M}_- = \{0, 3\}, \qquad \mathcal{M}_+ = \{1, 2\}, \tag{4.23}$$

$$\text{and} \quad \mathcal{I}_- = \emptyset, \qquad \mathcal{I}_+ = \{1, 2, 3\}. \tag{4.24}$$

Finally Eq. (4.6) is solved by choosing $c = (\beta, 0)^t$, $d = (0, \delta)^t$.

These four cases have 2 + 2 supersymmetries.

V. ENUMERATING THE CASES WITH FOUR OR EIGHT SUPERCHARGES

Let us generalize the results obtained in the last section. We would first need to rewrite supersymmetry transformations in the new variables A_μ, B_i, ψ_0, and ψ_i, but we will not spell out its explicit form here. As we have seen in the last section, the condition $[\hat{\rho}^{(\mu)}_{b\mp}, \delta^{(1)}]A_\mu = 0$ yields

$$\begin{aligned} &(\bar{\epsilon}_0 \Gamma^{\mu_-})_{A_+} = (\bar{\epsilon}_i \Gamma^{\mu_-})_{A_+} = (\bar{\epsilon}^c_0 \Gamma^{\mu_-})_{A_+} = (\bar{\epsilon}^c_i \Gamma^{\mu_-})_{A_+} = 0, \\ &(\bar{\epsilon}_0 \Gamma^{\mu_-})_{A_-} = (\bar{\epsilon}_i \Gamma^{\nu_+})_{A_-} = (\bar{\epsilon}^c_0 \Gamma^{\mu_+})_{A_-} = (\bar{\epsilon}^c_i \Gamma^{\mu_+})_{A_-} = 0, \end{aligned} \tag{5.1}$$

and the condition $[\hat{\rho}^{(i)}_{b\mp}, \delta^{(1)}]B_i = 0$ leads to

$$\begin{aligned} &(\bar{\epsilon}_0 \bar{\Gamma}^{(i_-)})_{A_+} = (\bar{\epsilon}_{i_-} \bar{\Gamma}^{(i_-)})_{A_+} = (\bar{\epsilon}^c_j \bar{\Gamma}^{(i_-)})_{A_+} = 0, \\ &(\bar{\epsilon}_0 \bar{\Gamma}^{(i_+)})_{A_-} = (\bar{\epsilon}_i + \bar{\Gamma}^{(i_+)})_{A_-} = (\bar{\epsilon}^c_j \bar{\Gamma}^{(i_+)})_{A_-} = 0, \end{aligned} \tag{5.2}$$

where $j \neq i$. Here the repeated indices are not to be summed over unless stated explicitly. Similarly, from $[\hat{\rho}^{(i)}_{b\mp}, \delta^{(1)}]B_i^\dagger = 0$ we obtain

$$\begin{aligned} &(\bar{\epsilon}^c_0 \Gamma^{(i_-)})_{A_+} = (\bar{\epsilon}^c_{i_-} \Gamma^{(i_-)})_{A_+} = (\bar{\epsilon}_j \Gamma^{(i_-)})_{A_+} = 0, \\ &(\bar{\epsilon}^c_0 \Gamma^{(i_+)})_{A_-} = (\bar{\epsilon}^c_{i_+} \Gamma^{(i_+)})_{A_-} = (\bar{\epsilon}_j \Gamma^{(i_+)})_{A_-} = 0. \end{aligned} \tag{5.3}$$

The condition $[\hat{\rho}^{(0)(A)}_{f\mp}, \delta^{(1)}](\psi_0)_A = 0$ yields

$$(\Gamma^{\mu_-\nu_+}\epsilon_0)_{A_-} = 0, \qquad (\Gamma^{\mu_-\nu_-}\epsilon_0)_{A_+} = (\Gamma^{\mu_+\nu_+}\epsilon_0)_{A_+} = 0, \tag{5.4}$$

$$\begin{aligned} &|\epsilon_{i_-j_+k}|(\bar\Gamma^{(i_-)}\bar\Gamma^{(j_+)}\epsilon_k)_{A_-} = 0, \\ &|\epsilon_{i_-j_-k}|(\bar\Gamma^{(i_-)}\bar\Gamma^{(j_-)}\epsilon_k)_{A_+} = |\epsilon_{i_+j_+k}|(\bar\Gamma^{(i_+)}\bar\Gamma^{(j_+)}\epsilon_k)_{A_+} = 0, \end{aligned} \tag{5.5}$$

$$\begin{aligned} (\Gamma^{\mu_-}\bar\Gamma^{(i_+)}\epsilon^c_{i_+})_{A_-} = (\Gamma^{\mu_+}\bar\Gamma^{(i_-)}\epsilon^c_{i_-})_{A_-} = 0, \\ (\Gamma^{\mu_-}\bar\Gamma^{(i_-)}\epsilon^c_{i_-})_{A_+} = (\Gamma^{\mu_+}\bar\Gamma^{(i_+)}\epsilon^c_{i_+})_{A_+} = 0, \end{aligned} \tag{5.6}$$

$$(\epsilon_0)_{A_+} = 0, \tag{5.7}$$

while $[\hat\rho^{(0)(A)}_{f\mp}, \delta^{(1)}](\psi^c_0)_A = 0$

$$(\Gamma^{\mu_-\nu_+}\epsilon^c_0)_{A_-} = 0, \qquad (\Gamma^{\mu_-\nu_-}\epsilon^c_0)_{A_+} = (\Gamma^{\mu_+\nu_+}\epsilon^c_0)_{A_+} = 0, \tag{5.8}$$

$$\begin{aligned} &|\epsilon_{i_-j_+k}|(\Gamma^{(i_-)}\Gamma^{(j_+)}\epsilon^c_k)_{A_-} = 0, \\ &|\epsilon_{i_-j_-k}|(\Gamma^{(i_-)}\Gamma^{(j_-)}\epsilon^c_k)_{A_+} = |\epsilon_{i_+j_+k}|(\Gamma^{(i_+)}\Gamma^{(j_+)}\epsilon^c_k)_{A_+} = 0, \end{aligned} \tag{5.9}$$

$$\begin{aligned} (\Gamma^{\mu_-}\Gamma^{(i_+)}\epsilon_{i_+})_{A_-} = (\Gamma^{\mu_+}\Gamma^{(i_-)}\epsilon_{i_-})_{A_-} = 0, \\ (\Gamma^{\mu_-}\Gamma^{(i_-)}\epsilon_{i_-})_{A_+} = (\Gamma^{\mu_+}\Gamma^{(i_+)}\epsilon_{i_+})_{A_+} = 0, \end{aligned} \tag{5.10}$$

$$(\epsilon^c_0)_{A_+} = 0. \tag{5.11}$$

The condition $[\hat\rho^{(i)(A)}_{f\mp}, \delta^{(1)}](\psi_i)_A = 0$ leads to

$$(\Gamma^{\mu_-\nu_+}\epsilon_i)_{A_-} = 0, \qquad (\Gamma^{\mu_-\nu_-}\epsilon_i)_{A_+} = (\Gamma^{\mu_+\nu_+}\epsilon_i)_{A_+} = 0, \tag{5.12}$$

$$\begin{aligned} &|\epsilon_{ij_+k}|(\Gamma^{\mu_-}\Gamma^{(j_+)}\epsilon^c_k)_{A_-} = |\epsilon_{ij_-k}|(\Gamma^{\mu_+}\Gamma^{(j_-)}\epsilon^c_k)_{A_-} = 0, \\ &|\epsilon_{ij_-k}|(\Gamma^{\mu_-}\Gamma^{(j_-)}\epsilon^c_k)_{A_+} = |\epsilon_{ij_+k}|(\Gamma^{\mu_+}\Gamma^{(j_+)}\epsilon^c_k)_{A_+} = 0, \end{aligned} \tag{5.13}$$

$$\begin{aligned} &|\epsilon_{ij_+k_-}|(\Gamma^{(j_+)}\Gamma^{(k_-)}\epsilon_0)_{A_-} = 0, \\ &|\epsilon_{ij_+k_+}|(\Gamma^{(j_+)}\Gamma^{(k_+)}\epsilon_0)_{A_+} = |\epsilon_{ij_-k_-}|(\Gamma^{(j_-)}\Gamma^{(k_-)}\epsilon_0)_{A_+} = 0, \end{aligned} \tag{5.14}$$

$$\begin{aligned} (\Gamma^{\mu_-}\bar\Gamma^{(i_+)}\epsilon^c_0)_{A_-} = (\Gamma^{\mu_+}\bar\Gamma^{(i_-)}\epsilon^c_0)_{A_-} = 0, \\ (\Gamma^{\mu_-}\bar\Gamma^{(i_-)}\epsilon^c_0)_{A_+} = (\Gamma^{\mu_+}\bar\Gamma^{(i_+)}\epsilon^c_0)_{A_+} = 0, \end{aligned} \tag{5.15}$$

$$\begin{aligned} &|\epsilon_{i_-j_+k}|(\Gamma^{(i_-)}\bar\Gamma^{(j_+)}\epsilon_{i_-})_{A_-} = |\epsilon_{i_+j_-k}|(\Gamma^{(i_+)}\bar\Gamma^{(j_-)}\epsilon_{i_+})_{A_-} = 0, \\ &|\epsilon_{i_-j_-k}|(\Gamma^{(i_-)}\bar\Gamma^{(j_-)}\epsilon_{i_-})_{A_+} = |\epsilon_{i_+j_+k}|(\Gamma^{(i_+)}\bar\Gamma^{(j_+)}\epsilon_{i_+})_{A_+} = 0, \end{aligned} \tag{5.16}$$

$$(\epsilon_i)_{A_+} = 0. \tag{5.17}$$

The condition $[\hat\rho^{(i)(A)}_{f\mp}, \delta^{(1)}](\psi^c_i)_A = 0$ leads to

$$(\Gamma^{\mu_-\nu_+}\epsilon^c_i)_{A_-} = 0, \qquad (\Gamma^{\mu_-\nu_-}\epsilon^c_i)_{A_+} = (\Gamma^{\mu_+\nu_+}\epsilon^c_i)_{A_+} = 0, \tag{5.18}$$

$$\begin{aligned} &|\epsilon_{ij_+k}|(\Gamma^{\mu_-}\bar\Gamma^{(j_+)}\epsilon_k)_{A_-} = |\epsilon_{ij_-k}|(\Gamma^{\mu_+}\bar\Gamma^{(j_-)}\epsilon_k)_{A_-} = 0, \\ &|\epsilon_{ij_-k}|(\Gamma^{\mu_-}\bar\Gamma^{(j_-)}\epsilon_k)_{A_+} = |\epsilon_{ij_+k}|(\Gamma^{\mu_+}\bar\Gamma^{(j_+)}\epsilon_k)_{A_+} = 0, \end{aligned} \tag{5.19}$$

$$\begin{aligned} &|\epsilon_{ij_+k_-}|(\bar\Gamma^{(j_+)}\bar\Gamma^{(k_-)}\epsilon^c_0)_{A_-} = 0, \\ &|\epsilon_{ij_+k_+}|(\bar\Gamma^{(j_+)}\bar\Gamma^{(k_+)}\epsilon^c_0)_{A_+} = |\epsilon_{ij_-k_-}|(\bar\Gamma^{(j_-)}\bar\Gamma^{(k_-)}\epsilon^c_0)_{A_+} = 0, \end{aligned} \tag{5.20}$$

$$\begin{aligned} (\Gamma^{\mu_-}\Gamma^{(i_+)}\epsilon_0)_{A_-} = (\Gamma^{\mu_+}\Gamma^{(i_-)}\epsilon_0)_{A_-} = 0, \\ (\Gamma^{\mu_-}\Gamma^{(i_-)}\epsilon_0)_{A_+} = (\Gamma^{\mu_+}\Gamma^{(i_+)}\epsilon_0)_{A_+} = 0, \end{aligned} \tag{5.21}$$

$$\begin{aligned} &|\epsilon_{i_-j_+k}|(\bar\Gamma^{(i_-)}\Gamma^{(j_+)}\epsilon^c_{i_-})_{A_-} = |\epsilon_{i_+j_-k}|(\bar\Gamma^{(i_+)}\Gamma^{(j_-)}\epsilon^c_{i_+})_{A_-} = 0, \\ &|\epsilon_{i_-j_-k}|(\bar\Gamma^{(i_-)}\Gamma^{(j_-)}\epsilon^c_{i_-})_{A_+} = |\epsilon_{i_+j_+k}|(\bar\Gamma^{(i_+)}\Gamma^{(j_+)}\epsilon^c_{i_+})_{A_+} = 0, \end{aligned} \tag{5.22}$$

$$(\epsilon^c_i)_{A_+} = 0. \tag{5.23}$$

In addition, from $[\hat\rho^{(A)}_{f\mp}, \delta^{(2)}](\psi_0)_A = 0$ and $[\hat\rho^{(A)}_{f\mp}, \delta^{(2)}]\times(\psi_i)_A = 0$, we obtain

$$(\xi_0)_{A_-} = (\xi_i)_{A_-} = 0. \tag{5.24}$$

Upon $\mathbf{Z}_3$ orbifolding, the number of surviving supersymmetries is related to the number of b_i such that $b_i = 0$ is satisfied. The cases with 4 + 4 supercharges have only one such b_i, and we obtain the following four possibilities with 4 + 4 supercharges:

(a) $b_0 = 0,\ b_1 = -a_1,\ b_2 = -a_2,\ b_3 = -a_3\ (a_1 + a_2 + a_3 = 0)$
(b) $b_0 = -a_1,\ b_1 = 0,\ b_2 = a_3,\ b_3 = a_2\ (a_1 - a_2 - a_3 = 0)$
(c) $b_0 = -a_2,\ b_1 = a_3,\ b_2 = 0,\ b_3 = a_1\ (a_1 - a_2 + a_3 = 0)$
(d) $b_0 = -a_3,\ b_1 = a_2,\ b_2 = a_1,\ b_3 = 0\ (a_1 + a_2 - a_3 = 0)$.

The first one is the model which we already treated in the last section.

Similarly we construct the models with 8 + 8 supercharges, which have two of vanishing b_i. There are six possibilities:

(a) $b_0 = b_1 = 0,\ a_1 = 0,\ b_2 = -b_3 = -a_2 = a_3$
(b) $b_0 = b_2 = 0,\ a_2 = 0,\ b_1 = -b_3 = -a_1 = a_3$
(c) $b_0 = b_3 = 0,\ a_3 = 0,\ b_1 = -b_2 = a_2 = -a_1$
(d) $b_1 = b_2 = 0,\ a_3 = 0,\ b_0 = -b_3 = -a_1 = -a_2$
(e) $b_1 = b_3 = 0,\ a_2 = 0,\ b_0 = -b_2 = -a_1 = -a_3$
(f) $b_2 = b_3 = 0,\ a_1 = 0,\ b_0 = -b_1 = -a_2 = -a_3$.

We collect these possibilities in the table:

Supersymmetry	b_0	b_1	b_2	b_3
4 + 4	0	$-a_1$	$-a_2$	$-a_3$
	$-a_1$	0	a_3	a_2
	$-a_2$	a_3	0	a_1
	$-a_3$	a_2	a_1	0
8 + 8	0	0	$-a_2$	a_2
	0	$-a_1$	0	a_1
	0	$-a_1$	a_1	0
	$-a_1$	0	0	a_1
	$-a_1$	0	a_1	0
	$-a_2$	a_2	0	0

In each possibility, we need only to keep ϵ_i such that $b_i = 0$ is satisfied. Note that the individual forms of ϵ_i are written as

$$\epsilon_0 = (a, 0, a, 0, ia, 0, -a, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \quad (5.25)$$

$$\epsilon_1 = (b, 0, b, 0, -ib, 0, -b, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \quad (5.26)$$

$$\epsilon_2 = (c, 0, -c, 0, ic, 0, c, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \quad (5.27)$$

$$\epsilon_3 = (d, 0, d, 0, -id, 0, d, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \quad (5.28)$$

$$\epsilon_0^c = (0, 0, 0, 0, 0, 0, 0, 0, e, 0, e, 0, -ie, 0, -e)^t, \quad (5.29)$$

$$\epsilon_1^c = (0, 0, 0, 0, 0, 0, 0, 0, f, 0, -f, 0, if, 0, -f)^t, \quad (5.30)$$

$$\epsilon_2^c = (0, 0, 0, 0, 0, 0, 0, 0, g, 0, -g, 0, -ig, 0, g)^t, \quad (5.31)$$

$$\epsilon_3^c = (0, 0, 0, 0, 0, 0, 0, 0, h, 0, h, 0, ih, 0, h)^t, \quad (5.32)$$

where a, b, c, d, e, f, g, h are two component real vectors. Consequently we consider the conditions on these remaining parameters.

It is noted that Eqs. (5.26), (5.27), and (5.28) become proportional to Eq. (5.25) once we flip signs in one or two entries. The same is true for Eqs. (5.30), (5.31), and (5.32), which become proportional to Eq. (5.29) with one or two sign flips. This means that the calculation in the last section is also applicable to the remaining possibilities. From each of the four possibilities of $\mathbf{Z}_3$ orbifolding with 4 + 4 supercharges, we obtain one case with 4 + 0 supercharges and four cases with 2 + 2 supercharges upon matrix orientifolding. There are in total 20 such cases.

Likewise, to each of the six possibilities of $\mathbf{Z}_3$ orbifolding with 8 + 8 supercharges, we first find an appropriate intersection of above Eqs. (5.25), (5.26), (5.27), and (5.28), (5.29), (5.30), (5.31), and (5.32) and impose the conditions of matrix orientifolding. In this way, we are able to exhaust all cases with either 8 + 0 supersymmetries or 4 + 4 supersymmetries upon $\mathbf{Z}_3$ orbifolding followed by matrix orientifolding. To each of the six possibilities, there exists one case with 8 + 0 supersymmetries and four cases with 4 + 4 supersymmetries. There are 30 such cases in total.

We conclude that there are in total 50 cases carrying four or eight supercharges upon $\mathbf{Z}_3$ orbifolding followed by matrix orientifolding.

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