# Nonperturbative quark-antiquark production from a constant chromoelectric field via the Schwinger mechanism

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We obtain an exact result for the nonperturbative quark (antiquark) production rate and its $p_T$ distribution from a constant SU(3) chromoelectric field $E^a$ with arbitrary color index $a$ by directly evaluating the path integral. Unlike the WKB tunneling result, which depends only on one gauge invariant quantity $|E|$, the strength of the chromoelectric field, we find that the exact result for the $p_T$ distribution for quark (antiquark) production rate depends on two independent Casimir (gauge) invariants, $E^aE^a$ and $[d_{abc}E^aE^bE^c]^2$.

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Nonperturbative quark-antiquark pair production from a constant chromoelectric field is widely employed to study hadronization at low $p_T$ in high energy $e^+e^-$ and $pp$ collisions [1]. In these approaches, the color flux-tube energy density or the string tension is related to the constant chromoelectric field strength $|E|$. In high energy heavy-ion collisions at the RHIC and the LHC [2], a classical chromo field might be formed just after two nuclei pass through each other [3,4]. In order to study the production of a quark-gluon plasma from a classical chromo field, it is necessary to know how quarks and gluons are formed from the latter. In a recent paper [5], we have derived a formula for the rate for nonperturbative gluon pair production and its $p_T$ distribution from a constant SU(3) chromoelectric field with arbitrary color index $a$ via vacuum polarization. In this paper we will extend our study to quark-antiquark pair production.

We will not employ Schwinger's proper time method [6] in our calculation. Although this method is widely used to obtain the total fermion production rate $dN/d^4x$ [6,7] (for a review see [8]), this method cannot be used to obtain the $p_T$ distribution of the rate $dN/d^4xd^2p_T$. For this purpose the WKB tunneling method [9,10] has been used in the past to approximate the $p_T$ distribution for the quark (antiquark) production rate. Although the WKB tunneling result for the $p_T$ distribution is correct in QED, it is not necessarily true in QCD because of the presence of the nontrivial color generators in the fundamental and adjoint representations of SU(3). For this reason we will directly evaluate the path integral in this paper and obtain an exact result for the $p_T$ distribution of the nonperturbative quark (antiquark) production rate from a constant chromoelectric field with arbitrary color in the gauge group SU(3). We find that, unlike the WKB tunneling result, which depends on one gauge invariant quantity $|E|$, the strength of the chromoelectric field [9,10], the exact result for the $p_T$ distribution for the quark (antiquark) production rate depends on two independent gauge invariants, $E^aE^a$ and $[d_{abc}E^aE^bE^c]^2$.

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We obtain the following formula for the number of nonperturbative quarks (antiquarks) produced per unit time, per unit volume, and per unit transverse momentum from a given constant chromoelectric field $E^a$:

$$\frac{dN_{q,\bar{q}}}{dtd^3xd^2p_T} = -\frac{1}{4\pi^3}\sum_{j=1}^{3}|g\lambda_j|\ln[1-e^{-\{[\pi(p_T^2+m^2)]/|g\lambda_j|\}}], \tag{1}$$

where $m$ is the mass of the quark. This result is gauge invariant because it depends on the following gauge invariant eigenvalues:

$$\lambda_1 = \sqrt{\frac{C_1}{3}}\cos\theta, \qquad \lambda_2 = \sqrt{\frac{C_1}{3}}\cos(2\pi/3-\theta),$$
$$\lambda_3 = \sqrt{\frac{C_1}{3}}\cos(2\pi/3+\theta), \tag{2}$$

where $\theta$ is given by

$$\cos^2 3\theta = 3C_2/C_1^3. \tag{3}$$

These eigenvalues only depend on two independent Casimir invariants for SU(3),

$$C_1 = E^aE^a, \qquad C_2 = [d_{abc}E^aE^bE^c]^2, \tag{4}$$

where $a, b, c = 1, \ldots, 8$. Note that $0 \leq \cos^2 3\theta \leq 1$ because $C_1^3 \geq 3C_2$ and both $C_1$ and $C_2$ are positive. The integration over $p_T$ in Eq. (1) reproduces Schwinger's result for total production rate $dN/d^4x$ [7].

The exact result in Eq. (1) can be contrasted with the following formula obtained by the WKB tunneling method [9]:

$$\frac{dN_{q,\bar{q}}}{dtd^3xd^2p_T} = \frac{-|gE|}{4\pi^3}\ln[1-e^{-\{[\pi(p_T^2+m^2)]/|gE|\}}]. \tag{5}$$

In our result in Eq. (1) the symmetric tensor $d_{abc}$ appears. Hence the WKB tunneling method does not reproduce the correct result for the $p_T$ distribution of the quark (antiquark) production rate from a constant chromoelectric field $E^a$. We now present a derivation of Eq. (1).

The Lagrangian density for a quark in a non-Abelian background field $A^a_\mu$ is given by

$$\mathcal{L} = \bar{\psi}^i[(\delta_{ij}\hat{\not p} - gT^a_{ij}\not A^a) - m\delta_{ij}]\psi^j = \bar{\psi}^i M_{ij}[A]\psi^j, \quad (6)$$

where $\hat{p}_\mu$ is the momentum operator and $T^a_{ij}$ is the generator in the fundamental representation of gauge group SU(3) with $a = 1, 2 \ldots 8$ and $i, j = 1, 2, 3$. The vacuum to vacuum transition amplitude in the presence of the non-Abelian background field $A^a_\mu$ is given by

$$\langle 0|0\rangle = \frac{\int[d\bar{\psi}][d\psi]e^{i\int d^4x \bar{\psi}^j M_{jk}[A]\psi^k}}{\int[d\bar{\psi}][d\psi]e^{i\int d^4x \bar{\psi}^j M_{jk}[0]\psi^k}} = \mathrm{Det}[M[A]]/\mathrm{Det}[M[0]] = e^{iS^{(1)}}. \quad (7)$$

The one loop effective action becomes

$$S^{(1)}_{q\bar{q}} = -i\,\mathrm{Tr}\ln[(\delta_{ij}\hat{\not p} - gT^a_{ij}\not A^a) - m\delta_{ij}] + i\,\mathrm{Tr}\ln[\delta_{ij}\hat{\not p} - m\delta_{ij}]. \quad (8)$$

The trace Tr contains an integration over $d^4x$, a sum over color and Lorentz indices and a trace over Dirac matrices. Since the trace is invariant under transposition we also get

$$S^{(1)}_{q\bar{q}} = -i\,\mathrm{Tr}\ln[(\delta_{ij}\hat{\not p} - gT^a_{ij}\not A^a) + m\delta_{ij}] + i\,\mathrm{Tr}\ln[\delta_{ij}\hat{\not p} + m\delta_{ij}]. \quad (9)$$

Adding both of the above equations we get

$$2S^{(1)}_{q\bar{q}} = -i\,\mathrm{Tr}\ln[(\delta_{ij}\hat{\not p} - gT^a_{ij}\not A^a)^2 - m^2\delta_{ij}] + i\,\mathrm{Tr}\ln[\delta_{ij}\hat{\not p}^2 - m^2\delta_{ij}], \quad (10)$$

which can be written as

$$2S^{(1)}_{q\bar{q}} = -i\,\mathrm{Tr}\ln\left[(\delta_{ij}\hat{p} - gT^a_{ij}A^a)^2 + \frac{g}{2}\sigma_{\mu\nu}T^a_{ij}F^{a\mu\nu} - m^2\delta_{ij}\right] + i\,\mathrm{Tr}\ln[\delta_{ij}\hat{p}^2 - m^2\delta_{ij}]. \quad (11)$$

Since it is convenient to work with the trace of the exponential, we replace the logarithm by

$$\ln\frac{a}{b} = \int_0^\infty \frac{ds}{s}[e^{is(b+i\epsilon)} - e^{is(a+i\epsilon)}]. \quad (12)$$

We assume that the constant electric field is along the $z$-axis (the beam direction) and we choose the gauge $A^a_0 = 0$ so that $A^a_3 = -E^a\hat{x}^0$ [5]. The color indices ($a = 1, \ldots, 8$) are arbitrary. Since $\Lambda_{ij} = T^a_{ij}E^a$ has three eigenvalues we write after diagonalization

$$(\Lambda_d)_{ij} = (\lambda_1, \lambda_2, \lambda_3). \quad (13)$$

The trace over the Dirac matrices ($\mathrm{tr}_D$) gives

$$\mathrm{tr}_D[e^{is(g/2)\sigma_{\mu\nu}T^a_{ij}F^{a\mu\nu}}] = 4\cosh(sgT^a_{ij}E^a). \quad (14)$$

To reduce this problem to the motion of one harmonic oscillator, we make a similarity transformation [5,11] (we also make a similarity transformation in the group space) and obtain

$$\mathrm{tr}_D e^{is[(\delta_{ij}\hat{p} - gT^a_{ij}A^a)^2 + (g/2)\sigma_{\mu\nu}T^a_{ij}F^{a\mu\nu} - m^2\delta_{ij}]} = 4\cosh(sg(\Lambda_d)_{il}) \times [e^{ip^3p_0/g(\Lambda_d)}e^{is(\hat{p}_0{}^2 - \hat{p}_T{}^2 - m^2 - g^2(\Lambda_d^2)\hat{x}^{0^2})}e^{-ip^3p_0/g(\Lambda_d)}]_{lj}, \quad (15)$$

where $p_T = \sqrt{p_1^2 + p_2^2}$ is the transverse momentum of the quark or antiquark (transverse to the electric field direction). Hence, from Eqs. (11) and (12) we find

$$2S^{(1)}_{q\bar{q}} = i\int_0^\infty \frac{ds}{s}\sum_{j=1}^{3}\mathrm{tr}[e^{ip^3p_0/g\lambda_j}e^{is(\hat{p}_0{}^2 - \hat{p}_T{}^2 - m^2 - g^2\lambda_j^2\hat{x}_0{}^2 + i\epsilon)} \times e^{-ip^3p_0/g\lambda_j}[4\cosh sg\lambda_j] - 4e^{is(\hat{p}_0{}^2 - \hat{p}_T{}^2 - m^2 + i\epsilon)}]. \quad (16)$$

The trace tr denotes an integral over a complete set of $x$ eigenstates. We add complete sets of $p_j$ eigenstates, and obtain

$$2S^{(1)}_{q\bar{q}} = i\int_0^\infty \frac{ds}{s}\sum_{j=1}^{3}\frac{1}{4\pi^3}\int d^4x\int d^2p_T e^{-is(p_T^2+m^2)-s\epsilon} \times \left[|g\lambda_j|\frac{\cosh sg\lambda_j}{\sinh s|g\lambda_j|} - \frac{1}{s}\right]. \quad (17)$$

The $s$-integral at fixed $p_T$ is convergent at $s \to 0$, but the integration over $p_T$ yields an extra factor $1/s$ so now it seems divergent. However, charge renormalization cures this ultraviolet problem by subtracting also the term linear in $s$ in the expansion of $\cosh sg\lambda_j/\sinh s|g\lambda_j|$. The integral is well behaved as $s \to \infty$. To perform the $s$ contour integration, we use the well-known expansion

$$\frac{1}{\sinh x} = \frac{1}{x} + 2x\sum_{n=1}^{\infty}\frac{(-1)^n}{x^2 + n^2\pi^2}, \quad (18)$$

and then we formally replace $s$ by $-is$ (as first advocated by Schwinger in QED). The integral is now real, except for half-circles around the poles at $s|g\lambda_j| = -in\pi$ for $n = 1, 2, 3 \ldots$. The $1/x$ term in (18) cancels against the $1/s$ term in (17). This yields the probability for quark (antiquark) production per unit time and per unit volume

$$W_{q\bar{q}} = 2\,\mathrm{Im}\,\mathcal{L}^{(1)} = \frac{1}{4\pi^3}\int d^2p_T \sum_{n=1}^{\infty}\sum_{j=1}^{3}\frac{|g\lambda_j|}{n}e^{\{-[n\pi(p_T^2+m^2)]/|g\lambda_j|\}}. \quad (19)$$

Now all that is left is to determine the eigenvalues $\lambda_j$ ($j = 1, 2, 3$) of the matrix $\Lambda_{ij} = T^a_{ij}E^a$ in the fundamental representation of the gauge group SU(3). Evaluating the traces of $\Lambda_{ij}$, $\Lambda^2_{ij}$, and the determinant of $\Lambda_{ij}$, we find

$$\lambda_1 + \lambda_2 + \lambda_3 = 0, \quad (20)$$

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = \tfrac{1}{2}E^aE^a, \quad (21)$$

and

$$\lambda_1\lambda_2\lambda_3 = \frac{1}{12}[d_{abc}E^aE^bE^c], \quad (22)$$

the solution of which is given by Eq. (2).

In this paper we have obtained an exact result for the rate for nonperturbative quark (antiquark) production and its $p_T$ distribution in a constant chromoelectric field $E^a$ with arbitrary color index $a$ via vacuum polarization. We have used the background field method of QCD with the gauge group SU(3). The $p_T$ distribution for quark (antiquark) production can be applied at the RHIC and the LHC colliders. We find that, unlike the WKB tunneling method, the $p_T$ distribution of the quark or antiquark production rate depends on two independent Casimir (gauge) invariants, $E^aE^a$ and $[d_{abc}E^aE^bE^c]^2$.

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