Dark matter and the anthropic principle

Simeon Hellerman and Johannes Walcher

School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey, USA

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We evaluate the problem of galaxy formation in the landscape approach to phenomenology of the axion sector. With other parameters of standard ΛCDM cosmology held fixed, the density of cold dark matter is bounded below relative to the density of baryonic matter by the requirement that structure should form before the era of cosmological constant domination of the universe. Galaxies comparable to the Milky Way can only form if the ratio also satisfies an upper bound. The resulting constraint on the density of dark matter is too loose to select a low axion decay constant or small initial displacement angle on anthropic grounds.

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In the ''landscape'' approach to string phenomenology [1–4], one starts with an assumption of a large number, $\sim 10^{\text{several hundred}}$, of discrete vacua of the fundamental theory, a picture which is supported by several estimates based on counting solutions to the effective action of string theory on Calabi-Yau manifolds. The goal of the program is to explain low energy phenomena not as a unique and direct consequence of Planck scale physics, but as one realization among a rich set of possibilities. An important input in this approach is the ''anthropic cut'': The only potentially phenomenologically relevant parts of the landscape are the vacua whose effective physics allows for the occurrence of biologically complex life.

One fruitful application of the anthropic approach has been Weinberg's reasoning [5] leading to a bound on the cosmological constant from the minimal condition that galaxies must have enough time to form. From this apparent success we can derive a general lesson about the application of anthropic reasoning: the simplest anthropic constraints are those related to physics which is almost decoupled from phenomena at ordinary scales. Attempts to extract anthropic predictions for other (particle or cosmological) standard model parameters is hampered by our ignorance of the distribution of vacua, but also by the intricacy and interconnectedness of the effects of those parameters on the development of life, and on each other at different scales via RG evolution.

One highly decoupled sector where anthropic reasoning might usefully be applied is that of the Peccei-Quinn axion, a. Postulated as a mechanism for solving the strong *CP* problem, the axion is a pseudoscalar field with an approximate shift symmetry $a \to a + \theta$ which is broken only by the coupling $\Delta\mathcal{L} = an\mathrm{tr}(F \wedge F)_{SU(3)}$. If a is dimensionless, its kinetic term, $\Delta\mathcal{L}_{\text{kinetic}} = f^2(\partial a)^2$ contains a dimensionful parameter f, known as the axion decay constant. QCD instantons then induce an axion potential of the form

$$V(a) = c\Lambda_{\text{QCD}}^4 \sin^2(na)$$

where c is a numerical coefficient which can be set to 1 by a choice of definition of the dynamical scale Λ_{QCD}. An integer rescaling of f can also set n to 1. So the physical mass of the axion is given by Λ_{QCD}^2/f at tree level. The massive parameter f is determined by physics at scales above the standard model. In a generic high-scale model, such as any GUT or string model, the expectation would be that f should have its magnitude set by the scale at which new physics enters—say 10^{16} GeV or so. This expectation is borne out specifically in superstring models such as the heterotic string [6], as well as in many of the more recently studied perturbative superstring vacua [7].

As has been realized for a long time [8], such a value of f appears in conflict with the cosmological standard model, as relic axions produced from initial vacuum displacement in the early universe make a contribution to the dark matter density that exceeds the observed value by orders of magnitude. Linde has pioneered the approach [9] of using anthropic ideas to loosen this bound in inflationary cosmology, where the homogeneous initial misalignment angle, θ_0, of the axion is a free parameter, putatively ''environmental'', in the sense of varying from region to region. If f is fixed at its natural value, however, inflationary fluctuations are too large to make this work [10], a conclusion which possibly can be avoided in models of hybrid inflation [11]. More recently, Banks and Dine [12], and Banks, Dine, and Graesser [13] have emphasized that the cosmological axion problem is dominated in the supersymmetric context by the Saxion (and other moduli), and that a satisfactory resolution might require a much more drastic modification of the history of the universe between inflation and decoupling.

While this debate is by no means settled, it shows that if the strong *CP* problem is solved by a Peccei-Quinn axion in our universe, realizing observational results on dark matter density will most likely require some degree of fine adjustments of parameters and/or initial conditions. In the absence of a mechanism, but in the background of the landscape, we may ask if such adjustments can perhaps be justified anthropically, as has been done in [9] and elsewhere (see e.g., [14]).

We will here evaluate the extent to which the requirement that habitable structures form bounds the ratio of dark matter to baryonic matter in the universe. It is well accepted that in order for galaxies and similar astrophysical objects to form out of the primordial density perturbations seeded during inflation, the matter density $\rho_{\rm matter}$ should contain a predominant dark matter component, $\rho_{\rm DM}$, which drives the growth of structure between equality and decoupling. It is also clear that without any baryons, $\rho_b \to 0$, all structure would remain dark and uninhabited. A combination that works well is when the ratio $\zeta = \rho_{\rm DM}/\rho_b \approx 5$ as in our universe. It thus being clear that a universe with $\zeta = 5$ is habitable, and a universe with $\zeta^{-1} = 0$ is not, one wonders what range of values of ζ life can actually tolerate. Our chief interest here is to understand what general form such an anthropic bound may take and where in the allowed region our universe is situated, as well as what detailed astrophysics the tightness of the bound depends on.

For simplicity, we will fix all other cosmological parameters to the values they take in our universe. In particular, this means that all dimensionless parameters (except ζ) of the cosmological standard model, such as baryon to photon ration, $\eta \equiv n_b/n_\gamma$, scale and spectrum of initial perturbations, etc., have the values we have observed today. We will also assume that inflation has taken place, so that the curvature parameter k nearly vanishes.

One of the consequences of keeping all other parameters fixed and only varying ζ and the cosmological constant term ρ_Λ is that the radiation dominated era in the evolution of the universe, including important processes such as baryogenesis and nucleosynthesis, are essentially unaffected. Later stages of evolution and quantities such as the age of the universe at the time of galaxy formation, will of course change as we vary ζ. So we begin our discussion when matter starts dominating over radiation.

At equality of matter and radiation, $\rho_\gamma = \rho_{\rm matter} = \rho_b + \rho_{\rm DM} \approx \rho_{\rm DM} = \zeta\rho_b$. Using $\rho_\gamma = T_{\rm eq}^4 = T_{\rm eq} n_\gamma$, and $\rho_b = \mu n_b$, where $\mu = 1$ GeV is the mass of a baryon, this gives for the temperature at equality

$$T_{\rm eq} = \mu\eta\zeta \tag{1}$$

The density perturbations, which we assume to be of inflationary origin, can be divided roughly into two classes, depending on their mass scale, M. Since perturbations only grow logarithmically in the radiation dominated era, all density perturbations whose physical size is smaller than the horizon size at equality have their primordial strength $\delta \approx \delta_0 = 10^{-5}$. Perturbations which are superhorizon at equality will reach their scale-invariant amplitude when they enter the horizon and can be ascribed a strength $\delta \approx \delta_0(\lambda/H_{\rm eq}^{-1})^{-2}$ at equality. Here, $\lambda = (M/\rho_{\rm eq})^{1/3}$ gives the relation between the size, λ, of a perturbation and its mass scale, $\rho_{\rm eq} = T_{\rm eq}^4 = (\mu\eta\zeta)^4$ is the energy density at equality, and $H_{\rm eq}^{-1} = (G\rho_{\rm eq})^{-1/2}$ is the horizon size. Thus,

$$\delta_{M,\rm eq} \approx \begin{cases} \delta_0 & \lambda_{M,\rm eq} < H_{\rm eq}^{-1} \\ \delta_0(M^{1/3}(\mu\eta\zeta)^{2/3}/M_{\rm Pl})^{-2} & \lambda_{M,\rm eq} > H_{\rm eq}^{-1} \end{cases} \tag{2}$$

In the matter dominated era, the strength of the perturbations grows linearly with the scale factor of the universe, $\delta \propto a$. The nonlinear regime is reached when $a/a_{\rm eq} \approx 1/\delta_{M,\rm eq}$, after which the structure breaks away from the overall expansion of the universe. The celebrated Weinberg bound expresses the fact that this should happen before the universe is dominated by vacuum energy, ρ_Λ, lest acceleration disrupt the forming structure. So, structures of scale M have time to form between equality and cosmological constant domination if and only if [1]

$$\rho_\Lambda \lesssim \rho_{\rm eq}(\delta_{M,\rm eq})^3 \tag{3}$$

Now, we have to decide what scale of structure is required for life, and how this depends on ζ. Beyond its usual murkiness, this question is even more delicate in the universes that we are envisaging, because the structures may look quite different from those that form with our value of ζ, as we will describe in more detail below. As an example, we can consider the fate of a perturbation that has a chance of evolving to a galaxy like ours. This will give us our strongest bound on ζ, and useful expectations for a more careful study (see Fig. 1).

The Milky Way contains about $M_{\rm gal} = 10^{11} M_\odot$ worth of baryons and so corresponds to a total mass scale $M = \zeta M_{\rm gal}$. Inserting this into (3) implies the bounds,

$$\rho_\Lambda \lesssim \begin{cases} (\mu\eta)^4\delta_0^3\zeta^4 & \zeta M_{\rm gal}^{1/3}(\mu\eta)^{2/3}/M_{\rm Pl} < 1 \\ M_{\rm Pl}^6 M_{\rm gal}^{-2}\delta_0^3\zeta^{-2} & \zeta M_{\rm gal}^{1/3}(\mu\eta)^{2/3}/M_{\rm Pl} > 1 \end{cases} \tag{4}$$

In other words, if the perturbation giving rise to our galaxy is subhorizon at equality, it enters the nonlinear regime after the energy density has dropped by a fixed amount. Since the energy density at equality scales with the fourth power of ζ, the cosmological constant can be correspondingly larger. If our galaxy is superhorizon size at equality, the strength of the corresponding perturbation is down by a factor of ζ^{-2}, and it takes correspondingly longer to grow to nonlinearity. The cosmological constant cannot be too large.

Besides its simplicity, the interest of this derivation is that, for fixed Λ, it yields both a lower and an upper bound on ζ. Numerically, in our universe, $\eta = 10^{-9}$, $\zeta = 5$, $\delta_0 = 10^{-5}$, $\lambda_{\rm gal}/H_{\rm eq}^{-1} \approx 5 \times 10^{-2}$, while the vacuum energy

[1] More precisely, the bound derived in [5] from a nonlinear collapse criterion is $\rho_\Lambda \lesssim \frac{500}{729}\rho_{\rm eq}\delta_{M,\rm eq}^3$. Dropping the factor of order 1 is a much smaller modification than several other imprecisions in our argument.

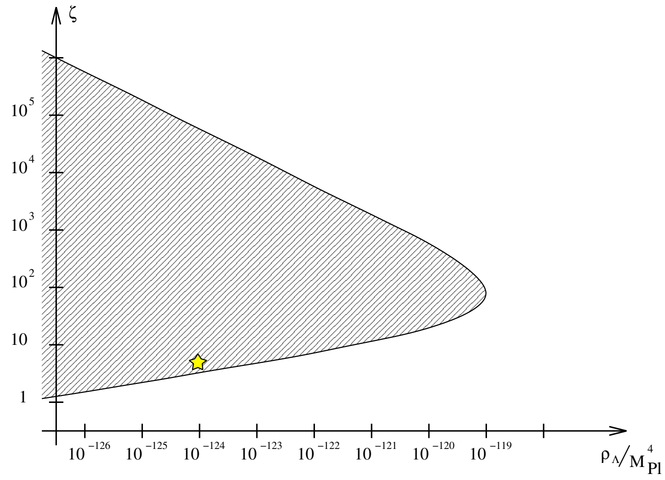

FIG. 1 (color online). A schematic picture of the region in Λ-ζ space for which gravitationally bound structures containing $10^{11}M_\odot$ of baryons form. The star represents our universe.

density is comparable to its upper bound. (We are using numbers from, e.g., [15].) Therefore, if we increased ζ by a factor of 20, the perturbation corresponding to our galaxy would have extended up to the horizon at equality, and Λ could have been $\sim 10^5$ times larger. Increasing ζ by another factor of 400 brings back the bound on Λ to the familiar value. Thus, for fixed Λ, the existence of our Milky Way can tolerate a value of ζ in the range $5 \lesssim \zeta \lesssim 8 \times 10^4$.

Conditioning on structures containing $10^{11}M_\odot$ of baryons ignores the possibility that observers could evolve in a galaxy with far fewer baryons. Moreover, we have to address the effects of changing ζ on the evolution of structures after they break away from the overall expansion of the universe. The first of the subsequent events is the cooling of the baryonic gas that has fallen into the collapsed dark matter halos. This cooling is considered efficient if its time scale $\tau_{\rm cool}$ is much shorter than the dyamical timescale $\tau_{\rm dyn} = (G\rho_{\rm vir})^{-1/2}$ [16]. By following the standard treatment, discussed, for example, in [17], we can estimate how the cooling efficiency depends on ζ.

The dominant mechanisms which have contributed to the cooling of the structures in our universe are atomic and molecular line cooling of hydrogen and heavier elements as well as bremsstrahlung resulting from collisions of constituents of the charged plasma in the potential well of the dark matter halo. Both these mechanisms depend on the density, temperature, and the ionization level of the gas, and hence on ζ. The virialization density $\rho_{\rm vir}$ is proportional to the total matter density at the time of collapse, $\rho_{\rm coll} \approx \rho_{\rm eq}\delta^3_{M,\rm eq}$ and the virial temperature $T_{\rm vir} = GM\mu/r = M_{\rm Pl}^{-2}\mu M^{2/3}\rho_{\rm vir}^{1/3}$. As we have seen, the density at collapse increases independent of M as ζ^4 until $M \sim M_{\rm Pl}^3(\mu\eta\zeta)^{-2}$, and thereafter drops as M^{-2}, independent of ζ. As a result, $T_{\rm vir} \propto M^{2/3}\zeta^{4/3}$ and $T_{\rm vir} \approx \mu\delta_0$ in the two regimes, respectively. [2] Therefore, if we increase ζ, the virial temperature will soon exceed 10^4K, and line cooling will cease to be relevant for most structures. Cooling by bremsstrahlung will dominate, and we find

$$\frac{\tau_{\rm brems}}{\tau_{\rm dyn}} \propto \begin{cases} \zeta^{-1/3}M^{1/3} & M \text{ subhorizon at equality} \\ \zeta M & M \text{ super horizon at equality} \end{cases} \tag{5}$$

with a kinked power law dependence in the two regimes, similar to (6). The main effect if we increase ζ, however, will be that the structures will soon form so early that the dominant cooling is from Compton scattering off the cosmic microwave background. Compton cooling is (in some regime) independent of the temperature and density of the baryons, but depends quite sensitively on the temperature of the CMB at the time when the structures have formed. Quantitatively, $\tau_{\rm comp} \propto T_\gamma^{-4}$, and hence

$$\frac{\tau_{\rm comp}}{\tau_{\rm dyn}} \propto \begin{cases} \zeta^{-2} \\ \zeta^{4/3}M^{5/3} \end{cases} \tag{6}$$

In the regime of dominant Compton cooling, the effect of the CMB is to act as friction for the charged components in the plasma. In contrast to other cooling mechanisms, it is quite efficient in absorbing angular momentum. The baryons should therefore lose their angular momentum and radial kinetic energy in typical time $\tau_{\rm comp}$ and slide down into the minimum of the potential in a radially symmetric way. At the bottom of the potential, they will have little angular momentum support, and fragmentation into stars is also likely to be inhibited if the collapse is sufficiently isothermal so that the Jeans mass does not decrease too rapidly.

It is plausible that the final state of such a collapse is one where the bulk of the baryonic matter forms a supermassive black hole (SMBH), possibly with a brief intermediate stage of life as a supermassive star. Indeed, it is believed that most larger galaxies in our universe have an SMBH at their center. As has recently become clear, for instance in the celebrated M-σ relation [18] linking the hole mass to the velocity dispersion of the central region of the galaxy, the history of these black holes is intrinsically linked to the formation of the galaxy itself. The SMBH can grow by accretion or mergers but most models assume a sizable seed black hole whose likely origin is the collapse of gas under conditions with inhibited star formation. (See [19] for a short list of references.) If Compton cooling in addition withdraws angular momentum support, collapse to a black hole is a very likely outcome. Clearly, if increasing ζ would confine baryons into black holes, this would be a strong anthropic basis for selecting universes with

[2] These estimates also imply that the mean density of a collapsed structure has a maximum as a function of the mass for fixed ζ. It also has a maximum as a function of ζ for fixed mass. The collapse density alone therefore does not seem suited as anthropic gauge as in [9].

roughly equal proportions of baryons and dark matter. At this stage, however, it seems that numerical simulations would be needed to confirm whether SMBHs are a reasonable scenario.[3]

One argument that is often cited to justify the claim that galaxies with fewer that $10^6 M_\odot$ of baryons are unlikely to support life is that in our universe, such ''galaxies'' reside inside of halos of roughly the same size and have a much more shallow gravitational potential. As a consequence, when the first stars are formed, pressure created by supernova explosions are powerful enough to eject gas (as well as the heavy elements produced in the supernovæ, which are plausibly necessary for life dependent on an interesting chemistry) from the galaxy, thus reducing the prospects of forming a second generation of stars, with planets around them.

A similar mechanism would probably provide a lower cutoff on the mass of baryon-containing galaxies for larger values of ζ, but the formation of initial baryonic structure is not under good analytic control, so we do not know how to compute the dependence of this effect on ζ. The all-important feedback processes are also likely to differ. This would be another good direction for future study in the subject of anthropic constraints on dark matter.

The general question of whether very diffuse baryon clouds are able to form gravitationally bound structures inside the halo at all has recently been answered in the affirmative by the discovery of a ''dark galaxy'' in the Virgo cluster [20]—a gravitationally bound structure of $4 \times 10^7 M_\odot$ inside of a $2 \times 10^{10} M_\odot$ dark matter halo. This galaxy, known as VIRGOHI 21, has a density and temperature too low to cool efficiently by the available mechanism of hydrogen-line cooling, but nonetheless the baryons have been able to separate themselves from the ambient halo enough to form a disk whose structure is determined by its own gravity.

In conclusion, we can be quite confident that with all other parameters of ΛCDM cosmology held fixes, moderately larger values of the dark matter to baryon ratio allow the formation of structure with astrophysical conditions similar to those in our own galaxy. Anthropic constraints appear to weak to force ζ as low as we observe it. Even the quite stringent requirement of having $10^{11} M_\odot$ of baryons does not force the upper bound on ζ lower than $\sim 10^5$.

Finally, we discuss the implications for the axion sector. Vilenkin's ''mediocrity principle'' [21] can be interpreted as saying that when a parameter has a range of values which would allow life to exist, we should expect the parameter to lie at the point within that range which is most favored by conventional notions of naturalness, or perhaps by the statistics of discrete vacua in a fundamental theory. In a model in which all dark matter is axionic, $\zeta \propto f^{3/2}\theta_0^2$ (see, e.g., [22]), so a bound on ζ can be interpreted either as a bound on the axion decay constant or on the initial displacement angle. Conventional naturalness and statistical arguments would both seem to favor large values of f.[4] Likewise, the statistics of initial values of the axion push θ_0 towards values of $o(1)$. The key point is that the pressures of mediocrity on f and θ_0 should both push ζ towards the higher end of its anthropic window. Unless further studies reveals an unexpected dysanthropic astrophysical effect, it appears difficult to claim an anthropic explanation for the observed size of ζ, f, or θ_0.

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[3]One of the bigger uncertainties is what fraction of baryons would initially collapse to the SMBH. If a significant fraction remains in the halo, it would be subject to the usual accretion and feedback processes.

[4]It has been pointed out [7] that all known weakly coupled regions of string theory have values of f of at least 10^{16} GeV, which leads us to suspect a statistical enhancement at large values.

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