Running non-Gaussianities in Dirac-Born-Infeld inflation

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We study the non-Gaussianity in the simplest infrared model of the Dirac-Born-Infeld (DBI) inflation. We show that the non-Gaussianity in such a model is compatible with the current observational bound and is within the sensitivity of future experiments. We also discuss the scale dependence of the non-Gaussianity. In DBI inflation, such a feature can be used as a probe to the properties of the background geometry of the extra dimensions or internal space.

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I. INTRODUCTION

Recently, Dirac-Born-Infeld (DBI) inflation [1–4] was proposed as an alternative to slow-roll inflation. The motivation comes from the nongenericness of flat potentials. In slow-roll inflation, the required flatness is described by the slow-roll conditions and is necessary to hold the inflaton on the top of the potential for a sufficiently long time, at the same time producing a scale-invariant spectrum for density perturbations. In DBI inflation, one starts with a generic steep potential but, in the meanwhile, considers a warped background. The inflaton can move slowly on the steep potential because a large warping can give rise to a stringent causality constraint on the speed limit of the inflaton. Despite the slowness of the inflaton coordinate speed, it is highly relativistic. Such inflationary models are interesting in situations where warped space is common but not flat potentials.

There are two types of DBI inflation models. In the UV model [1,2], the inflaton slides down the potential from the UV side of the warped space to the IR end. This results in a power law inflation when the scale of the potential is high enough. In the IR model [3,4], the inflaton is originally trapped in the IR region through some sort of phase transition and then rolls out from the IR to the UV side. The resulting inflation is exponential and the potential scale is flexible.

As for many other inflationary models, when confronted with experiments, it is generally a combination of different properties that will pin down some particular models. These properties include the scalar and tensor spectral indices, the running of these indices, and the primordial non-Gaussianity. In this paper, we will be most interested in the non-Gaussianity from the three-point correlation functions of the scalar fluctuations. This is a function of three momenta forming a triangle. The property includes the overall order of magnitude of the non-Gaussianity and its dependence on the shape of the momentum triangle and on the overall size of the triangle.

The slow-roll inflation usually gives very low non-Gaussianity [5–7], because in the leading order the quantum fluctuations are generated by free fields in the dS background. However, in the DBI inflation, the causality constraint in the kinetic term introduces nonlinear interactions among different momentum modes of the scalar field. It is, therefore, important to study the level of non-Gaussianities of such models.

The dependence on the shape of the momentum triangle is generally complicated and model-dependent [8]. If such information can be extracted from cosmic microwave background (CMB) observations, it can be very useful to distinguish different models. The overall magnitude of the non-Gaussianity can usually be defined when the triangle takes a specific shape, for example, when the triangle becomes equilateral.

The dependence on the size of the triangle is relatively weaker. This follows from the scale invariance of the inflation. However, in realistic inflation models, such a scale invariance is generally broken. So the non-Gaussianity is also titled. As we will see, this running behavior can also be used to distinguish different models, especially for models having similar dependence on the triangle shape.

In Ref. [2], the non-Gaussianity of a UV DBI inflation model has been studied. There the observational bound constrains both the non-Gaussianity and the value of the inflaton field. The required field range and the corresponding non-Gaussianity and tensor modes were discussed in Ref. [2]. They can be tested in future experiments. In Sec. II, we will study an IR model and show that the non-Gaussianity here is almost independent of the inflationary energy scale. The predicted range of the non-Gaussianity is also within the observational ability. In Sec. III, we make comparisons with the UV model and some slow-roll models on several interesting aspects. In these two sections, we will also emphasize the scale dependence of the non-Gaussianity, namely, the dependence on the size of the momentum triangle. This dependence is determined by the geometry of the warped space scanned by the inflaton during inflation and, thus, carries information of the background geometry of the internal space or extra dimensions.

In this paper, we restrict ourselves to the simplest model. By simplest, we mean that we choose parameters so that the effective field theory works. Complications, or some-times improvements, may come at least in two occasions [4]—where the redshifted string scale is too low so that stringy effects become significant or where the relativistic reheating is happening in a relatively deep warped space so the cosmological rescaling [9] takes effect. We discuss the first in Sec. IV.

II. THE IR MODEL

In this section, we study the non-Gaussianity in the simplest IR DBI inflation model. We begin with a brief review on the model. Details can be found in Refs. [3,4].

The inflaton potential is parametrized as

$$V = V_0 - \tfrac{1}{2}m^2\phi^2 = V_0 - \tfrac{1}{2}\beta H^2\phi^2, \tag{2.1}$$

where the Hubble parameter H is approximately a constant. In many inflationary models, there is always naturally a contribution to the potential with $|\beta| \sim 1$. In these models, such a potential is too steep to support a long period of slow-roll inflation. This is the well-known η problem which plagues slow-roll inflation [10].

However, it is shown [4] that, with warped space, the DBI inflation can happen for both small and large β ($\beta > 0$). It generates a scale-invariant spectrum for the density perturbations with a tilt independent of the parameter β. In this case, the steepness of the potential does not play such an important role. The inflaton stays on the potential due to a warping in the internal space

$$ds^2 \propto \frac{\phi^2}{\lambda}ds_4^2 + \frac{\lambda}{\phi^2}d\phi^2, \tag{2.2}$$

where $ds_4^2 \equiv g_{\mu\nu}dx^\mu dx^\nu$ is the metric of the four-dimensional space-time and λ is a dimensionless parameter. The inflaton ϕ and the parameter λ are related to the notations of Refs. [3,4] by $\phi \equiv r\sqrt{T_3}$ and $\lambda \equiv T_3R^4 \sim N$. The λ has the same order of magnitude as the effective background charge N of the warped space and characterizes the strength of the background. The low-energy dynamics is described by the DBI–Chern-Simons action

$$\begin{aligned} S = {}& \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}R \\ & - \int d^4x\sqrt{-g}\bigg(\frac{\phi^4}{\lambda}\sqrt{1 + \frac{\lambda}{\phi^4}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} \\ & - \frac{\phi^4}{\lambda} + V(\phi)\bigg). \end{aligned} \tag{2.3}$$

In the nonrelativistic limit, this action reduces to the usual minimal form.

We start the inflaton near $\phi \sim 0$ through a phase transition.[1] Without the warped space, the scalar will quickly roll down the steep potential ($\beta \gtrsim 1$) and make the inflation impossible. To obtain inflation, it is natural to expect that the speed limit should be nearly saturated. Indeed, solving the equations of motion, we find

$$\phi = -\frac{\sqrt{\lambda}}{t} + \frac{9\sqrt{\lambda}}{2\beta^2H^2}\frac{1}{t^3} + \cdots, \qquad t \ll -H^{-1}, \tag{2.4}$$

where the time t is chosen to run from $-\infty$. The inflaton travels ultrarelativistically with a Lorentz contraction factor

$$\gamma = (1 - \lambda\dot{\phi}^2/\phi^4)^{-1/2}. \tag{2.5}$$

Nonetheless, the coordinate speed of light is very small due to the large warping near $\phi \approx 0$, and in such a way the inflaton achieves "slow rolling." The potential stays nearly constant during the inflation, and we have a period of exponential expansion with the Hubble constant $H = \dot{a}/a = \sqrt{V_0}/\sqrt{3}M_{\rm Pl}$. There is no lower bound on the inflationary scale, and the approximations that H is constant and dominated by the potential energy during inflation require an upper bound on V [4],

$$\frac{V}{M_{\rm Pl}^4} \ll \frac{1}{\beta\lambda N_e}. \tag{2.6}$$

Generally speaking, this bound is not significant, since, to get enough e-foldings, we need only $\lambda \gtrsim 10^4$. However, for some specific models, such as the simplest one that we focus on in this paper, λ is determined by density perturbations and can be much larger.

For the case $\beta \gtrsim 1$ that we are most interested in, the behavior (2.4) is valid for $t \ll -H^{-1}$. The lower bound on t comes from backreactions of the relativistic inflaton [1,4,9] and the de Sitter (dS) space [4] on the warped space. These effects will smooth out the effective geometry of a certain IR region of the warped space. Therefore, even if we start the inflaton from that region, the inflationary period cannot be further increased in terms of the order of magnitude. For the case that we consider here,[2] the strongest lower bound is the closed string creation from the dS background. This gives $t > -\sqrt{\lambda}H^{-1}$. So the maximum number of e-foldings in this model is of order $\sqrt{\lambda}$. The latest e-fold N_e is given by

$$N_e = \sqrt{\lambda}H/\phi. \tag{2.7}$$

It has an interesting relation to the Lorentz contraction factor of the inflaton,

$$\gamma \approx \beta N_e/3. \tag{2.8}$$

Since the sound speed $c_s = \gamma^{-1}$, during inflation, $c_s \ll 1$.[3]

[1] This initial condition can be naturally obtained without tuning in e.g. a scenario of Refs. [3,4].

[2] This corresponds to the case of a single brane rolling out of a throat in Refs. [3,4].

[3] So we cannot use the results of Ref. [11], where the assumption has been made that c_s departs from unity by a quantity much less than one.

To study the perturbations, we expand the inflaton around the above background,

$$\phi = -\frac{\sqrt{\lambda}}{t} + \alpha(\mathbf{x}, t). \tag{2.9}$$

It is useful to define the parameter ζ:

$$\zeta \equiv H\frac{\alpha}{\dot{\phi}} + \Phi. \tag{2.10}$$

(It is shown [4] that the scalar metric perturbation Φ is negligible in this model.) The parameter ζ is useful because it remains constant after the corresponding mode exits the horizon [12]. This can be seen by looking at the exact equation of motion for the linear perturbations [13]

$$v'' - \gamma^{-2}\nabla^2 v - \frac{z''}{z}v = 0. \tag{2.11}$$

The variable v is defined by $v \equiv z\zeta$, with $z \approx a\dot{\phi}\gamma^{3/2}H^{-1}$ in this case. The prime in this equation denotes the derivative with respect to the conformal time η defined by $d\eta \equiv dt/a(t)$. The horizon exit for a mode $\mathbf{k}$ happens when $k/a \ll H\gamma$. The γ factor comes in due to the relativistic effect. It reduces the horizon size by a factor of the sound speed γ^{-1}. It is easy to see that, in this limit, Eq. (2.11) becomes $v''_{\mathbf{k}}/v_{\mathbf{k}} = z''/z$ and has the solution $v_{\mathbf{k}} = z \cdot \text{const}$, so that $\zeta_{\mathbf{k}} = \text{const}$. Therefore, we will calculate the primordial scalar correlation functions in terms of the correlation functions of ζ and evaluate it after the horizon crossing. Decompose the Fourier modes of ζ as

$$\zeta_{\mathbf{k}} = \zeta^{\rm cl}_{\mathbf{k}} a_{\mathbf{k}} + \zeta^{\rm cl*}_{-\mathbf{k}} a^{\dagger}_{-\mathbf{k}} \tag{2.12}$$

with the commutation relations

$$[a_{\mathbf{k}}, a^{\dagger}_{\mathbf{k}'}] = (2\pi)^3\delta^3(\mathbf{k} - \mathbf{k}'). \tag{2.13}$$

The explicit form of the classical solution $\zeta^{\rm cl}_{\mathbf{k}}$ can be worked out by examining two different limits in Eq. (2.11). When modes are well within the horizon, $k/a \gg H\gamma$,

$$\zeta^{\rm cl}_{\mathbf{k}} \approx -\frac{H^2}{\dot{\phi}}\frac{1}{\sqrt{2k^3}}\frac{k\eta}{\gamma}e^{-ik\eta/\gamma}. \tag{2.14}$$

When modes are far outside of the horizon, $k/a \ll H\gamma$,

$$\zeta^{\rm cl}_{\mathbf{k}} \approx i\frac{H_*^2}{\dot{\phi}_*\sqrt{2k^3}}, \tag{2.15}$$

where the subscript $*$ indicates that the variable be evaluated at the horizon crossing.

Equivalently, we can also first calculate the correlation functions of α using the decomposition

$$\alpha_k = u_{\mathbf{k}}a_{\mathbf{k}} + u^*_{-\mathbf{k}}a^{\dagger}_{-\mathbf{k}}. \tag{2.16}$$

The function $u_{\mathbf{k}}(\eta)$ is the usual classical solution of the scalar fluctuations in the dS background,

$$u_{\mathbf{k}}(\eta) = \frac{H}{\sqrt{2k^3}}e^{-ik\eta/\gamma}\left(i - \frac{k\eta}{\gamma}\right), \tag{2.17}$$

except for the presence of the γ factors for the reason that we have mentioned below (2.11). We then use the relation $\zeta \approx H\alpha/\dot{\phi}$ to convert α to ζ, evaluating H and $\dot{\phi}$ at the horizon crossing. Since the non-Gaussianity in this model will turn out to be much larger than that in the minimal slow-roll model [5], we need consider only the leading order of ζ [2,14]. With this prescription, it is easy to see that these two methods are the same since Eqs. (2.14), (2.15), and (2.17) are connected by the same relation.

We first calculate the two-point function. This function contains information on the density perturbations.

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\rangle = (2\pi)^5\delta^3(\mathbf{k}_1 + \mathbf{k}_2)\mathcal{P}_{\mathcal{R}}\frac{1}{2k_1^3}, \tag{2.18}$$

where $\mathcal{P}_{\mathcal{R}}$ is the spectral density

$$\mathcal{P}_{\mathcal{R}} = \frac{H_*^4}{(2\pi)^2\dot{\phi}_*^2}. \tag{2.19}$$

The calculation of the three-point function is the same as that in the UV model [2]. We plug the expansion (2.9) into the Lagrangian and read off the cubic terms of the scalar field fluctuations,

$$\begin{aligned}\mathcal{L}_3 = a^3\bigg[&\frac{\lambda\dot{\phi}\gamma^5}{2\phi^4}\dot{\alpha}^3 - \frac{\lambda\dot{\phi}\gamma^3}{2\phi^4a^2}(\nabla\alpha)^2\dot{\alpha} \\ &+ \frac{\lambda\dot{\phi}^2\gamma^3}{\phi^5a^2}(\nabla\alpha)^2\alpha + \left(\frac{5\lambda\dot{\phi}^3\gamma^3}{\phi^6} + \frac{6\lambda^2\dot{\phi}^5\gamma^5}{\phi^{10}}\right)\dot{\alpha}\alpha^2 \\ &- \frac{3\lambda\dot{\phi}^2\gamma^5}{\phi^5}\dot{\alpha}^2\alpha \\ &- \left(\frac{2\dot{\phi}^2\gamma^3}{\phi^3} + \frac{4\lambda^2\dot{\phi}^6\gamma^5}{\phi^{11}} - \left(1 - \frac{1}{\gamma}\right)\frac{4\phi}{\lambda}\right)\alpha^3\bigg].\end{aligned} \tag{2.20}$$

Using (2.4), we can estimate that the first line in (2.20) is the leading terms for $t \ll -H^{-1}$.[4] So to the first order [5],

$$\langle \zeta^3(t)\rangle = -i\int_{t_0}^{t} dt'\langle[\zeta^3(t), H_{\rm int}(t')]\rangle, \tag{2.21}$$

where the leading terms of the interaction Hamiltonian are

$$H_{\rm int} = -a^3\frac{\lambda\dot{\phi}\gamma^5}{2\phi^4}\int d^3\mathbf{x}\left[\dot{\alpha}^3 - \frac{(\nabla\alpha)^2\dot{\alpha}}{\gamma^2a^2}\right]. \tag{2.22}$$

The lower limit t_0 in the integration (2.21) is some early time when the modes are still well within the Hubble horizon. The modes are rapidly oscillating at that time and average to zero. This effect can also be captured by

[4]We can also check the magnitude of the expansion parameter used for $\mathcal{L}$, which is $\gamma^2\lambda\dot{\phi}\,\dot{\alpha}/\phi^4 \sim \gamma^2(H^2/\dot{\phi})$ near the horizon crossing. This is small since the second factor is related to the density perturbations and has to be very small according to the Cosmic Background Explorer (COBE) normalization.

adding a damping term. The upper limit t is taken to be the time of several e-folds after the horizon crossing. In terms of the conformal time $\eta \equiv -a^{-1}H^{-1}$, the integration range can be taken from $-\infty$ to a value where $\eta k/\gamma \ll 1$ or effectively to 0.

With all the prescription given above, using (2.16), (2.17), (2.21), and (2.22), it is straightforward to work out the three-point correlation function

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = \frac{i}{16}(2\pi)^7\delta^3\left(\sum_i \mathbf{k}_i\right)\mathcal{P}_{\mathcal{R}}^2\frac{1}{\prod_i k_i^3} \times \int_{-\infty}^0 d\eta e^{ik_t\eta/\gamma}\left[\frac{6\eta^2}{\gamma}k_1^2k_2^2k_3^2 - 2\gamma k_3^2(\mathbf{k}_1\cdot\mathbf{k}_2)\left(i+\frac{k_1\eta}{\gamma}\right)\left(i+\frac{k_2\eta}{\gamma}\right) + \text{perm.}\right] + \text{c.c.}, \tag{2.23}$$

where $k_t = k_1 + k_2 + k_3$ and Eq. (2.4) has been used to combine factors of ϕ and $\dot{\phi}$ into factors of $\mathcal{P}_{\mathcal{R}}$. Since $\mathcal{P}_{\mathcal{R}}$ remains constant after the horizon crossing, we pull it to the front of the integration. The "perm." indicates two other terms with the same structure as the last term but permutation of indices 1, 2, and 3. The "c.c." stands for the complex conjugate of all terms. The damping effect for the early time can be added by replacing $\eta \to \eta(1-i\epsilon)$. Performing the integral, we get[5]

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = (2\pi)^7\delta^3\left(\sum_i \mathbf{k}_i\right)\mathcal{P}_{\mathcal{R}}^2 F(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3), \tag{2.24}$$

where the form factor is

$$F = \frac{\gamma^2}{4k_t^3\prod_i k_i^3}[-6k_1^2k_2^2k_3^2 + k_3^2(\mathbf{k}_1\cdot\mathbf{k}_2) \times (2k_1k_2 - k_3k_t + 2k_t^2) + \text{perm.}]. \tag{2.25}$$

There are two interesting limits of the shape of the momentum triangle. When one side of the triangle is very small, for example, $k_1 \approx 0$ and $k_2 = k_3$, the form factor goes as

$$F \approx -\frac{11\gamma^2}{16k_1k_2^5}. \tag{2.26}$$

When the triangle becomes equilateral, $k_1 = k_2 = k_3 = k$,

$$F = -\frac{7\gamma^2}{24k^6}. \tag{2.27}$$

The non-Gaussianity of the CMB in the Wilkinson microwave anisotropy probe (WMAP) observations is analyzed by assuming the simple ansatz [15,16]

$$\zeta = \zeta_L - \tfrac{3}{5}f_{NL}(\zeta_L^2 - \langle\zeta_L^2\rangle), \tag{2.28}$$

where ζ_L is the linear Gaussian part of the perturbations, and f_{NL} parametrizes the level of the non-Gaussianity. Using $\zeta_L^2(\mathbf{k}) = \int[d^3\mathbf{p}/(2\pi)^3]\zeta_L(\mathbf{p})\zeta_L(\mathbf{k}-\mathbf{p})$ and $\langle\zeta_L^2(\mathbf{x})\rangle = (H_*^2/\dot{\phi}_*^2)\int[d^3\mathbf{p}/(2\pi)^3]|u_{\mathbf{p}}|^2$, this assumption leads to

$$F = -f_{NL}\frac{3\sum_i k_i^3}{10\prod_i k_i^3}. \tag{2.29}$$

The triangle shape dependence is very different from that in the DBI inflation case. For example, for $k_1 \approx 0$ and $k_2 = k_3$,

$$F \approx -f_{NL}\frac{3}{5k_1^3k_2^3}. \tag{2.30}$$

As pointed out in Ref. [8], this feature can be used to distinguish different models, for example, by plotting the function $k_1^2k_2^2k_3^2F$ with one side fixed ($k_3 = 1$). However, the momentum dependence for the equilateral case is the same:

$$F = -f_{NL}\frac{9}{10k^6}. \tag{2.31}$$

We can, therefore, use the f_{NL} in the equilateral case to compare with the existing experimental analyses. From (2.27), we get

$$f_{NL} \approx 0.32\gamma^2. \tag{2.32}$$

The momentum dependence in (2.25) and the relation (2.32) are the same as in the UV model [2]. The differences lie in its evaluation under the zero-mode background evolution discussed in the beginning of this section. Using the relation (2.8), we find

$$f_{NL} \approx 0.036\beta^2N_e^2 \tag{2.33}$$

for this simplest IR model. Depending on the energy scale of the inflation, the CMB can correspond to an N_e ranging from 30 to 60. For $1/2 \lesssim \beta \lesssim 2$, we have $8 \lesssim f_{NL} \lesssim 518$. For example, for $N_e \approx 50$ and $\beta \approx 1$, we have $f_{NL} \approx 90$. This is compatible with the current observational bound [16]

$$|f_{NL}| \lesssim 100. \tag{2.34}$$

The WMAP will eventually reach a sensitivity $|f_{NL}| \lesssim 20$, and the Planck $|f_{NL}| \lesssim 5$. So the non-Gaussianity in this model is also within the sensitivity of future experiments. We also note here that the dependence of Eq. (2.33) on the inflationary energy scale is very weak (only through N_e) compared to the UV model that we will discuss in the next section.

The triangle size dependence of the non-Gaussianity also provides interesting information. We are interested in the equilateral case since the momentum dependence is the same for different models. The size dependence in Eq. (2.24) comes from two different factors. The first is from $\mathcal{P}_{\mathcal{R}}$. This is due to the tilt of the density perturba-

[5] A more general analysis is in progress [27].

tions, so we are more interested in the second factor coming from $F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$, namely, (2.33). This has a logarithmic dependence on the wave number. Analogous to the spectral index, we can define an index parametrizing the running of the overall non-Gaussianity,

$$n_{\rm NG} - 1 \equiv \frac{\partial \ln|f_{NL}|}{\partial \ln k} = -\frac{\partial \ln|f_{NL}|}{\partial N_e} \approx -\frac{2}{N_e}. \quad (2.35)$$

This tilt is directly related to the background geometry of the internal space or extra dimensions which the inflaton scans through during the inflation. To see this, it is instructive to consider a different background geometry.

Suppose that a portion of a warped space in the IR side has a constant warp factor,

$$ds^2 \propto f_0^2 ds_4^2 + f_0^{-2} d\phi^2, \quad (2.36)$$

where f_0 is a constant. Consider that the inflation is caused by the speed-limit constraint when the inflaton moves out of this part of the warped space. Note that the existing explicit examples of the string theory flux compactification usually give geometries of the type of (2.2), as in Refs. [17–19], but not (2.36). But we consider this anyway just for the sake of comparison.

The inflaton dynamics can be similarly solved,

$$\phi = f_0^2 t + \frac{9 f_0^2}{2\beta^2 H^2 t} + \cdots, \qquad t \gg H^{-1}, \quad (2.37)$$

with the Lorentz contraction factor

$$\gamma \approx \frac{\beta H t}{3} \approx \frac{\beta}{3}(N_{\rm tot} - N_e), \quad (2.38)$$

where $N_{\rm tot}$ is the total number of e-folds. Repeating the previous procedures, we get a similar interaction Hamiltonian

$$H_{\rm int} = -a^3 \frac{\dot\phi \gamma^5}{2 f_0^4} \int d^3\mathbf{x} \left[\dot\alpha^3 - \frac{(\nabla\alpha)^2 \dot\alpha}{\gamma^2 a^2} \right] \quad (2.39)$$

and the same three-point function (2.24) and (2.25). But the relation between γ and N_e is different from the previous case. For the equilateral case, we have

$$f_{NL} \approx 0.036 \beta^2 (N_{\rm tot} - N_e)^2, \quad (2.40)$$

and the running index for the non-Gaussianity is

$$n_{\rm NG} - 1 \approx \frac{2}{N_{\rm tot} - N_e}. \quad (2.41)$$

This has an opposite sign to Eq. (2.35).

So we see that, in the DBI inflation, the measurement of the dependence of the non-Gaussianity on N_e encodes the relation between γ and N_e and, thus, can provide information on the background geometry of the internal space or extra dimensions. There are two factors that can change γ. The first is the falling-down of the potential and it always tends to increase γ as N_e decreases. The second is the shape of the background geometry. For the case of (2.2), the inflaton moves from the IR to the UV side of the warped space; the causality constraint is weaker for later time. This effect wins over the effect coming from the potential, resulting in a negative $n_{\rm NG} - 1$. For the case of the flat geometry (2.36), we have only the first factor, and, hence, we get a positive $n_{\rm NG} - 1$. In the next section, we will discuss the UV model, where the two factors add up and the running of the non-Gaussianity is much stronger.

III. THE UV MODEL AND SOME SLOW-ROLL MODELS

In this section, we review aspects of the UV DBI inflation model and slow-roll inflation models and make some comparison.

In the UV model [1,2], the inflaton travels from the UV side of the warped space to the IR side under the potential

$$V = \tfrac{1}{2} m^2 \phi^2. \quad (3.1)$$

The inflaton dynamics can be found as

$$\phi = \frac{\sqrt{\lambda}}{t} - \frac{\lambda^{3/2}}{4 m^2 M_{\rm Pl}^2 t^5} + \cdots. \quad (3.2)$$

The Hubble parameter $H \approx p/t$ is no longer a constant, and the inflation is in a power law $a(t) \propto t^p$, where $p \approx \sqrt{\lambda/6}\, m/M_{\rm Pl}$. The inflation happens if the inflaton mass satisfies $m \gg M_{\rm Pl}/\sqrt{\lambda}$, i.e. when $p \gg 1$.

Using (2.5) and (3.2), we can express the Lorentz contraction factor as

$$\gamma \approx 2p \frac{M_{\rm Pl}^2}{\phi^2}. \quad (3.3)$$

From this equation, one can see that there is a tension between having a small γ and a large p. According to (2.32) and (2.34), in the CMB region, $\gamma \lesssim 18$. Normally, $\phi \ll M_{\rm Pl}$ for large λ.[6] In this case, Eq. (3.3) indicates that the non-Gaussianity is too big to be compatible with the observation. However, if one considers a large scalar vacuum expectation value $\phi \gtrsim M_{\rm Pl}$, it is still possible to satisfy the observational bound [2].[7] For example, taking $\phi \sim M_{\rm Pl}$, the resulting non-Gaussianity together with the tensor modes can, in principle, be detected in future experiments.

We denote the end point of the inflation as ϕ_f, which can come from having a warped space with a modest total warping or can be caused by the backreaction of the relativistic inflaton. The relation between γ and N_e is now

[6]For example, if we require $r < R$ in terms of a brane moving in a warped space with a characteristic length scale R.

[7]For example, if we consider that a large number, $\sqrt{\lambda}$, of branes stick together. This increases ϕ by increasing the effective brane tension T_3 through $\phi = r\sqrt{T_3}$ and may be possible to make $\phi \gtrsim M_{\rm Pl}$.

$$\gamma \approx \frac{2pM_{\rm Pl}^2}{\phi_f^2} e^{-2N_e/p}. \tag{3.4}$$

So we see that the non-Gaussianity

$$f_{NL} \approx 1.3 \frac{p^2 M_{\rm Pl}^4}{\phi^4} \approx 1.3 \frac{p^2 M_{\rm Pl}^4}{\phi_f^4} e^{-4N_e/p} \tag{3.5}$$

grows exponentially as N_e decreases. As we mentioned, this reflects the fact that the inflaton is traveling towards a region of higher warping. In terms of the index $n_{\rm NG}$,

$$n_{\rm NG} - 1 \approx \frac{4}{p}. \tag{3.6}$$

Comparing to the IR case (2.35), it has an opposite sign and bigger magnitude (for $\phi \sim M_{\rm Pl}$).

Unlike the direct scalar interactions that we have seen in the DBI inflation, the leading non-Gaussianity in the simplest model of the slow-roll inflation originates from the nonlinearities of the Einstein action and the flat potential. Calculations [5–7] show that f_{NL} is in the same order of the slow-roll parameters and, therefore, is unobservably small. One can consider nonminimal cases of the slow-roll models, for example, by adding a correction term $1/8M^4(\partial_\mu \phi \partial^\mu \phi)^2$ [20], where M is the energy scale of new physics. Interestingly, this term gives a non-Gaussianity [14] with the same shape dependence as in the DBI inflation models that we just discussed. The difference is the overall magnitude—the γ^2 in (2.25) is replaced by $\dot\phi^2/M^4$. The running of f_{NL} depends on the shape of the potential. For example, using the slow-roll relations $\dot\phi \approx -V'(\phi)/3H$ and $\phi = \phi_m e^{\mp \beta N_e/3}$ (ϕ_m is the end point of the inflation), we have $n_{\rm NG} - 1 = \pm 2\beta/3$ for the potential $V = V_0 \mp \frac{1}{2}\beta H^2 \phi^2$. But in any case, since $\dot\phi^2/M^4$ has to be less than one to justify the neglect of higher order corrections, f_{NL} has to be less than one and is also too small to be observed.

IV. DISCUSSIONS

We have studied the non-Gaussianity in the simplest IR model of the DBI inflation and shown that it satisfies the current experimental bound and provides interesting predictions for future test. In this section, we discuss a nonminimal case [4].

Such a case deserves further investigations because there are at least two aspects of the simplest model that may not be fully satisfying. The first comes from the fitting of the density perturbations. Using (2.7) and (2.19), we find the density perturbations on the CMB scale

$$\delta_H \approx \frac{N_e^2}{5\pi\sqrt{\lambda}}. \tag{4.1}$$

The COBE normalization $\delta_H \approx 1.9 \times 10^{-5}$ at $N_e \sim 60$ requires $\lambda \sim 10^{14}$.

Whether such a number is natural depends on the fundamental physics which realizes the model. The brane inflation [21] in warped string compactification [17–19] is a natural place to realize the DBI inflation [3,4], where the brane position moving in the warped extra dimensions plays the role of the scalar inflaton moving in the warped internal space, as in the slow-roll models [22–25]. The inflationary energy can be provided by antibranes or simply by a moduli potential. In this type of scenario, λ is in the order of the effective background charge of the warped throat N. In this context, this required number is extremely large[8] [2–4,26].

The second concern is that the tilt of the density perturbations

$$n_s - 1 \approx -\frac{4}{N_e} \tag{4.2}$$

may be too large to fit the observations.

Interestingly, these two concerns may be addressed at the same time without adding any new features to the model. We need to look more carefully at the validity region of our field theory analyses of the quantum fluctuations in dS space. There are three regions of N_e that are interesting given a λ (which now is not necessarily large). For a large N_e, the brane (or inflaton) is in the deep infrared region of the warped space. The energy density of the scalar quantum fluctuations has to be less than the redshifted string scale in order for the field theory analyses to hold. The former can be estimated as (for an instantaneous observer moving with the brane)

$$\gamma^2 \Delta\phi^2/\Delta x^2, \tag{4.3}$$

where the horizon size $\Delta x \sim \gamma^{-1} H^{-1}$ and the quantum fluctuation $\Delta\phi \sim H$. The factor of γ comes from the restoration of the Lorentz contraction factor for the moving observer on the brane. The latter is $T_3 h^4 = \phi^4/\lambda$, where the h is the warp factor at ϕ. Using (2.7) and (2.8), we get the critical e-folding

$$N_c \sim \frac{\lambda^{1/8}}{\beta^{1/2}}. \tag{4.4}$$

The first region is $N_e < N_c$, where the field theory holds. This is the region we have been studying in this paper. In the second region, $N_e > N_c$, the stringy fluctuations become significant. We can no longer use the DBI action, which is a low-energy field theory approximation, to calculate the quantum fluctuations. But the inflation still

[8] Although a somewhat different point of view is to regard it as a requirement that the characteristic length scale of the throat be $\mathcal{O}(10^3–10^4)$ in string units, and so, in general, the tuning of λ may be different from the tuning of N. I thank Eva Silverstein for pointing it out to me. Other discussions can be found in Refs. [2–4]. For example, the effect of the cosmological rescaling can greatly reduce δ_H. This happens if the reheating happens in a warped space with a relatively large warping [4,9].

proceeds as long as the warped background is strong enough, since the only thing important for the zero-mode inflaton dynamics is the speed-limit causality constraint. We need a different way to estimate the quantum fluctuations. A rough estimate goes as follows. We assume the part of energy that goes into the scalar excitations to be $\mathcal{O}(T_3 h^4)$, and the rest is used to excite strings. From (4.3), this gives $\Delta\phi \sim \sqrt{\lambda}H/N_e^2\gamma^2$. While the strings get diluted in the later spatial expansion, the position dependent time delay δt caused by the scalar fluctuations remains constant. Therefore, at the reheating, we can still approximate the density perturbations using the time delay without considering the diluted stringy excitations. So we have

$$\delta_H = \frac{2}{5}H\delta t \approx \frac{2}{5}H\Delta\phi/\dot{\phi} \sim \frac{2}{5\gamma^2} \sim \frac{18}{5\beta^2 N_e^2}. \tag{4.5}$$

Note that this is very different from (4.1). It is independent of λ (therefore, no need to have a large λ) and has an opposite running

$$n_s - 1 \sim \frac{4}{N_e}. \tag{4.6}$$

The third region of N_e is, therefore, the transition region between (4.2) and (4.6). This region has two interesting properties—it should have smaller $|n_s - 1|$ but larger $|dn_s/d\ln k|$ because it has to connect (4.2) and (4.6) in a short range. The non-Gaussianity feature for $N_e \gtrsim N_c$, including its dependence on the background geometry, should be very interesting and remains a challenge for future studies.

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