

Ghosts in the self-accelerating brane universeKazuya Koyama^{1,2}¹*Department of Physics, University of Tokyo 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan*²*Institute of Cosmology and Gravitation, Portsmouth University, Portsmouth, PO1 2EG, United Kingdom*
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We study the spectrum of gravitational perturbations about a vacuum de Sitter brane with the induced 4D Einstein-Hilbert term, in a 5D Minkowski spacetime (DGP model). We consider solutions that include a self-accelerating universe, where the accelerating expansion of the universe is realized without introducing a cosmological constant on the brane. The mass of the discrete mode for the spin-2 graviton is calculated for various Hr_c , where H is the Hubble parameter and r_c is the crossover scale determined by the ratio between the 5D Newton constant and the 4D Newton constant. We show that, if we introduce a positive cosmological constant on the brane ($Hr_c > 1$), the spin-2 graviton has mass in the range $0 < m^2 < 2H^2$ and there is a normalizable brane fluctuation mode with mass $m^2 = 2H^2$. Although the brane fluctuation mode is healthy, the spin-2 graviton has a helicity-0 excitation that is a ghost. If we allow a negative cosmological constant on the brane, the brane fluctuation mode becomes a ghost for $1/2 < Hr_c < 1$. This confirms the results obtained by the boundary effective action that there exists a scalar ghost mode for $Hr_c > 1/2$. In a self-accelerating universe $Hr_c = 1$, the spin-2 graviton has mass $m^2 = 2H^2$, which coincides with the mass of the brane fluctuation mode. Then there arises a mixing between the brane fluctuation mode and the spin-2 graviton. We argue that this mixing presumably gives a ghost in the self-accelerating universe by continuity across $Hr_c = 1$, although a careful calculation of the effective action is required to verify this rigorously.

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I. INTRODUCTION

The cosmological constant problem is one of the most difficult and important problems in particle physics and cosmology [1]. The old problem is the smallness of the cosmological constant compared with the fundamental scales of particle physics. A natural solution for this problem has been thought to be an exact cancellation of the cosmological constant. However, recent discoveries of the accelerated expansion of the universe make the problem more complicated [2]. Within the 4D Einstein theory of gravity, we do need a tiny cosmological constant in order to explain the acceleration of the universe. Then we must introduce a tiny cosmological constant by hand and accept a fine-tuning [3].

There have been many attempts to modify the 4D Einstein theory of gravity to explain the acceleration of the universe instead of introducing the cosmological constant [4]. One such attempt was made in the context of the brane world models, where our universe is realized as a 4D hypersurface (brane) in a higher dimensional spacetime (bulk). Dvali, Gabadadze, and Porrati (DGP) proposed a model where the 4D Einstein-Hilbert term is assumed to be induced on the brane [5]. In this model, the accelerated expansion of the universe can be realized by a modification of Einstein gravity on large scales and we do not need a cosmological constant (self-accelerating universe) [6].

Recently several authors claimed that there exist ghost-like excitations in the self-accelerating universe [7,8]. If this is true, it becomes difficult to consider the DGP model as a consistent model to explain the acceleration of the universe. In this paper, we reexamine the existence of the

ghost in the DGP model based on a direct calculation of the spectrum of the perturbations.

II. BACKGROUND SPACETIME AND PERTURBATIONS

The 5D action describing the DGP model is given by

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} R + \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-\gamma} {}^{(4)}R - \sigma \int d^4x \sqrt{-\gamma} + \frac{1}{\kappa^2} \int d^4x \sqrt{-\gamma} K, \quad (1)$$

where σ is the tension of the brane, $K_{\mu\nu}$ is the extrinsic curvature and $K = K^\mu{}_\mu$. In this letter, we only consider a vacuum energy contribution from matter fields on the brane for simplicity. We also assume reflection symmetry across the brane. Then the junction condition must be imposed at the brane as

$$K^\mu{}_\nu = \frac{\kappa^2}{2} \left(-\frac{\sigma}{3} \delta^\mu{}_\nu + \frac{1}{\kappa_4^2} \tilde{G}^\mu{}_\nu \right), \quad (2)$$

where ${}^{(4)}G^\mu{}_\nu$ is the Einstein tensor on the brane and ${}^{(4)}\tilde{G}^\mu{}_\nu = {}^{(4)}G^\mu{}_\nu - (1/3){}^{(4)}G\delta^\mu{}_\nu$. The 4D Einstein tensor comes from the induced Einstein-Hilbert term. The Friedmann equation on the brane is given by

$$\pm H = r_c H^2 - \frac{\kappa^2}{6} \sigma, \quad r_c = \frac{\kappa^2}{2\kappa_4^2}. \quad (3)$$

The 5D solution for the metric with the 4D de Sitter brane can be obtained as [6]

$$ds^2 = dy^2 + N(y)^2 \gamma_{\mu\nu} dx^\mu dx^\nu, \quad N(y) = 1 \pm Hy, \quad (4)$$

where $\gamma_{\mu\nu}$ is the metric for the de Sitter spacetime and the brane is located at $y = 0$. If we take the + branch solutions, there is a solution for the de Sitter spacetime without σ ,

$$H = \frac{1}{r_c}. \quad (5)$$

We call this solution the self-accelerating universe.

Let us investigate the perturbations $N(y)^2 \gamma_{\mu\nu} + h_{\mu\nu}$ about the background de Sitter spacetime. In the following, we assume $Hr_c \neq 1$ and treat the case $Hr_c = 1$ separately. In addition to the gravitational perturbations $h_{\mu\nu}$, we must take into account a perturbation of the position of the brane $y = \varphi(x)$ [9]. Using the transverse-traceless gauge $\nabla^\mu h_{\mu\nu} = h = 0$, the perturbed junction condition is given by

$$k_{\mu\nu} - \mathcal{H}h_{\mu\nu} - r_c \left[X_{\mu\nu}(h) - \kappa_4^2 \left(T_{\mu\nu} - \frac{1}{3} \gamma_{\mu\nu} T \right) \right] = -(1 - 2\mathcal{H}r_c)(\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu})\varphi, \quad (6)$$

where $k_{\mu\nu} = (1/2)\partial_y h_{\mu\nu}$, $\mathcal{H} = \partial_y N/N$ on the brane and $X_{\mu\nu}$ is given by

$$\begin{aligned} X_{\mu\nu} &= \delta^{(4)}G_{\mu\nu} + 3H^2 h_{\mu\nu} \\ &= -\frac{1}{2}(\square_4 h_{\mu\nu} - \nabla_\mu \nabla_\alpha h_\nu^\alpha - \nabla_\nu \nabla_\alpha h_\mu^\alpha + \nabla_\mu \nabla_\nu h) \\ &\quad - \frac{1}{2} \gamma_{\mu\nu} (\nabla_\alpha \nabla_\beta h^{\alpha\beta} - \square_4 h) + H^2 \left(h_{\mu\nu} + \frac{1}{2} \gamma_{\mu\nu} h \right). \end{aligned} \quad (7)$$

The equation of motion for φ is obtained from the traceless condition $h = 0$;

$$(1 - 2\mathcal{H}r_c)(\square_4 + 4H^2)\varphi = \frac{\kappa^2 T}{6}. \quad (8)$$

Let us find solutions for the vacuum brane $T_{\mu\nu} = 0$. Using the separation of variables $h_{\mu\nu} = \int dm e_{\mu\nu}(x) F_m(y)$, the equation of motion in the bulk is written as

$$F_m'' + \frac{1}{N^2}(m^2 - 2H^2)F_m = 0, \quad (9)$$

where prime denotes a derivative with respect to y . There are two types of solutions. One type of solution is an inhomogeneous solution sourced by the scalar mode φ . We call this solution the spin-0 perturbation [10]. The other solution is a homogeneous solution with $\varphi = 0$, which is called the spin-2 perturbation. The spin-2 perturbations $\chi_{\mu\nu}$ satisfy the junction condition without φ

$$\chi'_{\mu\nu} - 2\mathcal{H}\chi_{\mu\nu} = -m^2 r_c \chi_{\mu\nu}. \quad (10)$$

We find a tower of continuous Kaluza-Klein (KK) modes starting from $m^2 = (9/4)H^2$ as well as a normalizable discrete mode $m_d^2 = 0$ in the - branch and with

$$\frac{m_d^2}{H^2} = \frac{1}{(Hr_c)^2} (3Hr_c - 1), \quad (11)$$

in the + branch for $Hr_c > 2/3$ [11]. For $Hr_c > 1$, the mass is in the range $0 < m_d^2 \leq 2H^2$ where $m_d^2 = 2H^2$ for the self-accelerating universe $Hr_c = 1$ and $m_d^2 \rightarrow 0$ for $Hr_c \rightarrow \infty$ [12].

In the - branch, there are no normalizable solutions for the spin-0 perturbations. In the + branch, there is a normalizable solution given by

$$h_{\mu\nu} = \frac{1 - 2Hr_c}{H(1 - Hr_c)} (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu})\varphi. \quad (12)$$

This is a solution with $m^2 = 2H^2$.

III. BOUNDARY EFFECTIVE ACTION

We can construct the 2nd order action for $h_{\mu\nu}$ and φ from the 5D action by extending the result of Ref. [13]. The result is given by

$$\begin{aligned} \delta_2 S &= -\frac{1}{4\kappa^2} \int d^5 x \sqrt{-g} N^{-4} h^{\mu\nu} \delta^{(5)} G_{\mu\nu} \\ &\quad + \frac{1}{\kappa^2} \int d^4 x \sqrt{-\gamma} \mathcal{L}_B, \end{aligned} \quad (13)$$

where $\delta^{(5)}G_{\mu\nu}$ is the 5D perturbed Einstein tensor and

$$\begin{aligned} \mathcal{L}_B &= k^{\mu\nu} h_{\mu\nu} - kh + \frac{1}{2} \mathcal{H}(h^2 - h^{\mu\nu} h_{\mu\nu}) \\ &\quad + (1 - 2\mathcal{H}r_c)(h_{\mu\nu} \nabla^\mu \nabla^\nu \varphi - h \nabla^\rho \nabla_\rho \varphi - 3H^2 h \varphi) \\ &\quad - 3\mathcal{H} \left(-(1 - 2\mathcal{H}r_c)\varphi(\square_4 + 4H^2)\varphi + \frac{\kappa^2}{3} T\varphi \right) \\ &\quad + \frac{1}{2} \kappa^2 h^{\mu\nu} T_{\mu\nu} - \frac{r_c}{2} h^{\mu\nu} X_{\mu\nu}(h). \end{aligned} \quad (14)$$

This action gives the correct equation of motion and the junction condition for $h_{\mu\nu}$ and the equation of motion for φ .

We can derive an effective action for the brane fluctuation φ by substituting the 5D solution for $h_{\mu\nu}$ given by φ (12) into the 5D action and get the off-shell action for φ by integrating out only with respect to the extra coordinate y [14]. This yields the action for φ in the + branch as

$$S_\varphi = \frac{3H}{2\kappa^2} \left(\frac{1 - 2Hr_c}{1 - Hr_c} \right) \int d^4 x \sqrt{-\gamma} \varphi (\square_4 + 4H^2) \varphi. \quad (15)$$

We find that, for $Hr_c > 1$, the kinetic term is always positive, so the brane fluctuation mode φ is not a ghost.

However, there is a problem for the spin-2 perturbations. We consider only the + branch solutions that include the

self-accelerating universe. The 4D effective action for the spin-2 perturbations is also obtained in a similar way. For the discrete mode with m_d^2 , we get

$$S_x = \frac{r_c(3Hr_c - 1)}{4\kappa^2(3Hr_c - 2)} \int d^4x \sqrt{-\gamma} \chi^{\mu\nu} (\square_4 - 2H^2 - m_d^2) \chi_{\mu\nu}, \quad (16)$$

where transverse-traceless gauge fixing conditions $\nabla^\mu \chi_{\mu\nu} = \chi^\mu{}_\mu = 0$ are imposed. This is exactly the same action for the spin-2 perturbations in the 4D massive gravity theory where the Pauli-Fierz (PF) mass term is added to the Einstein-Hilbert action by hand [15]

$$S_M = -\frac{M^2}{8\kappa_4^2} \int d^4x \sqrt{-\gamma} (h^{\mu\nu} h_{\mu\nu} - h^2). \quad (17)$$

Note that in the 4D PF theory, the mass term breaks the gauge symmetry for the perturbation and the transverse-traceless conditions are imposed by the equation of motion as constraints. In the limit $Hr_c \rightarrow \infty$, the action approaches the one for massless spin-2 perturbations. However, there is a discontinuity between the massless perturbations and the massive perturbations that is known as the van Dam-Veltman-Zakharov discontinuity [16]. Because of the lack of gauge symmetry, the massive spin-2 perturbations contain a helicity-0 excitation. Moreover, it has been shown that this helicity-0 excitation becomes a ghost if $0 < M^2 < 2H^2$ [17,18]. This is exactly the same mass range for the discrete mode in the + branch for $Hr_c > 1$. Thus we expect that there appears a ghost when $Hr_c > 1$, that is, the universe with a positive cosmological constant is unstable.

IV. 4D EFFECTIVE ACTION FOR HELICITY-0 EXCITATIONS

In summary, we expect that the helicity-0 excitation for the spin-2 perturbations is a ghost for $Hr_c > 1$ in the + branch. In order to confirm this result, we construct the effective action for the helicity-0 excitations in the DGP model. It is convenient to take the metric as

$$ds^2 = (1 + 2A_{yy})dy^2 + 2N(y)A_y dt dy + N(y)^2(-1 + 2A)dt^2 + a^2(1 + 2\mathcal{R})\delta_{ij}dx^i dx^j,$$

where $a(t) = \exp(Ht)$. In the absence of bulk matter, perturbations are solved using a ‘‘master variable,’’ Ω [19]. In the special case of a de Sitter brane in a Minkowski bulk, the metric variables are written via the master variable Ω as

$$\begin{aligned} A &= -\frac{1}{6a} \left(2\Omega'' - \frac{N'}{N} \Omega' + \frac{1}{N^2} \ddot{\Omega} \right), \\ A_y &= \frac{1}{aN} \left(\dot{\Omega}' - \frac{N'}{N} \dot{\Omega} \right), \\ A_{yy} &= \frac{1}{6a} \left(\Omega'' - 2\frac{N'}{N} \Omega' + \frac{2}{N^2} \ddot{\Omega} \right), \\ R &= \frac{1}{6a} \left(\Omega'' + \frac{N'}{N} \Omega' - \frac{1}{N^2} \ddot{\Omega} \right), \end{aligned} \quad (18)$$

where dot denotes a derivative with respect to t . In the bulk, the master equation for Ω can be solved by the separation of variables $\Omega = \int dm g_{mk}(t) f_m(y) e^{ikx}$ and the equation of motion in the bulk is given by

$$\begin{aligned} f_m'' - 2\frac{N'}{N} f_m' + \frac{m^2}{N^2} f_m &= 0, \\ \ddot{g}_{mk} - 3H\dot{g}_{mk} + \frac{k^2}{a^2} g_{mk} &= -m^2 g_{mk}. \end{aligned} \quad (19)$$

The junction conditions for Ω were derived in Ref. [20,21]. For the vacuum brane, the boundary condition for Ω is given by

$$\Omega' - H\Omega - \frac{r_c}{1 - 2Hr_c} (2H^2 - m^2)\Omega = 0. \quad (20)$$

We again find a tower of continuous massive modes starting from $m^2 = (9/4)H^2$, which are the KK modes for the spin-2 perturbations. In addition, there are two discrete modes. One is the mode with $m^2 = 2H^2$, which is the spin-0 perturbation and the other is the mode with $m^2 = m_d^2$, which is the helicity-0 excitation of the spin-2 perturbations. The 2nd order action for the master variable can be calculated by extending the result of Ref. [22]. Then we can construct the 4D effective action for the discrete modes with mass m_i^2 by integrating out with respect to y as

$$S_i = \frac{k^4 \mathcal{N}_{m_i^2}}{6H\kappa^2} \int d^4x a^3 \psi_{mk} (\square_4 - m_i^2) \psi_{mk}, \quad (21)$$

where

$$\mathcal{N}_{m_i^2} = \frac{Hr_c}{1 - 2Hr_c} + \frac{1}{2\sqrt{\frac{9}{4} - \frac{m_i^2}{H^2}}}, \quad (22)$$

and $\psi_{mk} = a^{-3} g_{mk}$. We should note that there is no mixing between two discrete modes for $Hr_c \neq 1$. For the spin-0 perturbation with $m^2 = 2H^2$, $\mathcal{N}_{m_i^2}$ becomes

$$\mathcal{N}_{2H^2} = \frac{1 - Hr_c}{1 - 2Hr_c}. \quad (23)$$

\mathcal{N}_{2H^2} is positive for $H > 1/r_c$, so it is not a ghost. For the spin-2 perturbation with $m^2 = m_d^2$, $\mathcal{N}_{m_d^2}$ becomes

$$\mathcal{N}_{m_d^2} = -\left(\frac{1 - Hr_c}{1 - 2Hr_c} \right) \left(\frac{Hr_c}{3Hr_c - 2} \right). \quad (24)$$

$\mathcal{N}_{m_d^2}$ is negative for $H > 1/r_c$, which confirms that the spin-2 perturbations contain a ghost.

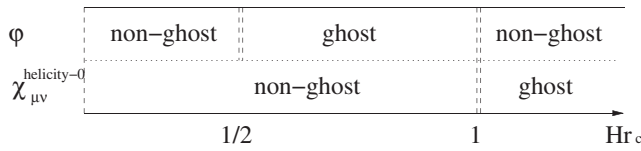


FIG. 1. Summary of the existence of the scalar ghost for various Hr_c .

V. SELF-ACCELERATING UNIVERSE

In a self-accelerating universe $Hr_c = 1$, the spin-2 graviton has mass $m^2 = 2H^2$, which coincides with the mass of the brane fluctuation mode. For $Hr_c = 1$, there is no longer a solution of the form Eq. (12), but there is a solution of the form $h_{\mu\nu} = A_{\mu\nu}(x) + B_{\mu\nu}(x) \log N(y)$ where $A_{\mu\nu}$ and $B_{\mu\nu}$ are determined by φ [23]. Then there is a mixing between the spin-0 mode and the helicity-0 excitation of the spin-2 graviton, which can give a ghost [23,24]. The derivation of the effective action is more subtle in this case and a careful calculation of the effective action is required.

However, we can argue the existence of the ghost in the following way [24]. From the effective action for the brane bending mode, it is clear that the brane bending mode becomes a ghost for $1/2 < Hr_c < 1$ Eq. (23). On the other hand, for $Hr_c > 1$, the spin-2 graviton becomes a ghost (see Fig. 1). Thus, by continuity, it is likely to have a ghost for $Hr_c = 1$, in this case, from a mixing between the brane bending mode and the spin-2 graviton. It should be mentioned that this is consistent with the result obtained by the boundary effective action in Refs. [7,8], where a scalar mode is found to be a ghost if $Hr_c > 1/2$.

VI. SUMMARY

In this paper, we studied the spectrum of gravitational perturbations in the DGP model. In the self-accelerating

branch, we showed that the spin-2 graviton has a discrete mode with mass in the range $0 < m^2 \leq 2H^2$ for $Hr_c > 1$ where $m^2 = 2H^2$ for the self-accelerating universe $Hr_c = 1$ and $m^2 \rightarrow 0$ for $Hr_c \rightarrow \infty$. Then the spin-2 graviton acquires a helicity-0 excitation that is a ghost for $Hr_c > 1$, which confirms earlier results [7,8]. In addition, there is a normalisable brane fluctuation mode with mass $m^2 = 2H^2$ that is not a ghost.

The self-accelerating universe is special in the sense that the spin-2 graviton has a mass $m^2 = 2H^2$, which is known to be a special case in 4D PF massive gravity [17]. In the DGP model, there is a brane fluctuation mode with the same mass, which can have a mixing with the spin-2 graviton. We argued that this mixing presumably gives a ghost in the self-accelerating universe. Recently, a new instability is found in the limit of vanishing cosmological constant in the analysis of the gravitational shock wave fields generated by a source on the brane [25]. This may be a signal of the appearance of the physical ghost in this limit. In order to verify the existence of the ghost in this special case rigorously, a more careful calculation of the effective action is required and this will be presented in a future publication [26].

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