

Heavy element production in inhomogeneous big bang nucleosynthesisShunji Matsuura,^{1,*} Shin-ichirou Fujimoto,² Sunao Nishimura,³ Masa-aki Hashimoto,³ and Katsuhiko Sato^{1,4}¹*Department of Physics, School of Science, University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan*²*Department of Electronic Control, Kumamoto National College of Technology, Kumamoto 861-1102, Japan*³*Department of Physics, School of Sciences, Kyushu University, Fukuoka 810-8560, Japan*⁴*Research Center for the Early Universe, University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan*

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We present a new astrophysical site of the big bang nucleosynthesis (BBN) that are very peculiar compared with the standard BBN. Some models of the baryogenesis suggest that very high baryon density regions were formed in the early universe. On the other hand, recent observations suggest that heavy elements already exist in high red-shifts and the origin of these elements become a big puzzle. Motivated by these, we investigate BBN in very high baryon density regions. BBN proceeds in proton-rich environment, which is known to be like the p-process. However, by taking very heavy nuclei into account, we find that BBN proceeds through both the p-process and the r-process simultaneously. P-nuclei such as ⁹²Mo, ⁹⁴Mo, ⁹⁶Ru, ⁹⁸Ru whose origin is not well known are also synthesized.

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I. INTRODUCTION

What happened in the early universe has a great influence on the history of the universe because they determined the initial conditions. It is very important to check whether our standard model of cosmology is correct or not as theories and observations develop. Baryogenesis and BBN should be checked because they determine the history of the chemical evolution.

In the standard model of elementary particle physics, the baryogenesis is possible only through the electro-weak sphaleron process. In the supersymmetric standard model, it is much easier to explain the baryon number asymmetry because there are many scalar fields which have baryon number. One of the most striking properties of supersymmetric theories is that they have flat directions in potentials. Some of them have baryon number and if fields condensate in these directions, it is possible to produce large baryon number. This is the basic idea of the Affleck-Dine baryogenesis [1]. Usually baryon number production is assumed to take place homogeneously all over the space. This is natural because we know that the universe is homogeneous and if baryogenesis is inhomogeneous in large scale it contradicts observations [2]. Of course resolution ability of observations is limited and small scale inhomogeneity is not excluded by observations. Though it seems to be unnatural to consider such small scale inhomogeneity, recent observations force us to reconsider the possibility of inhomogeneous baryogenesis.

It has become clear that the evolution of matter started earlier than we have known before. For example, Wilkinson Microwave Anisotropy Probe (WMAP) data suggests that reionization began when $z \sim 20$ [3]. According to Refs. [4,5], star formation activity started when $z \geq 10$. In addition, it is known that the quasar

metallicity did not significantly change from the time of high redshift to the present time [6]. Recently a galaxy at $z = 10.0$ was observed [7]. Other evidences of heavy elements from the high redshifts are given in [8–13]

Motivated by these observational evidences, we investigate the possibility that inhomogeneous baryogenesis produced very high baryon density in small fraction of the universe and in these regions some fraction of heavy elements were already synthesized during BBN.

Heavy elements production during BBN itself is not a new idea. Previous researches on the inhomogeneous big bang nucleosynthesis are given in [14,15]. Heavy elements production is also mentioned in [16]. These works, however, do not include very heavy elements [14] or they create neutron-rich regions and calculate the nucleosynthesis in those regions [15]. These are not suitable for our present purpose.

For the production of heavy elements during BBN, high baryon density is necessary. However if we simply increase the baryon-photon ratio all over the universe homogeneously, it would apparently contradict the observed light element abundances [17] and CMBR [2]. Instead, we assume that the baryon density of the universe is inhomogeneous before and during BBN. In most part of the universe η is small ($\eta \sim 6 \times 10^{-10}$) as observed while small fraction of the universe is occupied with very high baryon density, $\eta \sim \mathcal{O}(1)$. Because our aim is to see how BBN goes in the high baryon density regions and not make the precise adjustment between BBN and CMBR, we neglect the baryon diffusion. In this case, the baryon density in high density regions can be treated almost free parameter without contradicting observations. (It is a complicated problem whether we can treat η as a free parameter in realistic models. See, for example, [18].)

In Sec. II we explain the theoretical aspects of our model [19]. In Sec. III, we explain our network and what kind of effects we take into account. Section IV is the main results

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of our numerical study. In this section we explain BBN as the p-process like and simultaneously the r-process like. And also BBN can produce very heavy elements including proton-rich nuclei such as ^{92}Mo , ^{94}Mo , ^{96}Ru , and ^{98}Ru .

II. THEORETICAL BACKGROUND

Theoretical background of this model is inhomogeneous baryogenesis [19]. We are going to explain basic aspects of this model. For more detail, see [19,20].

The basic idea of the model [19] is a modified version of the Affleck-Dine baryogenesis [1]. Assume that the interaction Lagrangian has the general renormalizable form

$$\begin{aligned}\mathcal{L}_{\text{int}} &= \lambda|\phi|^2\Phi^2 + g|\phi|^2\Phi \\ &= \lambda(\Phi - \Phi_1)^2|\phi|^2 - \lambda\Phi_1^2|\phi|^2,\end{aligned}\quad (1)$$

where ϕ is Affleck-Dine (AD) field, Φ is the inflaton field, g and λ are the coupling constants and $\Phi_1 = -g/2\lambda$.

In a simplest case, the effective mass of the AD field ϕ can be written by

$$(m_{\text{eff}}^\phi)^2 = m_0^2 + \lambda(\Phi - \Phi_1)^2,\quad (2)$$

where m_0^2 is the vacuum mass of ϕ .

The vacuum expectation value of Φ is assumed to evolve from very large value, i.e., $\Phi \geq \Phi_1$, decreases to zero. As Φ goes down to $\sim\Phi_1$, the effective mass square of ϕ becomes negative and the phase transition takes place. When Φ is far from Φ_1 the mass square is positive. If the duration of $\Phi \sim \Phi_1$ is short, the transition would take place only in a small fraction of space. Consequently in the dominant part of the universe baryon asymmetry is small as observed $\eta = \mathcal{O}(10^{-9})$, while in a small part of the universe the baryon asymmetry can be very large, even close to unity. In this simple model, the signature of barionic charge is not fixed. Baryonic charge can become both positive and negative [21]. However, the high density regions are very small compared to cosmological scale, high density antimatter regions would disappear by pair-annihilation while late time inflation can prevent the annihilation [22].

Because we are not very interested in the detail of the shape of the bubbles and the effect of diffusion in this paper, we assume that the bubble sizes are large enough to neglect the diffusion effects. Also, bubbles are not large so as to contradict the observations [2].

In this case, the BBN calculation can be treated as that of homogeneous big bang nucleosynthesis. In the following section, we present the results of the calculations and their physical interpretations.

At first sight, in the high baryon density regions the reaction seems to proceed along the proton-rich side because BBN occurs in proton-rich environment [20]. However, surprisingly it is not correct. BBN proceeds along the *proton-rich* and the *neutron-rich* side.

III. NUMERICAL CALCULATIONS

The basic method of our calculation is the same as that of the homogeneous big bang nucleosynthesis.

We solve the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}\quad (3)$$

where $\rho = \rho_\gamma + (\rho_{e^-} + \rho_{e^+}) + \rho_\nu + \rho_b$, and a is the scale factor. The energy conservation law is

$$\frac{d}{dt}(\rho a^3) + \frac{p}{c^2} \frac{d}{dt}(a^3) = 0\quad (4)$$

for the time evolution of the temperature and the baryon density.

Abundance change in the region is evaluated with a nuclear reaction network, which includes 4463 nuclei from neutron, proton to Americium ($Z = 95$, $A = 292$). Nuclear data, such as reaction rates, nuclear masses, and partition functions, are same as in [23]. It should be emphasized that both proton-rich and neutron-rich nuclei are produced in a high density region (Fig. 1). Therefore it is required using a large network to calculate abundances in the high η region.

IV. RESULTS

We have calculated BBN for various values of η , from 10^{-10} to 10^{-2} . It is known that in the standard, low baryon density BBN, nuclei heavier than Boron are hardly synthesized. Figure 1 represents the synthesized nuclei for $\eta = 1 \times 10^{-6}$ at the epoch $T = 1 \times 10^7\text{K}$. We can see that heavier nuclei such as Ca (10^{-14} in mass fraction) are synthesized.

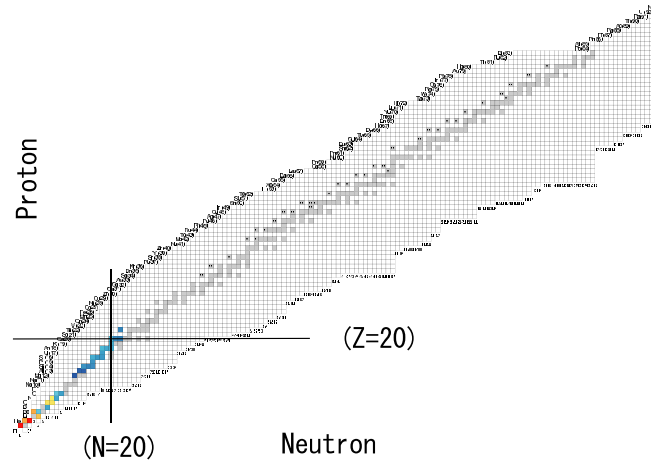


FIG. 1 (color online). Abundance distribution at $T = 1 \times 10^7\text{K}$ ($\eta = 10^{-6}$) is plotted on the nuclear chart. The gray color regions represent stable nuclei. The dark color represents more synthesized nuclei and bright color represents less synthesized nuclei. Those nuclei that are not synthesized in the standard BBN scenario, such as Ca are synthesized.

Naturally as η becomes large, heavier nuclei are synthesized. However we find there is drastic change in the nucleosynthesis around $\eta \sim 3 \times 10^{-4}$. To see this transition, we pick up two values of $\eta = 10^{-4}$ and 10^{-3} , and investigate what is going on during the nucleosynthesis.

Figure 2 represents how many nuclei are synthesized at the temperature $T = 3 \times 10^9$ K, and $\eta = 10^{-4}$.

Dark (bright) color represents more (less) synthesized nuclei. (Stable nuclei also plotted in Figs. 2, 3, and 5, with gray color.) We can see that the reaction goes along stable line. It is well known that nuclei whose neutron and proton numbers are special values (for example 20, 28, 50, 82 etc.), the magic numbers, are especially stable. At these points, the reactions are stagnated and the reaction paths are bent. Especially, the stagnation at $N = 82$ is one of the biggest factors that prevent the reaction to proceed further beyond the mass number 190. As the temperature goes down, the locus of the reaction begins to bent to different directions (Fig. 3).

For lighter nuclei (mass number $A \leq 100$), proton captures are very active and the locus moves to proton-rich direction. For nuclei whose mass numbers are between 100 and 120, the locus is across the stable nuclei from proton-rich side to neutron-rich side. For heavier nuclei for $A \geq 120$, neutron capture is more efficient. This suggests that both the r-process and p-process occur simultaneously in BBN.

Physical interpretation of this situation is as follows. The environment in BBN is proton rich, i.e., the electron fraction Y_e ranges from 0.8 to 0.9 [20]. Naive expectation of BBN is the p-process. For relatively light heavy nuclei, proton capture is active. However proton capture processes become exponentially difficult as the proton number increases because of their coulomb barrier. On the other hand, neutrons are still not consumed out during heavy

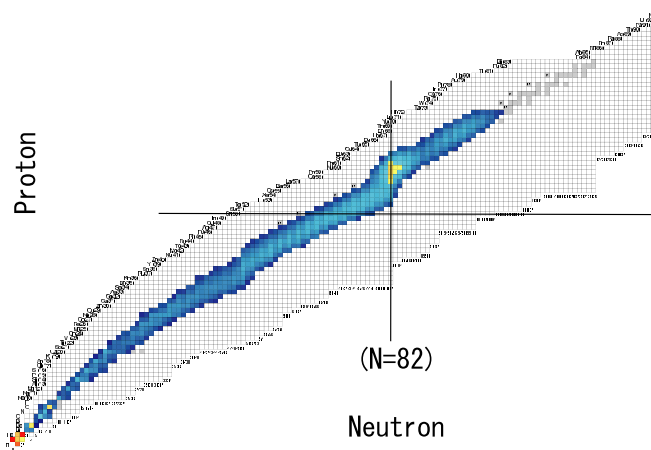


FIG. 2 (color online). Abundance distribution at $T = 3 \times 10^9$ K ($\eta = 10^{-4}$) are plotted on the nuclear chart. The dark color represents more synthesized nuclei. Produced nuclei distribute along the stable nuclei.

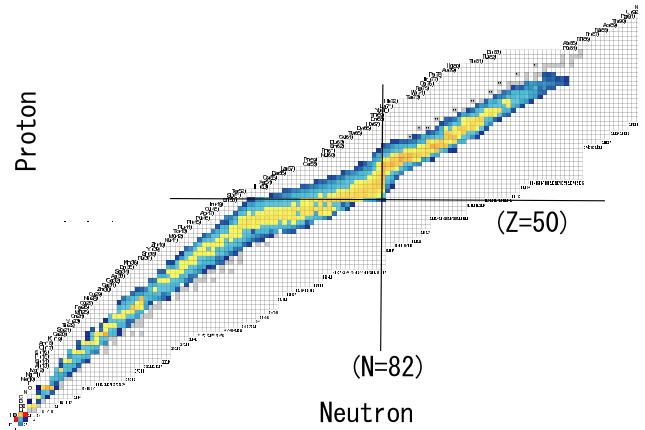


FIG. 3 (color online). Abundance distribution at $T = 1 \times 10^9$ K ($\eta = 10^{-4}$). For nuclei with $A \leq 100$ the synthesis is the p-process while for heavier nuclei the synthesis is the r-process.

nuclei are synthesized as shown in Fig. 4. Very heavy nuclei captures neutrons and the locus of the reaction changes toward the neutron-rich side. Transition point from proton-rich side to neutron side depends on the baryon-photon ratio η . The transition occurs at larger mass number for larger η . The reactions depend on the abundances of the seed nuclei. The higher baryon density follows many seeds which lead to proton captures on heavier nuclei. Figure 4 shows the time evolution of mass fraction for nuclei whose mass number is 90 and 158. Let us see the time evolution of the mass number 90. ^{90}Zr is a stable nucleus and ^{90}Mo is a proton-rich one. First, ^{90}Zr is synthesized and later ^{90}Mo is synthesized while the

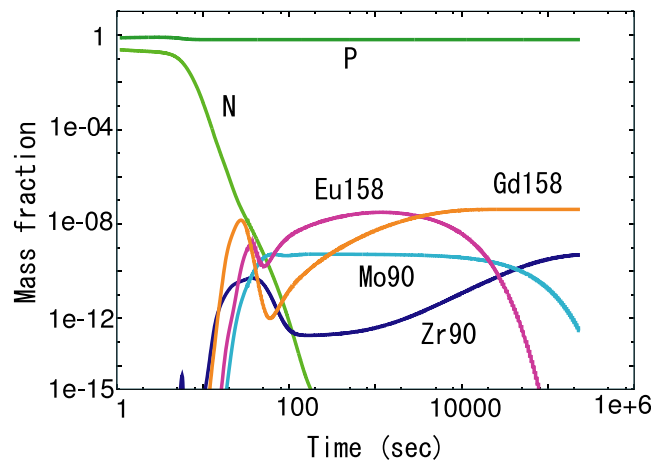


FIG. 4 (color online). Time evolution of mass fraction ($\eta = 1 \times 10^{-4}$). Neutrons are still left when heavy elements are synthesized. For nuclei with mass number 90, stable ones are synthesized first followed by proton-rich nuclei. On the other hand, for nuclei with mass number 158 stable nuclei are synthesized first followed by r-rich nuclei.

amount of ^{90}Zr decreases. This represents that the reaction first proceeds along the stable line and later move to the proton-rich side. At the late stage, ^{90}Zr increases while ^{90}Mo decreases because unstable proton-rich nuclei decay to stable nuclei such as $^{90}\text{Mo} \rightarrow ^{90}\text{Nb} \rightarrow ^{90}\text{Zr}$. The lifetimes of ^{90}Nb and ^{90}Mo are $14.6 \text{ h} = 5.33 \times 10^4 \text{ sec}$ and $5.67 \text{ h} = 2.04 \times 10^4 \text{ sec}$, respectively.

For heavier nuclei of the mass number 158, the situation is different. ^{158}Gd is a stable nucleus and ^{158}Eu is a neutron-rich nucleus, instead of proton-rich one. First the stable nucleus ^{158}Gd is synthesized and later neutron-rich ^{158}Eu is synthesized. This shows that in heavier nuclei region, the stable nuclei are produced first as ^{90}Zr , but later the neutron-rich nuclei are produced instead of proton-rich nuclei. The decrease in ^{158}Eu in the late stage is the same as ^{90}Mo , β decay to stable nuclei. The abundances of neutron-rich nuclei of the mass number 90 and those of proton-rich nuclei of the mass number 158 are very small and not drawn in this figure. We can also see that neutrons are still left when heavy nuclei are synthesized.

Now let us see the case $\eta = 10^{-3}$. Figure 5 shows the locus of the reaction at the temperature $T = 1.8 \times 10^9 \text{ K}$. It is apparently different from the results of $\eta = 10^{-4}$. For, in this case, the reactions first proceeds along the stable line. However, the reactions directly proceeds to the proton-rich region. Another important difference is that very heavy nuclei of $A \geq 80$ are not synthesized. The physical interpretation is as follows. In a high baryon density region, the seeds for the reactions to proceed are abundant. The nuclear reaction proceeds promptly and all neutrons are consumed by light nuclei as shown in Fig. 6. This prevents the nucleosynthesis from proceeding to the large mass number region. In Fig. 6 we only draw the abundance having $A = 90$. Heavier nuclei are not synthesized enough. When heavy nuclei are synthesized, neutrons are almost consumed out.

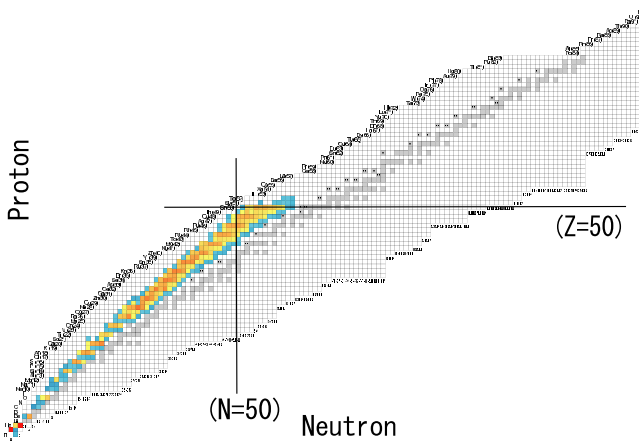


FIG. 5 (color online). Abundance distribution at $T = 1.8 \times 10^9 \text{ K} (\eta = 10^{-3})$. Nucleosynthesis occurs through the p-process and very heavy nuclei are not synthesized.

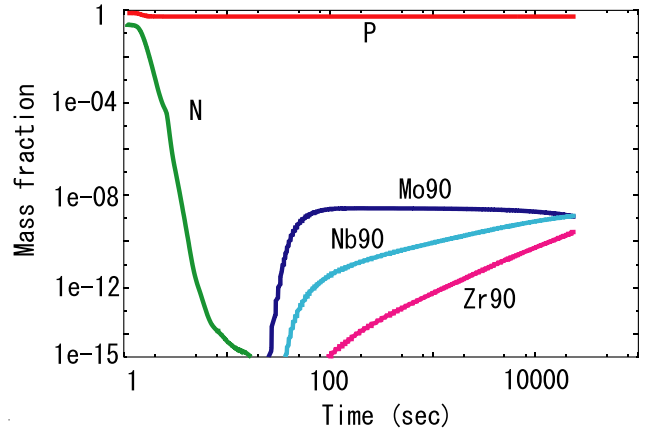


FIG. 6 (color online). Time evolution of mass fraction ($\eta = 1 \times 10^{-3}$). Neutrons have already been consumed when the abundances of heavy elements are increasing.

In Fig. 7, we show the relation between the mass number and the number fraction relative to the solar abundances. As the baryon density becomes larger, the heavier nuclei are synthesized for η less than 1×10^{-4} . However, when $\eta \geq 1 \times 10^{-4}$, the maximum mass number decreases as η becomes larger.

The number fraction ratios of p-nuclei to the solar abundances is listed in Table I.

For $\eta = 10^{-3}$, ^{92}Mo , ^{94}Mo , ^{96}Ru and ^{98}Ru drastically increase, due to the change of the loci of the reaction flows. This suggests that highly inhomogeneous BBN would have a large influence on the abundances of solar p-nuclei.

The observed abundances seem not to be explained by BBN with only a single η . However this does not exclude the possibility of our assumption. It is unnecessary for BBN abundances to match exactly to the solar abundances because produced nuclei in high η regions would have mixed with nuclei synthesized in low density regions and also there should be nuclei synthesized in star activities.

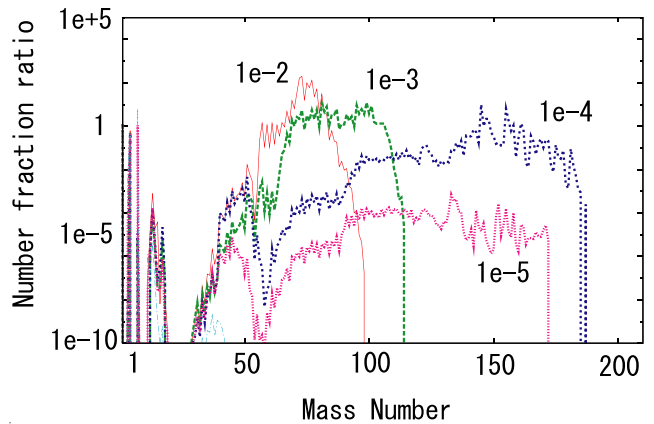


FIG. 7 (color online). The number fraction relative to the solar abundances. As η becomes larger, the maximum A becomes larger for $\eta \leq 1 \times 10^{-4}$, while for $\eta \geq 1 \times 10^{-4}$, the maximum A becomes smaller.

TABLE I. The number fraction of p-nuclei relative to the solar abundances.

$\eta =$	10^{-4}	10^{-3}	10^{-2}
^{92}Mo	1.0×10^{-2}	1.1×10	1.1×10^{-2}
^{94}Mo	4.2×10^{-2}	0.9×10	9.5×10^{-4}
^{96}Ru	9.6×10^{-2}	3.1×10	8.1×10^{-5}
^{98}Ru	3.1×10^{-1}	6.6×10	3.5×10^{-6}

Basic feature of BBN at each η are classified as follows. For $\eta \leq 10^{-6}$, synthesized nuclei are limited to $A \leq 40$. Nuclei whose A are around 20 are less synthesized even at large value of η . For η from 10^{-5} to 10^{-4} , the abundances of nuclei whose mass number of $30 \leq A \leq 56$ grow rapidly with A . Abundances of nuclei $A \geq 56$ suddenly decrease but again slowly increase. At around $A = 140$, they turn to decrease.

After the rapid decrease in the abundances of $A \geq 56$ for $\eta = 10^{-3}$ and 10^{-2} , the abundance profiles are rather different.

For $\eta = 10^{-3}$, the abundances do not drastically change from $A = 64$ to around 86. They rapidly decrease for A above 100 and maximum A synthesized is 114. For $\eta = 10^{-2}$, right side wing of Fe peak is similar to the solar abundances. There is a peak around $A = 72$ and the abundance production decreases rapidly above beyond the peak until $A \sim 98$. The maximum A synthesized is 98.

To compare our results with observations such as metal poor star abundances, we need to take into account dynamical mixing after the epoch of BBN. This depends on a model significantly and will be a future work.

We should examine the idea presented in this paper with more realistic model, and determine whether heavy elements were really synthesized in BBN or not. The former is to take into account the diffusion effects before and during BBN and also lepton asymmetry. The latter is to calculate the nucleosynthesis in supermassive stars. This is because supermassive stars are generally thought to have synthesized first heavy elements in the universe. We need to know whether heavy elements observed in high redshift were

synthesized in BBN or supermassive stars. Nucleosynthesis in supermassive stars and BBN in high baryon density region is similar. It would be a problem how to distinguish these two nucleosynthesis from observations.

V. CONCLUSION

We have investigated BBN in high baryon density region. In these regions, not only light elements which are synthesized in standard BBN but also very heavy elements are produced. We found BBN is both the p-process like and the r-process like. The transition from the p-process to the r-process is due to the Coulomb barriers of proton-rich nuclei and the amounts of neutrons when heavy elements begin to be synthesized. The loci of the reaction flows change drastically above $\eta = 10^{-3}$. Above $\eta = 10^{-3}$, a lot of seed nuclei cause active proton capture and the reaction flows end before very heavy elements are synthesized.

Our calculations demonstrate that very heavy elements can be synthesized in BBN, including proton-rich nuclei. These nuclei will be related to the origin of the solar abundances, heavy elements observed in high redshifts and early star formations via cooling effects.

For more realistic models in BBN, we need to include diffusion effects. Comparison with the nucleosynthesis in supermassive stars is also important. We leave these issues for future study.

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