

**Fermions as sources of accelerated regimes in cosmology**

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In this work it is investigated if fermionic sources could be responsible for accelerated periods during the evolution of a universe where a matter field would answer for the decelerated period. The self-interaction potential of the fermionic field is considered as a function of the scalar and pseudoscalar invariants. Irreversible processes of energy transfer between the matter and gravitational fields are also considered. It is shown that the fermionic field could behave like an inflaton field in the early universe and as dark energy for an old universe.

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**I. INTRODUCTION**

The search for constituents responsible for accelerated periods in the evolution of the universe is a fundamental topic in cosmology. Usually the formulations include elements of general relativity, field theory, and thermodynamics, putting under analysis the evolution of space-time variables like the scale factor, its acceleration, and energy densities. Several candidates have been tested for describing both the inflationary period and the present accelerated era: scalar fields, exotic equations of state, and cosmological constants.

Another possibility is to consider fermionic fields as gravitational sources for an expanding universe. Fermionic sources have been investigated using several approaches, with results including exact solutions, anisotropy-to-isotropy scenarios, and cyclic cosmologies (see, for example, [1–3]).

In the present work the connection between general relativity and the Dirac equation is done via the tetrad formalism, where the components of the tetrad or “vierbein” play the role of the gravitational degrees of freedom. The interactions between the constituents are modeled through the presence of a nonequilibrium pressure term in the source’s energy-momentum tensor. Besides, it is considered a self-interaction term for the fermionic constituent, in the form of a potential that can assume several forms (including the Nambu-Jona-Lasinio case [4]).

While testing fermionic sources as responsible for accelerated periods, different regimes are possible. In a young universe scenario, the fermion produces a fast expansion where matter (included via a barotropic equation of state) is created until it starts to predominate and the initial accelerated expansion is followed by a decelerated

era, dominated by a matter field, which ends when the fermionic field predominates again leading to an accelerated era. In this case, the fermionic field plays the role of the inflaton in the early period of the universe and of dark energy for the old universe, without the need for a cosmological constant term or a scalar field. In an old universe scenario an initially matter dominated period gradually turns into a dark (fermionic) energy period when an accelerated regime starts and remains for the rest of the evolution of the system.

This manuscript is structured as follows. In Sec. II we make a brief review of the elements of the tetrad formalism used to include fermionic and matter fields in a dynamical curved space-time. The field equations for an isotropic, homogeneous, and spatially flat universe are derived in Sec. III. In Sec. IV we present the analysis of the different scenarios in which the fermionic constituent answers for accelerated eras and the transitions from accelerated to decelerated periods and vice-versa. Finally, in Sec. V we display our conclusions. The metric signature used is  $(+, -, -, -)$  and units have been chosen so that  $8\pi G = c = \hbar = k = 1$ .

**II. DIRAC AND EINSTEIN EQUATIONS**

In this section we present briefly the techniques that are used to include fermionic sources in the Einstein theory of gravitation, and for a more detailed analysis the reader is referred to [5–8]. The starting point is that the gauge group of general relativity does not admit a spinor representation and the tetrad formalism is invoked to solve the problem. Following the general covariance principle, a connection between the tetrad and the metric tensor  $g_{\mu\nu}$  is established through the relation

$$g_{\mu\gamma} = e^a_\mu e^b_\gamma \eta_{ab}, \quad a = 0, 1, 2, 3, \quad (1)$$

where  $e^a_\mu$  denotes the tetrad or vierbein and  $\eta_{ab}$  is the Minkowski metric tensor. Here Latin indices refer to the

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local inertial frame, whereas Greek indices refer to the general system.

As it was said above, the main objective of this work is to describe the behavior of fermions in the presence of a gravitational field, and the next step is to construct an action for this system. The Dirac Lagrangian density in Minkowski space-time is

$$L_D = \frac{i}{2} [\bar{\psi} \gamma^a \partial_a \psi - (\partial_a \bar{\psi}) \gamma^a \psi] - m \bar{\psi} \psi - V, \quad (2)$$

where the spinors are treated as classically commuting fields [6]. Above,  $m$  is the fermionic mass,  $\bar{\psi} = \psi^\dagger \gamma^0$  denotes the adjoint spinor field, and the term  $V$ , which is a function of  $\psi$  and  $\bar{\psi}$ , describes the potential density of self-interaction between fermions. The general covariance principle imposes that the Dirac-Pauli matrices  $\gamma^a$  must be replaced by their generalized counterparts  $\Gamma^\mu = e_a^\mu \gamma^a$ , whereas the generalized Dirac-Pauli matrices satisfy the extended Clifford algebra, i.e.,  $\{\Gamma^\mu, \Gamma^\nu\} = 2g^{\mu\nu}$ .

In a second step we need to substitute the ordinary derivatives by their covariant versions

$$\begin{aligned} \partial_\mu \psi &\rightarrow D_\mu \psi = \partial_\mu \psi - \Omega_\mu \psi, \\ \partial_\mu \bar{\psi} &\rightarrow D_\mu \bar{\psi} = \partial_\mu \bar{\psi} + \bar{\psi} \Omega_\mu, \end{aligned} \quad (3)$$

where the spin connection  $\Omega_\mu$  is given by

$$\Omega_\mu = -\frac{i}{4} g_{\mu\nu} [\Gamma_{\sigma\lambda}^\nu - e_b^\nu (\partial_\sigma e_\lambda^b)] \gamma^\sigma \gamma^\lambda, \quad (4)$$

with  $\Gamma_{\sigma\lambda}^\nu$  denoting the Christoffel symbols. Hence, the generally covariant Dirac Lagrangian becomes

$$L_D = \frac{i}{2} [\bar{\psi} \Gamma^\mu D_\mu \psi - (D_\mu \bar{\psi}) \Gamma^\mu \psi] - m \bar{\psi} \psi - V. \quad (5)$$

The field equations are obtained from the total action

$$S(g, \psi, \bar{\psi}) = \int \sqrt{-g} L_t d^4x, \quad (6)$$

where  $L_t = L_g + L_D + L_m$  is the total Lagrangian density.  $L_g = R/2$ , with  $R$  denoting the curvature scalar, is the gravitational Einstein Lagrangian density for fermions which are minimally coupled to the gravitational field.  $L_D$  is the Dirac Lagrangian density (5) and  $L_m$  is the Lagrangian density of the matter field.

From the Lagrangian density (5), through Euler-Lagrange equations, we obtain the Dirac equations for the spinor field and its adjoint coupled to the gravitational field,

$$\begin{aligned} i\Gamma^\mu D_\mu \psi - m\psi - \frac{dV}{d\psi} &= 0, \\ iD_\mu \bar{\psi} \Gamma^\mu + m\bar{\psi} + \frac{dV}{d\bar{\psi}} &= 0. \end{aligned} \quad (7)$$

The variation of the action (6) with respect to the tetrad leads to Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -T_{\mu\nu}, \quad (8)$$

where  $T_{\mu\nu}$  is the total energy-momentum tensor which is a sum of the energy-momentum tensor of the fermionic field  $T_f^{\mu\nu}$  and of the matter field  $T_m^{\mu\nu}$ , i.e.,  $T^{\mu\nu} = T_m^{\mu\nu} + T_f^{\mu\nu}$ . Furthermore, the symmetric form of the energy-momentum tensor of the fermionic field is given by

$$\begin{aligned} T_f^{\mu\nu} &= \frac{i}{4} [\bar{\psi} \Gamma^\mu D^\nu \psi + \bar{\psi} \Gamma^\nu D^\mu \psi - D^\nu \bar{\psi} \Gamma^\mu \psi - D^\mu \bar{\psi} \Gamma^\nu \psi] \\ &\quad - g^{\mu\nu} L_D. \end{aligned} \quad (9)$$

### III. FIELD EQUATIONS

The Robertson-Walker metric incorporates the homogeneity and isotropy hypotheses for the universe. Here we consider a spatially flat universe described by the metric

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2), \quad (10)$$

where  $a(t)$  refers to the cosmic scale factor.

The total energy-momentum tensor for an isotropic and homogeneous universe—which is composed of fermionic and matter fields and where dissipative processes are taken into account—is written as

$$(T_\nu^\mu) = \text{diag}(\rho, -p - \varpi, -p - \varpi, -p - \varpi). \quad (11)$$

Above, the total energy density  $\rho$  and the total pressure  $p$  are given as a sum of the corresponding terms of the fermionic and matter fields, i.e.,  $\rho = \rho_f + \rho_m$  and  $p = p_f + p_m$ . Moreover, the quantity  $\varpi$  refers to a nonequilibrium pressure which is related to dissipative processes during the evolution of the universe and represents an irreversible process of energy transfer between the matter and the gravitational field [9].

Thanks to the Bianchi identities, the covariant differentiation of Einstein field equations (8) leads to the conservation law of the total energy-momentum tensor  $T^{\mu\nu}{}_{;\nu} = 0$ , hence it follows by using the representation (11) that the evolution equation for the total energy density is

$$\dot{\rho} + 3H(\rho + p + \varpi) = 0, \quad (12)$$

where the dot refers to a differentiation with respect to time and  $H = \dot{a}(t)/a(t)$  denotes the Hubble parameter. Furthermore, from the Einstein field equations (8), the Friedman and acceleration equations are

$$H^2 = \frac{1}{3}\rho, \quad \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p + 3\varpi), \quad (13)$$

respectively. Only two equations from (12) and (13) are linearly independent.

For the metric (10) the tetrad components read

$$e_0^\mu = \delta_0^\mu, \quad e_i^\mu = \frac{1}{a(t)} \delta_i^\mu. \quad (14)$$

Also, the Dirac matrices become

$$\begin{aligned}\Gamma^0 &= \gamma^0, & \Gamma^i &= \frac{1}{a(t)} \gamma^i, \\ \Gamma^5 &= -i\sqrt{-g}\Gamma^0\Gamma^1\Gamma^2\Gamma^3 = \gamma^5,\end{aligned}\quad (15)$$

from which the spin connection components are obtained, yielding

$$\Omega_0 = 0, \quad \Omega_i = \frac{1}{2}\dot{a}(t)\gamma^i\gamma^0. \quad (16)$$

For an isotropic and homogeneous universe the fermionic field is an exclusive function of time. So the Dirac equations (7) become

$$\dot{\psi} + \frac{3}{2}H\psi + im\gamma^0\psi + i\gamma^0\frac{dV}{d\psi} = 0, \quad (17)$$

$$\dot{\bar{\psi}} + \frac{3}{2}H\bar{\psi} - im\bar{\psi}\gamma^0 - i\frac{dV}{d\bar{\psi}}\gamma^0 = 0, \quad (18)$$

thanks to Eqs. (15) and (16).

The nonvanishing components of the energy-momentum tensor of the fermionic field follow from (9) together with (5) and (15)–(18), yielding

$$(T_f)_0^0 = m(\bar{\psi}\psi) + V, \quad (19)$$

$$(T_f)^1_1 = (T_f)^2_2 = (T_f)^3_3 = V - \frac{dV}{d\psi} \frac{\psi}{2} - \frac{\bar{\psi}}{2} \frac{dV}{d\bar{\psi}}, \quad (20)$$

which are only functions of  $\psi$  and  $\bar{\psi}$ . By identifying the components of the energy-momentum tensor of the fermionic field as  $(T_f)^\mu_\nu = \text{diag}(\rho_f, -p_f, -p_f, -p_f)$ , one can obtain from (17) and (18) by the use of (19) and (20) the conservation law for the energy density of the fermionic field:

$$\dot{\rho}_f + 3H(\rho_f + p_f) = 0. \quad (21)$$

Hence, the evolution equation for the energy density of the fermionic field decouples from the energy density of the matter field, and we have from (12) and (21)

$$\dot{\rho}_m + 3H(\rho_m + p_m) = -3H\varpi, \quad (22)$$

where the term  $-3H\varpi$  could be interpreted as the energy density production rate of the matter field (see, e.g., [9]).

According to the extended (causal or second-order) thermodynamic theory, the nonequilibrium pressure  $\varpi(t)$  obeys an evolution equation, whose linearized form reads

$$\tau\dot{\varpi} + \varpi = -3\eta H, \quad (23)$$

where  $\tau$  denotes a characteristic time and  $\eta$  is the so-called bulk viscosity coefficient.

#### IV. COSMOLOGICAL SOLUTIONS

In order to analyze cosmological solutions for the field equations of the previous section, we have to specify the

potential density of self-interaction between the fermions  $V$ . According to the Pauli-Fierz theorem  $V$  is an exclusive function of the scalar invariant  $(\bar{\psi}\psi)^2$  and on the pseudo-scalar invariant  $(i\bar{\psi}\gamma^5\psi)^2$ , i.e.,  $V = V((\bar{\psi}\psi)^2, (i\bar{\psi}\gamma^5\psi)^2)$ . Here we are interested in analyzing self-interaction potentials of the form

$$V = \lambda[\beta_1(\bar{\psi}\psi)^2 + \beta_2(i\bar{\psi}\gamma^5\psi)^2]^n, \quad (24)$$

where the coupling constant  $\lambda$  is a non-negative quantity and  $n$  is a constant exponent. We shall analyze three cases, namely, (i)  $\beta_1 = 1$  and  $\beta_2 = 0$  where  $V$  is a function only of the scalar invariant; (ii)  $\beta_1 = 0$  and  $\beta_2 = 1$  where  $V$  depends only on the pseudoscalar invariant, and (iii)  $\beta_1 = \beta_2 = 1$  where  $V$  is a combination of the scalar and pseudoscalar invariants. The Nambu-Jona-Lasinio potential [4] is represented by the last case with  $n = 1$ .

The energy density and the pressure of the fermions for the potential (24) are given by

$$\rho_f = m(\bar{\psi}\psi) + \lambda[\beta_1(\bar{\psi}\psi)^2 + \beta_2(i\bar{\psi}\gamma^5\psi)^2]^n, \quad (25)$$

$$p_f = (2n - 1)\lambda[\beta_1(\bar{\psi}\psi)^2 + \beta_2(i\bar{\psi}\gamma^5\psi)^2]^n, \quad (26)$$

respectively, thanks to (20).

We infer from (26) that the fermions could be classified according to the value of the exponent  $n$ . Indeed, for  $n \geq 1/2$  the fermions represent a matter field with positive pressure ( $n > 1/2$ ) or a pressureless fluid ( $n = 1/2$ ), whereas for  $n < 1/2$  the pressure of the fermions is negative and they could represent either the inflaton or the dark energy.

For massless fermions, the pressure is connected with the energy density by a simple barotropic equation of state  $p_f = (2n - 1)\rho_f$  and it follows from the conservation equation (21) for the fermions that  $\rho_f \propto 1/a^{6n}$ . We shall not analyze this case here, since the behavior of the fermionic field does not differ from that of a matter field when  $n > 1/2$  or from that of a bosonic field when  $n < 1/2$ . Moreover, for the massive case, we shall deal with only the case where the fermionic field behaves as an inflaton or dark energy, i.e., the case where  $n < 1/2$ .

For the pressure of the matter field we shall adopt a barotropic equation of state, i.e.,  $p_m = w_m\rho_m$  with  $0 \leq w_m \leq 1$ . Furthermore, the coefficient of bulk viscosity  $\eta$  and the characteristic time  $\tau$  are assumed to be related with the energy density  $\rho$  by  $\eta = \alpha\rho$  and  $\tau = \eta/\rho$ , where  $\alpha$  is a constant [9].

The system of field equations we shall investigate in order to find the cosmological solutions are (i) the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho_f + \rho_m + 3p_f + 3p_m + 3\varpi); \quad (27)$$

(ii) the evolution equation for the energy density of the matter field,

$$\dot{\rho}_m + 3H(\rho_m + p_m + \varpi) = 0; \quad (28)$$

(iii) the evolution equation for the nonequilibrium pressure,

$$\tau \dot{\varpi} + \varpi = -3\eta H; \quad (29)$$

(iv) the Dirac equation (17), which in terms of the spinor components  $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ , becomes

$$\begin{pmatrix} \dot{\psi}_1 \\ \dot{\psi}_2 \\ \dot{\psi}_3 \\ \dot{\psi}_4 \end{pmatrix} + \frac{3}{2}H \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} + im \begin{pmatrix} \psi_1 \\ \psi_2 \\ -\psi_3 \\ -\psi_4 \end{pmatrix} - 2i(\psi_1^\dagger \psi_1 + \psi_2^\dagger \psi_2 - \psi_3^\dagger \psi_3 - \psi_4^\dagger \psi_4) \begin{pmatrix} \psi_1 \\ \psi_2 \\ -\psi_3 \\ -\psi_4 \end{pmatrix} V' - 2i(\psi_3^\dagger \psi_1 + \psi_4^\dagger \psi_2 - \psi_1^\dagger \psi_3 - \psi_2^\dagger \psi_4) \begin{pmatrix} \psi_3 \\ \psi_4 \\ -\psi_1 \\ -\psi_2 \end{pmatrix} V^* = 0. \quad (30)$$

In Eq. (16) we have introduced the following abbreviations:

$$V' = \frac{\partial V}{\partial(\bar{\psi}\psi)^2}, \quad V^* = \frac{\partial V}{\partial(\bar{\psi}\gamma^5\psi)^2}. \quad (31)$$

Equations (27)–(30) consist of a system of seven coupled ordinary differential equations for the fields  $a(t)$ ,  $\rho_m(t)$ ,  $\varpi(t)$ ,  $\psi_1(t)$ ,  $\psi_2(t)$ ,  $\psi_3(t)$ , and  $\psi_4(t)$ , and in the following subsections we shall find solutions for this system of equations for given initial conditions.

### A. Accelerated-decelerated-accelerated regime

Let us first analyze the case that corresponds to the evolution of the early universe, where, at the beginning, the fermionic field plays the role of an inflaton and the matter is created by an irreversible process through the presence of a nonequilibrium pressure  $\varpi$ . The initial conditions we have chosen for  $t = 0$  (by adjusting clocks) are

$$\begin{cases} a(0) = 1, & \psi_1(0) = 0.1i, & \psi_2(0) = 1, & \psi_3(0) = 0.3, \\ \psi_4(0) = i, & \rho_m(0) = 0, & \varpi(0) = 0. \end{cases} \quad (32)$$

The last two initial conditions correspond to a vanishing energy density of the matter field and a vanishing nonequilibrium pressure at  $t = 0$ . The conditions chosen here characterize qualitatively an initial proportion between the constituents in the corresponding era (i.e., in this case we have a predominating fermionic field over the matter, indicating the initial inflationary state).

Since Eq. (27) is a second-order differential equation we need to specify an initial condition for  $\dot{a}(0)$ . This condition follows from the Friedmann equation (13), i.e.,

$$\dot{a}(0) = a(0) \sqrt{\frac{\rho_f(0) + \rho_m(0)}{3}}, \quad (33)$$

with  $\rho_f(0)$  determined from Eq. (25). However, we have to

specify some parameters in order to obtain numerical solutions of the coupled system of differential equations (27)–(30). These parameters are (i)  $\lambda$ ,  $\beta_1$ ,  $\beta_2$ , and  $n$ , which define the self-interacting potential (24); (ii)  $m$ , which is related to the mass of the fermionic field; (iii)  $w_m$ , which defines the matter field through its barotropic equation of state  $p_m = w_m \rho_m$ , and (iv)  $\alpha$ , which is connected with the bulk viscosity term  $\eta = \alpha \rho$ . In order to plot Figs. 1 and 2 we have chosen the following values for these parameters:

$$\begin{cases} \lambda = 0.1, & \beta_1 = \beta_2 = 1, & n = 0.3, \\ m = 0.01, & w_m = 1/3, & \alpha = 1.0, \quad \text{and} \quad \alpha = 1.2, \end{cases} \quad (34)$$

which correspond to a fermionic field with a negative pressure, described by a self-interacting potential that depends on the scalar and pseudoscalar invariants and a matter field of massless particles that could describe a radiation field.

In Fig. 1 the acceleration field  $\ddot{a}$  is plotted as a function of time  $t$  for two different values of  $\alpha = 1.0$  and  $1.2$ ,

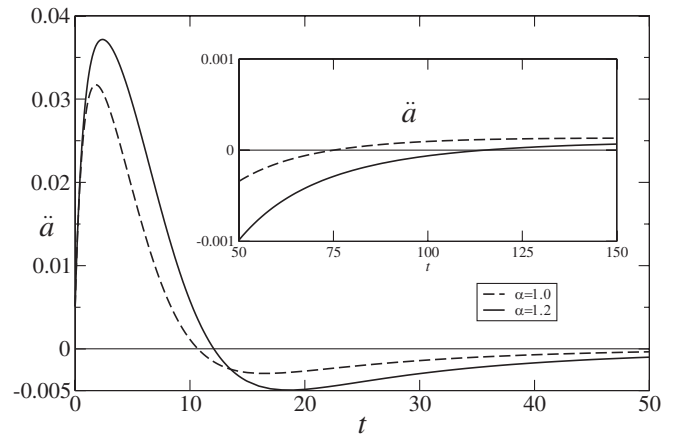


FIG. 1. Acceleration field  $\ddot{a}$  vs time  $t$ .

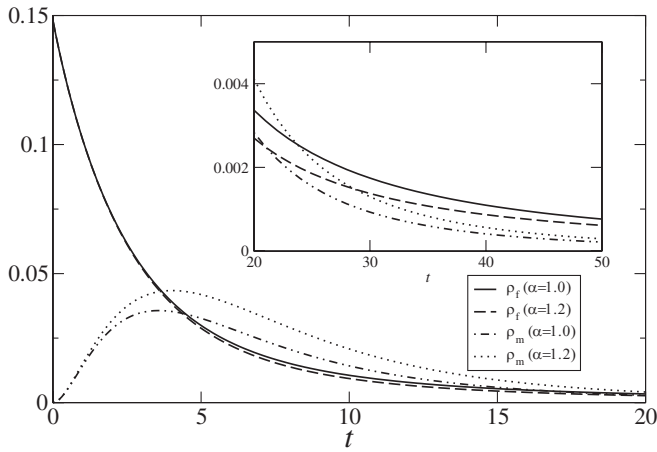


FIG. 2. Energy densities of fermionic  $\rho_f$  and matter  $\rho_m$  fields vs time  $t$ .

whereas in Fig. 2 the behavior of the energy densities of the fermionic  $\rho_f$  and matter  $\rho_m$  fields is shown as a function of time  $t$ . We infer from these figures that there exists an accelerated period where the fermionic field dominates and the matter field is created at the expense of an irreversible process of energy transfer between the matter and the gravitational field. The accelerated period is followed by a decelerated era which is dominated by the matter field. Because of the fact that the self-interaction potential tends to a constant value for large values of time, the energy density of the fermionic field overcomes the energy density of the matter field and the universe goes into another accelerated period. It is noteworthy that for the bosonic case where the potentials are exponentials or inverse power laws, one has to add a constant value—which is similar to introduce a cosmological constant—in order to return to an accelerated era after the accelerated-decelerated period (see, for example, [10]). Here the same self-potential interaction, which plays the role of an inflaton field at the beginning, plays the role of a cosmological constant for later times and could be identified as dark energy. We note from the figures that for large values of the coefficient  $\alpha$  it follows that (i) the energy density of the fermionic field decays more rapidly causing a larger accelerated period and (ii) the energy density of the matter field has a more significant growth and leads to a larger decelerated period.

The same conclusions above could be obtained for a self-interacting potential, which is only a function of the scalar invariant ( $\beta_1 = 1, \beta_2 = 0$ ) or of the scalar pseudoinvariant ( $\beta_1 = 0, \beta_2 = 1$ ). However, all cases are strongly dependent on the exponent  $n$  of the self-interacting potential and one can obtain different behaviors in which there exists only an accelerated period for the universe or the universe begins with a decelerated period. This last case will be analyzed in the next subsection.

It is worth mentioning that the behavior of the fields found in our analysis is not restricted to the initial con-

ditions given in (32) or the values for the parameters given in (34). In fact, these represent typical values that describe qualitatively the transitions under investigation in this case, i.e., the accelerated-decelerated case when matter emerges as the predominating constituent and a second situation where dark energy, represented by the fermionic field, overcomes the matter field and leads to a final accelerated period.

### B. Decelerated-accelerated regime

Let us now investigate an old universe dominated by nonrelativistic matter or dust ( $w_m = 0$  so that  $p_m = 0$ ) where a fermionic field is present but with an energy density smaller than that of the matter field. We shall consider the same initial conditions (by adjusting clocks) as those of the previous subsection for  $a(0)$  and  $\psi_1(0)$  through  $\psi_4(0)$ , but for the energy density of the matter field we regard it as twice that of the fermionic field, i.e.,  $\rho_m(0) = 2\rho_f(0)$ . With respect to the parameters, we shall choose the same values as above for  $\lambda, \beta_1, \beta_2, m,$  and  $n$ , but different values for  $\alpha$ , namely  $\alpha = 0.1$  and  $0.05$ . We have also considered the case where the irreversible processes during the evolution of the universe are absent. For this last case the evolution equation of the nonequilibrium pressure (29) was not taken into account. The acceleration and the energy densities as functions of time are plotted in Figs. 3 and 4, respectively. We observe from these figures that there exists a transition from a high deceleration—where the universe is dominated by the nonrelativistic matter field—to a small acceleration—where the universe is dominated by the fermionic field which plays the role of dark energy. By considering irreversible processes there is no sensitive change in the energy density of the fermionic field, but the matter field decays more slowly so that the accelerated period begins later.

The same conclusions of last subsection regarding the self-interaction potential are also valid for this case.

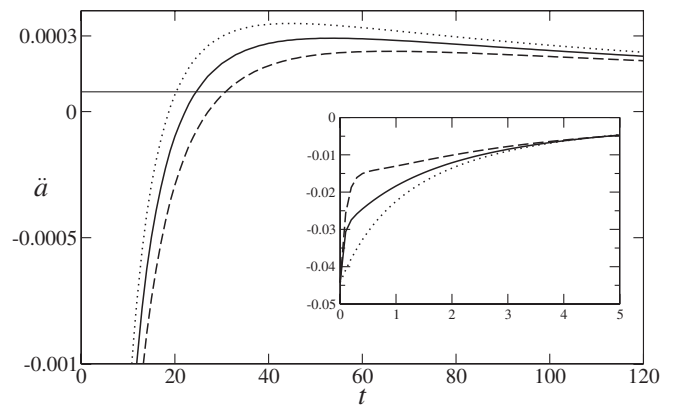


FIG. 3. Acceleration field  $\ddot{a}$  vs time  $t$ . Dashed line:  $\alpha = 0.1$ ; straight line:  $\alpha = 0.05$ ; dotted line: without irreversible processes.

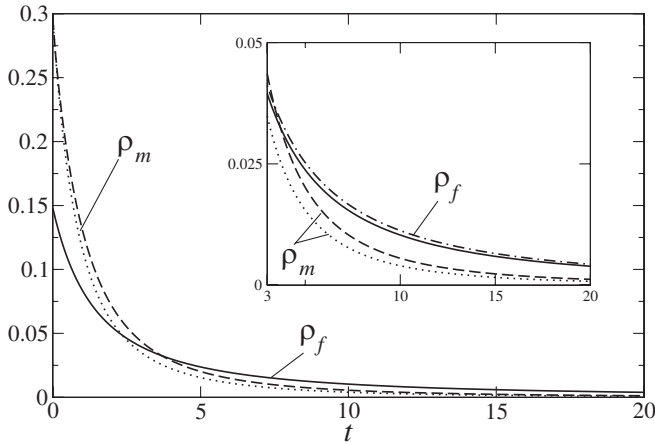


FIG. 4. Energy densities of fermionic  $\rho_f$  and matter  $\rho_m$  fields vs time  $t$ . Dotted and dotted-dashed lines: without irreversible processes; straight and dashed lines:  $\alpha = 0.1$ .

## V. FINAL REMARKS AND CONCLUSIONS

One question that could be formulated is about the importance of the irreversible processes during the evolution of the universe. To answer this question one should

discuss the relevance of the bulk viscosity coefficient in the different periods of the universe's evolution. In the above calculations a dimensionless coefficient has been used, but if one goes back to its dimensional expression, it is found that it has to be multiplied by a factor proportional to the initial value of the Hubble parameter. Since we know that the present value of that parameter is very small, this coefficient would be playing an important role only for early periods of the universe. On the other hand, for the late period, although its contribution is very small, the presence of the viscosity is necessary for describing the thermodynamic dissipative effects in an expanding universe.

We have investigated the possibility that a fermionic field—with a self-interacting potential that depends on the scalar and pseudoscalar invariants—could be the responsible for accelerated regimes in the evolution of the universe. We have shown that the fermionic field behaves like an inflaton field for the early universe and, later on, as a dark energy field, whereas the matter field was created by an irreversible process connected with a nonequilibrium pressure. Moreover, for an old decelerated universe dominated by nonrelativistic matter the fermionic field plays again the role of dark energy and drives the universe to an accelerated regime.

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