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Enhanced $K_L \to \pi^0 \nu\bar{\nu}$ from direct CP violation in $B \to K\pi$ with four generations

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Recent CP violation results in B decays suggest that Z penguins may have large weak phase. This can be realized by the four generation (standard) model. Concurrently, $B \to X_s \ell^+ \ell^-$ and B_s mixing allow for sizable $V_{t's}^* V_{t'b}$ only if it is nearly imaginary. Such large effects in $b \leftrightarrow s$ transitions would affect $s \leftrightarrow d$ transitions, as kaon constraints would demand $V_{t'd} \neq 0$. Using $\Gamma(Z \to b\bar{b})$ to bound $|V_{t'b}|$, we infer sizable $|V_{t's}| \lesssim |V_{t'b}| \lesssim |V_{us}|$. Imposing ε_K, $K^+ \to \pi^+ \nu\bar{\nu}$ and ε'/ε constraints, we find $V_{t'd}^* V_{t's} \sim$ few $\times 10^{-4}$ with large phase, enhancing $K_L \to \pi^0 \nu\bar{\nu}$ to 5×10^{-10} or even higher. Interestingly, Δm_{B_d} and $\sin 2\Phi_{B_d}$ are not much affected, as $|V_{t'd}^* V_{t'b}| \ll |V_{td}^* V_{tb}| \sim 0.01$.

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Just three years after CP violation (CPV) in the B system was established, direct CP violation (DCPV) was also observed in $B^0 \to K^+ \pi^-$ decay, $\mathcal{A}_{K^+\pi^-} \sim -0.12$. A puzzle emerged, however, that the charged $B^+ \to K^+ \pi^0$ mode gave no indication of DCPV, and is in fact a little positive, $\mathcal{A}_{K^+\pi^0} \gtrsim 0$. Currently, $\mathcal{A}_{K^+\pi^0} - \mathcal{A}_{K^+\pi^-} \simeq 0.16$, and differs from zero with 3.8σ significance [1].

The $B \to K^+ \pi^-$ amplitude $\mathcal{M}_{K^+\pi^-} \simeq P + T$ is dominated by the strong penguin (P) and tree (T) contributions, while the main difference $\sqrt{2}\mathcal{M}_{K^+\pi^0} - \mathcal{M}_{K^+\pi^-} \simeq P_{\rm EW} + C$ is from electroweak penguin (EWP, or $P_{\rm EW}$) and color-suppressed tree (C) contributions which are subdominant. Thus, $\mathcal{A}_{K^+\pi^0} \sim \mathcal{A}_{K^+\pi^-}$ was anticipated by all models. As data indicated otherwise, it has been stressed [2] that the C term could be much larger than previously thought, effectively cancelling against the CPV phase in T, leading to $\mathcal{A}_{K^+\pi^0} \to 0$. While this may well be realized, a very large C (especially if $\mathcal{A}_{K\pi^0} > 0$) would be a surprise in itself.

In a previous paper [3], we explored the possibility of new physics (NP) effects in $P_{\rm EW}$, in particular, in the 4 generation standard model (SM4, with SM3 for 3 generations). A sequential t' quark could affect $P_{\rm EW}$ most naturally for two reasons. On one hand, the associated Cabibbo-Kobayashi-Maskawa (CKM) matrix element product $V_{t's}^* V_{t'b}$ could be large and imaginary; on the other hand, it is well known that $P_{\rm EW}$ is sensitive to $m_{t'}^2$ in amplitude, and heavy t' does not decouple.

Using the PQCD factorization approach at leading order [4], which successfully predicted $\mathcal{A}_{K^+\pi^-} < -0.1$ (and C was not inordinately large), we showed that $\mathcal{A}_{K^+\pi^0} \gtrsim 0$ called for sizable $m_{t'} \gtrsim 300$ GeV and large, nearly imaginary $V_{t's}^* V_{t'b}$. As the $m_{t'}$ dependence is similar, we also showed that data on $B \to X_s \ell^+ \ell^-$ and B_s mixing concurred, in the sense that large t' effect is allowed *only if* $V_{t's}^* V_{t'b}$ is nearly imaginary. Applying the latter two constraints, however, $m_{t'}$ and $V_{t's}^* V_{t'b}$ become highly constrained. In the following, we will take [3]

$$m_{t'} \cong 300 \text{ GeV}, \qquad V_{t's}^* V_{t'b} \equiv r_{sb} e^{i\phi_{sb}} \simeq 0.025 e^{i70^\circ}, \tag{1}$$

as exemplary values for realizing $\mathcal{A}_{K^+\pi^0} - \mathcal{A}_{K^+\pi^-} \gtrsim 0.10$, without recourse to a large C contribution.

Comparing with $|V_{cs} V_{cb}| \simeq 0.04$, $r_{sb} \sim 0.025$ is quite sizable. In our $b \to s$ study, we had assumed [3] $V_{t'd} \to 0$ out of convenience, so as to decouple from $b \to d$ and $s \to d$ concerns. The main purpose of this note, however, is to show that, in view of the large r_{sb} and ϕ_{sb} values given in Eq. (1), $V_{t'd} = 0$ is untenable. The reason is as follows. Since a rather large impact on $V_{ts}^* V_{tb}$ is implied by Eq. (1), if one sets $V_{t'd} = 0$, then $V_{td}^* V_{ts}$ would still be rather different from SM3 case. With our current knowledge of m_t, the ε_K parameter would deviate from the well measured experimental value. Thus, a finite $V_{t'd}$ is needed to tune for ε_K.

We find that the kaon constraints that are sensitive to t' (i.e. $P_{\rm EW}$-like), viz. $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \mu^+ \mu^-$, ε_K, and ε'/ε can all be satisfied. Interestingly, once kaon constraints are satisfied, little impact is implied for $b \leftrightarrow d$ transitions, such as Δm_{B_d} and $\sin 2\Phi_{B_d}$, and $V_{t'd} \to 0$ works approximately for $b \to d$ transitions at current sensitivities. The main outcome is the enhancement of $K_L \to \pi^0 \nu\bar{\nu}$ mode by an order of magnitude or more, to beyond 5×10^{-10}.

With four generations, adding $V_{t's}^* V_{t'b}$ extends the familiar unitarity triangle relation into a quadrangle,

$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} + V_{t's}^* V_{t'b} = 0. \tag{2}$$

Using SM3 values for $V_{us}^* V_{ub}$, $V_{cs}^* V_{cb}$ (validated later by our $b \to d$ study), since they are probed in multiple ways already, and taking $V_{t's}^* V_{t'b}$ as given in Eq. (1), we depict Eq. (2) in Fig. 1(a). The solid, rather squashed triangle is the usual $V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = 0$ in SM3. Given the size and phase of $V_{t's}^* V_{t'b}$, one sees that the invariant phase represented by the area of the quadrangle is rather

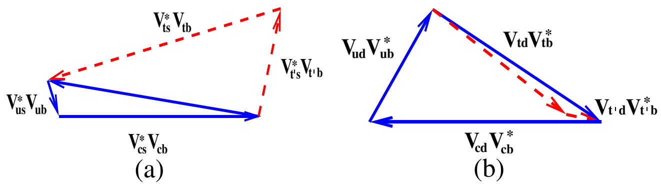

FIG. 1 (color online). Unitarity quadrangles of (a) Eq. (2), with $|V_{us}^* V_{ub}|$ exaggerated; (b) Eq. (17), where actual scale is $\sim 1/4$ of (a). From Eq. (1), $V_{t's}^* V_{t'b}$ (dashed) drastically changes the invariant phase and $V_{ts}^* V_{tb}$ from the SM3 triangle (solid), but from Eq. (16), the dashed lines for $V_{td} V_{tb}^*$ and $V_{t'd} V_{t'b}^*$ can hardly be distinguished from SM3 case.

large, and $V_{ts}^* V_{tb}$ picks up a large imaginary part, which is very different from SM3 case. Such large effect in $b \to s$ would likely spill over into $s \to d$ transitions, since taking V_{tb} as real and of order 1, one immediately finds the strength and complexity of $V_{td}^* V_{ts}$ would be rather different from SM3, and $V_{t'd}^* V_{t's} \neq 0$ is needed to reach the well measured value for ε_K.

To face $s \to d$ and $b \to d$ transitions, one should respect unitarity of the 4×4 CKM matrix $V_{\rm CKM}$. We adopt the parametrization in Ref. [5] where the third column and fourth row is kept simple. This is suitable for B physics, as well as for loop effects in kaon sector. With V_{cb}, V_{tb} and $V_{t'b}$ defined as real, one keeps the SM3 phase convention for V_{ub}, now defined as

$$\arg V_{ub}^* = \phi_{ub}, \tag{3}$$

which is usually called ϕ_3 or γ in SM3. We take $\phi_{ub} = 60°$ as our nominal value [6]. This can in principle be measured through tree level processes such as the $B \to DK$ Dalitz method [7]. The two additional phases are associated with $V_{t's}$ and $V_{t'd}$, and for the rotation angles we follow the PDG notation [8]. To wit, we have

$$V_{t'd} = -c_{24} c_{34} s_{14} e^{-i\phi_{db}}, \tag{4}$$

$$V_{t's} = -c_{34} s_{24} e^{-i\phi_{sb}}, \tag{5}$$

$$V_{t'b} = -s_{34}, \tag{6}$$

while $V_{t'b'} = c_{14} c_{24} c_{34}$, $V_{tb} = c_{13} c_{23} c_{34}$, $V_{cb} = c_{13} c_{34} s_{23}$ are all real. With this convention for rotation angles, from Eq. (3) we have $V_{ub} = c_{34} s_{13} e^{-i\phi_{ub}}$.

Analogous to Eq. (1), we also make the heuristic but redundant definition of

$$V_{t'd}^* V_{t'b} \equiv r_{db} e^{i\phi_{db}}, \qquad V_{t'd}^* V_{t's} \equiv r_{ds} e^{i\phi_{ds}}, \tag{7}$$

as these combinations enter $b \to d$ and $s \to d$ transitions. Inspection of Eqs. (1) and (4)–(6) gives the relations

$$r_{db} r_{sb} = r_{ds} s_{34}^2, \qquad \phi_{ds} = \phi_{db} - \phi_{sb}. \tag{8}$$

As we shall see, $s \to d$ transitions are much more stringent than $b \to d$ transitions, hence we shall turn to constraining r_{ds} and ϕ_{ds}.

Before turning to the kaon sector, we need to infer what value to use for $s_{34} = |V_{t'b}|$, as this can still affect the relevant physics through unitarity. We have some constraint on s_{34} from $Z \to b\bar{b}$ width, which receives special t (and hence t') contribution compared to other $Z \to q\bar{q}$, and is now suitably well measured.

Following Ref. [9] and using $m_{t'} = 300$ GeV, we find

$$|V_{tb}|^2 + 3.4|V_{t'b}|^2 < 1.14. \tag{9}$$

Since all c_{ij}s except perhaps c_{34} would still likely be close to 1, we infer that $s_{34} \lesssim 0.25$. We take the liberty to nearly saturate this bound ($\Gamma(Z \to b\bar{b})$ is close to 1σ above SM3 expectation), by *imposing*

$$s_{34} \simeq 0.22, \tag{10}$$

to be close to the Cabibbo angle, $\lambda \equiv |V_{us}| \cong 0.22$. Note that Eq. (10) is somewhat below the expectation of "maximal mixing" of $s_{34}^2 \sim 1/2$ between third and fourth generations. Combining it with Eq. (1) gives $|V_{t's}| \sim 0.11 \sim \lambda/2$. Its strength would grow if a lower value of $s_{34} \lesssim \lambda$ is chosen, and would make even greater impact on $s \to d$ transitions.

Using current values [8] of V_{cb} and V_{ub} as input and respecting full unitarity, we now turn to the kaon constraints of $K^+ \to \pi^+ \nu\bar{\nu}$, ε_K, $K_L \to \mu^+\mu^-$, and ε'/ε. The first two are short-distance (SD) dominated, while the last two suffer from long-distance (LD) effects.

Let us start with $K^+ \to \pi^+ \nu\bar{\nu}$. The first observed event [10] by E787 suggested a sizable rate hence hinted at NP. The fourth generation would be a good candidate, since the process is dominated by the Z penguin. Continued running, including E949 data, has yielded overall 3 events, and the rate is now $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) = (1.47^{+1.30}_{-0.89}) \times 10^{-10}$ [11]. This is still somewhat higher than the SM3 expectation of order 0.8×10^{-10}.

Defining $\lambda_q^{ds} \equiv V_{qd} V_{qs}^*$ and using the formula [12]

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) = \kappa_+ \left| \frac{\lambda_c^{ds}}{|V_{us}|} P_c + \frac{\lambda_t^{ds}}{|V_{us}|^5} \eta_t X_0(x_t) + \frac{\lambda_{t'}^{ds}}{|V_{us}|^5} \eta_{t'} X_0(x_{t'}) \right|^2, \tag{11}$$

we plot in Fig. 2 the allowed range (valley shaped shaded region) of r_{ds}–ϕ_{ds} for the 90% confidence level (C.L.) bound of $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) < 3.6 \times 10^{-10}$. We have used [12] $\kappa_+ = (4.84 \pm 0.06) \times 10^{-11} \times (0.224/|V_{us}|)^8$ and $P_c = (0.39 \pm 0.07) \times (0.224/|V_{us}|)^4$. We take the QCD correction factors $\eta_{t^{(\prime)}} \sim 1$, and $X_0(x_{t^{(\prime)}})$ evaluated for $m_t = 166$ GeV and $m_{t'} = 300$ GeV. We see that r_{ds} up to 7×10^{-4} is possible, which is not smaller than the SM3 value of 4×10^{-4} for $|V_{td}^* V_{ts}|$.

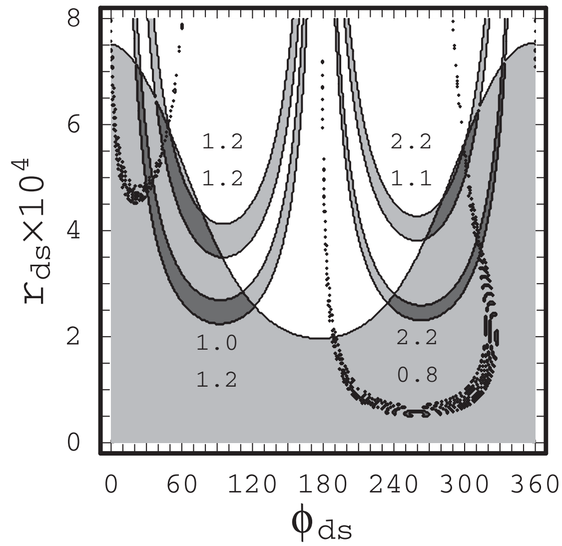

FIG. 2. Allowed region from $K^+ \to \pi^+ \nu \bar{\nu}$ (valley shaped shaded region), ε_K (simulated dots) and ε'/ε (elliptic rings) in r_{ds} and ϕ_{ds} plane, as described in text, where $V^*_{t'd} V_{t's} \equiv r_{ds} e^{i\phi_{ds}}$. For ε'/ε, the rings on upper right correspond to $R_6 = 2.2$, and $R_8 = 0.8$, 1.1 (bottom to top), and on upper left, $R_6 = 1.0$, 1.2 (bottom to top), $R_8 = 1.2$.

The SD contribution to $K_L \to \mu^+ \mu^-$ is also of interest. The $K_L \to \mu^+ \mu^-$ rate is saturated by the absorptive $K_L \to \gamma\gamma \to \mu^+ \mu^-$, while the off-shell photon contribution makes the SD contribution hard to constrain. To be conservative, we use the experimental bound of $\mathcal{B}(K_L \to \mu^+ \mu^-)_{\rm SD} < 3.7 \times 10^{-9}$ [13]. It is then in general less stringent than $K^+ \to \pi^+ \nu \bar{\nu}$, although the generic constraint on r_{ds} drops slightly. We do not plot this constraint in Fig. 2.

The rather precisely measured CPV parameter $\varepsilon_K = (2.284 \pm 0.014) \times 10^{-3}$ [8] is predominantly SD. It maps out rather thin slices of allowed regions on the r_{ds}–ϕ_{ds} plane, as illustrated by dots in Fig. 2, where we use the formula of Ref. [9] and follow the treatment. Note that r_{ds} up to 7×10^{-4} is still possible, for several range of values for ϕ_{ds}. This is the aforementioned effect that extra CPV effects due to large ϕ_{sb} and r_{sb} now have to be tuned by t' effect to reach the correct ε_K value. We have checked that Δm_K makes no additional new constraint, especially since LD effect is of order $\Delta m_K^{\rm exp}$.

The DCPV parameter, ${\rm Re}(\varepsilon'/\varepsilon)$, was first measured in 1999 [14], with current value at $(1.67 \pm 0.26) \times 10^{-3}$ [8]. It depends on a myriad of hadronic parameters, such as m_s, $\Omega_{\rm IB}$ (isospin breaking), and especially the nonperturbative parameters R_6 and R_8, which are related to the hadronic matrix elements of the dominant strong and electroweak penguin operators. With associated large uncertainties, we expect ε'/ε to be rather accommodating, but for specific values of R_6 and R_8, some range for r_{ds} and ϕ_{ds} is determined.

We use the formula

$$ {\rm Re}\,\frac{\varepsilon'}{\varepsilon} = {\rm Im}(\lambda_c^{ds}) P_0 + {\rm Im}(\lambda_t^{ds}) F(x_t) + {\rm Im}(\lambda_{t'}^{ds}) F(x_{t'}), \tag{12} $$

where $F(x)$ is given by

$$ F(x) = P_X X_0(x) + P_Y Y_0(x) + P_Z Z_0(x) + P_E E_0(x). \tag{13} $$

The SD functions X_0, Y_0, Z_0 and E_0 can be found, for example, in Ref. [15], and the coefficients P_i are given in terms of R_6 and R_8 as

$$ P_i = r_i^{(0)} + r_i^{(6)} R_6 + r_i^{(8)} R_8, \tag{14} $$

which depends on LD physics. We differ from Ref. [15] by placing P_0, multiplied by ${\rm Im}(\lambda_c^{ds})$, explicitly in Eq. (12). In SM4, one no longer has the relation ${\rm Im}\lambda_c^{ds} = -{\rm Im}\lambda_t^{ds}$ that makes ${\rm Re}(\varepsilon'/\varepsilon)$ proportional to ${\rm Im}(\lambda_t^{ds})$. We take the $r_i^{(j)}$ values from Ref. [15] for $\Lambda_{\overline{MS}}^{(4)} = 310$ MeV, but reverse the sign of $r_0^{(j)}$ for above mentioned reason. Note that ${\rm Re}(\varepsilon'/\varepsilon)$ depends linearly on R_6 and R_8. For fixed SD parameters $m_{t'}$ and $\lambda_{t'}^{ds} = V_{t'd} V^*_{t's}$, one may adjust for solutions to $K^+ \to \pi^+ \nu \bar{\nu}$ and ε_K.

For the "standard" [15] parameter range of $R_6 = 1.23 \pm 0.16$ and $R_8 = 1.0 \pm 0.2$, we find $R_8 \sim 1.2$ and $R_6 \sim 1.0$–1.2 allows for solutions at $r_{ds} \sim (5$–$6) \times 10^{-4}$ with $\phi_{ds} \sim +(35°$–$50°)$, as illustrated by the elliptic rings on upper left part of Fig. 2. For $R_6 = 2.2 \pm 0.4$ found [16] in $1/N_C$ expansion at next-to-leading order (and chiral perturbation theory at leading order), within SM3 one has trouble giving the correct ${\rm Re}(\varepsilon'/\varepsilon)$ value. However, for SM4, solutions exist for $R_6 \sim 2.2$ and $R_8 = 0.8$–1.1, for $r_{ds} \sim (3.5$–$5) \times 10^{-4}$ and $\phi_{ds} \sim -(45°$–$60)°$, as illustrated by the elliptic rings on upper right part of Fig. 2. We will take

$$ r_{ds} \sim 5 \times 10^{-4}, \qquad \phi_{ds} \sim -60° \,{\rm or}\, + 35°, \tag{15} $$

as our two nominal cases that satisfy all kaon constraints. The corresponding values for R_6 and R_8 can be roughly read off from Fig. 2. We stress again that these values should be taken as exemplary.

To illustrate in a different way, we plot ε_K, $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})$ and ${\rm Re}(\varepsilon'/\varepsilon)$ vs. ϕ_{ds} in Figs. 3(a)–3(c), respectively, for $r_{ds} = 4$ and 6×10^{-4}. The current 1σ experimental range is also illustrated. In Fig. 3(c), we have illustrated with $R_6 = 1.1$, $R_8 = 1.2$ [15] and $R_6 = 2.2$, $R_8 = 1.1$ [16]. For the former (latter) case, the variation is enhanced as R_6 (R_8) drops.

It is interesting to see what are the implications for the CPV decay $K_L \to \pi^0 \nu \bar{\nu}$. The formula for $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})$ is analogous to Eq. (11), except [15] the change of κ_+ to $\kappa_L = (2.12 \pm 0.03) \times 10^{-10} \times (|V_{us}|/0.224)^8$, and taking only the imaginary part for the various CKM products. Since $\phi_{ds} \sim -60°$ or $+35°$ have large imaginary parts, while $r_{ds} \equiv |V^*_{t'd} V_{t's}| \sim 5 \times 10^{-4}$ is stronger than the SM3

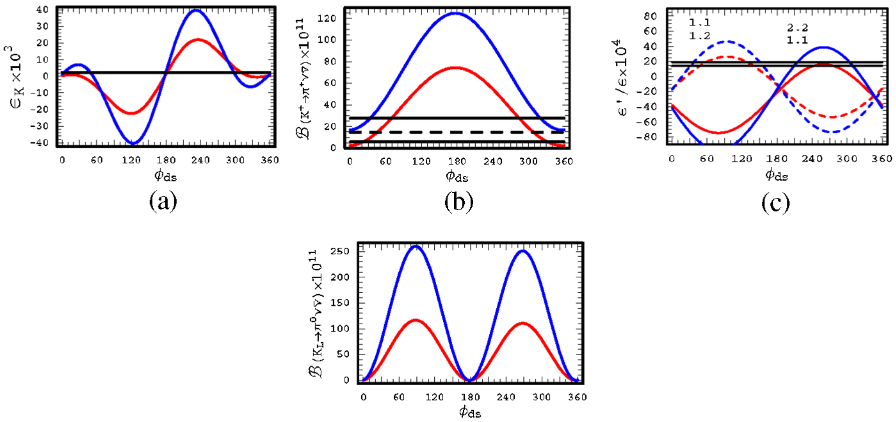

FIG. 3 (color online). (a) ε_K, (b) $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$, (c) $\mathrm{Re}(\varepsilon'/\varepsilon)$ and (d) $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})$ vs. ϕ_{ds}, for $r_{ds} = 4$ and 6×10^{-4} and $m_{t'} = 300$ GeV. Larger r_{ds} gives stronger variation, and horizontal bands are current (1σ) experimental range [8] (the bound for (d) is outside the plot). For (c), solid (dashed) lines are for $R_6 = 2.2$, $R_8 = 1.1$ ($R_6 = 1.1$, $R_8 = 1.2$).

expectation of $\mathrm{Im}V_{td}^* V_{ts} \sim 10^{-4}$, we expect $K_L \to \pi^0 \nu\bar{\nu}$ to be much enhanced.

We plot $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})$ vs. ϕ_{ds} in Fig. 3(d), for $r_{ds} = 4$ and 6×10^{-4}. Reading off from the figure, we see that *the $K_L \to \pi^0 \nu\bar{\nu}$ rate can reach above* 10^{-9}, almost 2 orders of magnitude above SM3 expectation of 0.3×10^{-10}. It is likely above 5×10^{-10}, and in general larger than $K^+ \to \pi^+ \nu\bar{\nu}$. Specifically, for our nominal $r_{ds} \sim 5 \times 10^{-4}$ and $\phi_{ds} \sim +35°$, $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})$ and $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$ are 6.5 and 2×10^{-10}, respectively, while for $\phi_{ds} \sim -60°$, they are 12 and 3×10^{-10}, respectively. The latter case is closer to the Grossman-Nir bound [17], i.e. $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})/\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) \sim \tau_{K_L}/\tau_{K^+} \sim 4.2$, because $V_{t'd}V_{t's}^*$ is more imaginary. Thus, both $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ should be very interesting at the next round of experiments. We note that the ongoing E391A experiment could [18] attain single event sensitivity with the Grossman-Nir bound based on the current $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$ measurement. However, for $r_{ds} \sim 3.5 \times 10^{-4}$ and $\phi_{ds} \sim -45°$, which is still a solution for $R_6 \sim 2.2$, one has $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) \sim 4 \times 10^{-10}$ with $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$ at lower end of current range.

With $\phi_{sb} \sim 70°$ and $\phi_{ds} \sim -60°$ (and $+35°$) both sizable while the associated CKM product is larger than the corresponding SM3 top contribution, there is large impact on $b \to s$ and $s \to d$ transitions from Z penguin and box diagrams. It is therefore imperative to check that one does not run into difficulty with $b \to d$ transitions. Remarkably, we find that the impact on $b \to d$ is mild. From Eqs. (1), (8), (10), and (15),, we infer

$$r_{db} \sim 1 \times 10^{-3}, \qquad \phi_{db} \sim 10°(105°). \tag{16}$$

Since r_{db} is much smaller than $|V_{td}^* V_{tb}| \sim \lambda^3 \sim 0.01$ in SM3, the impact on $b \to d$ is expected to be milder, i.e. we are not far from the $V_{t'd} \to 0$ limit. We stress that this is *nontrivial* since there is a large effect in $b \to s$; it is a consequence of imposing $s \to d$ and $Z \to b\bar{b}$ constraints. We illustrate in Fig. 1(b) the unitarity quadrangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* + V_{t'd}V_{t'b}^* = 0. \tag{17}$$

In contrast to Fig. 1(a), $(V_{td}V_{tb}^* + V_{t'd}V_{t'b}^*)_{\mathrm{SM4}}$ and $(V_{td}V_{tb}^*)_{\mathrm{SM3}}$ can hardly be distinguished.

The B_d^0-$\bar{B}_d^0$ mass difference and CP violation phase in mixing are, respectively, given by $\Delta m_{B_d} \equiv 2|M_{12}|$ and $\sin 2\Phi_{B_d} \equiv \mathrm{Im}(M_{12}/|M_{12}|)$, where

$$M_{12} = \kappa_{B_d}[(\lambda_t^{db})^2 \eta_t S(x_t) + (\lambda_{t'}^{db})^2 \eta_{t'} S(x_{t'}) + 2\lambda_t^{db}\lambda_{t'}^{db}\eta_{tt'} S(x_t, x_{t'})], \tag{18}$$

with $\kappa_{B_d} = (G_F^2/12\pi^2) m_W^2 m_{B_d} B_{B_d} f_{B_d}^2$. The functions $S(x)$ and $S(x, y)$ can be found in [19]. We take $\eta_t = 0.55$, $\eta_{t'} = 0.58$ and $\eta_{tt'} = 0.50$, and plot in Fig. 4(a) Δm_{B_d} vs. ϕ_{db}, for $r_{db} = 8$ and 12×10^{-4} (corresponding to $r_{ds} = 4$ and 6×10^{-4}). We have taken the experimental value of $\Delta m_{B_d} = (0.505 \pm 0.005)\ \mathrm{ps}^{-1}$ from PDG 2005 [8], and illustrated with the lower range of $f_{B_d}\sqrt{B_{B_d}} = (246 \pm 38)$ MeV [20]. We have scaled up the error for the latter

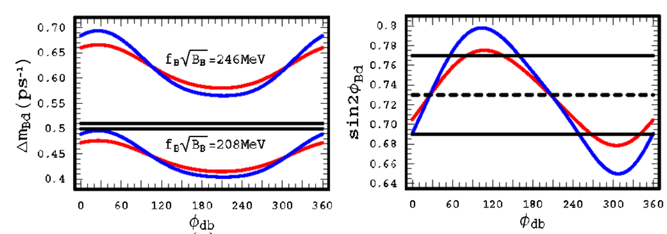

FIG. 4 (color online). (a) Δm_{B_d} and (b) $\sin 2\Phi_{B_d}$ vs. ϕ_{db} for $r_{db} = 8$ and 12×10^{-4}, with $V_{t'd}^* V_{t'b} \equiv r_{db} e^{i\phi_{db}}$. Larger r_{db} gives stronger variation, and horizontal bands are the experimental range [8].

by 1.4, since it comes from the new result on f_{B_d} with unquenched lattice QCD [21], but B_{B_d} is not yet updated. We see from Fig. 4(a) that Δm_{B_d} does not rule out the parameter space around Eq. (16) (equivalent to Eq. (15)). The overall dependence on r_{db} and ϕ_{db} is mild, and error on $f_{B_d}\sqrt{B_{B_d}}$ dominates. Seemingly, a lower value of $f_{B_d}\sqrt{B_{B_d}} \sim 215$ MeV is preferred. SM3 would give $\Delta m_{B_d} = 0.44 - 0.62\ \mathrm{ps}^{-1}$ for $f_{B_d}\sqrt{B_{B_d}} = 208$ MeV $-$ 246 MeV, so the problem is not with SM4.

We plot $\sin 2\Phi_{B_d}$ vs. ϕ_{db} in Fig. 4(b), for $r_{db} = 8$ and 12×10^{-4}. One can see that $\sin 2\Phi_{B_d}$, which is not sensitive to hadronic parameters such as $f_{B_d}\sqrt{B_{B_d}}$, is well within experimental range of "$\sin 2\phi_1$" $= 0.73 \pm 0.04$ from PDG 2005 [8] for the $\phi_{db} \sim 10°$ case. However, for $\phi_{db} \sim 105°$ case, which is much more imaginary, $\sin 2\Phi_{B_d}$ is on the high side [22], and it seems that CPV in B physics prefers $R_6 \sim 2.2$ over $R_6 \sim 1$. As another check, we find the semileptonic asymmetry $A_{SL} = -0.7 \times 10^{-3}$ (-0.2×10^{-3}) for $\phi_{db} \sim 10°$ (105°), which is also well within range of $A_{SL}^{\exp} = (-1.1 \pm 7.9 \pm 7.0) \times 10^{-3}$ [23].

With Eqs. (1), (10), and (16), together with standard (SM3) values for V_{cb} and V_{ub}, we can get a glimpse of the typical 4×4 CKM matrix, which appears like

$$\begin{pmatrix} 0.9745 & 0.2225 & 0.0038e^{-i60°} & 0.0281e^{i61°} \\ -0.2241 & 0.9667 & 0.0415 & 0.1164e^{i66°} \\ 0.0073e^{-i25°} & -0.0555e^{-i25°} & 0.9746 & 0.2168e^{-i1°} \\ -0.0044e^{-i10°} & -0.1136e^{-i70°} & -0.2200 & 0.9688 \end{pmatrix} \tag{19}$$

for $\phi_{db} \sim 10°$ case (V_{cd} and V_{cs} pick up tiny imaginary parts, which are too small to show in angles). For the $\phi_{db} \sim 105°$ case, the appearance is almost the same, except $V_{td} \simeq 0.0082e^{-i17°}$ and $V_{ub'} \simeq 0.029e^{i74°}$. Note the "double Cabibbo" nature, i.e. the 12 and 34 diagonal 2×2 submatrices appear almost the same. This is a consequence of our choice of Eq. (10). To keep Eq. (1) intact, however, weakening s_{34} would result in even large $V_{t's}$, but it would still be close to imaginary. Since $V_{t^{(\prime)}d}^{*}V_{t^{(\prime)}s}$ are tiny compared to $V_{ud}^{*}V_{us} \simeq -V_{cd}^{*}V_{cs}$, the unitarity quadrangle for $s \to d$ cannot be plotted as in Fig. 1. However, note that $V_{td}^{*}V_{ts}$ is almost real, and CPV in $s \to d$ comes mostly from t'.

The entries for $V_{ib'}$, $i = u, c, t$ are all sizable. $|V_{ub'}| \sim 0.03$ satisfies the unitarity constraint $|V_{ub'}| < 0.08$ [8] from the first row, but it is almost as large as V_{cb}. However, the long standing puzzle of unitarity of the first row could be taken as a hint for finite $|V_{ub'}| \sim 0.03$ [24].

The element $V_{cb'} \simeq -V_{t's}^{*}$ is larger than V_{cb} and close to imaginary. Together with $V_{ub'}$, $V_{ub'}V_{cb'}^{*} \simeq 0.0033e^{-i5°}$ ($0.0034e^{i9°}$) is not negligible, and one may worry about D^0-$\bar{D}^0$ mixing. Fortunately the D decay rate is fully Cabibbo allowed. Using $f_D\sqrt{B_D} = 200$ MeV, we find $\Delta m_{D^0} \lesssim 0.05\ \mathrm{ps}^{-1}$ for $m_{b'} \lesssim 280$ GeV, for both nominal cases of Eq. (16). Thus, the current bound of $\Delta m_{D^0} < 0.07\ \mathrm{ps}^{-1}$ is satisfied, and the search for D^0 mixing is of great interest. The bound weakens by factor of 2 if there is strong phase between $D^0 \to K^-\pi^+$ and $K^+\pi^-$ [8].

If $m_{b'} < m_{t'}$, as slightly preferred by D^0-$\bar{D}^0$ mixing constraint, the direct search for b' just above 200 GeV at the Tevatron Run II could be rather interesting. Since $V_{cb'}$ is not suppressed, the b' quark would decay via charged current. Both b' and t', regardless of which one is lighter, with $m_{t'} \sim 300$ GeV and $|m_{t'} - m_{b'}| \lesssim 85$ GeV [8], can be easily discovered at the LHC.

The large and mainly imaginary element $V_{t's} \simeq -V_{cb'}^{*}$ in Eq. (19), being larger than V_{ts} and V_{cb}, may appear unnatural (likewise for $V_{ub'}$ vs. V_{ub}). However, it is allowed, since the main frontier that we are just starting to explore is in fact $b \to s$ transitions. The current situation that $\mathcal{A}_{K^+\pi^-} \sim -0.12$ while $\mathcal{A}_{K^+\pi^0} \gtrsim 0$ in $B \to K\pi$ decays may actually be hinting at the need for such large $b \to s$ CPV effects. The litmus test would be finding Δm_{B_s} not far above current bound, *but with sizable* $\sin 2\Phi_{B_s} < 0$ [3], which may even emerge at Tevatron Run II. Our results studied here are for illustration purpose, but the main result, that $K_L \to \pi^0 \nu\bar{\nu}$ may be rather enhanced, is a generic consequence of Eq. (1), which is a possible solution to the $B^+ \to K^+\pi^0$ DCPV puzzle.

In a series of papers, Buras *et al.* [25] have suggested possible NP effects in $P_{\rm EW}$ by considering ratios of various $B \to K\pi$ decay rates, and a new complex phase was also introduced in light of CPV data. By assuming "minimal flavor violation", this was transferred to rare kaon decays. We note that in SM4, the link between B and K is automatic, and no extra assumption is needed. With the emergence of $B^+ \to K^+\pi^0$ DCPV puzzle, Baek *et al.* [26] have also favored a NP modification of $P_{\rm EW}$, where our SM4 study is a particularly predictive realization. For other specific models, Barger *et al.* [27] have proposed a Z' model with left-handed flavor-changing couplings, which could accommodate current data. But the predictive power of Z' models are somewhat limited, and depends on assumptions made. For example, with right-handed nonuniversal couplings, Ref. [28] found somewhat limited enhancement for $K_L \to \pi^0 \nu\nu$.

In summary, the deviation of direct CPV measurements between neutral and charged B decays, $\mathcal{A}_{K^+\pi^0} - \mathcal{A}_{K^+\pi^-} \simeq 0.16$ while $\mathcal{A}_{K^+\pi^-} \simeq -0.12$, is a puzzle that could be hinting at new physics. A plausible solution is the existence of a 4th generation with $m_{t'} \sim 300$ GeV and $V_{t's}^* V_{t'b} \sim 0.025 e^{i70^\circ}$. If so, we find special solution space is carved out by stringent kaon constraints, and the 4×4 CKM matrix is almost fully determined. $K^+ \to \pi^+ \nu\bar{\nu}$ may well be of order $(1 - 3) \times 10^{-10}$, while $K_L \to \pi^0 \nu\bar{\nu} \sim (4 - 12) \times 10^{-10}$ is greatly enhanced by the large phase in $V_{t'd}^* V_{t's}$. With kaon constraints satisfied, B_d mixing and $\sin 2\Phi_{B_d}$ are consistent with experiment, which is remarkable. Our results are generic. If the effect weakens in $b \to s$ transitions, the effect on $K \to \pi \nu\bar{\nu}$ would also weaken. But a large CPV effect in electroweak $b \to s$ penguins would translate into an enhanced $K_L \to \pi^0 \nu\bar{\nu}$ (and $\sin 2\Phi_{B_s} < 0$).

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